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Exact solution of the (1+2)-dimensional generalized Kemmer oscillator in the cosmic string background with the magnetic field*

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We study a two-dimensional generalized Kemmer oscillator in the cosmic string spacetime with the magnetic field to better understand the contribution from gravitational field caused by topology defects, and present the exact solutions to the generalized Kemmer equation in the cosmic string with the Morse potential and Coulomb-liked potential through using the Nikiforov–Uvarov (NU) method and biconfluent Heun equation method, respectively. Our results give the topological defect's correction for the wave function, energy spectrum and motion equation, and show that the energy levels of the generalized Kemmer oscillator rely on the angular deficit α connected with the linear mass density m of the cosmic string and characterized the metric's structure in the cosmic string spacetime.

Keywords: generalized Kemmer oscillator, cosmic string, exact solutions

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1. Introduction

The Dirac equation with the linear interactions is known as the Dirac oscillator[1,2] and the interaction of Dirac oscillators can be explained as the interaction between linear electric fields and abnormal magnetic moments.[3,4] There are very few quantum systems that can be accurately solved in quantum mechanics, and the Dirac oscillator (DO) is one of them.[5] The reason why Dirac oscillators have attracted much attention is not only because it is solvable[6–9] but also because it has many physical applications, such as in condensed matter physics,[10,11] high energy physics[12,13] and quantum optics.[14,15] Furthermore, in relativistic quantum mechanics the Kemmer equation is Dirac-liked relativistic wave equation,[16] which was first proposed by Kmmer in 1939. Bednar *et al.* consider that the massive spin-1 particles form a two-particle system with spin $1/2$, rather than a single particle with spin 1. As a consequence, the Kemmer equation is a two-body Dirac-liked equation.[17] Recently, this equation has particularly obtained a lot of interests. It has been investigated under the background of five-dimensional Galilean invariance,[18] in the context of quantum chromodynamics[19] and in the existence of some form interactions.[20,21]

The wave function's exact solutions for relativistic quantum mechanics are quite significant to comprehending the physics meaning about such solutions. They are a meritorious tool for determining the radiative contribution for energy. In many different cases, different techniques have been used to study the quantum mechanics of spin-1, massed and charged particles in the external field. These works specifically studied the solution to wave equation in a magnetic field.[22–24] There are a host of ways that can be used to study the solution to the Kemmer equation, such as the functional Bethe ansatz method,[25,26] the supersymmetric quantum mechanics method[27,28] and the Nikiforov–Uvarov method.[29,30] Particularly, the Nikiforov–Uvarov method has been used extensively to solve many quasi-accurate models in theoretical physics research.[31–33]

Recent years, the quantum systems in curved spacetime background has motivated a great number of investigations.[34–39] The cosmic string, which is a linear defect, will transform the medium's topology as globally observed. The general relativity deems that gravitation is a curvature of spacetime and the Riemann tensor is used to describe the curvature. As is well-known, due to the fact that the atom interact with curvature spacetime, the atom's energy levels in gravitational field can be transformed. Hence, for the sake of completely describing the physical system, the topology of the spacetime should be considered. The impact of topology for the two-dimensional Dirac oscillator, the Dirac field and the Dirac particle restricted in the Aharonov–Bohm ring has been studied.[40–42] In Ref. [43] the authors have studied the one-dimensional generalized Kemmer oscillator in the flat spacetime. In Ref. [44], the authors have studied the two-dimensional Kemmer oscillator in the curve spacetime. In this work, we extend the two-dimensional Kemmer oscillator to the two-dimensional generalized Kemmer oscillator in the

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curved spacetime to explore the topology defects' effects on the generalized Kemmer oscillator.

In this work, the relativistic quantum dynamics for the generalized Kemmer oscillator in the cosmic string background with the magnetic field is studied. According to the corresponding Kemmer equation, the topological defects on the motion equation, the wave function and the energy spectrum are analyzed. The paper is organized as follows. In Section 2, we present the generalized two-dimensional Kemmer oscillator in cosmic string background with the magnetic field. In Section 3, we devote the precise solutions to the generalized Kemmer equation under the Morse potential, using the Nikiforov–Uvarov method, and Coulomb-liked potential, using the biconfluent Heun equation method. Finally, the conclusion is given in Section 4.

2. The generalized Kemmer oscillator in the cosmic string background with the magnetic field

The line element, in cylindrical coordinates, of cosmic string spacetime can be written as[45–47]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \tag{1}$$

where $-\infty < (t,z) < +\infty$, $r \geq 0$ and $0 \leq \phi \leq 2\pi$. The angular deficit parameter α has the ranges $(0,1]$, which has the following relation with linear mass density (m) of the cosmic string: $\alpha = 1 - 4m$. The Kemmer equation in curved spacetime can be written as[44]

$$\left[\mathrm{i}\tilde{\beta}^{\mu}\nabla_{\mu} - M\right]\psi = 0, \tag{2}$$

where covariant derivative is $\nabla_{\mu} = \partial_{\mu} - \Sigma_{\mu}$, $\Sigma_{\mu} = \lim\limits_{\gamma\to\sigma}\Sigma_{\mu} = \left(\Gamma_{\mu}\otimes\hat{I} + \hat{I}\otimes\Gamma_{\mu}\right)$,[48] Γ_{μ} are the spinor affine connections, and $\hat{I}$ is the 4×4 identity matrix. Moreover, $\tilde{\beta}$, M and $\otimes$ denote the Kemmer matrices decided by the line element, the mass of spin-1/2 particles and direct product, respectively. The Kemmer matrices under the cosmic string background satisfy the commutation relation[44]

$$\tilde{\beta}^{\mu}\tilde{\beta}^{\nu}\tilde{\beta}^{\lambda} + \tilde{\beta}^{\lambda}\tilde{\beta}^{\nu}\tilde{\beta}^{\mu} = g^{\mu\nu}\tilde{\beta}^{\lambda} + g^{\lambda\nu}\tilde{\beta}^{\mu}, \tag{3}$$

with $\tilde{\beta}^{\mu} = \gamma^{\mu}(x)\otimes\hat{I} + \hat{I}\otimes\gamma^{\mu}(x)$. Here $\gamma^{\mu}(x)$ are the generalized Dirac matrices that satiate the relations[49]

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu} \tag{4}$$

and it is defined on the basis of a set of tetrad $\gamma^{\mu} = e^{\mu}_{(a)}\gamma^{a}$. Meanwhile, $e^{\mu}_{(a)}$ satisfies the relation $g^{\mu\nu} = e^{\mu}_{(a)}e^{\nu}_{(b)}\eta^{ab}$, γ^{a} represents the Dirac matrices in the flat spacetime, and the tensor η^{ab} is the Minkowski spacetime metric tensor. The spinorial affine connection is given by[48]

$$\Gamma_{\mu} = \frac{1}{8}w_{\mu\mathrm{ab}}\left[\gamma^{\mathrm{a}},\gamma^{\mathrm{b}}\right]. \tag{5}$$

The Dirac matrices γ^{a} in the two-dimensional situation is determined by the Pauli matrices $\gamma^{a} = \left(\sigma^{3},\ \mathrm{i}\sigma^{1},\ \mathrm{i}s\sigma^{2}\right)$,[50] where the parameter s denotes the symbol $\pm$ (+ denotes spin up and − denotes spin down). According to the line element, the tetrads can be written as

$$\hat{e}^{0} = \mathrm{d}t;\quad \hat{e} = \mathrm{d}r;\quad \hat{e}^{2} = r\mathrm{d}\varphi;\quad \hat{e}^{3} = \mathrm{d}z. \tag{6}$$

Solving the Maurer–Cartan structure equation, we obtain $\gamma^{\mu}\Gamma_{\mu} = -\gamma^{1}/2r$. The stationary state of the Kemmer equation is a four-constituent wave function, and it can be written as[44]

$$\psi = \psi_{\mathrm{D}}\otimes\psi_{\mathrm{D}} = \left(\begin{array}{cccc}\psi_1 & \psi_2 & \psi_3 & \psi_4\end{array}\right)^{\mathrm{T}}, \tag{7}$$

where ψ_{D} is the solution to the Dirac equation. In addition, the two-dimensional Kemmer equation with a magnetic field can be defined through the magnetic vector potential $\boldsymbol{A}_{\varphi} = \mathrm{i}\frac{\Phi_{\mathrm{B}}}{2\pi\alpha r}e_{\varphi}$ in the cosmic string spacetime. Therefore, the Kemmer equation considering the magnetic vector potential in the cosmic string background can be written as[44]

$$\left[\mathrm{i}\tilde{\beta}^{0}\partial_0 + \mathrm{i}\tilde{\beta}^{1}\partial_1 + \mathrm{i}\tilde{\beta}^{2}\left(\partial_2 - \Sigma_2 + \mathrm{i}\frac{e\Phi_{\mathrm{B}}}{2\pi\alpha r}\right) - M\right]\psi = 0. \tag{8}$$

To solve the generalized Kemmer oscillator, we replace ∂_1 with $\partial_1 + Mwf_1(\mu_1)\hat{B}$. The operator $\hat{B}$ is chosen as $\hat{B} = \gamma^0\otimes\gamma^0$. Hence, the Kemmer equation under the Dirac oscillator interaction can be written as

$$\begin{aligned}&\left[\mathrm{i}\left(\gamma^0\otimes\hat{I} + \hat{I}\otimes\gamma^0\right)\partial_0 + \mathrm{i}\left(\gamma^1\otimes\hat{I} + \hat{I}\otimes\gamma^1\right)\right.\\ &\times\left.\left(\partial_1 + Mwf(r)\hat{B}\right)\right]\psi + \mathrm{i}\left(\gamma^2\otimes\hat{I} + \hat{I}\otimes\gamma^2\right)\\ &\times\left[\left(\partial_2 - \Sigma_2 + \mathrm{i}\frac{e\Phi_{\mathrm{B}}}{2\pi\alpha r}\right) - M\right]\psi = 0.\end{aligned} \tag{9}$$

When we substitute Eq. (2) into Eq. (3), we gain the generalized Kemmer oscillator coupled equation

$$(2E - M)\psi_1 = \left[\partial_1 - Mwf(r) + \frac{e\Phi_{\mathrm{B}}}{2\pi\alpha r} - \mathrm{i}s\frac{\partial_2}{\alpha r}\right](\psi_2 + \psi_3), \tag{10}$$

$$\begin{aligned}&2\left[\partial_1 + Mwf(r) + \frac{1}{r} - \frac{e\Phi_{\mathrm{B}}}{2\pi\alpha r} + \mathrm{i}s\frac{\partial_2}{\alpha r}\right]\psi_1 + M(\psi_2 + \psi_3)\\ &+2\left[\partial_1 + Mwf(r) + \frac{1}{r} + \frac{e\Phi_{\mathrm{B}}}{2\pi\alpha r} - \mathrm{i}s\frac{\partial_2}{\alpha r}\right]\psi_4 = 0,\end{aligned} \tag{11}$$

$$(2E + M)\psi_4 = \left[-\partial_1 + Mwf(r) + \frac{e\Phi_{\mathrm{B}}}{2\pi\alpha r} - \mathrm{i}s\frac{\partial_2}{\alpha r}\right](\psi_2 + \psi_3). \tag{12}$$

It is obvious that the component of wave function satisfy the relation $\psi_2 = \psi_3$. In this case, combining with the relevant equation we can obtain

$$\begin{aligned}&\left[\partial_1^2 + \frac{1}{r}\partial_1 - Mw\frac{\partial f(r)}{\partial r} - M^2w^2f^2(r)\right.\\ &-\frac{Mwf(r)}{r} + \frac{1}{\alpha^2r^2}\left(s^2\partial_2^2 + \frac{\mathrm{i}\,\Phi_{\mathrm{B}}\,\partial_2}{\pi} - \frac{e^2\Phi_{\mathrm{B}}^2}{4\pi^2}\right)\\ &\left.+\frac{f(r)}{r}4Ew\left(\frac{e\Phi_{\mathrm{B}}}{2\pi\alpha} - \mathrm{i}s\partial_2\right)\right]\psi_2\end{aligned}$$

$$+\left(E^2-\frac{M^2}{4}\right)\psi_2=0. \tag{13}$$

Meanwhile, we assume the wave function $\psi_2=\mathrm{e}^{-\mathrm{i}\lambda\varphi}\zeta(r)$. Therefore, we have

$$\begin{aligned}&\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-Mw\frac{\mathrm{d}f(r)}{\mathrm{d}r}-M^2w^2f^2(r)\right.\\&-\frac{Mwf(r)}{r}-\frac{1}{r^2}\left(\frac{s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}}{\alpha}\right)^2\\&\left.+\frac{f(r)}{r}4Ew\left(\frac{s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}}{\alpha}\right)\right]\zeta(r)\\&+\left(E^2-\frac{M^2}{4}\right)\zeta(r)=0.\end{aligned} \tag{14}$$

In this present work, our main purpose is to solve Eq. (14) for some appropriate potential $f(r)$.

3. The exact solution to the radial equation

3.1. The Kemmer equation's solutions under the Morse potential

The Morse potential is a crucial molecular potential, which can describe the interaction between two atoms. In this section, we discuss that the potential function $f(r)$ is Morse potential,[51,52]

$$f(r)=\frac{\Delta_0}{r^2}\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right), \tag{15}$$

where Δ_0 and β are real constants describing property of the generalized Morse potential. Then the equation describing the generalized Kemmer oscillator in the cosmic string background with the magnetic field can be written as

$$\begin{aligned}&\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-M^2w^2\frac{\Delta_0^2}{r^4}\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right)^2\right.\\&\left.-\frac{1}{r^2}\left[\left(\frac{s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}}{\alpha}\right)^2-Mw\beta\Delta_0\left(\mathrm{e}^{-\beta r}-2\mathrm{e}^{-2\beta r}\right)\right]\right\}\zeta(r)\\&+\left\{\frac{w\Delta_0\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right)}{r^3}\left[2M+\frac{4E}{\alpha}\left(s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}\right)\right]\right.\\&\left.+E^2-\frac{M^2}{4}\right\}\zeta(r)=0.\end{aligned} \tag{16}$$

We assume the wave function in the form of

$$\zeta(r)=\frac{\mathrm{X}(r)}{\sqrt{r}}. \tag{17}$$

Thus, equation (16) becomes

$$\begin{aligned}&\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2}-M^2w^2\frac{\Delta_0^2}{r^4}\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right)^2-\frac{1}{r^2}\left[\left(\frac{s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}}{\alpha}\right)^2\right.\right.\\&\left.\left.-Mw\beta\Delta_0\left(\mathrm{e}^{-\beta r}-2\mathrm{e}^{-2\beta r}\right)-\frac{1}{4}\right]\right\}X(r)\\&+\left\{\frac{w\Delta_0\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right)}{r^3}\left[2M+\frac{4E}{\alpha}\left(s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}\right)\right]\right.\\&\left.+E^2-\frac{M^2}{4}\right\}X(r)=0.\end{aligned} \tag{18}$$

There is no doubt that the equation cannot be exactly solved due to existing the centrifugal term. To obtain the analytical solution, we can use some approximation approaches for the centrifugal term potential. We make the centrifugal term read

$$\frac{1}{r}\approx\frac{\beta}{\left(1-\mathrm{e}^{-\beta r}\right)},\quad \frac{1}{r^2}\approx\frac{\beta^2}{\left(1-\mathrm{e}^{-\beta r}\right)^2}. \tag{19}$$

We also introduce a new variable $\rho=\mathrm{e}^{-\beta r}$, then equation (18) becomes

$$\begin{aligned}&\left[\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}+\frac{1-\rho}{\rho(1-\rho)}\frac{\mathrm{d}}{\mathrm{d}\rho}\right.\\&\left.+\frac{1}{\rho^2(1-\rho)^2}\left(-\Sigma_1\rho^2+\Sigma_2\rho-\Sigma_3\right)\right]X(\rho)=0,\end{aligned} \tag{20}$$

where

$$\Sigma_1=M^2w^2\beta^2\Delta_0^2+2Mw\beta\Delta_0-\frac{E^2}{\beta^2}+\frac{M^2}{4\beta^2}, \tag{21}$$

$$\Sigma_2=\frac{M^2}{2\beta^2}-2\frac{E^2}{\beta^2}+w\beta\Delta_0\left[3M+\frac{4E}{\alpha}\left(s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}\right)\right],$$

$$\Sigma_3=\frac{M^2}{4\beta^2}+\left(\frac{s\lambda-\frac{se\Phi_\mathrm{B}}{2\pi}}{\alpha}\right)^2-\frac{1}{4}-\frac{E^2}{\beta^2}. \tag{22}$$

Equation (20) is a very user-friendly form, which can be solved through the so-called parametric NU method. Comparing it with the NU method, we can obtain the parameters

$$\begin{aligned}&\alpha_4=0,\quad \alpha_5=-\frac{1}{2},\quad \alpha_6=\frac{1}{4}+\Sigma_1,\quad \alpha_7=-\Sigma_2,\\&\alpha_8=\Sigma_3,\quad \alpha_9=\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4},\end{aligned} \tag{23}$$

$$\alpha_{10}=1+2\sqrt{\Sigma_3},\quad \alpha_{11}=2+\left(\sqrt{\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4}}+\sqrt{\Sigma_3}\right),$$

$$\alpha_{12}=\sqrt{\Sigma_3}, \tag{24}$$

$$\alpha_{13}=-\frac{1}{2}-\left(\sqrt{\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4}}+\sqrt{\Sigma_3}\right). \tag{25}$$

Using the NU method, the energy spectrum can be written as

$$\begin{aligned}&n^2+2-\Sigma_2-2\Sigma_3+(2n+1)\left(\sqrt{\Sigma_3}+\sqrt{\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4}}\right)\\&+2\sqrt{\Sigma_3\left(\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4}\right)}=0.\end{aligned} \tag{26}$$

At this moment, inserting detailed $\Sigma_1,\Sigma_2,\Sigma_3$, the energy spectrum can be rewritten as

$$(2n+1)\sqrt{\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)^2+M^2w^2\beta^2\Delta_0^2+2Mw\beta\Delta_0-w\beta\Delta_0\left[3M+\frac{4E}{\alpha}\left(s\lambda-\frac{se\Phi_{\rm B}}{2\pi}\right)\right]}-w\beta\Delta_0\left[3M+\frac{4E}{\alpha}\left(s\lambda-\frac{se\Phi_{\rm B}}{2\pi}\right)\right]$$
$$2\sqrt{\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)^2+M^2w^2\beta^2\Delta_0^2+2Mw\beta\Delta_0-w\beta\Delta_0\left[3M+\frac{4E}{\alpha}\left(s\lambda-\frac{se\Phi_{\rm B}}{2\pi}\right)\right]}\sqrt{\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)^2+\frac{M^2}{4\beta^2}-\frac{1}{4}-\frac{E^2}{\beta^2}}$$
$$+(2n+1)\left[\sqrt{\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)^2+\frac{M^2}{4\beta^2}-\frac{1}{4}-\frac{E^2}{\beta^2}}\right]+n^2+\frac{5}{2}+4\frac{E^2}{\beta^2}-\frac{M^2}{\beta^2}-2\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)^2=0. \quad (27)$$

It should be noted that the magnetic field strength may eliminate the impact from the cosmic string spacetime and the magnetic field satisfies the condition $\Phi_{\rm B}=\frac{2\pi\lambda}{e}$. Therefore, the energy spectrum and wave function can read

$$(2n+1)\left(\sqrt{\frac{M^2}{4\beta^2}-\frac{1}{4}-\frac{E^2}{\beta^2}}+\sqrt{M^2w^2\beta^2\Delta_0^2-Mw\beta\Delta_0}\right)$$
$$+2\sqrt{\left(\frac{M^2}{4\beta^2}-\frac{1}{4}-\frac{E^2}{\beta^2}\right)\left(M^2w^2\beta^2\Delta_0^2-Mw\beta\Delta_0\right)}$$
$$n^2+\frac{5}{2}-3Mw\beta\Delta_0+4\frac{E^2}{\beta^2}-\frac{M^2}{\beta^2}=0, \quad (28)$$

$$\Psi(r)=\begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{pmatrix}$$
$$=N\begin{pmatrix}\frac{2\left[\partial_1-Mw\frac{\Delta_0}{r^2}\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right)+\frac{e\Phi_{\rm B}}{2\pi\alpha r}+\mathrm{i}s\frac{\partial_2}{\alpha r}\right]}{(2E-M)}\\ 1\\ 1\\ \frac{2\left[\partial_1-Mw\frac{\Delta_0}{r^2}\left(\mathrm{e}^{-\beta r}-\mathrm{e}^{-2\beta r}\right)-\frac{e\Phi_{\rm B}}{2\pi\alpha r}+\mathrm{i}s\frac{\partial_2}{\alpha r}\right]}{(2E+M)}\end{pmatrix}$$
$$\times\frac{\mathrm{e}^{\mathrm{i}\lambda\varphi}}{\sqrt{r}}\mathrm{e}^{-\beta\sqrt{\Sigma_3}r}(1-\mathrm{e}^{-\beta r})^{2\sqrt{\Sigma_3+\Sigma_2-\Sigma_2+\frac{1}{4}}}$$
$$\times P_n^{(2\sqrt{\Sigma_3},2\sqrt{\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4}})}(1-2\mathrm{e}^{-\beta r}), \quad (29)$$

where N is the normalization constant, and $P_n^{(2\sqrt{\Sigma_3},2\sqrt{\Sigma_3+\Sigma_1-\Sigma_2+\frac{1}{4}})}$ is Jacobi polynomials.

3.2. The Kemmer equation's solutions under the Coulomb-liked potential

Let us consider the $f(r)$ being the Coulomb-liked potential,[53–55]

$$f(r)=\Delta_1+\Delta_2 r-\frac{\Delta_3}{r}, \quad (30)$$

where Δ_1, Δ_2, and Δ_3 are real parameters. Inserting the Coulomb-liked potential into the equation, we can obtain

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-M^2w^2\Delta_2^2r^2-2M^2w^2\Delta_1\Delta_2 r-\frac{\nabla_3}{r}-\frac{\nabla_2}{r^2}+\nabla_1\right]\zeta(r)=0, \quad (31)$$

where

$$\nabla_1 = 2M^2w^2\Delta_2\Delta_3-M^2w^2\Delta_1^2-2Mw\Delta_2+4Ew\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)\Delta_2+E^2-\frac{M^2}{4}, \quad (32)$$
$$\nabla_2 = \left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)^2+M^2w^2\Delta_3^2+4Ew\left(\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right)\Delta_3,$$
$$\nabla_3 = Mw\Delta_1-4Ew\Delta_1-2M^2w^2\Delta_1\Delta_3. \quad (33)$$

Further we choose the following ansatz:

$$\zeta(r)=r^{\sqrt{\nabla_2}}\exp\left(-Mw\frac{\Delta_2}{2}r^2-Mw\Delta_1 r\right)\xi(r). \quad (34)$$

Using the redefinition $r=\frac{\rho}{\sqrt{Mw\Delta_2}}$, equation (31) can be transformed to

$$\frac{\mathrm{d}^2}{\mathrm{d}\rho^2}\zeta(\rho)+\left(\frac{1+2\sqrt{\nabla_2}}{\rho}-\frac{2\Delta_1}{Mw\Delta_2}-2\rho\right)\frac{\mathrm{d}}{\mathrm{d}\rho}\zeta(\rho)$$
$$+\left\{\frac{Mw\Delta_1^2}{\Delta_2}+\frac{2\Delta_1}{Mw\Delta_2}-2-2\sqrt{\nabla_2}\right.$$
$$\left.-\frac{1}{2\rho}\left[(1+2\sqrt{\nabla_2})\frac{2\Delta_1}{Mw\Delta_2}\right]\right\}\zeta(\rho)$$
$$-\frac{\nabla_3}{\sqrt{Mw\Delta_2}\rho}\zeta(\rho)=0. \quad (35)$$

The equation has the same form as the biconfluent Heun equation.[56–61] Therefore, $\zeta(\rho)$, through the Frobenius method, can be written as

$$\zeta(\rho)=\zeta_{\rm B}\left(2\sqrt{\nabla_2},\frac{2\Delta_1}{Mw\Delta_2},\frac{Mw\Delta_1^2}{\Delta_2}+\frac{\Delta_1}{Mw\Delta_2},\frac{2\nabla_3}{\sqrt{Mw\Delta_2}};\rho\right),$$
$$\zeta(\rho)=\Sigma C_j\rho^j. \quad (36)$$

We can also obtain the following recurrence relations:

$$C_{j+2}=\frac{1}{(j+2)(j+2+2\sqrt{\nabla_2})}\left[\frac{2\Delta_1}{Mw\Delta_2}\left(2j+3+2\sqrt{\nabla_2}+\frac{\nabla_3}{Mw\Delta_2}\right)\right]C_{j+1}+\left(\frac{M^2w^2\Delta_1^2}{\Delta_2}+2\sqrt{\nabla_2}-\frac{\Delta_1}{Mw\Delta_2}\right)C_j, \quad (37)$$

$$C_0=1,\quad C_1=\frac{\Delta_1}{Mw\Delta_2^2}+\frac{\nabla_3}{(1+2\sqrt{\nabla_2})\sqrt{Mw\Delta_2}}, \tag{38}$$

$$C_2=\frac{1}{4(1+2\sqrt{\nabla_2})}\left[\left(2-\frac{Mw\Delta_1^2}{\Delta_2}+2\sqrt{\nabla_2}-\frac{\nabla_3}{Mw\Delta_2}\right)+\frac{3Mw\Delta_1+2Mw\Delta_1\sqrt{\nabla_2}}{\sqrt{Mw\Delta_2}}+\frac{\nabla_3}{\sqrt{Mw\Delta_2}}\right]. \tag{39}$$

Furthermore, according to the Ref. [62], the biconfluent Heun function can be expressed as

$$\zeta_{\rm B}\left(2\sqrt{\nabla_2},\quad \frac{2\Delta_1}{Mw\Delta_2},\quad \frac{Mw\Delta_1^2}{\Delta_2}+\frac{\Delta_1}{Mw\Delta_2},\quad \frac{2\nabla_3}{\sqrt{Mw\Delta_2}};\quad \rho\right)=\sum_{j\geq 0}\frac{A_j\rho^j}{(1+\sqrt{\nabla_2})_j j!} \tag{40}$$

where $A_j=(1+\sqrt{\nabla_2})_j C_j$ and the coefficients A_j obey

$$\begin{aligned}A_{j+2}=&\left\{(j+1)\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}}+\frac{1}{2}\left[\frac{2\nabla_3}{\sqrt{Mw\Delta_2}}+(1+2\sqrt{\nabla_2})\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}}\right]\right\}A_{j+1}\\&+(j+1)(j+1-2\sqrt{\nabla_2})\left(\frac{Mw\Delta_1^2}{\Delta_2}+\frac{\nabla_1}{Mw\Delta_2}-2-2j\right)A_j.\end{aligned} \tag{41}$$

In this case, the wave function can be written as

$$\begin{aligned}\Psi(r)=&\begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{pmatrix}=\begin{pmatrix}\frac{2\left[\partial_1-Mw\left(\Delta_1+\Delta_2 r-\frac{\Delta_3}{r}\right)+\frac{e\Phi_{\rm B}}{2\pi\alpha r}+{\rm i}s\frac{\partial_2}{\alpha r}\right]}{(2E-M)}\\ 1\\ 1\\ \frac{2\left[\partial_1-Mw\left(\Delta_1+\Delta_2 r-\frac{\Delta_3}{r}\right)-\frac{e\Phi_{\rm B}}{2\pi\alpha r}+{\rm i}s\frac{\partial_2}{\alpha r}\right]}{2M+E}\end{pmatrix}r^{\sqrt{\sqrt{2}}-\frac{1}{2}}\exp\left({\rm i}\lambda\varphi-Mw\frac{\Delta_2}{2}r^2-Mw\Delta_1 r\right)\varsigma_{\rm B}\\&\times\left(2\sqrt{\nabla_2},\quad\frac{2\Delta_1}{Mw\Delta_2},\quad\frac{Mw\Delta_1^2}{\Delta_2}+\frac{\Delta_1}{Mw\Delta_2},\quad\frac{2\nabla_3}{\sqrt{Mw\Delta_2}},\quad\sqrt{Mw\Delta_2}r\right).\end{aligned} \tag{42}$$

We can see that the biconfluent Heun functions at infinity are divergent. As a result, in order to enable the function having the n-degree multinomial form, it must meet the following relations:

$$A_{n+1}=0,\quad \frac{Mw\Delta_1^2}{\Delta_2}+\frac{2\Delta_1}{Mw\Delta_2}-2-2\sqrt{\nabla_2}=2n, \tag{43}$$

where the multinomial A_{n+1} has $n+1$ real roots when $1+2\sqrt{\nabla_2}>0$, and $\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}}\in R$. Meanwhile, the multinomial A_{n+1} is the $(n+1)$-dimensional determinant, which can be written as

$$\begin{vmatrix}\Gamma & 1 & 0 & 0 & \cdots & \cdots & 0\\ 2(1+2\sqrt{\nabla_2}) & \Gamma-\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}} & 1 & 0 & \cdots & \cdots & 0\\ 0 & 4(2+2\sqrt{\nabla_2})(n-1) & \Gamma-\frac{4Mw\Delta_1}{\sqrt{Mw\Delta_2}} & 1 & 0 & \cdots & 0\\ 0 & 0 & \Pi_2 & \Gamma-\frac{6Mw\Delta_1}{\sqrt{Mw\Delta_2}} & 1 & \cdots & 0\\ \vdots & \vdots & 0 & \ddots & \ddots & \ddots & \cdots\\ \vdots & \vdots & \vdots & \vdots & \Pi_{j-1} & \Gamma_{j-1} & 1\\ 0 & 0 & 0 & 0 & 0 & \Pi_j & \Gamma_j\end{vmatrix}=0,$$

where

$$\Gamma=\frac{1}{2}\left(\frac{2\nabla_3}{\sqrt{Mw\Delta_2}}+(1+2\sqrt{\nabla_2})\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}}\right),\quad \Pi_j=2(j+1)(j+1-2\sqrt{\nabla_2})(n-j), \tag{44}$$

$$\Gamma_j=\frac{1}{2}\left(\frac{2\nabla_3}{\sqrt{Mw\Delta_2}}+(1+2\sqrt{\nabla_2})\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}}\right)-(j+1)\frac{2Mw\Delta_1}{\sqrt{Mw\Delta_2}}. \tag{45}$$

In addition, we can obtain the energy spectrum

$$\begin{aligned}E^2=&\frac{M^2}{4}-2M^2w^2\Delta_2\Delta_3+M^2w^2\Delta_1^2+2Mw\Delta_2-4Ew\left|\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right|\Delta_2+2Mw\Delta_2+2Mw\Delta_2\\&\times\sqrt{\left|\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right|^2+M^2w^2\Delta_3^2+4Ew\left|\frac{s\lambda-\frac{se\Phi_{\rm B}}{2\pi}}{\alpha}\right|\Delta_3}+2nMw\Delta_2-M^2w^2\Delta_1^2\Delta_2.\end{aligned} \tag{46}$$

From the above spectrum, we can see that changing the intensity $\Phi_{\rm B}$ of the external magnetic field can eliminate the cosmic string spacetime effect, and the corresponding energy spectrum can be written as

$$E^2 = \frac{M^2}{4} - 2M^2w^2\Delta_2\Delta_3 + M^2w^2\Delta_1^2 + 2Mw\Delta_2\Delta_2 + 2Mw\Delta_2 + 2M^2w^2\Delta_2\Delta_3 + 2nMw\Delta_2 - M^2w^2\Delta_1^2\Delta_2. \quad (47)$$

Further, when the potential parameter $\Delta_1 = 0$, the system reduces to Cornell potential which is Coulomb potential plus linear potential, and the corresponding energy spectrum is similar to Ref. [43].

4. Conclusion

We have introduced a $(1+2)$-dimensional generalized Kemmer oscillator under uniform magnetic cosmic string background. The $(1+2)$-dimensional generalized Kemmer oscillator under uniform magnetic cosmic string background has been solved. The wave functions and energy eigenvalues have been presented using the NU method and the biconfluent Heun functions for the Morse potential and the Coulomb-liked potential, respectively. It is demonstrated that the wave function and energy spectrum have the dependence on angular deficit α, which is associated with the linear mass density of the cosmic string.

References

[1] Moshinsky M and Szczepaniak A 1989 *J. Phys. A: Math. Gen.* **22** L817
[2] Ito D, Mori K and Carriere E 1967 *Nuov. Cim. A* **51** 1119
[3] Moreno M and Zentella A 1989 *J. Phys. A: Math. Gen.* **22** L821
[4] Martinez-y-Moreno R P and Salas-Brito A L 1992 *J. Math. Phys.* **33** 1831
[5] Boumali A and Hassanabadi H 2015 *Z. Naturforsch. A* **70** 619
[6] Sari R A, Suparmi A and Cari C 2016 *Chin. Phys. B* **25** 010301
[7] Kurniawan A, Suparmi A and Cari C 2015 *Chin. Phys. B* **24** 030302
[8] Suparmi A, Cari C and Deta U A 2014 *Chin. Phys. B* **23** 090304
[9] Maghsoodi E, Hassanabadi H and Zarrinkamar S 2013 *Chin. Phys. B* **22** 030302
[10] Chargui Y, Trabelsi A and Chetouani L 2010 *Phys. Lett. A* **374** 2907
[11] Franco-Villafane J A, Sadurni E, Barkhofen S, Kuhl U, Mortessagne F and Seligman T H 2013 *Phys. Rev. Lett.* **111** 170405
[12] Grineviciute J and Halderson D 2012 *Phys. Rev. C* **85** 054617
[13] Munarriz J, Dominguez-Adamea F and Limab R P A 2012 *Phys. Lett. A* **376** 3475
[14] Bermudez A, Martin-Delgado M A and Solano E 2007 *Phys. Rev. Lett.* **99** 123602
[15] Lamata L, Leon J, Schatz T and Solano E 2007 *Phys. Rev. Lett.* **98** 253005
[16] Kemmer N 1939 *Proc. R. Soc. Lond. A* **173** 91
[17] Bednar M, Ndimubandi J and Nikitin A G 1997 *Can. J. Phys.* **75** 283
[18] Montigny M D, Khanna F C, Santana A E, Santos E S and Vianna J D M 2000 *J. Phys. A: Math. Gen.* **33** L273
[19] Gribov V 1999 *Eur. Phys. J. C* **10** 71
[20] Boutabia-Cheraitia B and Boudjedaa T 2005 *Phys. Lett. A* **338** 97
[21] Fernandes M C B, Santana A E and Vianna J D M 2003 *J. Phys. A: Math. Gen.* **36** 3841
[22] Krase L D, Pao L and Good R H 1971 *Phys. Rev. D* **3** 1275
[23] Goldman J T and Tsai W Y 1971 *Phys. Rev. D* **4** 3648
[24] Tsai W Y 1971 *Phys. Rev. D* **4** 3652
[25] Hamzavi M, Ikhdair S M and Thylwe K E 2012 *Z. Naturforsch. A* **67** 567
[26] Hamzavi M, Rajabi A A 2011 *Commun. Theor. Phys.* **55** 35
[27] Moghadam S A, Eshghi M and Mehraban H 2014 *Phys. Scr.* **89** 095202
[28] Ikot A N, Maghsoodi E, Ibanga E J, Zarrinkamar S and Hassanabadi H 2013 *Chin. Phys. B* **22** 120302
[29] Aydogdu O, Sever R 2009 *Phys. Scr.* **80** 015001
[30] Hassanabadi H, Maghsoodi E and Zarrinkamar S 2012 *J. Math. Phys.* **53** 022104
[31] Sedaghatnia P, Hassanabadi H and Ahmed F 2019 *Eur. Phys. J. C* **79** 541
[32] Rahimov H, Nikoofard H, Zarrinkamar S and Hassanabadi H 2013 *Appl. Math. Comput.* **219** 4710
[33] Hassanabadi H, Zarrinkamar S and Rajabi A A 2011 *Commun. Theor. Phys.* **55** 541
[34] Bakke K 2013 *Gen. Rel. Grav.* **45** 1847
[35] Oliveira R R S 2019 *Gen. Rel. Grav.* **51** 120
[36] Da Silva W C F and Bakke K 2019 *Class. Quantum Grav.* **36** 235002
[37] Cuzinatto R R, De Montigny M and Pompeia P J 2019 *Gen. Rel. Grav.* **51** 107
[38] Ikot A N, Abbey T M, Chukwuocha E O and Onyeaju M C 2016 *Can. J. Phys.* **94** 517
[39] Bakke K and Mota H 2018 *Eur. Phys. J. Plus* **133** 409
[40] Oliveira R R S 2019 *Eur. Phys. J. C* **79** 725
[41] Maia A V D M and Bakke K 2019 *Eur. Phys. J. C* **79** 551
[42] Oliveira R R S 2019 *Eur. Phys. J. C* **79** 725
[43] Zhao Z L, Long C Y, Long Z W and Xu T 2017 *Chin. Phys. B* **26** 080301
[44] Messai N and Boumali A 2015 *Eur. Phys. J. Plus* **130** 140
[45] Hosseinpour M, Hassanabadi H and Andrade F M 2018 *Eur. Phys. J. C* **78** 93
[46] Linet B 1985 *Gen. Rel. Grav.* **17** 1109
[47] Vilenkin A 1985 *Phys. Rept.* **121** 263
[48] Sogut K and Havare A 2006 *Class. Quantum Grav.* **23** 7129
[49] Carvalho J, Furtado C and Moraes F 2011 *Phys. Rev. A* **84** 032109
[50] Unal N 2005 *Concepts Phys.* II 273
[51] Deng L F, Long C Y, Long Z W and Xu T 2018 *Adv. High Energy Phys.* **2018** 2741694
[52] Qiang W C, Zhou R S and Gao Y 2007 *J. Phys. A: Math. Theor.* **40** 1677
[53] Vitoria R L L, Belich H and Bakke K 2017 *Eur. Phys. J. Plus* **132** 25
[54] Vitoria R L L and Bakke K 2018 *Eur. Phys. J. Plus* **133** 490
[55] Bakke K and Furtado C 2015 *Ann. Phys.* **355** 48
[56] Vitoria R L L and Bakke K 2016 *Eur. Phys. J. Plus* **131** 36
[57] Figueiredo-Medeiros E R and De Mello E R B 2012 *Eur. Phys. J. C* **72** 2051
[58] Sobhani H, Ikot A N and Hassanabadi H 2017 *Eur. Phys. J. Plus* **132** 240
[59] Sobhani H, Hassanabadi H and Chung W S 2018 *Nucl. Phys. A* **973** 33
[60] Hosseini M, Hassanabadi H, Hassanabadi S and Sedaghatnia P 2019 *Int. J. Geom. Methods Mod. Phys.* **16** 1950054
[61] Sargolzaeipor S, Hassanabadi H and Chung W S 2019 *Mod. Phys. Lett. A* **34** 1950183
[62] Vieira H S and Bezerra V B 2015 *J. Math. Phys.* **56** 092501