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Li-Li Wang(王丽丽), Wen-Jun Liu(刘文军)

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Stable soliton propagation in a coupled (2+1) dimensional Ginzburg–Landau system*

Li-Li Wang(王丽丽)[1] and Wen-Jun Liu(刘文军)[1,2,†]

[1] State Key Laboratory of Information Photonics and Optical Communications, and School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China

[2] Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

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A coupled $(2+1)$-dimensional variable coefficient Ginzburg–Landau equation is studied. By virtue of the modified Hirota bilinear method, the bright one-soliton solution of the equation is derived. Some phenomena of soliton propagation are analyzed by setting different dispersion terms. The influences of the corresponding parameters on the solitons are also discussed. The results can enrich the soliton theory, and may be helpful in the manufacture of optical devices.

Keywords: soliton, modified Hirota bilinear method, Ginzburg–Landau equation, bright soliton solution

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1. Introduction

Since the advent and development of fiber lasers, the concept of soliton has gradually been introduced into the field of optics. In 1973, Hasegawa and Tappert derived the solitons in an optical fiber modeled by the nonlinear Schrödinger equation (NLSE), and put forward the concept of optical solitons.[1,2] Maintaining the shape, amplitude, and speed during the propagation, optical solitons have gained much attention since proposed.[3–14] Relying on the stability, they can help to achieve long-distance, large-capacity communication.[15] In 1980, scientists from Bell Labs first observed optical solitons in the laboratory, proving that it is completely possible to put the optical soliton theory into practical application.[16] Noticing the great research value of solitons, scientists have done some researches on them both theoretically and experimentally.[3,17–30]

The propagation of solitons in optical fibers can be modeled by the NLSEs.[34] In previous work, some researchers have successfully derived the analytical solutions of NLSEs.[35–43] They have analyzed how to improve the quality of communication and develop different kinds of optical devices by controlling the soliton propagation. With research getting further, since actual systems often have energy exchange with the outside world, people noticed that traditional integrable systems are not enough to describe the soliton phenomena in reality.[45] After that, scientists began to study the dissipative systems and extended the traditional NLSEs. Segel, Newell, and Whitehead have made outstanding contributions in this regard, and they have done pioneering work on the Ginzburg–Landau equation (GLE).[46,47] GLE is a more general equation than NLSE.[48] It has a conservative form and a dissipative form, which can well describe the transmission of optical solitons in actual optical fiber systems. This equation can also be applied in such fields as liquid crystal, Bose–Einstein condensate, fluid and chemical turbulence, etc.[49–51]

Considering that optical solitons in higher-dimensional systems will be more complicated, and in order to obtain richer soliton propagation phenomena, we will study the following coupled $(2+1)$-dimensional variable coefficient GLE:

$$\begin{aligned} &\mathrm{i}u_t+\frac{1}{2}u_{xx}+\frac{\beta(t)}{2}u_{yy}+(1-\mathrm{i}\delta)|u|^2u+(1-\mathrm{i}\delta)|v|^2u-\mathrm{i}\gamma(t)u=0,\\ &\mathrm{i}v_t+\frac{1}{2}v_{xx}+\frac{\beta(t)}{2}v_{yy}+(1-\mathrm{i}\delta)|v|^2v+(1-\mathrm{i}\delta)|u|^2v-\mathrm{i}\gamma(t)v=0, \end{aligned} \tag{1}$$

where u and v are both complex functions defined in a $(2+1)$-dimensional space, including x and y dimensions, plus a time dimension. In Eq. (1), u and v can indicate the field amplitude. $\beta(t)$ is used to represent the dispersion term in an optical fiber. δ and $\gamma(t)$ represent the nonlinear gain and linear loss, respectively. The subscripts t, x, and y represent the derivative of the corresponding variable.

In Ref. [52], three different methods have been used to find the solution of Eq. (1) with constant coefficients. In order to obtain a richer soliton propagation phenomenon, we study Eq. (1) with the variable coefficients. We will use the modified Hirota bilinear method, which has not been used to solve this equation before, to obtain the one-soliton solution of Eq. (1). And we will analyze the solution from both physical application and mathematical theory aspects.

This paper is organized as follows. In Section 2, the one-

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†Corresponding author. E-mail: jungliu@bupt.edu.cn

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soliton solution of Eq. (1) is derived by the modified Hirota bilinear method. In Section 3, some different phenomena of the soliton propagation are obtained by varying the dispersion term, and the influences of some other parameters are also discussed. The conclusion will be given in Section 4.

2. Bright soliton solution of Eq. (1)

In this section, we will use the modified Hirota bilinear method to obtain the one-soliton solution of Eq. (1). In order to get its bilinear form, a transformation is introduced: $u = g/f^{1+\mathrm{i}\alpha}$ and $v = p/f^{1+\mathrm{i}\alpha}$, where $g = g(x,y,t)$ and $p = p(x,y,t)$ are both complex functions while $f = f(x,y,t)$ and α are assumed to be real. Bringing the specific forms of u and v into the original equation, with the help of symbolic calculations, we can get the bilinear form

$$\begin{aligned}
&\mathrm{i}D_{t,\alpha}g\cdot f+\frac{1}{2}D^2_{x,\alpha}g\cdot f+\frac{\beta(t)}{2}D^2_{y,\alpha}g\cdot f-\mathrm{i}\gamma(t)g\cdot f=0,\\
&\mathrm{i}D_{t,\alpha}p\cdot f+\frac{1}{2}D^2_{x,\alpha}p\cdot f+\frac{\beta(t)}{2}D^2_{y,\alpha}p\cdot f-\mathrm{i}\gamma(t)p\cdot f=0,\\
&\frac{1}{4}(1+\mathrm{i}\alpha)(2+\mathrm{i}\alpha)D^2_x f\cdot f+\frac{1}{4}(1+\mathrm{i}\alpha)(2+\mathrm{i}\alpha)\beta(t)D^2_y f\cdot f\\
&-(1-\mathrm{i}\delta)gh-(1-\mathrm{i}\delta)pq=0, \qquad (2)
\end{aligned}$$

with D^2_x, D^2_y and $D_{t,\alpha}$, $D^2_{x,\alpha}$, $D^2_{y,\alpha}$ being the generalized Hirota bilinear operator and the modified Hirota bilinear operator, respectively. h and q are the complex conjugate of g and p. Their definitions are as follows:

$$\begin{aligned}
D^m_{\vartheta}D^n_{\varsigma}(a\cdot b)&=\left(\frac{\partial}{\partial\vartheta}-\frac{\partial}{\partial\vartheta'}\right)^m\left(\frac{\partial}{\partial\varsigma}-\frac{\partial}{\partial\varsigma'}\right)^n\\
&\quad a(\vartheta,\varsigma)b(\vartheta',\varsigma')|_{\vartheta'=\vartheta,\varsigma'=\varsigma},\\
D^m_{\vartheta,\alpha}D^n_{\varsigma,\alpha}(a\cdot b)&=\left[\frac{\partial}{\partial\vartheta}-(1+\mathrm{i}\alpha)\frac{\partial}{\partial\vartheta'}\right]^m\left[\frac{\partial}{\partial\varsigma}-(1+\mathrm{i}\alpha)\frac{\partial}{\partial\varsigma'}\right]^n\\
&\quad a(\vartheta,\varsigma)b(\vartheta',\varsigma')|_{\vartheta'=\vartheta,\varsigma'=\varsigma},
\end{aligned}$$

where m and n are both positive integers, a is a function of ϑ and ς, and b is a function of ϑ' and ς'.

To gain the one-soliton solution of Eq. (1), g, p, and f need to be expanded as follows:

$$g=\varepsilon g_1(x,y,t),\quad p=\varepsilon p_1(x,y,t),\quad f=1+\varepsilon^2 f_2(x,y,t),$$

where ε is a form expansion parameter, $g_1(x,y,t)=A\mathrm{e}^{\theta_1}$, $p_1(x,y,t)=B\mathrm{e}^{\theta_1}$, and $f_2(x,y,t)=\phi_1(t)\mathrm{e}^{\theta_1+\theta_1^*}$. Substituting the expanded forms of g, p, and f into Eq. (2), and taking $\varepsilon=1$ without loss of generality, we can obtain the following one-soliton solution of Eq. (1):

$$\begin{aligned}
u&=\frac{g_1(x,y,t)}{[1+f_2(x,y,t)]^{1+\mathrm{i}\alpha}}=\frac{A\mathrm{e}^{\theta_1}}{[1+\phi_1(t)\mathrm{e}^{\theta_1+\theta_1^*}]^{1+\mathrm{i}\alpha}},\\
v&=\frac{p_1(x,y,t)}{[1+f_2(x,y,t)]^{1+\mathrm{i}\alpha}}=\frac{B\mathrm{e}^{\theta_1}}{[1+\phi_1(t)\mathrm{e}^{\theta_1+\theta_1^*}]^{1+\mathrm{i}\alpha}},
\end{aligned}$$

with

$$\begin{aligned}
\theta_1&=(k_1+\mathrm{i}k_{11})x+(\rho_1+\mathrm{i}\rho_{11})y+\omega_1(t)+\mathrm{i}\omega_{11}(t),\\
\theta_1^*&=(k_1-\mathrm{i}k_{11})x+(\rho_1-\mathrm{i}\rho_{11})y+\omega_1(t)-\mathrm{i}\omega_{11}(t),\\
\omega_1(t)&=\int[-k_1k_{11}-\rho_1\rho_{11}\beta(t)+\gamma(t)]\,\mathrm{d}t,\\
\omega_{11}(t)&=\frac{1}{2}\int[k_1^2-k_{11}^2+\rho_1^2\beta(t)-{\rho_{11}}^2\beta(t)]\,\mathrm{d}t,\\
\phi_1(t)&=\frac{\mathrm{i}(\delta+\mathrm{i})(AA^*+BB^*)}{2(\alpha^2-3\mathrm{i}\alpha-2)[k_1^2+\rho_1^2\beta(t)]},\\
\gamma(t)&=\frac{2\alpha k_1^4+4\alpha k_1^2\rho_1^2\beta(t)+2\alpha\rho_1^4\beta(t)^2+\rho_1^2\beta'(t)}{2[k_1^2+\rho_1^2\beta(t)]},
\end{aligned}$$

where k_i and ρ_i are real constants, while $\omega_i(t)$ is a real function about t ($i=1,11$), A and B are both complex ones, and "*" indicates a complex conjugate. From the above results we can notice that the linear loss $\gamma(t)$ can be expressed by the dispersion coefficient $\beta(t)$. Thus, in Eq. (1), the change of the dispersion term can affect the linear loss accordingly.

3. Discussion

In this section, we will study the effects of the parameters in Eq. (1) on the soliton propagation. In Eq. (1), we set the dispersion term as a variable function, and can observe more phenomena of soliton propagation by varying it. In addition, the effects of the corresponding parameters will be discussed here.

To observe the characteristics of soliton propagation, we draw three-dimensional images based on the y–t coordinates. Firstly, setting the other parameters at appropriate values, we take $\beta(t)$ as a constant. A stable soliton propagation is obtained as shown in Fig. 1(a). Then, we fix the other parameters and increase $\beta(t)$ from 1.2 (Fig. 1(a)) to 2 (Fig. 1(b)). By observing Fig. 1(b), we can find that as $\beta(t)$ increases, the amplitude of the soliton slightly increases, with its direction unchanged. From the introduction in Section 1, we know that δ in Eq. (1) represents a linear gain. So theoretically, the larger the value of δ is, the larger the soliton amplitude should be. The comparison between Figs. 1(a) and 1(c) can help to verify this theory: Fig. 1(a) differs from Fig. 1(c) only in δ value, and the δ value in Fig. 1(c) is bigger than that in Fig. 1(a). It is obvious that the amplitude of the soliton is greater in Fig. 1(c).

On the premise of stable propagation of soliton, we adjust the values of the free parameters to observe their effects. In Fig. 2(a), we can see that when $k_1=1$, the soliton stably transmits and tilts at a certain angle. Then, we increase the value of k_1 to 1.35. As shown in Fig. 2(b), the propagation direction of the soliton changes, deflecting along the counterclockwise direction, almost parallel to the t axis. In addition, the soliton's amplitude has increased and the bottom of the soliton has narrowed during the rotation. While keeping the other parameters in Fig. 2(b) unchanged, we reduce the value of k_{11} from 2 to 1.75. In Fig. 2(c), we can see that the soliton continues to whirl in counterclockwise direction and the amplitude keeps

increasing with the width of the soliton becoming narrower. When investigating the other two free parameters ρ_1 and ρ_{11}, we find that their effect on soliton transmission is similar to the one above, which also affects the amplitude and transmission direction of the soliton. The phenomenon is similar to the above, so the image display would not be given here.

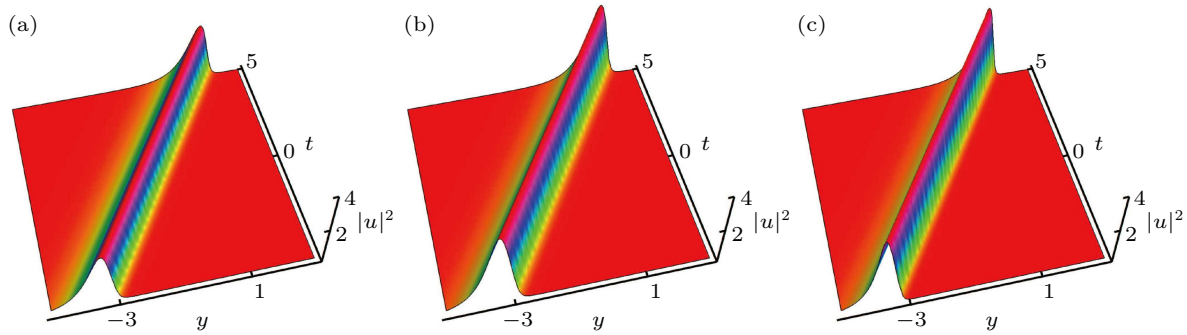


Fig. 1. Soliton propagation with parameters chosen as $x=1, A=0.5+2\mathrm{i}, B=1, \alpha=1.5, k_1=1, k_{11}=2, \rho_1=1, \rho_{11}=1.5$ with (a) $\beta(t)=1.2$, $\delta=1.2$; (b) $\beta(t)=2$, $\delta=1.2$; (c) $\beta(t)=1.2$, $\delta=2$.

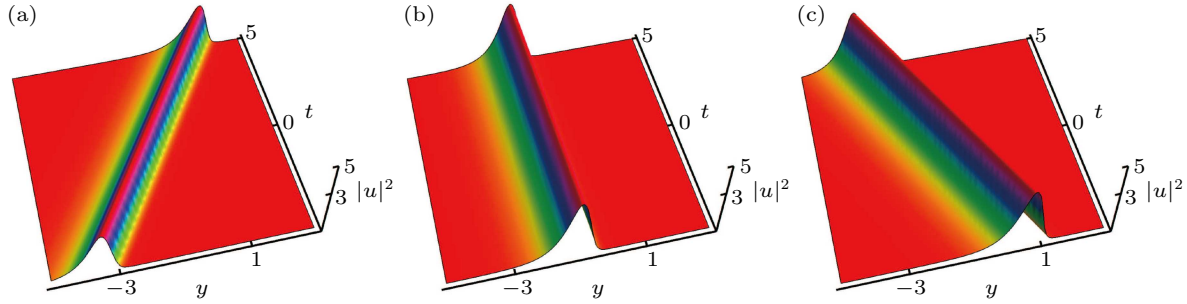


Fig. 2. Soliton propagation with parameters chosen as $x=1, A=0.5+2\mathrm{i}, B=1, \alpha=1.5, \rho_1=1, \rho_{11}=1.5,\ \beta(t)=1.2, \delta=1.2$ with (a) $k_1=1$, $k_{11}=2$; (b) $k_1=1.35$, $k_{11}=2$; (c) $k_1=1.35$, $k_{11}=1.75$.

From the above analysis, it can be found that when the dispersion term $\beta(t)$ is constant, the soliton transmits stably, with its envelope shape being flat. Besides, the free parameters in the solution will change the direction and amplitude of the soliton without affecting the envelope shape. Next, we will change the function type of $\beta(t)$ and observe its effect on the soliton propagation.

First, we set the dispersion term $\beta(t)$ as $\mathrm{e}^{0.25t}$. As shown in Fig. 3(a), when the dispersion term is taken as an exponential function, the soliton's amplitude increases as time t grows. The bigger the value of t is, the greater the amplification will be. From the above, we know that the soliton's amplitude will grow with the increase of $\beta(t)$. The possible explanation is that as t increases, the dispersion term increases. From Section 2, we realize that the change of $\beta(t)$ will affect the linear loss. Through the calculation, we find that when $\beta(t)$ is taken as an exponential function, the linear loss $\gamma(t)$ will also increase with the growth of time t. But it is not as large as that of $\beta(t)$, so the amplitude of the soliton is magnified overall. Thus, we can realize soliton amplification by setting the dispersion term $\beta(t)$ as an exponential function, and the degree of the amplification can be controlled by adjusting the size of the exponential function. Gaussian noise is the most common one in nature, so here we explore how the soliton will transmit when the dispersion term is a Gaussian function.

In Fig. 3(b), $\beta(t)$ is set to be $0.25\,\mathrm{e}^{0.1t^2}$. It can be seen through the figure that when the dispersion term is taken as a Gaussian function, the middle part of the soliton (where t is about in the range $[-2,3]$) transmits smoothly without any significant change in amplitude. Beyond this time range, the amplitude of the soliton increases, and the two ends are basically symmetrical. This phenomenon of soliton amplitudes with obvious differences and regularity in different regions may be helpful in optical devices such as optical switches.

In order to explore the properties of soliton propagation when the dispersion term is taken as a periodical function, we set $\beta(t)$ as a trigonometric function here. In Fig. 3(c), we take $\beta(t)=\cos(t)+0.5$. It can be seen that because the dispersion term changes periodically with t, the envelope shape of the soliton shows the same pattern. By changing the amplitudes of the trigonometric function, we can control the soliton's amplitude, and the phase can help to adjust the speed of the soliton during its propagation. When the phase of the trigonometric function is set to be a quadratic function about t, as shown in Fig. 3(d). Keeping other parameters fixed and taking $\beta(t)=\cos(t^2)+0.5$, we can find that the periodicity of the soliton envelope disappears, the soliton is cut into small pieces of different widths. Around the position of $t=0$, the width of the soliton piece intercepted turns to be the largest. As the absolute value of t increases, the length of the soliton pieces becomes shorter, which is symmetrical about $t=0$.

From the above analysis of the soliton transmission states

with different choices of $\beta(t)$, we can see that the dispersion term has a decisive effect on the amplitude and envelope shape of the soliton. When $\beta(t)$ is taken as a constant, the soliton propagates smoothly forward, and its amplitude does not change with time. When $\beta(t)$ is a function of t, the function type determines the shape of the soliton's envelope, and the magnification of the soliton can be decided by the amplitude of $\beta(t)$. Therefore, we can control the amplification of the solitons by selecting a reasonable dispersion in the fiber, which can also help to realize the reshape of the solitons.

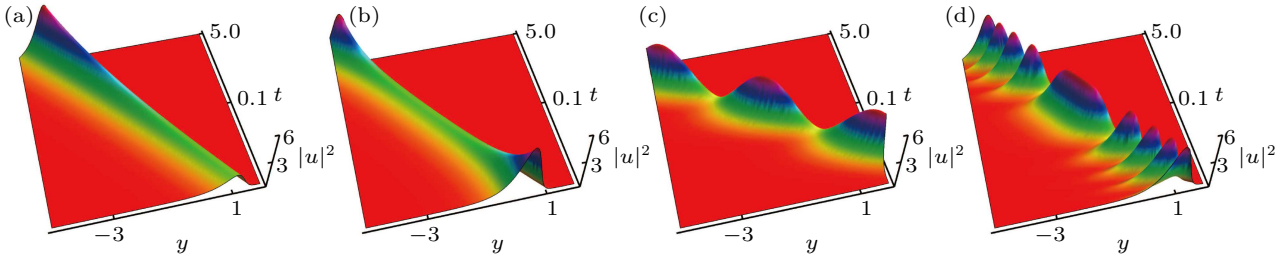


Fig. 3. Soliton propagation with parameters chosen as $x = 1$, $A = 0.5 + 2\mathrm{i}$, $B = 1$, $\alpha = 1.5$, $\rho_1 = 1$, $\rho_{11} = 1.5$, $k_1 = 1$, $k_{11} = 1$, $\delta = 1.2$ with (a) $\beta(t) = \mathrm{e}^{0.25t}$; (b) $\beta(t) = 0.25\,\mathrm{e}^{0.1t^2}$; (c) $\beta(t) = \cos(t) + 0.5$; (d) $\beta(t) = \cos(t^2) + 0.5$.

4. Conclusion

In this paper, we have obtained the bright one-soliton solution of Eq. (1) by the modified Hirota bilinear method. Based on the solution obtained, we realized the stable soliton propagation in the system, and the influence of different parameters on the soliton propagation was discussed. After the above discussion, we found that the free parameters can affect the amplitude and transmission direction of the solitons. In addition, the envelope shape and amplification degree of solitons can be controlled by adjusting the dispersion term $\beta(t)$. Thus, we can realize soliton amplification by setting the related parameters in the solution. And soliton reshaping has been realized by selecting the different values of $\beta(t)$. The results here may be helpful for amplification and direction control of optical solitons in multi-mode fibers.

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