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Design of passive filters for time-delay neural networks with quantized output*

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Passive filtering of neural networks with time-invariant delay and quantized output is considered. A criterion on the passivity of a filtering error system is proposed by means of the Lyapunov–Krasovskii functional and the Bessel–Legendre inequality. Based on the criterion, a design approach for desired passive filters is developed in terms of the feasible solution of a set of linear matrix inequalities. Then, analyses and syntheses are extended to the time-variant delay situation using the reciprocally convex combination inequality. Finally, a numerical example with simulations is used to illustrate the applicability and reduced conservatism of the present passive filter design approaches.

Keywords: neural networks, time delay, quantization, filtering

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1. Introduction

In 1982, Hopfield[1] designed a dynamic neural network model for content addressable memory based on knowledge of neurobiology. The network model, which has feedback connections between neurons, can store and retrieve memory like the human brain and is easily to be implemented by simple circuit components such as resistors, capacitors, and amplifiers. Since then, various models of neural networks with or without time delays including, but not limited to, cellular neural networks, memristor-based neural networks, quaternion-valued neural networks, and fractional-order neural networks, have been proposed and successfully applied in associative memory,[2] secure communication,[3,4] and image encryption.[5,6] In engineering applications of these neural networks, external disturbances always exist, which may induce performance degradation and even instability. Therefore, the dynamics analysis and control synthesis of neural networks with external disturbances have drawn increasing attention from the scientific and engineering communities and a lot of results have been published in literature, see Refs. [7–13] and references therein.

As can be seen from the literature, the neuron states are often used directly for controller designs. However, in a relatively large neural network, usually only a part of neuron state information can be obtained through network outputs. Thus, in order to effectively use neural networks, a natural choice is to estimate the neuron states of a neural network first from the available measurements and then make design decisions based on the estimates. It is unsurprising that some efforts have been undertaken to the state estimation and filtering of neural networks over the last two decades. For instance, in Ref. [14], a design procedure was proposed for solving the state estimation issue of neural networks using linear matrix inequalities (LMIs). Delayed neural networks with norm-bounded uncertainties were considered in Ref. [15], where a delay-dependent method for design of robust state estimators was presented. In Ref. [16], a filter is designed to ensure that the error system is passive from external disturbances to the output error by virtue of the Lyapunov–Krasovskii functional approach. Considering the importance of the passivity framework in classical control theory, which can result in general conclusions on internal stability by applying only input-output characteristics, design strategies concerning passive filters were proposed in Refs. [16,17] for the continuous- and discrete-time cases, respectively.

With the development of sciences and technologies, especially computer technology, networked control systems have become increasingly popular in engineering practice. In such systems, the control loops are closed through communication networks. In comparison with classical point-to-point control systems, networked control systems can provide advantages including resource sharing, remote operation, and increased maintainability.[18,19] However, network control systems also face new challenges, such as signal quantization, packet loss, and transmission time delay due to the limited bandwidth of communication networks. Among them, the issue of signal quantization, which can lead to a decrease in system performance, has drawn much attention in filter de-

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signs of neural networks during the past few years. As for the event-triggered $\mathcal{H}_\infty$ filtering, dissipativity-based filtering, and asynchronous filtering of neural networks subject to quantized measurements, one can see Refs. [20–22], respectively. To our best knowledge, however, the passive filtering for neural networks with time delay and signal quantization still remains open and challenging, partly due to the complexity in analyses and subsequent syntheses.

In this paper, we concern the quantized passive filtering issue of time-delay neural networks. Both the time-invariant and time-variant delay situations are investigated. The purpose is to design filters with quantized output to ensure that the resulting filtering error systems are passive from the external disturbance to the output error. The main contributions in the present study can be summarized as follows: (1) Unlike the existing studies concerning passive filtering of time-delay neural networks (see, e.g., Refs. [16,17]), the output measurements of the neural networks are assumed to be transmitted through networked communication channels and the quantization effects on the output are taken into consideration. (2) Two criteria on the passivity are proposed by means of Lyapunov–Krasovskii functionals, the Bessel–Legendre inequality, the reciprocally convex combination inequality, as well as the free-weighting method. (3) Design approaches for the desired quantized passive filters are developed in terms of LMIs, which are computationally tractable.

2. Preliminaries

In this study, $\mathrm{diag}\{M_1,\cdots,M_l\}$ denotes a $l\times l$ diagonal matrix whose j-th diagonal entry is M_j, $\mathbb{R}^l$ denotes the l-dimensional Euclidean space with norm $||\cdot||$, and $\mathbb{S}_+^l$ stands for the space of symmetric positive-definite matrices of dimension l. Moreover, we define $\mathscr{S}(M)=M+M^{\mathrm{T}}$ for any square matrix M and $col\{v_1,\ldots,v_l\}=[v_1^{\mathrm{T}},\ldots,v_l^{\mathrm{T}}]^{\mathrm{T}}$ for any column vectors $v_1,\ldots,v_l$.

Consider a general nonlinear time-delay system modeled as

$$\dot{X}(t) = F(X(t))+G(X(t))U(t), \tag{1a}$$
$$Y(t) = E(X(t))+H(X(t))U(t), \tag{1b}$$

where $X(t)\in\mathbb{R}^n$, $U(t)\in\mathbb{R}^m$, and $Y(t)\in\mathbb{R}^m$ denote the state, input, and output, respectively; $F(\cdot)$, $G(\cdot)$ denote smooth vector fields and $E(\cdot)$, $H(\cdot)$ are smooth mappings.

For the system, the passivity concept is defined as follows:

Definition 1 System (1) is known to be passive from input $U(t)$ to output $Y(t)$ if there are a scalar $\beta>0$ and a positive semi-definite function $\Phi(X(t))$ such that

$$\int_0^t U^{\mathrm{T}}(\tau)Y(\tau)\,\mathrm{d}\tau+\beta\geq\int_0^t\Phi(X(\tau))\,\mathrm{d}\tau,\ \forall t\geq 0.$$

Remark 1 For a passive system, the rate at which the energy flows into it is not less than the increase in storage; that is to say, it cannot store more energy than that provided from the outside.

Consider a neural network with time-invariant delay

$$\dot{x}(t) = Ax(t)+B_1\phi(x(t))+B_2\phi(x(t-h))+D_1w(t), \tag{2a}$$
$$y(t) = C_1x(t)+C_2x(t-h)+D_2w(t), \tag{2b}$$

where $x(t)=[x_1(t)\ldots x_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$, $y(t)=[y_1(t)\ \ldots\ y_m(t)]^{\mathrm{T}}\in\mathbb{R}^m$, and $w(t)\in\mathbb{R}^m$ are the neuron state, system output, and external disturbance input belong to $\mathcal{L}_2[0,\infty]$,[23] respectively; h denotes the time delay; $A=\mathrm{diag}\{-a_1,\ldots,-a_n\}\in\mathbb{R}^{n\times n}$ is a self-feedback matrix with $a_i>0$, $i=1,\ldots,n$; $B_1\in\mathbb{R}^{n\times n}$ and $B_2\in\mathbb{R}^{n\times n}$ denote the connection-weight matrix; $D_1\in\mathbb{R}^{n\times m}$, $C_1\in\mathbb{R}^{m\times n}$, $C_2\in\mathbb{R}^{m\times n}$, $D_2\in\mathbb{R}^{m\times m}$ are known parameter matrices; $\phi(x(t))=[\phi_1(x(t))\ \ldots\ \phi_n(x(t))]^{\mathrm{T}}:\mathbb{R}^n\longrightarrow\mathbb{R}^n$ denotes the activation function vector that meets the Lipschitz constraint as[24–28]

$$||\phi(v_2)-\phi(v_1)||\leq L_\phi||v_2-v_1||,\ \forall v_1,v_2\in\mathbb{R}^n, \tag{3}$$

where $L_\phi>0$ is the Lipschitz scalar. For more relax assumptions of the activation functions, one may refer to Refs. [29, 30].

In the present work, it is supposed that the outputs of the neural network are transmitted to the filter via an unreliable communication network, and the effects of quantization are considered. The form of the filter to be designed is given by

$$\begin{aligned}\dot{\check{x}}(t) &= A\check{x}(t)+B_1\phi(\check{x}(t))\\ &\quad +B_2\phi(\check{x}(t-h))+LQ\left(y(t)-\check{y}(t)\right),\end{aligned} \tag{4a}$$
$$\check{y}(t) = C_1\check{x}(t)+C_2\check{x}(t-h), \tag{4b}$$

where $\check{x}(t)=[\check{x}_1(t)\ \ldots\ \check{x}_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$ and $\check{y}(t)=[\check{y}_1(t)\ \ldots\ \check{y}_m(t)]^{\mathrm{T}}\in\mathbb{R}^m$ are the state and output of the filter, respectively; $L\in\mathbb{R}^{n\times m}$ is the filter gain that needs to be determined; $Q(r)=[Q_1(r_1)\ \ldots\ Q_m(r_m)]^{\mathrm{T}}$ is a logarithmic quantizer with

$$Q_k(r_k)=\begin{cases}\chi^l\psi_0, & \frac{1}{1+\rho}\chi^l\psi_0<r_k\leq\frac{1}{1-\rho}\chi^l\psi_0,\\ & l=0,\pm1,\pm2,\ldots\\ 0, & r_k=0,\\ -Q_k(-r_k), & r_k<0,\end{cases}$$

for any $k\in\{1,\ldots,m\}$, where $\psi_0>0$ is a scaling scalar, $\rho=\frac{1-\chi}{1+\chi}$ with χ $(0<\chi<1)$ denoting the quantization density.[18,31]

With the aid of the sector-bound method,[32] $Q(r)$ can be written as

$$Q(r)=(1+\delta)r, \tag{5}$$

where $\delta\in[-\rho,\rho]$. Noting Eq. (5), we can establish the following filtering error system (FES):

$$\dot{\lambda}(t) = \bar{A}_1\lambda(t)-\bar{A}_2\lambda(t-h)+B_1f(\lambda(t))$$

$$+B_2 f(\lambda(t-h))+\bar{D}w(t), \tag{6a}$$

$$\tilde{y}(t) = C_1\lambda(t)+C_2\lambda(t-h)+D_2 w(t), \tag{6b}$$

where

$$\begin{aligned}
\lambda(t) &= x(t)-\breve{x}(t), \quad \tilde{y}(t)=y(t)-\breve{y}(t),\\
f(\lambda(t)) &= \phi(\lambda(t)+\breve{x}(t))-\phi(\breve{x}(t)),\\
f(\lambda(t-h)) &= \phi(\lambda(t-h)+\breve{x}(t-h))-\phi(\breve{x}(t-h)),\\
\bar{A}_1 &= A-(1+\delta)LC_1,\\
\bar{A}_2 &= (1+\delta)LC_2, \quad \bar{D}=D_1-(1+\delta)LD_2.
\end{aligned}$$

For neural network (2), we are interested in finding a quantized filter as system (4) to ensure that the resulting FES in system (6) is passive from external disturbance $w(t)$ to output error $\tilde{y}(t)$; i.e.,

$$\int_0^t w^{\mathrm{T}}(\xi)\tilde{y}(t)\,\mathrm{d}\xi+\beta \geq \int_0^t \phi(e(\xi))\,\mathrm{d}\xi, \ \forall t\geq 0. \tag{7}$$

3. Quantized passive filtering

In this section, we will give our analysis and design methods for the quantized passive filter. To proceed further, we introduce two useful Lemmas as follows:

Lemma 1[33] (Bessel–Legendre inequality) For a given matrix $X\in\mathbb{S}_+^n$ and any smooth function η in $[a,b]\to\mathbb{R}^n$,

$$\int_a^b \dot{\eta}^{\mathrm{T}}(\xi)X\dot{\eta}(\xi)\,\mathrm{d}\xi \geq \frac{1}{b-a}\Omega^{\mathrm{T}}\,\mathrm{diag}\{X,3X,5X\}\,\Omega \tag{8}$$

holds, with

$$\begin{aligned}
\Omega &= \begin{bmatrix} \eta(b)-\eta(a) \\ \eta(b)+\eta(a)-\frac{2}{b-a}\int_a^b \eta(\xi)\,\mathrm{d}\xi \\ \eta(b)-\eta(a)-\frac{6}{b-a}\int_a^b \delta_{a,b}(\xi)\eta(\xi)\,\mathrm{d}\xi \end{bmatrix},\\
\delta_{a,b}(\xi) &= 2\Big(\frac{\xi-a}{b-a}\Big)-1.
\end{aligned}$$

Lemma 2[34] For any real constant $\varepsilon>0$ and real matrices X_1 and X_2 of proper dimensions,

$$X_1X_2^{\mathrm{T}}+X_2X_1^{\mathrm{T}} \leq \frac{1}{\varepsilon}X_1X_1^{\mathrm{T}}+\varepsilon X_2X_2^{\mathrm{T}}. \tag{9}$$

For the passivity analysis for system (6), we can propose the following condition:

Theorem 1 Suppose that there exist matrices $P=(P_{ij})_{3\times 3}\in\mathbb{S}_+^{3n}$, $S_1, R_1\in\mathbb{S}_+^n$, $U_1,U_2\in\mathbb{R}^{n\times n}$, and diagonal matrices $Q_1, Q_2\in\mathbb{S}_+^n$, such that

$$\Phi_0<0, \tag{10}$$

where

$$\begin{aligned}
\Phi_0 &= \mathscr{S}(G_1^{\mathrm{T}}PG_0+G_4^{\mathrm{T}}\hat{U}G_5)+\hat{S}_1\\
&\quad +d_8^{\mathrm{T}}h^2R_1d_8-d_7^{\mathrm{T}}D_2d_7-G_2^{\mathrm{T}}\tilde{R}_1G_2\\
&\quad +G_3^{\mathrm{T}}\hat{S}_2G_3-d_7^{\mathrm{T}}C_2d_2-d_7^{\mathrm{T}}C_1d_1,\\
\hat{S}_1 &= \mathrm{diag}\{2S_1,-S_1,0_{6n\times 6n}\},\\
\hat{S}_2 &= \mathrm{diag}\{L_\phi^2Q_1,L_\phi^2Q_2,-Q_1,-Q_2\},\\
\tilde{R}_1 &= \mathrm{diag}\{R_1,3R_1,5R_1\}, \hat{U}=\begin{bmatrix}U_1 & U_2\end{bmatrix}^{\mathrm{T}},\\
G_0 &= \begin{bmatrix} d_8^{\mathrm{T}} & d_1^{\mathrm{T}}-d_2^{\mathrm{T}} & d_1^{\mathrm{T}}+d_2^{\mathrm{T}}-2d_3^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},\\
G_1 &= \begin{bmatrix} d_1^{\mathrm{T}} & hd_3^{\mathrm{T}} & hd_4^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},\\
G_2 &= \begin{bmatrix} d_1^{\mathrm{T}}-d_2^{\mathrm{T}} & d_1^{\mathrm{T}}+d_2^{\mathrm{T}}-2d_3^{\mathrm{T}} & d_1^{\mathrm{T}}-d_2^{\mathrm{T}}-6d_4^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},\\
G_3 &= \begin{bmatrix} d_1^{\mathrm{T}} & d_2^{\mathrm{T}} & d_5^{\mathrm{T}} & d_6^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}, \quad G_4=\begin{bmatrix} d_8^{\mathrm{T}} & d_1^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},\\
G_5 &= d_8-\bar{A}_1d_1+\bar{A}_2d_2-B_1d_5-B_2d_6-\bar{D}d_7,\\
d_i &= \begin{bmatrix} 0_{n\times(i-1)n} & I_n & 0_{n\times(8-i)n}\end{bmatrix}, \quad i=1,\ldots,8.
\end{aligned}$$

Then FES (6) is passive from the external disturbance $w(t)$ to the output $\tilde{y}(t)$.

Proof Define

$$\zeta(t)=col\{\sigma_0(t),\sigma_1(t),\sigma_2(t)\},$$

where

$$\begin{aligned}
\sigma_0(t) &= \begin{bmatrix}\lambda^{\mathrm{T}}(t) & \lambda^{\mathrm{T}}(t-h)\end{bmatrix}^{\mathrm{T}},\\
\sigma_1(t) &= \frac{1}{h}\begin{bmatrix}\int_{-h}^0 \lambda_t^{\mathrm{T}}(s)\,\mathrm{d}s & \int_{-h}^0 \delta(s)\lambda_t^{\mathrm{T}}(s)\,\mathrm{d}s\end{bmatrix}^{\mathrm{T}},\\
\sigma_2(t) &= \begin{bmatrix} f^{\mathrm{T}}(\lambda(t)) & f^{\mathrm{T}}(\lambda(t-h)) & w^{\mathrm{T}}(t) & \dot{\lambda}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},\\
\lambda_t(s) &= \lambda(t+s), \quad \delta(s)=2\frac{s+h}{h}-1.
\end{aligned}$$

Then, by the free-weighting method,[35] for any matrices $U_1,U_2\in\mathbb{R}^{n\times n}$, it holds that

$$\begin{aligned}
&\zeta^{\mathrm{T}}(t)\mathscr{S}(G_4^{\mathrm{T}}\hat{U}G_5)\zeta(t)\\
&= 2\big(\dot{\lambda}^{\mathrm{T}}(t)U_1+\lambda^{\mathrm{T}}(t)U_2\big)\big(\dot{\lambda}(t)-\bar{A}_1\lambda(t)+\bar{A}_2\lambda(t-h)\\
&\quad -B_1f(\lambda(t))-B_2f(\lambda(t-h))-\bar{D}w(t)\big)\\
&= 0.
\end{aligned} \tag{11}$$

On the other hand, from Eq. (3), for any diagonal matrices Q_1, $Q_2\in\mathbb{S}_+^n$ we have

$$\begin{aligned}
&L_\phi^2\lambda(t)^{\mathrm{T}}Q_1\lambda(t)-f^{\mathrm{T}}(\lambda(t))Q_1f(\lambda(t))\geq 0,\\
&L_\phi^2\lambda^{\mathrm{T}}(t-h)Q_2\lambda(t-h)-f^{\mathrm{T}}(\lambda(t-h))Q_2f(\lambda(t-h))\geq 0,
\end{aligned}$$

which imply

$$\zeta^{\mathrm{T}}(t)G_3^{\mathrm{T}}\hat{S}_2G_3\zeta(t)\geq 0. \tag{12}$$

Let us choose a Lyapunov–Krasovskii functional as follows:

$$\begin{aligned}
V(\lambda_t,\dot{\lambda}_t) &= V_1(\lambda_t)+V_2(\lambda_t)+V_3(\lambda_t,\dot{\lambda}_t),\\
V_1(\lambda_t) &= \int_{t-h}^t \lambda^{\mathrm{T}}(s)S_1\lambda(s)\,\mathrm{d}s,\\
V_2(\lambda_t) &= \tilde{\lambda}^{\mathrm{T}}(t)P\tilde{\lambda}(t),\\
V_3(\lambda_t,\dot{\lambda}_t) &= h\int_{-h}^0\int_{t+\theta}^t \dot{e}^{\mathrm{T}}(s)R_1\dot{\lambda}(s)\,\mathrm{d}s\,\mathrm{d}\theta,
\end{aligned} \tag{13}$$

where

$$\tilde{\lambda}(t)=col\{\lambda(t),h\sigma_1(t)\}. \tag{14}$$

It can be calculated that

$$\tilde{\lambda}(t)=G_1\zeta(t),\ \ \dot{\tilde{\lambda}}(t)=G_0\zeta(t). \tag{15}$$

Differentiating Eq. (13) results in

$$\begin{aligned}\dot{V}_1(\lambda_t) &= \zeta^{\mathrm{T}}(t)\,\mathrm{diag}\{S_1,-S_1,0_{6n\times 6n}\}\zeta(t)\\ &= \zeta^{\mathrm{T}}(t)(\hat{S}_1-\mathrm{diag}\{S_1,0_{7n\times 7n}\})\zeta(t)\\ &= \zeta^{\mathrm{T}}(t)\hat{S}_1\zeta(t)-\lambda^{\mathrm{T}}(t)S_1\lambda(t),\\ \dot{V}_2(\lambda_t) &= 2\tilde{\lambda}^{\mathrm{T}}(t)P\dot{\tilde{\lambda}}(t),\\ \dot{V}_3(\lambda_t,\dot{\lambda}_t) &= h^2\dot{\lambda}^{\mathrm{T}}(t)R_1\dot{\lambda}(t)-h\int_{t-h}^{t}\dot{\lambda}^{\mathrm{T}}(s)R_1\dot{\lambda}(s)\mathrm{d}s.\end{aligned} \tag{16}$$

By means of Eq. (15) and Lemma 1, we have

$$\dot{V}_2(\lambda_t) = \zeta^{\mathrm{T}}(t)\mathscr{S}(G_1^{\mathrm{T}}PG_0)\zeta(t), \tag{17}$$

$$\dot{V}_3(\lambda_t,\dot{\lambda}_t) \le \zeta^{\mathrm{T}}(t)(d_8^{\mathrm{T}}h^2R_1d_8-G_2^{\mathrm{T}}\tilde{R}_1G_2)\zeta(t). \tag{18}$$

Now, using Eqs. (11), (12), and (16)–(18) we can reach

$$\begin{aligned}&\dot{V}(\lambda_t,\dot{\lambda}_t)\\ &\le \zeta^{\mathrm{T}}(t)(\mathscr{S}(G_1^{\mathrm{T}}PG_0+G_4^{\mathrm{T}}\hat{U}G_5)+\hat{S}_1+d_8^{\mathrm{T}}h^2R_1d_8\\ &\quad -G_2^{\mathrm{T}}\tilde{R}_1G_2+G_3^{\mathrm{T}}\hat{S}_2G_3)\zeta(t)-\lambda^{\mathrm{T}}(t)S_1\lambda(t)\\ &= \zeta^{\mathrm{T}}(t)\Phi_0\zeta(t)-\lambda^{\mathrm{T}}(t)S_1\lambda(t)+w^{\mathrm{T}}(t)[C_1\lambda(t)\\ &\quad +C_2\lambda(t-h)+D_2w(t)].\end{aligned}$$

From Eqs. (6b) and (10), we have

$$\dot{V}(\lambda_t,\dot{\lambda}_t)\le-\lambda^{\mathrm{T}}(t)S_1\lambda(t)+w^{\mathrm{T}}(t)\tilde{y}(t).$$

Integrating both sides of the above inequality from 0 to t gives

$$\begin{aligned}V(\lambda_t)-V(\lambda_0) \le &-\int_0^t\lambda^{\mathrm{T}}(\xi)S_1\lambda(\xi)\mathrm{d}\xi\\ &+\int_0^t w^{\mathrm{T}}(\xi)\tilde{y}(\xi)\mathrm{d}\xi.\end{aligned} \tag{19}$$

Noting $V(\lambda_t)\ge 0$, from Eq. (19) we have

$$\int_0^t w^{\mathrm{T}}(\xi)\tilde{y}(t)\mathrm{d}\xi+V(\lambda_0)\ge\int_0^t\phi(\lambda(\xi))\mathrm{d}\xi,$$

which corresponds to Eq. (7) by taking $\beta=V(\lambda_0)$ and, hence, FES in Eq. (6) is shown to be passive from external disturbance $w(t)$ to output error $\tilde{y}(t)$. □

Based on Theorem 1, it is able to develop an LMIs-based design strategy for the quantized passive filter, which is given as follows:

Theorem 2 For given scalars $\mu_1>0$, $\mu_2>0$, and $\rho>0$, if there are scalar $\varepsilon>0$, and matrices $P=(P_{ij})_{3\times 3}\in\mathbb{S}_+^{3n}$, S_1, $R_1\in\mathbb{S}_+^n$, $M\in\mathbb{R}^{n\times m}$, diagonal matrices Q_1, $Q_2\in\mathbb{S}_+^n$ such that

$$\begin{bmatrix}\Phi_1 & \rho W_1\\ * & -\varepsilon I\end{bmatrix}<0, \tag{20}$$

where

$$\begin{aligned}\Phi_1 &= \mathscr{S}(G_1^{\mathrm{T}}PG_0)+\bar{\Sigma}_1+\bar{\Sigma}_1+\hat{S}_1+d_8^{\mathrm{T}}h^2R_1d_8-G_2^{\mathrm{T}}\tilde{R}_1G_2\\ &\quad +G_3^{\mathrm{T}}\hat{S}_2G_3-d_7^{\mathrm{T}}C_2d_2-d_7^{\mathrm{T}}C_1d_1-d_7^{\mathrm{T}}D_2d_7+\varepsilon W_2W_2^{\mathrm{T}},\\ \bar{\Sigma}_1 &= \mathscr{S}(d_8^{\mathrm{T}}(-\mu_1MC_1+\mu_1P_{11}A)d_1-\mu_1d_8^{\mathrm{T}}MC_2d_2\\ &\quad +\mu_1d_8^{\mathrm{T}}P_{11}B_1d_5+\mu_1d_8^{\mathrm{T}}P_{11}B_2d_6\\ &\quad +d_8^{\mathrm{T}}(-\mu_1MD_2+\mu_1P_{11}D_1)d_7-\mu_1d_8^{\mathrm{T}}P_{11}d_8),\\ \bar{\Sigma}_2 &= \mathscr{S}(d_1^{\mathrm{T}}(-\mu_2MC_1+\mu_2P_{11}A)d_1-\mu_2d_1^{\mathrm{T}}MC_2d_2\\ &\quad +\mu_2d_1^{\mathrm{T}}P_{11}B_1d_5+\mu_2d_1^{\mathrm{T}}P_{11}B_2d_6\\ &\quad +d_1^{\mathrm{T}}(-\mu_2MD_2+\mu_2P_{11}D_1)d_7-\mu_2d_1^{\mathrm{T}}P_{11}d_8),\\ W_1 &= \begin{bmatrix}-\mu_2M^{\mathrm{T}} & 0_{6n\times 6n} & -\mu_1M^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},\\ W_2 &= \begin{bmatrix}C_1 & C_2 & 0_{4n\times 4n} & D_2 & 0\end{bmatrix}^{\mathrm{T}},\end{aligned}$$

and the other symbols are the same as those in Theorem 1, then system (4) with $L=P_{11}^{-1}M$ will be a desired quantized passive filter for neural network (2).

Proof Set $U_1=-\mu_1P_{11}$ and $U_2=-\mu_2P_{11}$. Then, it holds such that

$$\begin{aligned}G_4^{\mathrm{T}}\hat{U}G_5 &= \bar{\Sigma}_1+\bar{\Sigma}_2-\delta\mu_1d_8^{\mathrm{T}}MC_1d_1-\delta\mu_1d_8^{\mathrm{T}}MC_2d_2\\ &\quad -\delta\mu_1d_8^{\mathrm{T}}MD_2d_7-\delta\mu_2d_1^{\mathrm{T}}MC_1d_1\\ &\quad -\delta\mu_2d_1^{\mathrm{T}}MC_2d_2-\delta\mu_2d_1^{\mathrm{T}}MD_2.\end{aligned}$$

Using it, we can rewrite Eq. (10) as

$$\hat{\Phi}_0+\mathscr{S}(\delta W_1W_2^{\mathrm{T}})<0, \tag{21}$$

where

$$\begin{aligned}\hat{\Phi}_0 &= \mathscr{S}(G_1^{\mathrm{T}}PG_0)+\bar{\Sigma}_1+\bar{\Sigma}_2+\hat{S}_1\\ &\quad +d_8^{\mathrm{T}}h^2R_1d_8-d_7^{\mathrm{T}}D_2d_7-G_2^{\mathrm{T}}\tilde{R}_1G_2\\ &\quad +G_3^{\mathrm{T}}\hat{S}_2G_3-d_7^{\mathrm{T}}C_2d_2-d_7^{\mathrm{T}}C_1d_1.\end{aligned}$$

From Lemma 2, it follows

$$\mathscr{S}(\delta W_1W_2^{\mathrm{T}})\le\frac{\rho^2}{\varepsilon}W_1W_1^{\mathrm{T}}+\varepsilon W_2W_2^{\mathrm{T}},$$

by which it can be concluded that Eq. (21) will be held if

$$\hat{\Phi}_0+\frac{\rho^2}{\varepsilon}W_1W_1^{\mathrm{T}}+\varepsilon W_2W_2^{\mathrm{T}}<0 \tag{22}$$

is satisfied. Then, by noting $L=P_{11}^{-1}M$ and using Schur's complement, we can re-organize Eq. (22) as Eq. (20). In this way, it is shown that the condition in this theorem implies the one in Theorem 1, thereby ensuring the passivity of FES in system (6). □

Suppose that there is no quantization in the filter. Then, Eq. (4) reduces to

$$\begin{aligned}\dot{\breve{x}}(t) &= A\breve{x}(t)+B_1\phi(\breve{x}(t))+B_2\phi(\breve{x}(t-h))\\ &\quad +L(y(t)-\breve{y}(t)),\end{aligned} \tag{23a}$$

$$\breve{y}(t) = C_1\breve{x}(t)+C_2\breve{x}(t-h), \tag{23b}$$

and we can obtain the following result:

Corollary 1 For given scalars $\mu_1 > 0$, $\mu_2 > 0$, if there are matrices $P = (P_{ij})_{3\times 3} \in \mathbb{S}_+^{3n}$, $S_1, R_1 \in \mathbb{S}_+^n$, $M \in \mathbb{R}^{n\times m}$, diagonal matrices $Q_1, Q_2 \in \mathbb{S}_+^n$ such that

$$\Phi_2 < 0,$$

where

$$\begin{aligned}\Phi_2 =\ & \mathscr{S}(G_1^{\mathrm{T}} P G_0) + \bar{\Sigma}_1 + \bar{\Sigma}_2 + \hat{S}_1 + d_8^{\mathrm{T}} h^2 R_1 d_8 - G_2^{\mathrm{T}} \tilde{R}_1 G_2 \\ & + G_3^{\mathrm{T}} \hat{S}_2 G_3 - d_7^{\mathrm{T}} C_2 d_2 - d_7^{\mathrm{T}} C_1 d_1 - d_7^{\mathrm{T}} D_2 d_7,\end{aligned}$$

and the other notations are the same as those in Theorem 2, then for neural network (2), filter (23) with gain $L = P_{11}^{-1} M$ can ensure the resulting FES (6) with $\delta = 0$ to be passive from disturbance $w(t)$ to output error $\tilde{y}(t)$.

4. Extension to the time-variant delay situation

In a practical neural network, the time delay may be time-variant. In this section, we consider the following neural network:

$$\begin{aligned}\dot{x}(t) =\ & Ax(t) + B_1\phi(x(t)) \\ & + B_2\phi(x(t-h(t))) + D_1 w(t), \qquad (24a)\\ y(t) =\ & C_1 x(t) + C_2 x(t-h(t)) + D_2 w(t), \qquad (24b)\end{aligned}$$

where $h(t)$ denotes a time-variant delay, which meets

$$0 < h_1 < h(t) < h_2,\ h_{12} \triangleq h_2 - h_1. \qquad (25)$$

In this case, the filter to be designed and the corresponding FES change into

$$\begin{aligned}\dot{\check{x}}(t) =\ & A\check{x}(t) + B_1\phi(\check{x}(t)) + B_2\phi(\check{x}(t-h(t))) \\ & + LQ(y(t) - \check{y}(t)), \qquad (26a)\\ \check{y}(t) =\ & C_1\check{x}(t) + C_2\check{x}(t-h(t)), \qquad (26b)\end{aligned}$$

and

$$\begin{aligned}\dot{\lambda}(t) =\ & \bar{A}_1\lambda(t) - \bar{A}_2\lambda(t-h(t)) + B_1 f(\lambda(t)) \\ & + B_2 f(\lambda(t-h(t))) + \bar{D}w(t), \qquad (27a)\\ \tilde{y}(t) =\ & C_1\lambda(t) + C_2\lambda(t-h(t)) + D_2 w(t), \qquad (27b)\end{aligned}$$

respectively, where $\bar{A}_1$, $\bar{A}_2$ and $\bar{D}$ are the same as those in system (6).

To derive our main results, in addition to Lemmas 1 and 2, the following lemma is also required:

Lemma 3 (Reciprocally convex combination inequality)[36] For any given matrix $R \in \mathbb{S}_+^n$, suppose that there is the matrix $X \in \mathbb{R}^{n\times n}$ such that $\begin{bmatrix} R & X \\ X^{\mathrm{T}} & R \end{bmatrix} \geq 0$. Then one has

$$\begin{bmatrix} \frac{1}{\alpha}R & 0 \\ 0 & \frac{1}{1-\alpha}R \end{bmatrix} \geq \begin{bmatrix} R & X \\ X^{\mathrm{T}} & R \end{bmatrix}, \forall \alpha \in (0,1). \qquad (28)$$

The following condition regarding the passivity of the FES in Eq. (27) can be obtained without imposing any other additional requirements on time delay function $h(t)$ such as the differentiability (see, e.g., Refs. [37–44]):

Theorem 3 Suppose that there exist matrices $P = (P_{ij})_{5\times 5} \in \mathbb{S}_+^{5n}$, S_1, S_2, R_1, $R_2 \in \mathbb{S}_+^n$, $U_1, U_2 \in \mathbb{R}^{n\times n}$, N_1, $N_2 \in \mathbb{R}^{18n\times 2n}$, diagonal matrices Q_1, $Q_2 \in \mathbb{S}_+^n$, and a matrix $X \in \mathbb{R}^{3n\times 3n}$, such that the LMIs

$$\Psi = \begin{bmatrix} \tilde{R}_2 & X \\ X^{\mathrm{T}} & \tilde{R}_2 \end{bmatrix} \geq 0, \qquad (29)$$

$$\Phi(h(t)) = \Phi_0(h(t)) - \Gamma^{\mathrm{T}}\Psi\Gamma < 0 \qquad (30)$$

hold true for any $h(t) \in [h_1, h_2]$, where

$$\begin{aligned}\Phi_0(h(t)) =\ & \mathscr{S}(G_1^{\mathrm{T}}(h(t))PG_0 + N_1 g_1(h(t)) + N_2 g_2(h(t)) \\ & + G_6^{\mathrm{T}}\hat{U}G_7) + \hat{S}_1 + {d_{18}}^{\mathrm{T}}(h_1^2 R_1 + h_{12}^2 R_2) d_{18} \\ & - {d_{17}}^{\mathrm{T}} D_2 d_{17} - G_2^{\mathrm{T}}\tilde{R}_1 G_2 + G_5^{\mathrm{T}}\hat{S}_2 G_5 \\ & - {d_{17}}^{\mathrm{T}} C_2 d_3 - {d_{17}}^{\mathrm{T}} C_1 d_1, \\ \hat{S}_1 =\ & \mathrm{diag}\{2S_1, -S_1 + S_2, 0_{n\times n}, -S_2, 0_{14n\times 14n}\}, \\ \hat{S}_2 =\ & \mathrm{diag}\{L_\phi^2 Q_1, L_\phi^2 Q_2, -Q_1, -Q_2\}, \\ \tilde{R}_i =\ & \mathrm{diag}\{R_i, 3R_i, 5R_i\}, \quad i = 1,2, \hat{U} = \begin{bmatrix} U_1 & U_2 \end{bmatrix}^{\mathrm{T}}, \\ G_0 =\ & \begin{bmatrix} d_{18}^{\mathrm{T}} & d_1^{\mathrm{T}} - d_2^{\mathrm{T}} & d_1^{\mathrm{T}} + d_2^{\mathrm{T}} - 2d_5^{\mathrm{T}} & d_2^{\mathrm{T}} - d_4^{\mathrm{T}} & \hat{G}_0^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \hat{G}_0 =\ & h_{12}(d_2 + d_4) - 2(d_{11} + d_{13}), \\ G_1(h(t)) =\ & \begin{bmatrix} d_1^{\mathrm{T}} & h_1 d_5^{\mathrm{T}} & h_1 d_6^{\mathrm{T}} & d_{11}^{\mathrm{T}} + d_{13}^{\mathrm{T}} & \hat{G}_1^{\mathrm{T}}(h(t)) \end{bmatrix}^{\mathrm{T}}, \\ \hat{G}_1(h(t)) =\ & (h_2 - h(t))(d_{11} + d_{14}) + (h(t) - h_1)(d_{12} - d_{13}), \\ G_2 =\ & \begin{bmatrix} d_1^{\mathrm{T}} - d_2^{\mathrm{T}} & d_1^{\mathrm{T}} + d_2^{\mathrm{T}} - 2d_5^{\mathrm{T}} & d_1^{\mathrm{T}} - d_2^{\mathrm{T}} - 6d_6^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ G_3 =\ & \begin{bmatrix} d_2^{\mathrm{T}} - d_3^{\mathrm{T}} & d_2^{\mathrm{T}} + d_3^{\mathrm{T}} - 2d_7^{\mathrm{T}} & d_2^{\mathrm{T}} - d_3^{\mathrm{T}} - 6d_8^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ G_4 =\ & \begin{bmatrix} d_3^{\mathrm{T}} - d_4^{\mathrm{T}} & d_3^{\mathrm{T}} + d_4^{\mathrm{T}} - 2d_9^{\mathrm{T}} & d_3^{\mathrm{T}} - d_4^{\mathrm{T}} - 6d_{10}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ G_5 =\ & \begin{bmatrix} d_1^{\mathrm{T}} & d_3^{\mathrm{T}} & d_{15}^{\mathrm{T}} & d_{16}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, G_6 = \begin{bmatrix} d_{18}^{\mathrm{T}} & d_1^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ G_7 =\ & -\bar{A}_1 d_1 + \bar{A}_2 d_3 - B_1 d_{15} - B_2 d_{16} - \bar{D} d_{17} + d_{18}, \\ \Gamma =\ & \begin{bmatrix} G_3^{\mathrm{T}} & G_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ g_1(h(t)) =\ & (h(t) - h_1)\begin{bmatrix} d_7 \\ d_8 \end{bmatrix} - \begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix}, \\ g_2(h(t)) =\ & (h_2 - h(t))\begin{bmatrix} d_9 \\ d_{10} \end{bmatrix} - \begin{bmatrix} d_{13} \\ d_{14} \end{bmatrix}, \\ d_i =\ & \begin{bmatrix} 0_{n\times(i-1)n} & I_n & 0_{n\times(18-i)n} \end{bmatrix}, i = 1, \cdots, 18,\end{aligned}$$

then the FES (27) will be passive from disturbance $w(t)$ to output error $\tilde{y}(t)$.

Proof Define

$$\zeta(t) = col\{\sigma_0(t), \sigma_1(t), \sigma_2(t), \sigma_3(t), \sigma_4(t), \sigma_5(t), \sigma_6(t)\},$$

where

$$\begin{aligned}\sigma_0(t) =\ & \begin{bmatrix} \lambda^{\mathrm{T}}(t) & \lambda^{\mathrm{T}}(t-h_1) & \lambda^{\mathrm{T}}(t-h(t)) & \lambda^{\mathrm{T}}(t-h_2) \end{bmatrix}^{\mathrm{T}}, \\ \sigma_1(t) =\ & \frac{1}{h_1}\begin{bmatrix} \int_{-h_1}^0 \lambda_t^{\mathrm{T}}(s)ds & \int_{-h_1}^0 \delta_1(s)\lambda_t^{\mathrm{T}}(s)ds \end{bmatrix}^{\mathrm{T}}, \\ \sigma_2(t) =\ & \frac{1}{h(t) - h_1}\begin{bmatrix} \int_{-h(t)}^{-h_1} \lambda_t^{\mathrm{T}}(s)ds & \int_{-h(t)}^{-h_1} \delta_2(s)\lambda_t^{\mathrm{T}}(s)ds \end{bmatrix}^{\mathrm{T}},\end{aligned}$$

$$\begin{aligned}
\sigma_3(t) &= \frac{1}{h_2-h(t)}\left[\int_{-h_2}^{-h(t)}\lambda_t^{\mathrm{T}}(s)ds \ \int_{-h_2}^{-h(t)}\delta_3(s)\lambda_t^{\mathrm{T}}(s)ds\right]^{\mathrm{T}},\\
\sigma_4(t) &= (h(t)-h_1)\sigma_2(t), \sigma_5(t) = (h_2-h(t))\sigma_3(t),\\
\sigma_6(t) &= \left[f^{\mathrm{T}}(\lambda(t)) \ f^{\mathrm{T}}(\lambda(t-h(t))) \ w^{\mathrm{T}}(t) \ \dot{\lambda}^{\mathrm{T}}(t)\right]^{\mathrm{T}},\\
\lambda_t(s) &= \lambda(t+s),\\
\delta_1(s) &= 2\frac{s+h_1}{h_1}-1, \delta_2(s) = 2\frac{s+h(t)}{h(t)-h_1}-1,\\
\delta_3(s) &= 2\frac{s+h_2}{h_2-h(t)}-1, \delta_4(s) = 2\frac{s+h_2}{h_{12}}-1.
\end{aligned}$$

Then, noting that $g_1(h(t))\zeta(t) = g_2(h(t))\zeta(t) = 0$, for any matrices N_1, N_2 in $\mathbb{R}^{18n\times 2n}$, we have

$$2\zeta^{\mathrm{T}}(t)(N_1 g_1(h(t)) + N_2 g_2(h(t)))\zeta(t) = 0. \tag{31}$$

In addition, similar to the proof of Theorem 1, the following results can be written for any matrices $U_1, U_2 \in \mathbb{R}^{n\times n}$ and any diagonal matrices $Q_1, Q_2 \in \mathbb{S}_+^n$:

$$\zeta^{\mathrm{T}}(t)\mathscr{S}(G_6^{\mathrm{T}}\hat{U}G_7)\zeta(t) = 0, \tag{32}$$

$$\zeta^{\mathrm{T}}(t)G_5^{\mathrm{T}}\hat{S}_2G_5\zeta(t) \geq 0. \tag{33}$$

Consider a Lyapunov–Krasovskii functional given by

$$\begin{aligned}
V(\lambda_t,\dot{\lambda}_t) &= V_1(\lambda_t)+V_2(\lambda_t)+V_3(\lambda_t,\dot{\lambda}_t),\\
V_1(\lambda_t) &= \int_{t-h_1}^{t}\lambda^{\mathrm{T}}(s)S_1\lambda(s)\mathrm{d}s+\int_{t-h_2}^{t-h_1}\lambda^{\mathrm{T}}(s)S_2\lambda(s)\mathrm{d}s,\\
V_2(\lambda_t) &= \tilde{\lambda}^{\mathrm{T}}(t)P\tilde{\lambda}(t),\\
V_3(\lambda_t,\dot{\lambda}_t) &= h_1\int_{-h_1}^{0}\int_{t+\theta}^{t}\dot{\lambda}^{\mathrm{T}}(s)R_1\dot{\lambda}(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad +h_{12}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{\lambda}^{\mathrm{T}}(s)R_2\dot{\lambda}(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

in which

$$\begin{aligned}
\tilde{\lambda}(t) &= col\{\lambda(t), h_1\sigma_1(t), \sigma_7(t)\},\\
\sigma_7(t) &= \left[\int_{-h_2}^{-h_1}\lambda_t^{\mathrm{T}}(s)ds \ h_{12}\int_{-h_2}^{-h_1}\delta_4(s)\lambda_t^{\mathrm{T}}(s)ds\right]^{\mathrm{T}}.
\end{aligned}$$

It can be obtained that

$$\dot{V}_1(\lambda_t) = \zeta^{\mathrm{T}}(t)\hat{S}_1\zeta(t) - \lambda^{\mathrm{T}}(t)S_1\lambda(t). \tag{34}$$

Noting

$$\begin{aligned}
h_{12}\delta_4(s) &= (h(t)-h_1)\delta_2(s)+(h_2-h(t)),\\
h_{12}\delta_4(s) &= (h_2-h(t))\delta_3(s)-(h(t)-h_1),
\end{aligned}$$

we can write

$$\sigma_7(t) = \begin{bmatrix} d_{11}+d_{13} \\ \hat{G}_1(h(t))\end{bmatrix}\zeta(t).$$

By combining $\lambda(t) = d_1\zeta(t)$, $h_1\sigma_1(t) = h_1\left[d_5^{\mathrm{T}} \ d_6^{\mathrm{T}}\right]^{\mathrm{T}}\zeta(t)$ with the above expression, we have

$$\begin{aligned}
\dot{V}_2(\lambda_t) &= 2\tilde{\lambda}^{\mathrm{T}}(t)P\dot{\tilde{\lambda}}(t)\\
&= \zeta^{\mathrm{T}}(t)\mathscr{S}(G_1^{\mathrm{T}}(h(t))PG_0)\zeta(t).
\end{aligned} \tag{35}$$

Moreover, using Lemma 1 and Lemma 3 as[45] leads to

$$\begin{aligned}
&\dot{V}_3(\lambda_t,\dot{\lambda}_t)\\
&= h_1^2\dot{\lambda}^{\mathrm{T}}(t)R_1\dot{\lambda}(t)+h_{12}^2\dot{\lambda}^{\mathrm{T}}(t)R_2\dot{e}(t)\\
&\quad -h_1\int_{t-h_1}^{t}\dot{\lambda}^{\mathrm{T}}(s)R_1\dot{\lambda}(s)\mathrm{d}s-h_{12}\int_{t-h_2}^{t-h_1}\dot{\lambda}^{\mathrm{T}}(s)R_2\dot{\lambda}(s)\mathrm{d}s\\
&\leq \zeta^{\mathrm{T}}(t)(d_{18}{}^{\mathrm{T}}(h_1^2R_1+h_{12}^2R_2)d_{18}-G_2^{\mathrm{T}}\tilde{R}_1G_2-\Gamma^{\mathrm{T}}\Psi\Gamma)\zeta(t).
\end{aligned} \tag{36}$$

Now, from Eqs. (31)–(36) we can write

$$\begin{aligned}
\dot{V}(\lambda_t,\dot{\lambda}_t) &\leq \zeta^{\mathrm{T}}(t)(\mathscr{S}(G_1^{\mathrm{T}}(h(t))PG_0+N_1g_1(h(t))\\
&\quad +N_2g_2(h(t))+G_6^{\mathrm{T}}\hat{U}G_7)+\hat{S}_1\\
&\quad +d_{18}{}^{\mathrm{T}}(h_1^2R_1+h_{12}^2R_2)d_{18}-G_2^{\mathrm{T}}\tilde{R}_1G_2\\
&\quad -\Gamma^{\mathrm{T}}\Psi\Gamma+G_5^{\mathrm{T}}\hat{S}_2G_5)\zeta(t)-\lambda^{\mathrm{T}}(t)S_1\lambda(t)\\
&= \zeta^{\mathrm{T}}(t)\Phi(h(t))\zeta(t)-\lambda^{\mathrm{T}}(t)S_1\lambda(t)+w^{\mathrm{T}}(t)\tilde{y}(t).
\end{aligned}$$

Due to the convex property of $\Phi(\cdot)$, Eq. (30) implies that $\Phi(h(t)) < 0$, $\forall h(t) \in [h_1, h_2]$. It follows that

$$\dot{V}(\lambda_t,\dot{\lambda}_t) < -\lambda^{\mathrm{T}}(t)S_1\lambda(t)+w^{\mathrm{T}}(t)\tilde{y}(t).$$

Then, following the same line as in the proof of Theorem 1, we know that the FES in Eq. (27) is passive from external disturbance $w(t)$ to output error $\tilde{y}(t)$. □

Remark 2 When $w(t) \equiv 0$, from the proof of Theorems 1 and 3 we can find that $\dot{V}(\lambda_t,\dot{\lambda}_t) \leq -\lambda^{\mathrm{T}}(t)S_1\lambda(t)$. Therefore, it can be concluded from the Lyapunov stability theory that, under the condition of Theorem 1 (respectively, Theorem 3), the FES (6) (respectively, Eq. (27)) will be asymptotically stable if there is no external disturbance.

Similarly to Theorem 2, we can develop an LMIs-based design strategy as follows:

Theorem 4 For given scalars $\mu_1 > 0$, $\mu_2 > 0$, and $\rho > 0$, if there are scalar $\varepsilon > 0$, and matrices $P = (P_{ij})_{5\times 5} \in \mathbb{S}_+^{5n}$, S_1, S_2, R_1, $R_2 \in \mathbb{S}_+^n$, $N_1, N_2 \in \mathbb{R}^{18n\times 2n}$, $M \in \mathbb{R}^{n\times m}$, diagonal matrices Q_1, $Q_2 \in \mathbb{S}_+^n$, $X \in \mathbb{R}^{3n\times 3n}$, such that

$$\Psi = \begin{bmatrix}\tilde{R}_2 & X\\ X^{\mathrm{T}} & \tilde{R}_2\end{bmatrix} \geq 0, \tag{37}$$

$$\begin{bmatrix}\Phi_1(h(t))-\Gamma^{\mathrm{T}}\Psi\Gamma & \rho W_1\\ * & -\varepsilon I\end{bmatrix} < 0 \tag{38}$$

hold true for $h(t) \in [h_1, h_2]$, where

$$\begin{aligned}
\Phi_1(h(t)) &= \mathscr{S}(G_1^{\mathrm{T}}(h(t))PG_0+N_1g_1(h(t))+N_2g_2(h(t)))\\
&\quad +\hat{S}_1+d_{18}{}^{\mathrm{T}}(h_1^2R_1+h_{12}^2R_2)d_{18}\\
&\quad +\tilde{\Sigma}_1+\tilde{\Sigma}_2-d_{17}{}^{\mathrm{T}}D_2d_{17}-G_2^{\mathrm{T}}\tilde{R}_1G_2\\
&\quad +G_5^{\mathrm{T}}\hat{S}_2G_5-d_{17}{}^{\mathrm{T}}C_2d_3-d_{17}{}^{\mathrm{T}}C_1d_1+\varepsilon W_2W_2^{\mathrm{T}},\\
\tilde{\Sigma}_1 &= \mathscr{S}(d_{18}^{\mathrm{T}}(-\mu_1MC_1+\mu_1P_{11}A)d_1-\mu_1d_{18}^{\mathrm{T}}MC_2d_3\\
&\quad +\mu_1d_{18}^{\mathrm{T}}P_{11}B_1d_{15}+\mu_1d_{18}^{\mathrm{T}}P_{11}B_2d_{16}\\
&\quad +d_{18}^{\mathrm{T}}(-\mu_1MD_2+\mu_1P_{11}D_1)d_{17}-\mu_1d_{18}^{\mathrm{T}}P_{11}d_{18}),
\end{aligned}$$

$$\begin{aligned}\tilde{\Sigma}_2 &= \mathscr{S}(d_1^{\mathrm{T}}(-\mu_2 MC_1+\mu_2 P_{11}A)d_1-\mu_2 d_1^{\mathrm{T}}MC_2d_3\\ &\quad+\mu_2 d_1^{\mathrm{T}}P_{11}B_1d_{15}+\mu_2 d_1^{\mathrm{T}}P_{11}B_2d_{16}\\ &\quad+d_1^{\mathrm{T}}(-\mu_2 MD_2+\mu_2 P_{11}D_1)d_{17}-\mu_2 d_1^{\mathrm{T}}P_{11}d_{18}),\\ W_1 &= \left[-\mu_2 M^{\mathrm{T}}\ 0_{16n\times 16n}\ -\mu_1 M^{\mathrm{T}}\right]^{\mathrm{T}},\\ W_2 &= \left[C_1\ 0\ C_2\ 0_{13n\times 13n}\ D_2\ 0\right]^{\mathrm{T}},\end{aligned}$$

and the other notations are the same as those in Theorem 3, then system (26) with $L=P_{11}^{-1}M$ will be a desired quantized passive filter for neural network (24).

Remark 3 Theorems 2 and 4 present design approaches for the quantized passive filter of neural networks in the presence of time-invariant and time-variant delays, respectively. The required filter is able to be obtained via solving a series of LMIs, which is computationally tractable by means of the well-known interior-point method.

Suppose that there is no quantization, then Eq. (26) becomes

$$\begin{aligned}\dot{\check{x}}(t) &= A\check{x}(t)+B_1\phi(\check{x}(t))+B_2\phi(\check{x}(t-h(t)))\\ &\quad+L(y(t)-\check{y}(t)),\end{aligned}\tag{39a}$$

$$\check{y}(t) = C_1\check{x}(t)+C_2\check{x}(t-h(t)),\tag{39b}$$

and one can write the following result:

Corollary 2 For given scalars $\mu_1>0$, $\mu_2>0$, if there exist matrices $P=(P_{ij})_{5\times 5}\in\mathbb{S}_+^{5n}$, S_1, S_2, R_1, $R_2\in\mathbb{S}_+^n$, $N_1,N_2\in\mathbb{R}^{18n\times 2n}$, $M\in\mathbb{R}^{n\times m}$, diagonal matrices Q_1, $Q_2\in\mathbb{S}_+^n$, and a matrix $X\in\mathbb{R}^{3n\times 3n}$ such that Eq. (37), and the following

$$\Phi_2(h(t))-\Gamma^{\mathrm{T}}\Psi\Gamma<0$$

hold true for $h(t)=[h_1,h_2]$, where

$$\begin{aligned}\Phi_2(h(t)) &= \mathscr{S}(G_1^{\mathrm{T}}(h(t))PG_0+N_1g_1(h(t))\\ &\quad+N_2g_2(h(t)))+\hat{S}_1+{d_{18}}^{\mathrm{T}}(h_1^2R_1+h_{12}^2R_2)d_{18}\\ &\quad+\tilde{\Sigma}_1+\tilde{\Sigma}_2-{d_{17}}^{\mathrm{T}}D_2d_{17}-G_2^{\mathrm{T}}\tilde{R}_1G_2\\ &\quad+G_5^{\mathrm{T}}\hat{S}_2G_5-{d_{17}}^{\mathrm{T}}C_2d_3-{d_{17}}^{\mathrm{T}}C_1d_1\end{aligned}$$

and the other notations are the same as those in Theorem 4, then neural network (2), filter (39) with time-variant delay and gain $L=P_{11}^{-1}M$ can ensure the resulting FES to be passive from disturbance $w(t)$ to output error $\tilde{y}(t)$.

5. Numerical example

In this section, we provide one numerical example with simulations to illustrate the applicability and reduced conservatism of the present quantized filter design approaches.

Example 1 Consider a two-neuron neural network modeled by Eq. (2) subject to a time-invariant or time-variant delay, as well as the following parameters:

$$A=\begin{bmatrix}\alpha & 0\\ 0 & -3.5\end{bmatrix},\quad B_1=\begin{bmatrix}-1 & 0.4\\ 0 & -0.1\end{bmatrix},$$

$$B_2=\begin{bmatrix}0.2 & -0.8\\ 0.4 & 0.5\end{bmatrix},$$

$$C_1=\begin{bmatrix}1\ 0\end{bmatrix},\quad C_2=\begin{bmatrix}0.5\ 1\end{bmatrix},\quad D_1=\begin{bmatrix}1\\ -1\end{bmatrix},\quad D_2=1.$$

The external disturbance $w(t)$ is chosen to be a Gaussian noise with mean 0 and variance 1. The activation function is chosen as $\phi(x(t))=[\tanh(x_1(t)),\tanh(x_2(t))]^{\mathrm{T}}$. Note that $\phi(x(t))$ satisfies Eq. (3) with $L_\phi=1$.[46] Moreover, the initial condition is set to be

$$x(s)=\begin{bmatrix}-3.2\\ 1.5\end{bmatrix},\quad \check{x}(s)=\begin{bmatrix}2.4\\ -2.1\end{bmatrix},\quad s\in[-1,0].$$

Case 1 Time-invariant delay

When $\alpha>-1.46$, it is found that the filter design method in Ref. [16] is invalid, while Theorem 2 and Corollary 1 are applicable. Especially, suppose that the quantization density is $\rho=0.2$. Then, setting $\alpha=-1$, $h=1$, $\mu_1=0.3$ and $\mu_2=0.8$ and solving the LMIs in Eq. (20) yield

$$P_{11}=\begin{bmatrix}4.1567 & 0.7350\\ 0.7350 & 6.3392\end{bmatrix},\quad M=\begin{bmatrix}3.5862\\ -4.0557\end{bmatrix}.$$

Thus, by Theorem 2, system (4) with

$$L=P_{11}^{-1}M=\begin{bmatrix}0.9963\\ -0.7553\end{bmatrix}$$

is a desired quantized passive filter.

Case 2 Time-variant delay

Suppose that the time delay is given by $h(t)=1-0.5\exp(-t)$, which implies that $h_1=0.5$ and $h_2=1$. Set $\alpha=-1$, $\rho=0.2$, $\mu_1=0.3$ and $\mu_2=0.8$. Then, solving the LMIs in Eq. (38) yields

$$P_{11}=\begin{bmatrix}5.2597 & 0.9153\\ 0.9153 & 7.7925\end{bmatrix},\quad M=\begin{bmatrix}4.4418\\ -5.2999\end{bmatrix}.$$

Thus, by Theorem 4, system (26) with

$$L=P_{11}^{-1}M=\begin{bmatrix}0.9829\\ -0.7956\end{bmatrix}$$

is a desired quantized passive filter.

The simulations of the passive filter design are given in Figs. 1–3. Particularly, Figs. 1 and 2 depict the states $x_1(t)$ and $x_2(t)$ and their estimations $\check{x}_1(t)$ and $\check{x}_2(t)$, respectively, and Fig. 3 depicts the trajectories of the filtering error $\lambda(t)$. It can be seen from the simulations that the designed filter is effective.

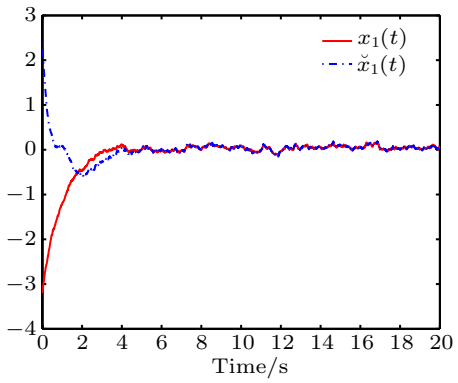


Fig. 1. State trajectories of $x_1(t)$ and its estimation $\breve{x}_1(t)$.

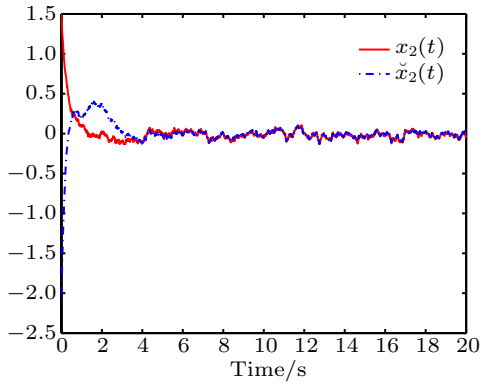


Fig. 2. State trajectories of $x_2(t)$ and its estimation $\breve{x}_2(t)$.

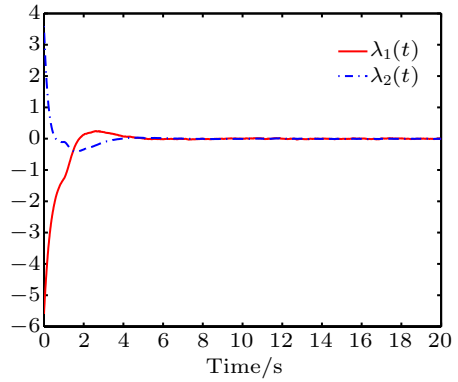


Fig. 3. State trajectories of filtering error $\lambda(t)$.

6. Conclusions

In the present work, the issue of passive filtering for time-delay neural networks has been revisited. Unlike the existing studies, the output measurements of the neural networks are assumed transmitted through networked communication channels and the quantization effects on the output have been considered. First, by means of the Lyapunov functional, the Bessel–Legendre inequality, as well as the free-weighting method, a criterion on the passivity of the FES has been obtained for the time-invariant delay case. Based on the criterion, an LMIs-based design approach for the desired quantized passive filter has been developed. Then, the analysis and synthesis have been extended to the time-variant delay situation with the aid of the reciprocally convex combination inequality. Finally, to illustrate the applicability and reduced conservatism of the present one filter design approaches, a numerical example with simulation results has been provided. Our future research will focus on improving and implementing the present filtering analysis and design approaches.

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