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Optical nonreciprocity in a piezo-optomechanical system*

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We theoretically study the optical nonreciprocity in a piezo-optomechanical microdisk resonator, in which the cavity modes and the mechanical mode are optically pumped and piezoelectrically driven, respectively. For asymmetric optical pumping and different piezoelectrical drivings, our system shows some nonreciprocal optical responses. We find that our system can function as an optical isolator, a nonreciprocal amplifier, or a nonreciprocal phase shifter.

Keywords: quantum optics, cavity optomechanical system, optical nonreciprocity, piezoelectric force

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1. Introduction

Nonreciprocal components such as isolators and circulators are indispensable in photonic systems.[1] So far, a number of schemes have been proposed to achieve the nonreciprocity using various physical effects or systems, such as the magneto-optical effect,[2] the interband photonic transitions,[3,4] the stimulated Brillouin scattering,[5–7] the parity-time-symmetric system,[8] and the rotating nonlinear devices.[9] Furthermore, scientists have found that the nonreciprocity can also be obtained by the optomechanical interaction,[10–17] and the realization of optomechanically induced nonreciprocity has been experimentally demonstrated.[18,19]

Cavity optomechanics studies the radiation-pressure interaction between electromagnetic field and the mechanical motion. A large number of physical effects have been demonstrated in cavity optomechanical systems, such as the optomechanically induced transparency,[20,21] the squeezing of light,[22,23] the parity-time-symmetric breaking,[24] the photon blockade,[25] and the cooling of mechanical resonators.[26,27] In recent years, more physical phenomena have been demonstrated in hybrid optomechanical systems, which combine the optomechanical system and other physical systems,[28–32] in which the piezo-optomechanical system is attractive since it provides a platform to investigate light-mechanical motion-electricity interaction.[33,34] In the latest studies, the double optomechanically induced transparency,[35] the microwave-optical frequency conversion[36] and the entangled microwave-optical photon pairs[37] in the piezo-optomechanical systems have been demonstrated. In our present work, we study the nonreciprocal transmission, amplification, and phase shift in a piezo-optomechanical system.

This paper is structured as follows: In Section 2, we describe the proposed scheme and give the Hamiltonian of the system and the dynamical equations. In Section 3, we study the nonreciprocal transmission and amplification phenomena. In Section 4, we show the nonreciprocal phase shift. In Section 5, we conclude by summarizing the results.

2. Model and dynamical equations

Our proposed scheme is shown in Fig. 1. We consider a piezo-optomechanical microresonator, in which a clockwise circulating cavity mode a and a counterclockwise circulating cavity mode c couple with the mechanical mode b via radiation pressure. Meanwhile, the mechanical mode is piezoelectrically driven by a microwave field. The mechanical motion of the microdisk is controlled by the piezoelectric effect, which has been described in many works.[38–40] The piezoelectric effect in microdisk naturally couples the microwave mode to the mechanical mode by introducing an electromechanical coupling energy given by $H_{\rm em} = \int (\boldsymbol{S}\cdot\boldsymbol{e}\cdot\boldsymbol{E})\mathrm{d}V$, where $\boldsymbol{S}$, $\boldsymbol{e}$ and $\boldsymbol{E}$ are the strain filed, piezoelectric coupling tensor and electric field. In addition, cavity mode a is simultaneously driven by a pump field $\varepsilon_{\rm l}$ with frequency $\omega_{\rm l}$ and a probe field $\varepsilon_{\rm pa}$ with frequency $\omega_{\rm pa}$. Cavity mode c is driven by a probe field $\varepsilon_{\rm pc}$ with frequency $\omega_{\rm pc}$. The Hamiltonian of the system can be expressed as

$$H = H_0 + H_1 + H_2 + H_3,$$

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$$
\begin{aligned}
H_0 &= \hbar\omega_0 a^\dagger a + \hbar\omega_0 c^\dagger c + \hbar\omega_{\rm m} b^\dagger b + \hbar\omega_{\rm d} d^\dagger d,\\
H_1 &= -\hbar g_{\rm om} a^\dagger a (b^\dagger + b) - \hbar g_{\rm om} c^\dagger c (b^\dagger + b),\\
H_2 &= -\hbar g_{\rm em} (b^\dagger d + b d^\dagger),\\
H_3 &= {\rm i}\hbar\varepsilon_{\rm l}(a^\dagger {\rm e}^{-{\rm i}\omega_{\rm l} t} - {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm pa}(a^\dagger {\rm e}^{-{\rm i}\omega_{\rm pa} t}\, {\rm e}^{-{\rm i}\phi_{\rm pa}} - {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm pc}(c^\dagger {\rm e}^{-{\rm i}\omega_{\rm pc} t}\, {\rm e}^{-{\rm i}\phi_{\rm pc}} - {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm m}(d^\dagger {\rm e}^{-{\rm i}\omega_{\rm d} t}\, {\rm e}^{-{\rm i}\phi_{\rm m}} - {\rm H.c.}).
\end{aligned}
\tag{1}
$$

Here H_0 denotes the free Hamiltonian of the system, with a (c), b and d being the annihilation operators of the clockwise (counterclockwise) circulating cavity mode with frequency ω_0, the mechanical mode with frequency $\omega_{\rm m}$ and the microwave mode with frequency $\omega_{\rm d}$. H_1 describes the optomechanical interaction, with $g_{\rm om}$ denoting the single photon optomechanical coupling strength. H_2 describes the piezoelectric interaction between the mechanical mode b and the microwave mode d, with $g_{\rm em}$ denoting the piezoelectric coupling coefficient.[36–38] H_3 describes the interaction between the cavity mode a, the cavity mode c, the microwave mode d and the driving fields. The amplitudes of the pump field, the probe fields and the microwave pump are defined as $\varepsilon_{\rm l} = \sqrt{\kappa_{\rm e} P_{\rm l}/\hbar\omega_{\rm l}}$, $\varepsilon_{\rm pa} = \sqrt{\kappa_{\rm e} P_{\rm pa}/\hbar\omega_{\rm pa}}$, $\varepsilon_{\rm pc} = \sqrt{\kappa_{\rm e} P_{\rm pc}/\hbar\omega_{\rm pc}}$ and $\varepsilon_{\rm m} = \sqrt{\gamma_{\rm e} P_{\rm d}/\hbar\omega_{\rm d}}$, respectively, where $P_{\rm l}$, $P_{\rm pa}$, $P_{\rm pc}$ and $P_{\rm d}$ denote their powers, $\kappa_{\rm e}$ and $\gamma_{\rm e}$ are the extrinsic coupling losses. In Eqs. (1), $\phi_{\rm pa}$ ($\phi_{\rm pc}$) is the phase difference between the probe field $\varepsilon_{\rm pa}$ ($\varepsilon_{\rm pc}$) and the pump field, and $\phi_{\rm m}$ is the phase difference between the microwave pump and the pump field. The optical mode coupling between a and c results from the imperfections of the disk, for the device we studied, such mode coupling is very small and can be neglected.[38]

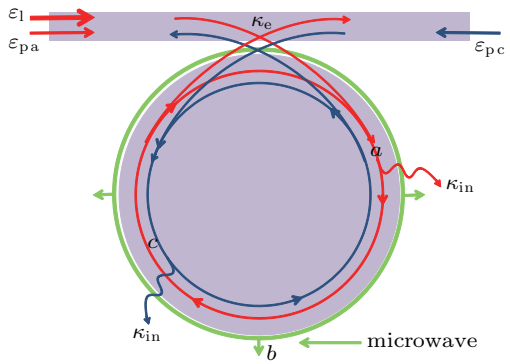


Fig. 1. Schematic diagram of our proposed scheme. The piezo-optomechanical microresonator is optically driven by a pump field (amplitude $\varepsilon_{\rm l}$, frequency $\omega_{\rm l}$) and two probe fields (amplitude $\varepsilon_{\rm pa}$, frequency $\omega_{\rm pa}$, amplitude $\varepsilon_{\rm pc}$, frequency $\omega_{\rm pc}$), and piezoelectrically driven by a microwave field with frequency $\omega_{\rm d}$.

In the rotation frame with $\hbar\omega_{\rm l} a^\dagger a + \hbar\omega_{\rm l} c^\dagger c + \hbar\omega_{\rm d} d^\dagger d$, the Hamiltonian of the system is written as

$$
\begin{aligned}
H &= \hbar\Delta_0 a^\dagger a + \hbar\Delta_0 c^\dagger c + \hbar\omega_{\rm m} b^\dagger b - \hbar g_{\rm om} a^\dagger a (b^\dagger + b)\\
&\quad - \hbar g_{\rm om} c^\dagger c (b^\dagger + b) - \hbar g_{\rm em}(b^\dagger d\, {\rm e}^{-{\rm i}\omega_{\rm d} t} + {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm l}(a^\dagger - a) + {\rm i}\hbar\varepsilon_{\rm pa}(a^\dagger {\rm e}^{-{\rm i}\Delta_{\rm pa} t}\, {\rm e}^{-{\rm i}\phi_{\rm pa}} - {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm pc}(c^\dagger {\rm e}^{-{\rm i}\Delta_{\rm pc} t}\, {\rm e}^{-{\rm i}\phi_{\rm pc}} - {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm m}(d^\dagger {\rm e}^{-{\rm i}\phi_{\rm m}} - {\rm H.c.}),
\end{aligned}
\tag{2}
$$

where $\Delta_0 = \omega_0 - \omega_{\rm l}$ is the frequency detuning between the cavity mode and the pump field, $\Delta_{\rm pa} = \omega_{\rm pa} - \omega_{\rm l}$ ($\Delta_{\rm pc} = \omega_{\rm pc} - \omega_{\rm l}$) is the frequency detuning between the probe field $\varepsilon_{\rm pa}$ ($\varepsilon_{\rm pc}$) and the pump field. When the microwave pump is strong,[38] the microwave mode d can be treated as a constant number $|d|\,{\rm e}^{-{\rm i}\phi_{\rm m}}$. We take this constant-number assumption, and the effective Hamiltonian of the system can be obtained as

$$
\begin{aligned}
H_{\rm eff} &= \hbar\Delta_0 a^\dagger a + \hbar\Delta_0 c^\dagger c + \hbar\omega_{\rm m} b^\dagger b - \hbar g_{\rm om} a^\dagger a (b^\dagger + b)\\
&\quad - \hbar g_{\rm om} c^\dagger c (b^\dagger + b) - \hbar g_{\rm em}|d|(b^\dagger {\rm e}^{-{\rm i}\omega_{\rm d} t}\, {\rm e}^{-{\rm i}\phi_{\rm m}} + {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm l}(a^\dagger - a) + {\rm i}\hbar\varepsilon_{\rm pa}(a^\dagger {\rm e}^{-{\rm i}\Delta_{\rm pa} t}\, {\rm e}^{-{\rm i}\phi_{\rm pa}} - {\rm H.c.})\\
&\quad + {\rm i}\hbar\varepsilon_{\rm pc}(c^\dagger {\rm e}^{-{\rm i}\Delta_{\rm pc} t}\, {\rm e}^{-{\rm i}\phi_{\rm pc}} - {\rm H.c.}).
\end{aligned}
\tag{3}
$$

The dynamics of the system can be described by a set of quantum Heisenberg–Langevin equations

$$
\begin{aligned}
\dot{a} &= -\left({\rm i}\Delta_0 + \frac{\kappa}{2}\right) a + {\rm i} g_{\rm om} a (b^\dagger + b)\\
&\quad + \varepsilon_{\rm l} + \varepsilon_{\rm pa}\, {\rm e}^{-{\rm i}\Delta_{\rm pa} t}\, {\rm e}^{-{\rm i}\phi_{\rm pa}} + \sqrt{\kappa_{\rm e}} a_{\rm in},\\
\dot{c} &= -\left({\rm i}\Delta_0 + \frac{\kappa}{2}\right) c + {\rm i} g_{\rm om} c (b^\dagger + b)\\
&\quad + \varepsilon_{\rm pc}\, {\rm e}^{-{\rm i}\Delta_{\rm pc} t}\, {\rm e}^{-{\rm i}\phi_{\rm pc}} + \sqrt{\kappa_{\rm e}} c_{\rm in},\\
\dot{b} &= -\left({\rm i}\omega_{\rm m} + \frac{\gamma}{2}\right) b + {\rm i} g_{\rm om}(a^\dagger a + c^\dagger c)\\
&\quad + {\rm i}\varepsilon_{\rm d}\, {\rm e}^{-{\rm i}\omega_{\rm d} t}\, {\rm e}^{-{\rm i}\phi_{\rm m}} + \sqrt{\gamma} b_{\rm in},
\end{aligned}
\tag{4}
$$

where $\varepsilon_{\rm d} = g_{\rm em}|d|$, $\kappa = \kappa_{\rm in} + \kappa_{\rm e}$ is the total loss rates of the cavity modes, with $\kappa_{\rm in}$ denoting the intrinsic loss, γ the loss rates of the mechanical mode, and $a_{\rm in}$ ($c_{\rm in}$) and $b_{\rm in}$ the δ-correlated operators of the input noises with zero-mean values for the cavity modes and the mechanical mode, respectively.

We have also assumed that $\varepsilon_{\rm pa}, \varepsilon_{\rm pc}, \varepsilon_{\rm d} \ll \varepsilon_{\rm l}$, and the microresonator is driven by the pump field at the red sideband ($\omega_l < \omega_0$). Then Eqs. (4) can be linearized by using the ansatz $a = \alpha + \delta a$, $c = \lambda + \delta c$, $b = \beta + \delta b$, where α, λ and β are the steady-state mean values, and δa, δc and δb are the fluctuations. By substituting the above ansatz into Eq. (4), and neglecting the nonlinear terms such as $\delta a \delta b$, $\delta c \delta b \cdots$ and using the rotating-wave approximation,[41] we can obtain the steady-state mean values

$$
\begin{aligned}
\alpha &= \frac{\varepsilon_{\rm l}}{{\rm i}\Delta + \kappa/2}, \quad \lambda = 0,\\
\beta &= \frac{{\rm i} g_{\rm om}|\alpha|^2}{{\rm i}\omega_{\rm m} + \gamma/2},
\end{aligned}
\tag{5}
$$

and the linearized quantum Heisenberg-Langevin equations

$$
\begin{aligned}
\delta\dot{a} &= -\left({\rm i}\Delta + \frac{\kappa}{2}\right)\delta a + {\rm i} g_{\rm om}\alpha\,\delta b\\
&\quad + \varepsilon_{\rm pa}\, {\rm e}^{-{\rm i}\Delta_{\rm pa} t}\, {\rm e}^{-{\rm i}\phi_{\rm pa}} + \sqrt{\kappa_{\rm e}} a_{\rm in},\\
\delta\dot{c} &= -\left({\rm i}\Delta + \frac{\kappa}{2}\right)\delta c + \varepsilon_{\rm pc}\, {\rm e}^{-{\rm i}\Delta_{\rm pc} t}\, {\rm e}^{-{\rm i}\phi_{\rm pc}} + \sqrt{\kappa_{\rm e}} c_{\rm in},
\end{aligned}
$$

$$\delta\dot{b} = -\left(\mathrm{i}\omega_{\mathrm{m}}+\frac{\gamma}{2}\right)\delta b+\mathrm{i}g_{\mathrm{om}}\alpha^{*}\delta a +\mathrm{i}\varepsilon_{\mathrm{d}}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{m}}}+\sqrt{\gamma}b_{\mathrm{in}}, \tag{6}$$

where $\Delta=\Delta_0-g_{\mathrm{om}}(\beta+\beta^{*})=\omega_{\mathrm{m}}$ is the effective frequency detuning between the cavity mode and the pump field. We have also assumed that $\Delta_{\mathrm{pa}}=\Delta_{\mathrm{pc}}=\omega_{\mathrm{d}}$. By using the ansatz $\delta O=O_{+}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}+O_{-}\,\mathrm{e}^{\mathrm{i}\omega_{\mathrm{d}}t}$ $(O=a,b,c)$, we can obtain the following solutions

$$\begin{aligned} a_{+} &= \frac{\varepsilon_{\mathrm{pa}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pa}}}+\mathrm{i}g_{\mathrm{om}}\alpha b_{+}}{\mathrm{i}\delta+\kappa/2},\\ c_{+} &= \frac{\varepsilon_{\mathrm{pc}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pc}}}}{\mathrm{i}\delta+\kappa/2},\\ b_{+} &= \frac{\mathrm{i}\varepsilon_{\mathrm{d}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{m}}}+\mathrm{i}g_{\mathrm{om}}\alpha^{*}a_{+}}{\mathrm{i}\delta+\gamma/2}, \end{aligned} \tag{7}$$

where $\delta=\Delta-\omega_{\mathrm{d}}=\omega_{\mathrm{m}}-\omega_{\mathrm{d}}$. We can see that the coherent mechanical displacement b_{+} consists of two terms which correspond to the contributions of the piezoelectric force and the radiation pressure, respectively.

The output fields are given by the input-output relations[42] $a_{\mathrm{out}}=\varepsilon_{\mathrm{l}}+\varepsilon_{\mathrm{pa}}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pa}}}-\kappa_{\mathrm{e}}a$, $c_{\mathrm{out}}=\varepsilon_{\mathrm{pc}}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pc}}}-\kappa_{\mathrm{e}}c$. By writing the output field as $a_{\mathrm{out}}=a^{0}_{\mathrm{out}}+\delta a_{\mathrm{out}}=a^{0}_{\mathrm{out}}+a^{+}_{\mathrm{out}}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}+a^{-}_{\mathrm{out}}\,\mathrm{e}^{\mathrm{i}\omega_{\mathrm{d}}t}$ and $c_{\mathrm{out}}=c^{0}_{\mathrm{out}}+\delta c_{\mathrm{out}}=c^{0}_{\mathrm{out}}+c^{+}_{\mathrm{out}}\,\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}+c^{-}_{\mathrm{out}}\,\mathrm{e}^{\mathrm{i}\omega_{\mathrm{d}}t}$, we can obtain

$$\begin{aligned} a^{+}_{\mathrm{out}} &= \varepsilon_{\mathrm{pa}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pa}}}-\kappa_{\mathrm{e}}a_{+},\\ c^{+}_{\mathrm{out}} &= \varepsilon_{\mathrm{pc}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pc}}}-\kappa_{\mathrm{e}}c_{+}. \end{aligned} \tag{8}$$

The nonreciprocal transmission is described by the normalized transmission spectra[43]

$$\begin{aligned} T_{\mathrm{a}} &= \left|\frac{a^{+}_{\mathrm{out}}}{\varepsilon_{\mathrm{pa}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pa}}}}\right|^{2}=\left|1-\frac{\kappa_{\mathrm{e}}(\mathrm{i}\delta+\gamma/2)-\kappa_{\mathrm{e}}G\eta\,\mathrm{e}^{-\mathrm{i}\phi}}{(\mathrm{i}\delta+\kappa/2)(\mathrm{i}\delta+\gamma/2)+|G|^{2}}\right|^{2},\\ T_{\mathrm{c}} &= \left|\frac{c^{+}_{\mathrm{out}}}{\varepsilon_{\mathrm{pc}}\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{pc}}}}\right|^{2}=\left|1-\frac{\kappa_{\mathrm{e}}}{\mathrm{i}\delta+\kappa/2}\right|^{2}, \end{aligned} \tag{9}$$

where $\eta=\varepsilon_{\mathrm{d}}/\varepsilon_{\mathrm{pa}}$, $\phi=\phi_{\mathrm{m}}-\phi_{\mathrm{pa}}$, and $G=g_{\mathrm{om}}\alpha$ is the effective optomechanical coupling strength. The phase shifts of the output fields are defined as

$$\begin{aligned} \theta_{\mathrm{a}} &= \arg\left(1-\frac{\kappa_{\mathrm{e}}(\mathrm{i}\delta+\gamma/2)-\kappa_{\mathrm{e}}G\eta\,\mathrm{e}^{-\mathrm{i}\phi}}{(\mathrm{i}\delta+\kappa/2)(\mathrm{i}\delta+\gamma/2)+|G|^{2}}\right),\\ \theta_{\mathrm{c}} &= \arg\left(1-\frac{\kappa_{\mathrm{e}}}{\mathrm{i}\delta+\kappa/2}\right). \end{aligned} \tag{10}$$

In the following sections, using above expressions, we numerically study the optical nonreciprocal responses of our system. The parameters are chosen based on a recent experiment:[38] $\omega_0=2\pi\times 192$ THz, $\kappa=2\pi\times 1.02$ GHz, $G=2\pi\times 0.16$ MHz, $\omega_{\mathrm{m}}=2\pi\times 3.235$ GHz, $\gamma=2\pi\times 0.961$ MHz.

3. Piezoelectrically induced nonreciprocal transmission and amplification

From Eqs. (9) we can learn that T_{a} can be controlled by adjusting the dimensionless amplitude η and the relative phase ϕ. However, T_{c} is independent of η and ϕ. Now we numerically evaluate T_{a} and T_{c} to show the parameter conditions of the piezoelectrically induced nonreciprocal transmission and amplification. Firstly we consider that our system works in the critical-coupled regime ($\kappa_{\mathrm{e}}=\kappa/2$). As shown in Fig. 2, we plot the transmission spectra T_{a} and T_{c} as a function of the detuning δ/κ. When $\eta=0$ [Fig. 2(a)], we find that the system exhibits a reciprocal transmission for the probe fields $\varepsilon_{\mathrm{pa}}$ and $\varepsilon_{\mathrm{pc}}$. When η is not equal to zero, there will appear a narrow peak in T_{a} near $\delta=0$ [Figs. 2(b)–2(d)], and the height of the peak increases with increasing η. The height of the peak can be 1 [Fig. 2(c)]. This means a complete transmission of the probe field $\varepsilon_{\mathrm{pa}}$. In this case our system can act as an optical isolator ($T_{\mathrm{a}}=1$, $T_{\mathrm{c}}=0$). The height of the peak can even be larger than 1 [Fig. 2(d)]. This means an amplification of the probe field $\varepsilon_{\mathrm{pa}}$. In this case our system can act as a nonreciprocal optical amplifier ($T_{\mathrm{a}}>1$, $T_{\mathrm{c}}=0$).

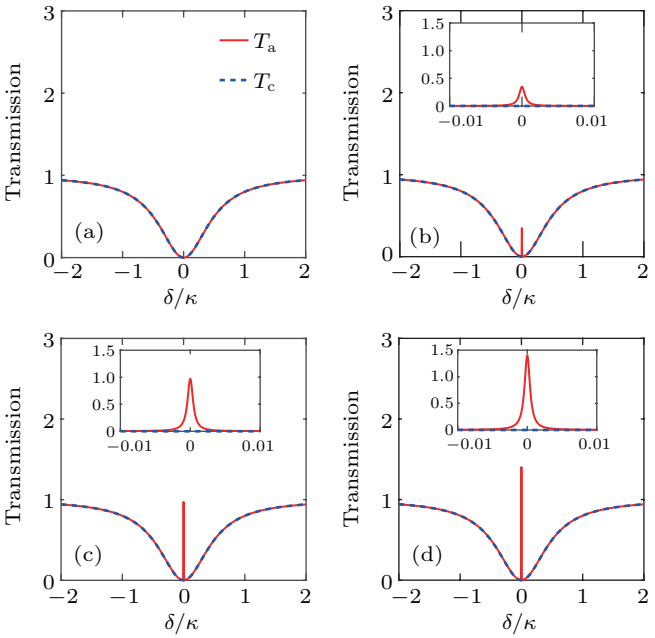


Fig. 2. The transmission spectra T_{a} and T_{c} as a function of δ/κ for $\phi=\pi$ in the critical-coupled regime ($\kappa_{\mathrm{e}}=\kappa/2$). The other parameters are (a) $\eta=0$, (b) $\eta=2$, (c) $\eta=3$, (d) $\eta=3.6$.

Then we consider that our system works in the overcoupled regime ($\kappa_{\mathrm{e}}\approx\kappa$). As shown in Fig. 3(a), the system exhibits a reciprocal transmission when $\eta=0$. When η is not equal to zero, depending on the value of η, T_{a} will show a narrow valley or peak near $\delta=0$. Firstly, the valley becomes deeper with increasing η [Figs. 3(b)–3(c)]. In Fig. 3(c) we adopt $\eta=1.5$, and we have $T_{\mathrm{a}}=0$ and $T_{\mathrm{c}}=1$ near $\delta=0$, i.e., the probe field $\varepsilon_{\mathrm{pc}}$ can be totally transmitted and the probe field $\varepsilon_{\mathrm{pa}}$ is completely absorbed. This means that our system

can function as an optical isolator. However, the transmitted direction is opposite with the case described in Fig. 2(c). After this value of η, near $\delta = 0$, T_a will increase with increasing η [Figs. 3(c)–3(e)]. When $\eta = 3$, the absorption valley of T_a almost vanishes and the system is transparent for both ε_{pa} and ε_{pc} [Fig. 3(e)]. When $\eta > 3$, the system exhibits a nonreciprocal amplification for the probe field ($T_a > 1$, $T_c = 1$), as shown in Fig. 3(f).

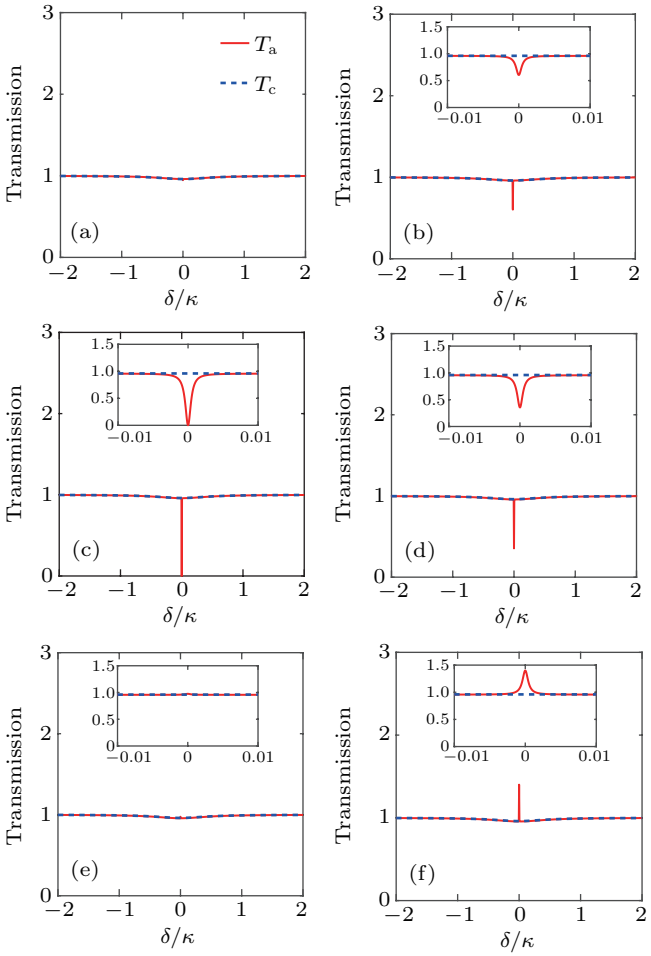


Fig. 3. The transmission spectra T_a and T_c as a function of δ/κ for $\phi = 0$ in the over-coupled regime ($\kappa_e \approx \kappa$). The other parameters are (a) $\eta = 0$, (b) $\eta = 0.3$, (c) $\eta = 1.5$, (d) $\eta = 2.5$, (e) $\eta = 3$, (f) $\eta = 3.3$.

In fact, the nonreciprocity of our system comes from the asymmetric optical pumping. Because the pump field ε_l is very strong, which result in the enhancement of the optomechanical coupling between the cavity mode a and the mechanical mode b, and the mechanical mode can be controlled by the mechanical driving, we can indirectly change the transmission of right-moving probe field by adjusting the mechanical driving. On the contrary, the cavity mode c is not driven, and the optomechanical coupling between c and b is too weak to be ignored. Thus the transmission of left-moving probe field cannot be controlled by the mechanical driving and remains unchanged. We note that in a membrane-in-the-middle optomechanical system, the nonreciprocity can also be achieved.[44] Compared with the work in Ref. [44], we show that the nonreciprocal transmission and amplification in a piezo-optomechanical microresonator can be achieved, and we also show that the transmissive direction of the isolator can be changed by adjusting the ratio between κ_e and κ.

4. Piezoelectrically induced nonreciprocal phase shift

In this section, we demonstrate that our system can perform as a nonreciprocal phase shifter. To achieve a nonreciprocal phase shifter, we should guarantee that both the left-moving probe field and the right-moving probe field are completely transmitted. Figure 4 shows that the probe fields ε_{pa} and ε_{pc} undergo different phase shifts over a finite range of detuning, and the used parameters are the same as that in Fig. 3(e). This means that our system can act as a nonreciprocal phase shifter. From the application perspective, the nonreciprocal phase shifter can be used to build circulators,[7] which demand the phase difference to be π. Figure 4 shows that $|\theta_a - \theta_c|$ can vary from π to 2π, and $|\theta_a - \theta_c| = \pi$ can be met near $\delta = 0$.

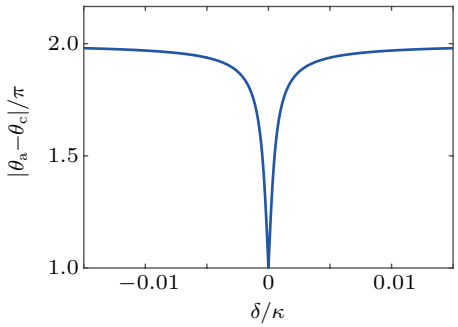


Fig. 4. The phase difference $|\theta_a - \theta_c|$ as a function of δ/κ. The parameters are the same as those in Fig. 3(e).

5. Conclusion and perspectives

In summary, we have investigated the optical nonreciprocity in a piezo-optomechanical system. The analytic expressions of the transmission spectra and the phase shifts have been given. We have shown the piezoelectrically induced nonreciprocal transmission and amplification in the critical-coupled and over-coupled regimes, respectively. The piezoelectrically induced nonreciprocal phase shift in the over-coupled regime has also been shown. Our works provide a new scheme for achieving the nonreciprocal devices and may have applications in the building of photonic network.

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