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# Dynamical interactions between higher-order rogue waves and various forms of $n$-soliton solutions ($n \to \infty$) of the (2+1)-dimensional ANNV equation

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We present new lemmas, theorem and corollaries to construct interactions among higher-order rogue waves, $n$-periodic waves and $n$-solitons solutions ($n \to \infty$) to the (2+1)-dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) equation. Several examples for theories are given by choosing definite interactions of the wave solutions for the model. In particular, we exhibit dynamical interactions between a rogue and a cross bright-dark bell wave, a rogue and a cross-bright bell wave, a rogue and a one-, two-, three-, four-periodic wave. In addition, we also present multi-types interactions between a rogue and a periodic cross-bright bell wave, a rogue and a periodic cross-bright-bark bell wave. Finally, we physically explain such interaction solutions of the model in the 3D and density plots.

**Keywords:** the (2+1)-dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) equation, higher-order rogue waves, $n$-solitons, periodic waves, bright-dark bell waves

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## 1. Introduction

Nonlinear partial differential equations (NLPDEs) have attracted a great deal of attention in various branches of sciences. They present many significant phenomena in physics, biology, applied mathematics as well as in engineering applications. NLPDEs lead to powerful methods for explaining states and evolution with time. Solitary waves (rogue/lump) of NLPDEs are a class of dynamical behaviors to describe properties of seismic nonlinear phenomena. Thus there has been an emerging research field to solve the solitary wave problems of the NLPDEs. The study of the solitary waves leads to periodic waves, lump solitons, rogue waves, breathers, semirational solutions as well as their interactions in Refs. [1–18].

Recently, the feather of rogue waves or killer waves has grown much popularity as they represent surreptitious oversize sea waves.[8–10] Such awful phenomena of rogue waves can make water wall larger than 20–30 m and create cataclysm for ships, coastal oil platform and marine industries.[9] A wave can be defined rogue category if its hight is two or more times taller than the background peak, and appears from nowhere and disappears without a trace.[19] Sometimes these massive rogue waves become Tsunamis, because of underwater disturbance by earthquakes or volcanic eruptions or typically with the aid of nuclear explosion or asteroids. During Tsunamis, this dissipated energy travels through water medium and when the depth of the ocean decreases, the wave height increases to higher. However, characteristics of rogue type waves have also been explored in optics,[11] superfluidity,[20] plasma physics,[21] finance,[22] and so on.

Moreover, interactions between higher-order rogue waves and $n$-solitons ($n \to \infty$) are a rear case in literature, and such calculations are still much more complicated involving the existing theories.[3,23] In this work, we present new theories with proper examples to built dynamical interactions between rogue waves and different forms of $n$-solitons solutions for the (2+1)-dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) equation via the Hirota bilinear scheme.[24–28] To our knowledge, this is a first step to study these type of dynamical interactions with the existing theories for the ANNV model.

## 2. The bilinear form of the ANNV model

Let us consider the (2+1)-dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) equation[27–29]

$$\mathcal{W}_t + \mathcal{W}_{xxx} + 3(\mathcal{W}\mathcal{V})_x = 0 \quad \text{and} \quad \mathcal{W}_x = \mathcal{V}_y, \tag{1}$$

where $\mathcal{W}(x,y,t)$ is evolution of the wave surface function. The model (1) plays a vital role in nonlinear incompressible fluids such as shallow-water, long internal and acoustic waves. It is a special case of the model studied in Ref. [16]. To investigate interaction soliton solutions of the model, we use the famous Hirota-bilinear scheme.[25] It is a classical approach to obtain multiple-front soliton solutions to the NLPDEs.[24–28] We now convert the model (1) in terms of bilinear form via the following transformations:

$$\mathcal{W} = 2(\ln \mathcal{H})_{xy} \quad \text{and} \quad \mathcal{V} = 2(\ln \mathcal{H})_{xx}. \tag{2}$$

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The model leads to

$$(D_yD_t + D_yD_x^3)(\mathcal{H}\cdot\mathcal{H}) = 0, \tag{3}$$

where $D$ is the bilinear differential operator defined in Refs. [25,27]. Now we can explicitly write the bilinear expression of Eq. (3) as follows:

$$\begin{aligned}&\mathcal{H}_{yt}\mathcal{H} - \mathcal{H}_y\mathcal{H}_t + \mathcal{H}_{yxxx}\mathcal{H} - 3\mathcal{H}_{yxx}\mathcal{H}_x\\ &+3\mathcal{H}_{yx}\mathcal{H}_{xx} - \mathcal{H}_y\mathcal{H}_{xxx} = 0.\end{aligned} \tag{4}$$

In the following sections, we will establish some theories to form interaction soliton solutions that will play various combinations of higher-order rogue waves, oscillating and $n$-soliton solutions of the model.

## 3. Interactions between higher-order rogue waves and $n$-solitons

In this section, we focus on constructing dynamical interactions of a general form in terms of quadratic polynomial functions (rogue) with $n$ periodic waves and $n$-solitons solutions to Eq. (1) involving certain assumptions on the parameters. We also present some relevant examples with their illustrative graphs. To the purpose, we first need to introduce the following lemmas and theorem.

**Lemma 1** Suppose that $\phi(x,y,t) = a_0 + \sum_{i=1}^{M}(l_ix + m_iy + n_it + r_i)^2$ is a quadratic polynomial. If $\phi(x,y,t)$ satisfies Eq. (3) with assumptions $a_0 > 0$, $\sum_i l_i^3 m_i = 0$ and $\sum_i m_i n_i = 0$, then $\phi(x,y,t)$ will be a rogue wave solution to Eq. (1) with the transformation (2).

**Proof** Since $D$ is a bilinear operator, we have

$$\begin{aligned}&(D_yD_t + D_yD_x^3)(\phi\cdot\phi)\\ &= 2(\phi_{yt}\phi - \phi_y\phi_t + \phi_{yxxx}\phi - 3\phi_{yxx}\phi_x\\ &\quad +3\phi_{yx}\phi_{xx} - \phi_y\phi_{xxx}), \quad [\text{by (4)}]\\ &= 2(\phi_{yt}\phi - \phi_y\phi_t + 3\phi_{yx}\phi_{xx}),\\ &\quad [\because \phi \text{ is a quadratic polynomial}]\\ &= -4\sum_i m_in_i(l_ix + m_iy + n_it + r_i)^2 + 24\sum l_i^3m_i.\end{aligned} \tag{5}$$

By the assumptions $\sum l_i^3 m_i = 0$ and $\sum_i m_i n_i = 0$, Eq. (3) becomes

$$(D_yD_t + D_yD_x^3)(\phi\cdot\phi) = 0. \tag{6}$$

Therefore $\phi(x,y,t)$ is a solution to Eq. (1) with the transformation (2).

**Lemma 2** If $\varphi(x,y,t) = b_0 + \sum_{j=1}^{N}\lambda_j e^{\nu_j(\rho_jx+\tau_jt+\sigma_j)}$ is an $N$ exponential function with $\varphi_y = 0$, then $\varphi(x,y,t)$ will be a solution to Eq. (1) with the transformation (2).

**Proof** For the bilinear operator $D$, we have

$$\begin{aligned}&(D_yD_t + D_yD_x^3)(\varphi\cdot\varphi)\\ &= 2(\varphi_{yt}\varphi - \varphi_y\varphi_t + \varphi_{yxxx}\varphi\\ &\quad -3\varphi_{yxx}\varphi_x + 3\varphi_{yx}\varphi_{xx} - \varphi_y\varphi_{xxx})\\ &= 0, \quad [\because \varphi_y = 0].\end{aligned} \tag{7}$$

Therefore $\varphi(x,y,t)$ is a solution to Eq. (1) with the transformation (2).

The above lemmas lead to the following theorem.

**Theorem 1** Suppose that

$$\begin{aligned}\mathcal{H}(x,y,t) &= l_0 + \sum_{i=1}^{M}(l_ix + m_iy + n_it + r_i)^2\\ &\quad + \sum_{j=1}^{N}\lambda_j e^{\nu_j(\rho_jx+\tau_jt+\sigma_j)}\end{aligned} \tag{8}$$

is a combination of a quadratic polynomial with $N$ exponential functions. If $\mathcal{H}(x,y,t)$ satisfies Eq. (3) with the assumptions $\sum_i l_i^3 m_i = 0$, $\sum_i l_i m_i = 0$, $\sum_i m_i n_i = 0$ and $\tau_j = -\rho_j^3$, then $\mathcal{H}(x,y,t)$ will be a solution to Eq. (1) with the transformation (2).

**Proof** Let

$$\phi(x,y,t) = l_0 + \sum_{i=1}^{M}(l_ix + m_iy + n_it + r_i)^2, \tag{9}$$

$$\varphi(x,y,t) = \sum_{j=1}^{N}\lambda_j e^{\nu_j(\rho_jx+\tau_jt+\sigma_j)} \tag{10}$$

with $\varphi_y = 0$. Then Eq. (8) takes the form

$$\mathcal{H}(x,y,t) = \phi(x,y,t) + \varphi(x,y,t). \tag{11}$$

We now have

$$\begin{aligned}&(D_yD_t + D_yD_x^3)(\mathcal{H}\cdot\mathcal{H})\\ &= (D_yD_t + D_yD_x^3)(\phi\cdot\phi + \phi\cdot\varphi + \varphi\cdot\phi + \varphi\cdot\varphi).\end{aligned} \tag{12}$$

In the proof, the first and fourth terms are similar to Lemmas 1 and 2, respectively. Now for the second term, we have

$$\begin{aligned}&(D_yD_t + D_yD_x^3)(\phi\cdot\varphi)\\ &= 2(\phi_{yt}\varphi - \phi_y\varphi_t + \phi_{yxxx}\varphi - 3\phi_{yxx}\varphi_x\\ &\quad +3\phi_{yx}\varphi_{xx} - \phi_y\varphi_{xxx})\\ &= 2(\phi_{yt}\varphi - \phi_y\varphi_t + 3\phi_{yx}\varphi_{xx} - \phi_y\varphi_{xxx}),\\ &\quad [\because \phi_{yxxx} = 0,\ \phi_{yxx} = 0]\\ &= 2\phi_{yt}\varphi + 6\phi_{yx}\varphi_{xx} - 2\phi_y(\varphi_t + \varphi_{xxx}).\end{aligned} \tag{13}$$

It can be easily seen that $\phi_{yt} = 2\sum m_in_i = 0$, $\phi_{yx} = 2\sum l_im_i = 0$ and $\varphi_t + \varphi_{xxx} = \sum\lambda_j\nu_j^3(\rho_j^3 + \tau_j)e^{\nu_j(\rho_jx+\tau_jt+\sigma_j)} = 0$, by using the assumptions. Hence

$$(D_yD_t + D_yD_x^3)(\phi\cdot\varphi) = 0. \tag{14}$$

It is obviously seen in the third term that

$$\begin{aligned}&(D_yD_t + D_yD_x^3)(\varphi\cdot\phi)\\ &= 2(\varphi_{yt}\phi - \varphi_y\phi_t + \varphi_{yxxx}\phi - 3\varphi_{yxx}\phi_x + 3\varphi_{yx}\phi_{xx} - \varphi_y\phi_{xxx})\\ &= 0, \quad [\because \varphi_y = 0].\end{aligned} \tag{15}$$

Thus from Lemma 1, Lemma 2 and Eqs. (14), (15) and (12), we can conclude that

$$(D_yD_t+D_yD_x^3)(\mathcal{H}\cdot\mathcal{H})=0. \tag{16}$$

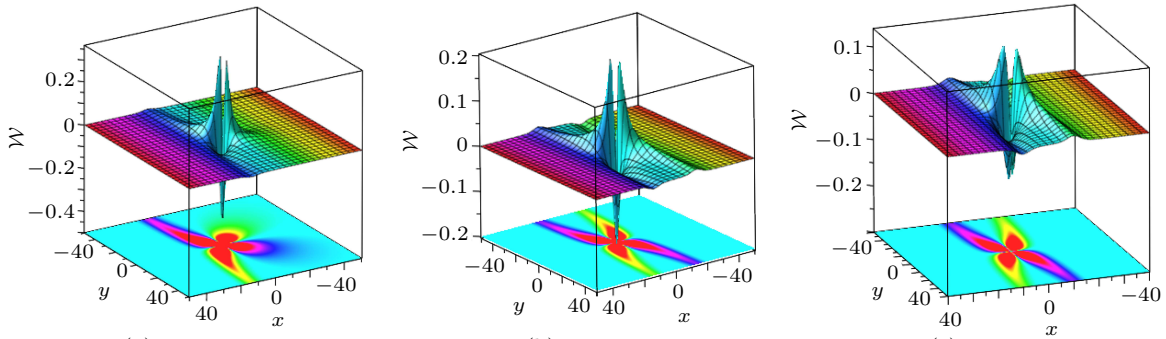
Therefore $\mathcal{H}(x,y,t)$ is a solution to Eq. (3) as well as Eq. (1) via the transformation (2). In addition, it is obviously seen that $\phi(x,y,t)$, $\varphi(y,t)$ and $\mathcal{H}(x,y,t)$ are also the solutions to Eq. (1). Hence the theorem is complete.

We now study the dynamical interactions of solitary wave solutions so as to provide particular values of parameters in higher-order rouge waves and in $N$-solitons, and their 3D and density graphical representations. It is noted that the value of $M$ should be 2 to study the lump solutions based on a general theory in Ref. [29].

**Example 1** First we choose $M=2$ with parameters $l_0=l_1=m_1=m_2=n_1=r_1=r_2=1$, $l_2=n_2=-1$, $\lambda_i=6$, $\nu_i=1$, $\rho_1=0.5$, $\rho_2=-0.5$, $\rho_3=0.6$, $\rho_4=-0.6$, $\sigma_i=0$, $i=1,2,3,4$ at $t=0$, then Theorem 1 leads to

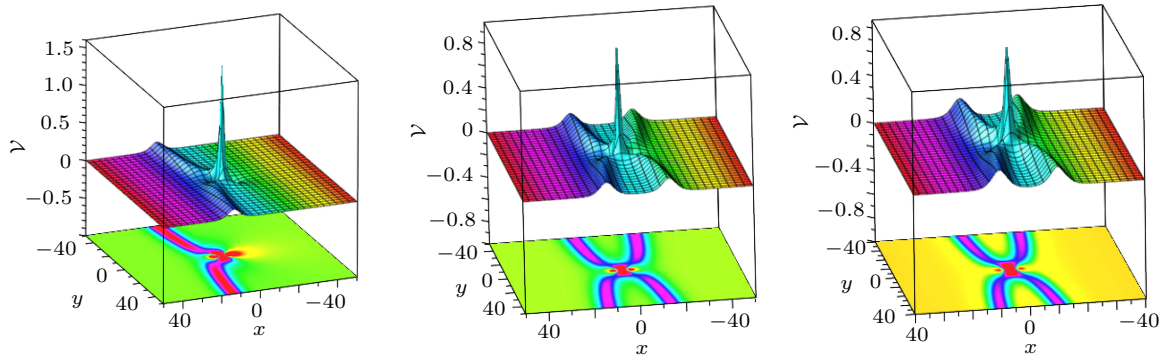
$$\begin{aligned}
\text{i.}\quad \mathcal{H} &= 1+(1+x+y)^2+(1-x+y)^2\\
&\quad +6e^{0.5x} \quad \text{for } N=1;\\
\text{ii.}\quad \mathcal{H} &= 1+(1+x+y)^2+(1-x+y)^2\\
&\quad +6e^{0.5x}+6e^{-0.5x} \quad \text{for } N=2;\\
\text{iii.}\quad \mathcal{H} &= 1+(1+x+y)^2+(1-x+y)^2+6e^{0.5x}\\
&\quad +6e^{-0.5x}+6e^{0.6x}+6e^{-0.6x} \quad \text{for } N=4;
\end{aligned}$$

and their profiles are displayed in Figs. 1 and 2.



(a) $M=2$, $N=1$ (b) $M=2$, $N=2$ (c) $M=2$, $N=4$

**Fig. 1.** The 3D (upper) and density (lower) profiles of Eq. (8) for $\mathcal{W}$: interaction between (a) a rogue and a bright bell wave, (b) a rogue and a two-bell (one bright and one dark) wave, and (c) a rogue and a four-bell (double bright and double dark) wave.



(a) $M=2$, $N=1$ (b) $M=2$, $N=2$ (c) $M=2$, $N=4$

**Fig. 2.** The 3D (upper) and density (lower) profiles of Eq. (8) for $\mathcal{V}$: interaction between (a) a rogue and a bright bell wave, (b) a rogue and a two-cross-bright-bell wave, and (c) a rogue and a four-cross-bright-bell wave.

To the observation, the Euler formulas $2\cosh\theta=e^{\theta}+e^{-\theta}$, $2\sinh\theta=e^{\theta}-e^{-\theta}$ and Theorem 1 lead to the following corollaries:

**Corollary 1** If

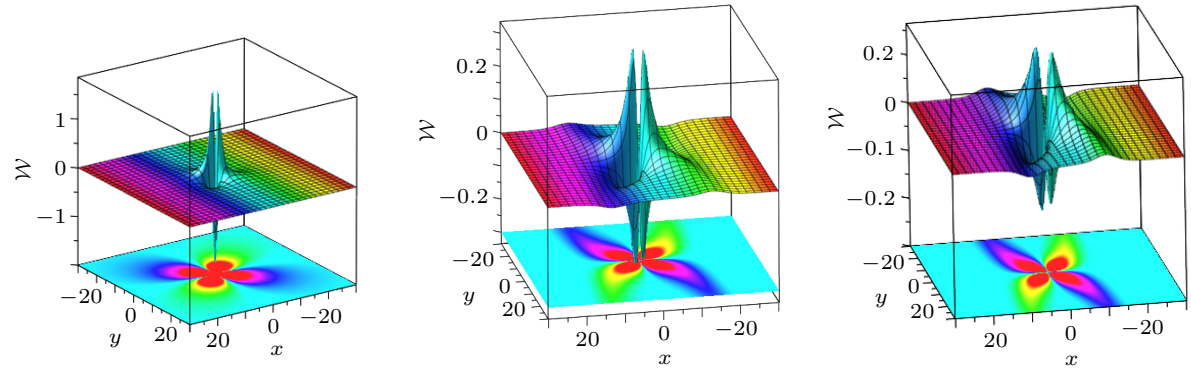
$$\begin{aligned}
\mathcal{H} &= l_0+\sum_{i=1}^{M}(l_ix+m_iy+n_it+r_i)^2\\
&\quad +\sum_{k=1}^{N}\omega_k\cosh(\nu_k(\rho_kx-\rho_k^3t+\sigma_k))
\end{aligned} \tag{17}$$

is a function with the assumptions $\sum l_i^3m_i=0$, $\sum_i l_im_i=0$ and $\sum m_in_i=0$, then $\mathcal{H}(x,y,t)$ will be a solution to Eq. (3).

**Example 2** If we choose $M=2$ with parameters $l_0=l_1=m_1=m_2=n_1=r_1=r_2=1$, $l_2=n_2=-1$, $\omega_i=6$, $\nu_i=1$, $\rho_1=0.5$, $\rho_2=-0.5$, $\rho_3=0.6$, $\sigma_i=0$, $i=1,2,3$ at $t=0$, then Corollary 1 will lead to

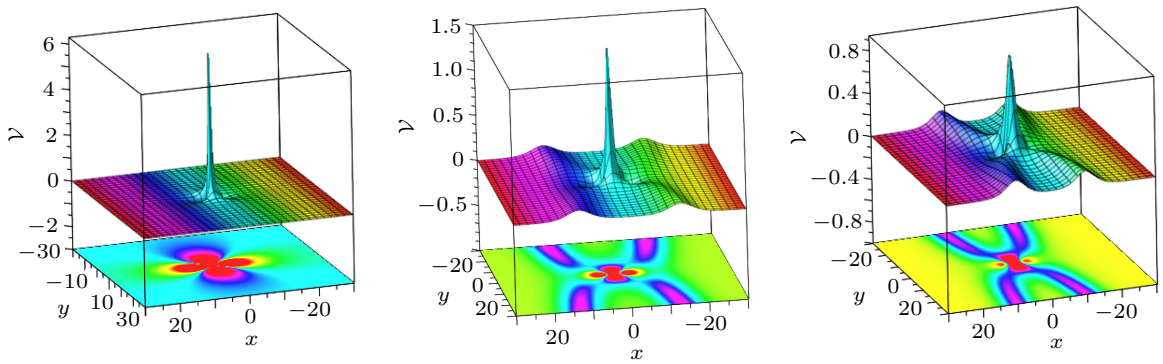
$$\begin{aligned}
\text{i.}\quad \mathcal{H} &= 1+(1+x+y)^2+(1-x+y)^2 \quad \text{for } N=0;\\
\text{ii.}\quad \mathcal{H} &= 1+(1+x+y)^2+(1-x+y)^2\\
&\quad +6\cosh(0.5x) \quad \text{for } N=1;\\
\text{iii.}\quad \mathcal{H} &= 1+(1+x+y)^2+(1-x+y)^2\\
&\quad +6\cosh(0.6x)+12\cosh(0.5x) \quad \text{for } N=3;
\end{aligned}$$

and their profiles are displayed in Figs. 3 and 4.



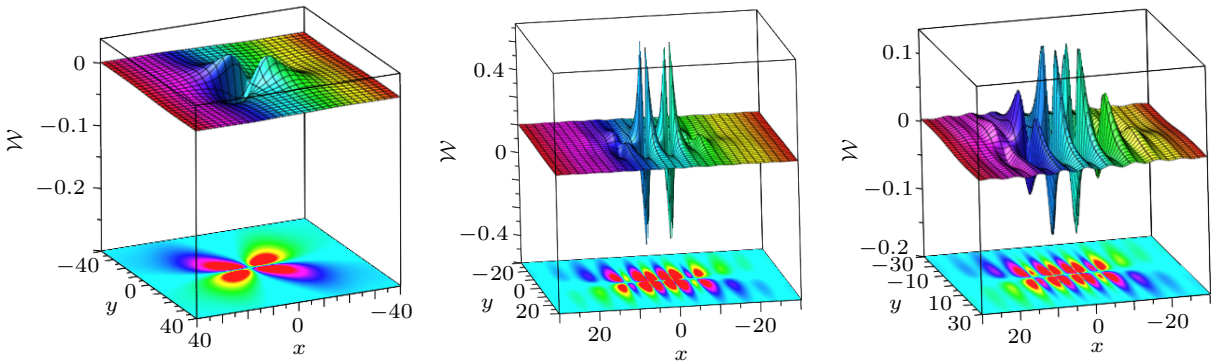
(a) $M=2,\ N=0$ (b) $M=2,\ N=1$ (c) $M=2,\ N=3$

**Fig. 3.** The 3D (upper) and density (lower) profiles of Eq. (17) for $\mathcal{W}$: (a) a single rogue wave; and interaction between (b) a rogue and a bright bell wave, and (c) a rogue and a three-bell (double bright and single dark) wave.



(a) $M=2,\ N=0$ (b) $M=2,\ N=1$ (c) $M=2,\ N=3$

**Fig. 4.** The 3D (upper) and density (lower) profiles of Eq. (17) for $\mathcal{V}$: (a) a single rogue wave; and interaction between (b) a rogue and a cross-bright bell wave, and (c) a rogue and a triple cross-bright bell wave.



(a) $M=2,\ N=1$ (b) $M=2,\ N=2$ (c) $M=2,\ N=3$

**Fig. 5.** The 3D (upper) and density (lower) profiles of Eq. (18) for $\mathcal{W}$: interaction between (a) a rogue and a periodic wave, (b) a rogue and a double periodic wave, and (c) a rogue and a triple periodic wave.

**Corollary 2** Setting $\mu_s = i\nu_k$ into Corollary 1 yields

$$\mathcal{H} = l_0 + \sum_{i=1}^{M}(l_i x + m_i y + n_i t + r_i)^2 + \sum_{s=1}^{N}\omega_s \cos(\mu_s(\rho_s x - \rho_s^3 t + \sigma_s)). \tag{18}$$

If $\mathcal{H}$ satisfies the assumptions $\sum l_i^3 m_i = 0$, $\sum_i l_i m_i = 0$ and $\sum m_i n_i = 0$, then $\mathcal{H}(x,y,t)$ will be a solution to Eq. (3).

**Example 3** If we choose $M=2$ with parameters $l_0 = l_1 = m_1 = m_2 = n_1 = r_1 = r_2 = 1$, $l_2 = n_2 = -1$, $\omega_i = 40$, $\mu_i = 1$, $\rho_1 = 0.25$, $\rho_2 = 1$, $\rho_3 = 0.2$, $\sigma_i = 0$, $i = 1,2,3$ at $t = 0$, then Corollary 2 will lead to

i. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2 + 40\cos(0.25x)$ for $N=1$;

ii. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2 + 40\cos(0.25x)+40\cos(x)$ for $N=2$;

iii. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2+40\cos(0.25x) + 40\cos(x)+40\cos(0.2x)$ for $N=3$;

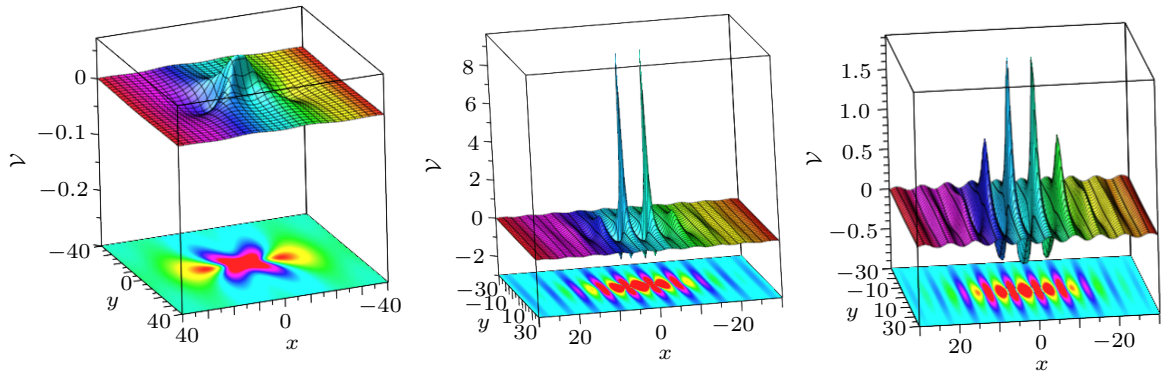
and their profiles are displayed in Figs. 5 and 6.

**Corollary 3** If

$$\mathcal{H} = l_0 + \sum_{i=1}^{M}(l_i x + m_i y + n_i t + r_i)^2$$

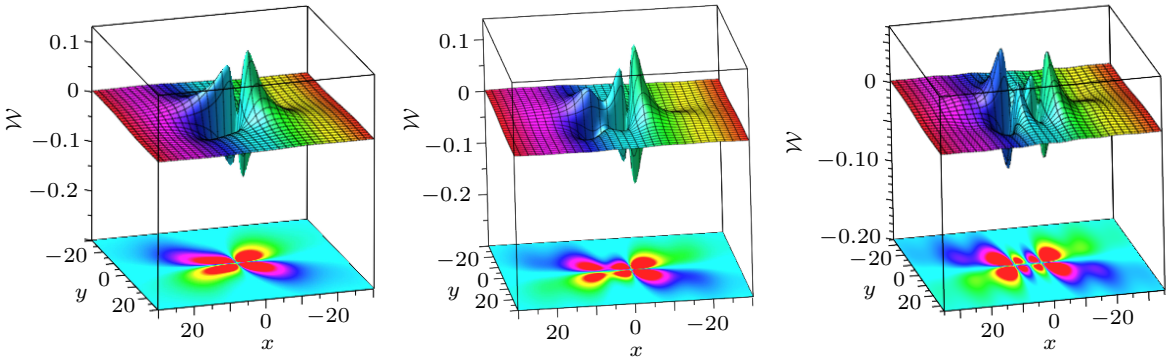
$$+\sum_{q=1}^{N}\omega_q\sin(\mu_q(\rho_q x-\rho_q^3 t+\sigma_q)) \tag{19}$$

satisfies the conditions $\sum l_i^3 m_i=0$, $\sum_i l_i m_i=0$ and $\sum m_i n_i=0$, then $\mathcal{H}(x,y,t)$ will be a solution to Eq. (3).



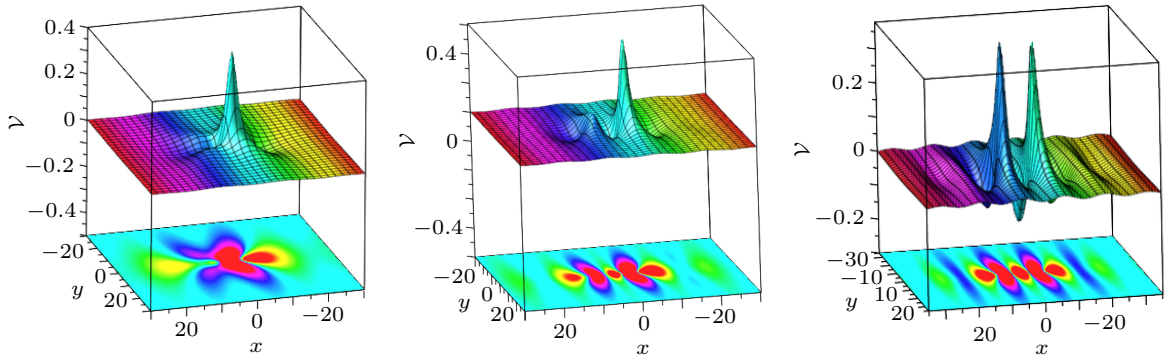
(a) $M=2,\ N=1$ (b) $M=2,\ N=2$ (c) $M=2,\ N=3$

**Fig. 6.** The 3D (upper) and density (lower) profiles of Eq. (18) for $\mathcal{V}$: interaction between (a) a rogue and a periodic wave, (b) a rogue and a double periodic wave, and (c) a rogue and a triple periodic wave.



(a) $M=2,\ N=1$ (b) $M=2,\ N=2$ (c) $M=2,\ N=4$

**Fig. 7.** The 3D (upper) and density (lower) profiles of Eq. (19) for $\mathcal{W}$: interaction between (a) a rogue and a periodic wave, (b) a rogue and a double periodic wave, and (c) a rogue and a four-periodic-bell wave.



(a) $M=2,\ N=1$ (b) $M=2,\ N=2$ (c) $M=2,\ N=4$

**Fig. 8.** The 3D (upper) and density (lower) profiles of Eq. (19) for $\mathcal{V}$: interaction between (a) a rogue and a periodic wave, (b) a rogue and a double-periodic-bell wave, and (c) a rogue and a four-periodic-bell wave.

**Example 4** First we choose $M=2$ with parameters $l_0=l_1=m_1=m_2=n_1=r_1=r_2=1$, $l_2=n_2=-1$, $\omega_i=40$, $\mu_i=1$, $\sigma_i=1$, $i=1,2,3,4$; for $\rho_1=1/3$ in (i), $\rho_1=1/2$, $\rho_2=1/7$ in (ii), $\rho_1=-0.5$, $\rho_2=0.5$, $\rho_3=1/7$, $\rho_4=-1/7$ in (iii) at $t=0$, then Corollary 3 will lead to

i. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2$
$+20\sin(x/3+1)$ for $N=1$;

ii. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2+20\sin(0.5x+1)$
$+20\sin(x/7+1)$ for $N=2$;

iii. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2-20\sin(0.5x-1)$
$+20\sin(0.5x+1)-20\sin(x/7-1)$
$+20\sin(x/7+1)$ for $N=4$;

and their profiles are displayed in Figs. 7 and 8.

**Corollary 4** If

$$\mathcal{H}(x,y,t) = l_0+\sum_{i=1}^{M}(l_i x+m_i y+n_i t+r_i)^2$$

$$+\sum_{j=1}^{N}\lambda_j e^{\nu_j(\rho_j x-\rho_j^3 t+\sigma_j)}$$
$$+\sum_{k=1}^{L}\omega_k\cosh(\varepsilon_k(\varepsilon_k x-\varepsilon_k^3 t+\varsigma_k))$$
$$+\sum_{s=1}^{P}\delta_s\cos(\mu_s(\iota_s x-\iota_s^3 t+\alpha_s))$$
$$+\sum_{q=1}^{Q}\xi_q\sin(\beta_q(\gamma_q x-\gamma_q^3 t+\kappa_q)) \qquad (20)$$

satisfies the conditions $\sum l_i^3 m_i=0$, $\sum_i l_i m_i=0$ and $\sum m_i n_i=0$, then $\mathcal{H}(x,y,t)$ will be a solution to Eq. (3).

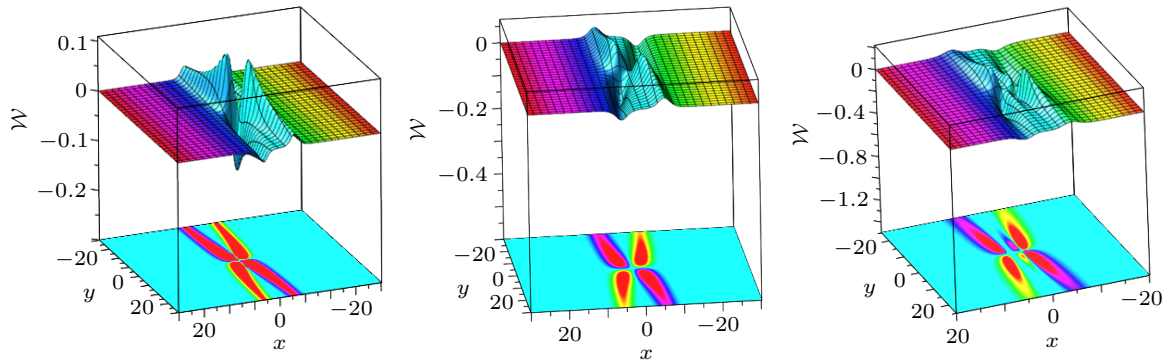
**Example 5** If we choose $M=2$ with parameters $l_0=l_1=m_1=m_2=n_1=r_1=r_2=1$, $l_2=n_2=-1$, $\lambda_1=5,\nu_1=1,\rho_1=1$, $\sigma_1=0$, $\omega_1=\omega_2=15,\varepsilon_1=\varepsilon_2=1$, $\varepsilon_1=1,\varepsilon_2=0.5$, $\varsigma_1=\varsigma_2=0$, $\delta_1=\delta_2=40,\mu_1=\mu_2=1,\iota_1=1/3,\iota_2=1/5$, $\alpha_1=\alpha_2=0$ at $t=0$, then Corollary 4 will lead to

i. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2$
$+40\cos(x/3)+15\cosh(x)$
for $N=0,\ L=1,P=1,\ Q=0$;

ii. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2+40\cos(x/3)$
$+40\cos(x/5)+15\cosh(x)+15\cosh(0.5x)$
for $N=0,\ L=2,\ P=2,\ Q=0$;

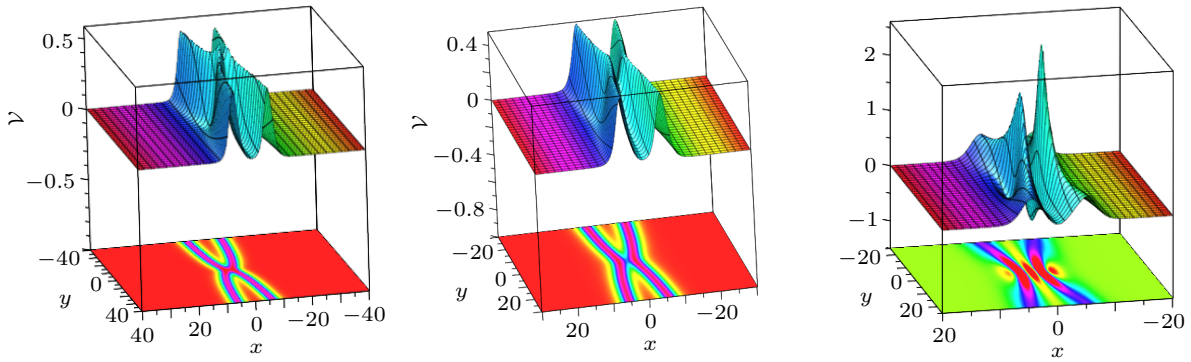
iii. $\mathcal{H} = 1+(1+x+y)^2+(1-x+y)^2+5e^x$
$+40\cos(x)+15\cosh(x)$
for $N=1,\ L=1,\ P=1,\ Q=0,\ \iota_1=1$;

and their profiles are displayed in Figs. 9 and 10.




(a) $M=2,\ N=0,\ L=1,\ P=1,\ Q=0$ (b) $M=2,\ N=0,\ L=2,\ P=2,\ Q=0$ (c) $M=2,\ N=1,\ L=1,\ P=1,\ Q=0$

**Fig. 9.** The 3D (upper) and density (lower) profiles of Eq. (20) for $\mathcal{W}$: multi-interaction among a rogue periodic wave and bright-bark bell soliton waves.




(a) $M=2,\ N=0,\ L=1,\ P=1,\ Q=0$ (b) $M=2,\ N=0,\ L=2,\ P=2,\ Q=0$ (c) $M=2,\ N=1,\ L=1,\ P=1,\ Q=0$

**Fig. 10.** The 3D (upper) and density (lower) profiles of Eq. (20) for $\mathcal{V}$: multi-interaction among a rogue periodic wave and bright-bark bell soliton waves.

## 4. Conclusion

The main results in this paper is to present new theories to allow multi-type interactions among higher-order rogues, $n$-periodic waves and $n$-soliton solutions to the (2+1)-dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) equation. To illustrate the existence of the theories, we present details of proofs of the theories with enough examples. We also explain dynamical behaviors of the corresponding interactions in graphically. Let us point out that more multi-types interaction solutions could be investigated by considering the following equation:

$$\mathcal{H}=l_0+\sum_{i=1}^{M}(l_i x+m_i y+n_i t+r_i)^2+\varphi(x,t), \qquad (21)$$

with the assumptions $\sum l_i^3 m_i=0$, $\sum_i l_i m_i=0$, $\sum m_i n_i=0$ and $\varphi_t+\varphi_{xxx}=0$ of the model (1). This illustrates a deep connection between higher-order rogue waves and $n$-soliton so-

lutions. Such computations would provide much interesting nonlinear physical phenomena in complex models.[30]

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