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SPECIAL TOPIC — Ultracold atom and its application in precision measurement

Michelson laser interferometer-based vibration noise contribution measurement method for cold atom interferometry gravimeter*

Ning Zhang(张宁)[1,2,†], Qingqing Hu(胡青青)[3,†], Qian Wang(王倩)[1,2], Qingchen Ji(姬清晨)[1,4], Weijing Zhao(赵伟靖)[1,2], Rong Wei(魏荣)[1,‡], and Yuzhu Wang(王育竹)[1]

[1] *Key Laboratory for Quantum Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai* 201800, *China*
[2] *Center of Materials Science and Optoelectronics Engineering, University of Chinese Academy of Sciences, Beijing* 100049, *China*
[3] *Advanced Interdisciplinary Technology Research Center, National Innovation Institute of Defense Technology, Beijing* 100010, *China*
[4] *Department of Physics, Shanghai University, Shanghai* 201800, *China*

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The measurement performance of the atom interferometry absolute gravimeter is strongly affected by the ground vibration noise. We propose a vibration noise evaluation scheme using a Michelson laser interferometer constructed by the intrinsic Raman laser of the atomic gravimeter. Theoretical analysis shows that the vibration phase measurement accuracy is better than 4 mrad, which corresponds to about 10^{-10} g accuracy for a single shot gravity measurement. Compared with the commercial seismometer or accelerometer, this method is a simple, low cost, direct, and fully synchronized measurement of the vibration phase which should benefit the development of the atomic gravimeter. On the other side, limited by equivalence principle, the result of the laser interferometer is not absolute but relative vibration measurement. Triangular cap method could be used to evaluation the noise contribution of vibration, which is a different method from others and should benefit the development of the atomic gravimeter.

Keywords: atomic gravimeter, Raman laser, vibration monitoring

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1. Introduction

Since 1990s, atomic interferometers based on matter wave interference theory and laser cooling technique have been applied widely in many aspects,[1–3] such as atomic gyroscope,[4,5] atomic gravimeter,[6,7] and gravity gradiometer.[8,9] As one of the most important applications, the atomic gravimeter has reached an ultrahigh-sensitivity of 42 nm/(s^2·$Hz^{1/2}$),[10] an accuracy of 39 nm/s^2 and a long-term stability of 0.5 nm/s,[11] which is compete with the state of the art classical absolute gravity measurement instruments.[12] However, it is far below the ultimate performances of atomic interference device because the noises come from the Raman laser phase and mirror vibration. The mirror vibration affects the Mach–Zehnder (MZ) atomic interferometric phase by creating an additional phase noise on the counter-propagated Raman lasers, in this way the interference fringe contrast will decrease and the measurement uncertainty will increase.[13,14] Therefore, several techniques are used to mitigate or isolate the effect of the vibration noise in atomic gravimeter. These vibration noise suppression techniques can be classified into two kinds: the first kind is reducing the absolute vibration of the mirror by putting it on a passive or active vibration isolation platform, in this way the vibration noise below 10 Hz could be reduced to a level of 10^{-8} g/$Hz^{1/2}$,[13,15] the second kind is monitoring the vibration of the mirror with high sensitive seismometer or accelerometer, and then correcting the vibration phase with real-time compensation or post-correction techniques.[16,17] The real-time compensation technique feedbacks the monitored vibration noise power spectral densities to the Raman laser phase-locked device to compensate vibration noise while the post-correction technique compensates the vibration noise induced phase fluctuations in the final atomic interference phase. However, from the perspective of implemental technology, these methods either need to add a cumbersome vibration isolation platform or an expensive seismometer to the atomic gravimeter. From the perspective of the noise suppression results, these methods are inconsistent with the vibration of the mirror.

Benefit from the ultrahigh measurement precision, the Michelson laser interferometer (MLI) has been widely used in many areas, such as gravitational wave detection,[18] distance measurement,[19,20] and vibration detection.[21,22] In this paper, we propose a scheme combing the MLI with atomic interferometer (AI) to monitor the vibration of the Raman reflection mirror, which is essential to estimate the vibration noise induced gravity measurement uncertainty in atomic

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†These authors contributed equally to this work.

‡Corresponding author. E-mail: weirong@siom.ac.cn

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gravimeter. This MLI is constructed of the intrinsic Raman laser of the atomic gravimeter and a four-channel phase shift detector.[21,23] Thanks to the ns-level response speed of the phototube and the high accuracy of the laser frequency,[24–26] this method could achieve a higher measurement speed and accuracy than the seismometer or accelerometer. We also discuss the absolute vibration measurement problem and give a modified method based on the "triangular cap method".[27]

2. Method

2.1. Vibration noise in the atomic gravimeter

The atomic interferometer gravimeter (AIG) is usually built using an atomic fountain or an atomic freefall inside a vacuum chamber device. We take the type of atomic fountain as an example. In one measurement cycle, cold atoms are prepared and launched in vertical direction firstly, and then three "$\pi/2$–π–$\pi/2$" Raman laser pulses are implemented to split–reflect–combine the cold atoms to construct the MZ interferometer in their trajectory of parabolic motion. The Raman lasers with wave vectors of $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ propagating downwards along the vertical axis of the device are reflected upwards by a mirror below the vacuum chamber, as shown in Fig. 1(a). By modulating the frequency difference of the Raman lasers, the atoms only interact with one pair of counter-propagation Raman lasers, and achieve a momentum variation of $\hbar\boldsymbol{k}_{\rm eff} = \hbar(\boldsymbol{k_1} - \boldsymbol{k_2}) \approx 2\hbar\boldsymbol{k_1}$. When neglecting the gravity gradient and noise, the phase difference Φ of the two MZ interferometer arms is expressed as[16]

$$\Phi = (k_{\rm eff} g - \alpha) T^2, \tag{1}$$

where T is the time interval between the Raman pulses, as shown in Fig. 1(b), α is the chirp rate of the effective Raman laser frequency used to compensate the Doppler shift. Interference phase Φ can be measured by detecting the relative population of atoms P_a in the initial state $|a\rangle$ as

$$P_a = \frac{1}{2}(1 + \cos\Phi). \tag{2}$$

The shift of the interferometer phase induced by the vibration of the mirror can be written as[9]

$$\delta\phi_{\rm vib} = k_{\rm eff} \int_{-T-\tau}^{T+\tau} g(t) v(t)\, {\rm d}t, \tag{3}$$

where $v(t)$ is the move velocity of the mirror, and $g(t)$ is the sensitivity function of the interferometer. If we choose the center of the three Raman laser pulses as the original time, the sensitivity function $g(t)$ can be written as

$$g(t) = \begin{cases} 0, & t < -T-\tau, \\ \sin\left[\Omega_{\rm R}(T+t+\tau)\right], & -T-\tau < t < -T-\tau/2, \\ -1, & -T-\tau/2 < t < -\tau/2, \\ \sin\left[\Omega_{\rm R}(t)\right], & -\tau/2 < t < \tau/2, \\ 1, & \tau/2 < t < T+\tau/2, \\ -\sin\left[\Omega_{\rm R}(T-t-\tau)\right], & T+\tau/2 < t < T+\tau, \\ 0, & \tau+T < t, \end{cases} \tag{4}$$

in which $\Omega_{\rm R}$ is the Rabi frequency of the Raman laser, T is the time interval between the Raman pulses and τ is the duration time of the π Raman pulse.

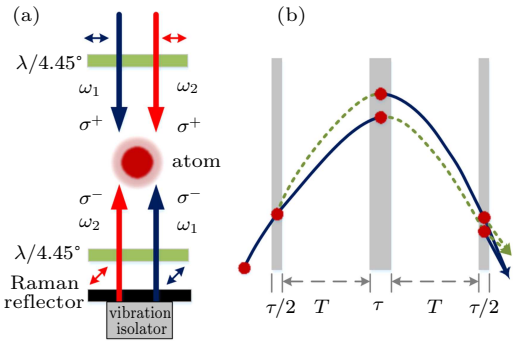


Fig. 1. (a) One Raman laser-polarized configuration for atomic gravimeter.[28] Red lines and blue lines represent linear polarized beams with frequencies of ω_1 and ω_2 respectively, the $\lambda/4$ plate is placed at 45° to generate circularly polarized lasers inside the vacuum chamber; (b) The blue solid line and the green dotted line represent the atomic $|a, \boldsymbol{P}\rangle$ and $|b, \boldsymbol{P} + \hbar\boldsymbol{k}_{\rm eff}\rangle$ states in the MZ atomic gravimeter, respectively. The gray areas represent the duration time of the Raman pulses.

2.2. Measurement principle of mirror vibration based on MIRL

The measurement principle of the mirror vibration using the MI of Raman lasers (MIRL) is shown in Fig. 2. This homodyne MIRL can be divided into three parts: an interferometer part, a detection part, and a signal processing part. In the interferometer part, the measurement arm L and the reference arm R are constituted by adding a half wave-plate (HWP1), a polarizing beam splitter (PBS1), a quarter wave-plate (QWP1), and a mirror to split the downwards propagated Raman laser before entering the vacuum chamber of the gravimeter. The measurement arm L is the atomic gravimeter intrinsic Raman laser used for interacting with atoms and constituting the MZ interferometer. These Raman beams are reflected by the bottom mirror2 and their polarizations are turned to circularly polarization or opposite circularly polarization by QWP1 and QWP2. The reference arm R is constructed by the transmitted laser of PBS1 which passed through a quarter-wave plate (QWP) twice to rotate its polarization state by 90° and finally reflected by a mirror. The measurement laser and reference laser are combined by a BS and propagated to the detection part. In order to ensure an appropriate Rabi frequency, HWP1 is used to make the Raman laser power in measurement arm much larger than that of the reference arm. Besides, considering this power unbalance between the two interferometric beams would decrease the fringe contrast of the MLI, we propose to add a signal power balance system, as shown in Fig. 2, at the detecting optical path to equalize the power of the two interferometric beams.

In the detection part, we adopt the four-channel phase shift detection method[21,23] to improve the measurable accuracy and range. Four channels of coherent signals with fixed additional phase of 0, $\pi/2$, π, $3\pi/2$ generated by the wave

plates and PBSs are detected by photodetectors of PD1–PD4. The detected signals are expressed as

$$I_i = A_i \cos\left(kL(t) + i \times \pi/2\right) + B_i, \quad i = 1,2,3,4, \tag{5}$$

where $i \times \pi/2$ is the phase added to channel i, A_i, and B_i are the signal amplitude and background offset of the detected interferometric signal in channel i, respectively.

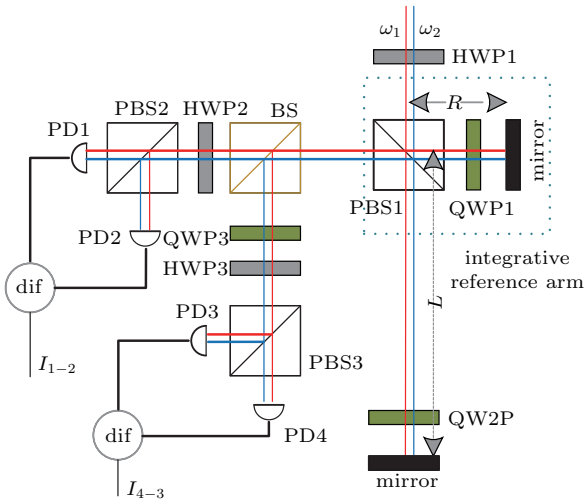


Fig. 2. Schematic diagram of the Michelson interferometer of Raman lasers (MIRL) vibration measurement device. The red and blue lines represent the two Raman beams with two frequencies, respectively. The L arm is the measurement part which interacting with atoms while the R arm is the reference arm (blue dotted frame). The reference arm can be integrated in the structure to reduce the effect of changes in environmental parameters in the measurement. The detection part adopts the biorthogonal-interference-method to eliminate the nonlinear response of the interference intensity to the phase.

In the signal processing part, two quadrature signals are obtained by subtracting I_1 from I_3 and I_2 from I_4. The real-time interference phase shift $\delta\phi(t)$ of the MIRL could be acquired by calculating the arc-tangent value of the two quadrature signals and distinguishing with a bidirectional counting and fringe subdivision part. Consider that the reference arm R of the MIRL usually is short and rigid enough, we assume it as a constant and attribute the variation of the interference phase $\delta\phi(t)$ to the measurement arm L. Therefore, the vibration displacement $\Delta L(t)$ and velocity $v(t)$ of the Raman mirror during the time period of $[t, t + \mathrm{d}t]$ is proportional to the difference of the phase shifts in interference signals by

$$\Delta L = v(t)\,\mathrm{d}t = \frac{\Delta\delta\phi}{2k} = \frac{\delta\phi(t+\mathrm{d}t) - \delta\phi(t)}{2k}, \tag{6}$$

where k is the wave number of the Raman laser and $\mathrm{d}t$ is the sampling interval of the MIRL, which could be smaller than 1 ns thanks to the high speed PD and fast electronics.

According to Eqs. (3) and (4), we know that the additional phase shift induced by the mirror vibration on the AI could be written as

$$\delta\phi_{\mathrm{vib}} = \delta\phi_{\mathrm{RL1}} + \delta\phi_{\mathrm{RL2}} + \delta\phi_{\mathrm{RL3}} + \delta\phi_{\mathrm{jumpT1}} + \delta\phi_{\mathrm{jumpT2}}, \tag{7}$$

in which $\delta\phi_{\mathrm{RL1}}$, $\delta\phi_{\mathrm{RL2}}$, $\delta\phi_{\mathrm{RL3}}$, $\delta\phi_{\mathrm{jumpT1}}$, and $\delta\phi_{\mathrm{jumpT2}}$ represent the integration of the mirror vibration phase shift during the first, second, third Raman laser pulses, as well as the first and second time intervals between the three Raman pulses, respectively. It is worth to mention that despite the MIRL does not work during the time intervals between the three Raman pulses, the phase jumps caused by the vibration of the Raman mirror during these periods could be derived from the difference of the real-time interference phase shift $\delta\phi(t)$ by Eq. (8). Therefore, all the five phase shifts in Eq. (7) could be derived from the synchronous, real-time vibration measurement results of the MIRL.

$$\begin{aligned}
\delta\phi_{\mathrm{jumpT1}} &= \int_{-\tau/2}^{-T-\tau/2} 2kv(t)\,\mathrm{d}t \\
&= \delta\phi(-\tau/2) - \delta\phi(-T-\tau/2), \\
\delta\phi_{\mathrm{jumpT2}} &= \int_{\tau/2}^{T+\tau/2} 2kv(t)\,\mathrm{d}t \\
&= \delta\phi(T+\tau/2) - \delta\phi(\tau/2).
\end{aligned} \tag{8}$$

3. The absolute measurement problem and mitigation strategy

3.1. The absolute measurement problem

As shown in Fig. 3(a), the interference phase shift of the MIRL comes from the optical path variation or the vibration displacement of the relative length between PBS1 and Raman mirror. Therefore, we should take the displacement of PBS1 into account when we talking the absolute vibration of the Raman mirror. To overcome this problem, we propose a modified "AI–MI–AI" tri-interferometer system as shown in Fig. 3(a). By adding another AI setup to implement synchronous measurement, the measured interference phases of $\Phi_{\mathrm{AI},1}$, $\Delta\Phi_{\mathrm{MI}}$, and $\Phi_{\mathrm{AI},2}$ are coupled with each other and can be expressed as follows:

$$\begin{aligned}
\Phi_{\mathrm{AI},1} &= (k_{\mathrm{eff_1}}g - \alpha_1)T^2 + \delta\phi_{\mathrm{vibM1}}, \\
\Phi_{\mathrm{AI},2} &= (k_{\mathrm{eff_2}}g - \alpha_2)T^2 + \delta\phi_{\mathrm{vibM2}}, \\
\Delta\Phi_{\mathrm{MI}} &= \delta\phi_{\mathrm{vibM1}} - \delta\phi_{\mathrm{vibM2}},
\end{aligned} \tag{9}$$

where k_{eff_i} and α_i are the effective wave vector or chirp rate of the i-th ($i = 1,2$) AI, respectively. $\delta\phi_{\mathrm{vibM1}}$ and $\delta\phi_{\mathrm{vibM2}}$ are the additional phase variations induced by the vibrations of the top and bottom Raman mirrors on AI_1 and AI_2, respectively. In Eq. (9) we take the assumption that the vibration of the top mirror1 equals the vibration of the PBS1, namely, $\delta\phi_{\mathrm{vibM1}} = \delta\phi_{\mathrm{vibPBS1}}$.

It seems that we could solve the three unknowns of g, $\delta\phi_{\mathrm{RL1}}$, and $\delta\phi_{\mathrm{RL2}}$ from Eq. (9) since it includes three equations. However, the three equations of Eq. (9) are not independent because the equivalence principle and the change of the reference frame. Normally when we measuring the absolute gravity of one body we always choose the earth fixed inertial reference system, as shown in Fig. 3(b). However, the reference point of the MIRL is PBS1, which is different to the reference system of the two AIs and inevitably affected by ground vibration for it is rigid connected with ground.

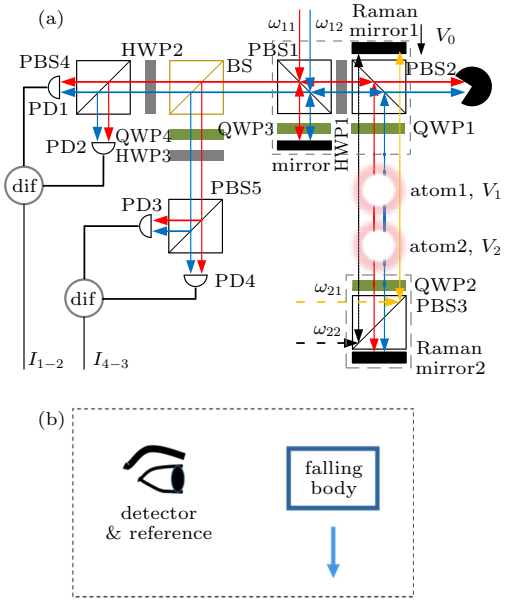


Fig. 3. (a) Scheme of a modified system of MIRL; (b) the earth fixed inertial reference system when we measuring the absolute gravity of one body.

3.2. The mitigation strategy

Though we cannot measure the absolute vibration of the mirror directly by MIRL, this method is still effective for the evaluation of the mirror vibration noise on AI. Denoting the standard deviation of the vibration noise induced phase instability on AI_i, Raman mirror_i, MI and gravity value as $\sigma_{\Phi_{\mathrm{AI},i}}$, $\sigma_{\Phi_{\mathrm{vibM},i}}$, $\sigma_{\Phi_{\mathrm{MI}}}$, and $\sigma_{\Phi_{\mathrm{g}}}$, respectively, we can derive the relationships between these Allan variances from Eq. (9) as

$$\begin{aligned}
\sigma^2_{\Phi_{\mathrm{AI},1}}(\tau) &= \sigma^2_{\Phi_{\mathrm{g}}}(\tau) + \sigma^2_{\Phi_{\mathrm{vibM},1}}(\tau),\\
\sigma^2_{\Phi_{\mathrm{AI},2}}(\tau) &= \sigma^2_{\Phi_{\mathrm{g}}}(\tau) + \sigma^2_{\Phi_{\mathrm{vibM},2}}(\tau),\\
\sigma^2_{\Phi_{\mathrm{MI}}}(\tau) &= \sigma^2_{\Phi_{\mathrm{vibM},1}}(\tau) + \sigma^2_{\Phi_{\mathrm{vibM},2}}(\tau). \qquad (10)
\end{aligned}$$

Assuming the phase instability is independent of each other, we can solve $\sigma^2_{\Phi_{\mathrm{g}}}(\tau)$, $\sigma^2_{\Phi_{\mathrm{vibM},1}}(\tau)$, and $\sigma^2_{\Phi_{\mathrm{vibM},2}}(\tau)$ from Eq. (10) by the "Triangular cap method". The "triangular cap method" is a widely used method in the theoretical and experimental research fields of atomic frequency standards and atomic interferometer.[27] Therefore, not only the contribution of the mirror vibration to the AI $\sigma^2_{\Phi_{\mathrm{vibM},1}}(\tau)$ but also the instability of gravity value without vibration noise $\sigma^2_{\Phi_{\mathrm{g}}}(\tau)$ could be derived from Eq. (10). These results are beneficial to the noise evaluation and performance improvement of the atomic gravimeter.

As to the independence of the phase instability, it could be verified by computing the tends of the Allan covariation in the following equation:[29]

$$\begin{aligned}
&A\mathrm{cov}\left(\delta\Phi_{\mathrm{AI}-1}, \delta\Phi_{\mathrm{AI}-2}\right)\\
&= \frac{1}{2(M-1)} \sum_{j=1}^{M-1} \left(\delta\Phi_{\mathrm{AI}-1,j+1} - \delta\Phi_{\mathrm{AI}-1,j}\right)\\
&\quad \times \left(\delta\Phi_{\mathrm{AI}-2,j+1} - \delta\Phi_{\mathrm{AI}-2,j}\right). \qquad (11)
\end{aligned}$$

In order to fulfill the noise independence condition, we could use a good vibration isolation to the Raman mirror or AI device to make Eq. (11) tend to zero.

4. Analysis of the measurement accuracy

We had introduced in the above section that the MIRL is an effective method for evaluating the contribution of vibration to the instability of the atomic gravimeter. Considering the MIRL is different from standard MI and has some unique characteristics, we will analyze its measurement accuracy quantitatively in the following section.

4.1. The effect of the two Raman laser frequencies

The Raman lasers used in an ^{87}Rb atomic gravimeter are composed of two beams with wavelength of 780.24 nm and frequency difference of about 6.8 GHz, thus the interference signal of the MIRL has a beat signal of 6.8 GHz which is out of the photodetectors' response range and can be neglected. Therefore, the MIRL can be approximately considered as an amplitude overlapping of two MI signals with tiny difference in laser frequencies. Furthermore, as shown in Fig. 4, the overlap of the interference signals could be simplified to a simple periodic signal if we use a single frequency approximate with a weighted average frequency instead of the two Raman lasers. By numerical simulation, we calculate the measurement error caused by this single frequency approximation is about 10^{-4} mrad.

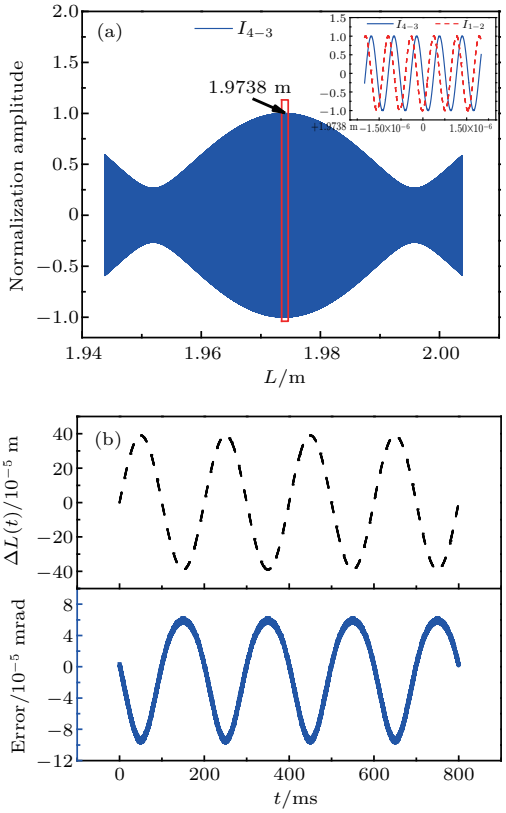


Fig. 4. (a) The beat frequency characteristic of interference signal formed by dual-frequency interference, where $A_{1,\omega_{12}}/A_{1,\omega_{11}} = 1.7$, $\Delta L_0 = 1.974$ m. Insert: Fine interference pattern at the peak of beat signal envelope. The blue solid line and the red dotted line represent I_{4-3} and I_{1-2}, respectively; (b) Phase error caused by single frequency approximation for the given vibration signal of $\Delta L(t) = 3.9 \times 10^{-5} \sin(2\pi \times 5t)$ with simulation time of 800 ms.

4.2. The effect of the Raman lasers' intensity ratio

In general, the intensity ratio η of the two Raman beams is typically around 1.5–2.5, which would lead to a periodic variation of interference signal envelope, as shown in Fig. 5(a). We quantitatively calculate the decrease rate of the fringe amplitude and the measurement error as a function of the power ratio η by numerical simulation, as shown in Fig. 5(b), in which the amplitude decrease rate represents the change rate between the maximum fringe amplitude and the minimum fringe amplitude among 100 fringes around the envelope peak. It can be seen that the amplitude attenuation rate decreases gradually with the increase of the power ratio while the measurement error increases first and decreases afterwards. The maximum measurement error is evaluated smaller than 10^{-4} mrad with the power ratio of $\eta = 1.5$–2.5.

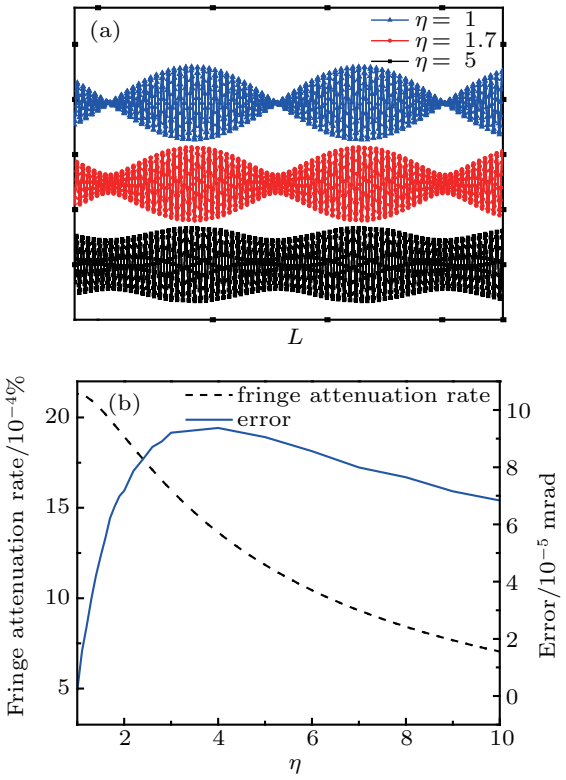


Fig. 5. (a) Beat frequency signal diagram under different lasers' power ratio ($\eta = 1$, $\eta = 1.7$, $\eta = 5$). The blue line with triangle symbol, red line with circle symbol, and black line with square symbol represent the beat fringes when the power intensity ratio is 1, 1.7, and 5 respectively; (b) The diagram of the signal's amplitude decrease rate (the black dotted line) and measurement error (the blue solid line) as a function of the power ratio η between the two beams.

4.3. The linewidth of the Raman laser

Generally, the Raman laser linewidth used in the atomic gravimeter is about 60 kHz and the relative frequency instability is about $\Delta\nu/\nu = 1.54 \times 10^{-10}$. Considering the relationship between phase and frequency is $\Delta\phi = \int \Delta\nu dt$, the phase measurement precision of the single-frequency laser interference measurement method is evaluated as $\delta\phi_{l,s} = 2\pi\Delta\nu\Delta L_0/c = 2.5$ mrad. For the MIRL measurement method, the measurement error is dependent on the frequency instability of both lasers, thus can be evaluated as $\delta\phi_l = \sqrt{2}\delta\phi_{l,s} = 3.54$ mrad.

4.4. The frequency chirp of the Raman laser

For atomic gravimeter, the frequency difference of Raman lasers is scanned at a specific chirp rate of $\alpha \approx 25.1$ MHz/s in order to compensate the Doppler shift.[11] The chirped frequency difference not only affects the envelope position, but also leads to the phase change of the fine interference fringes, as shown in Fig. 6.

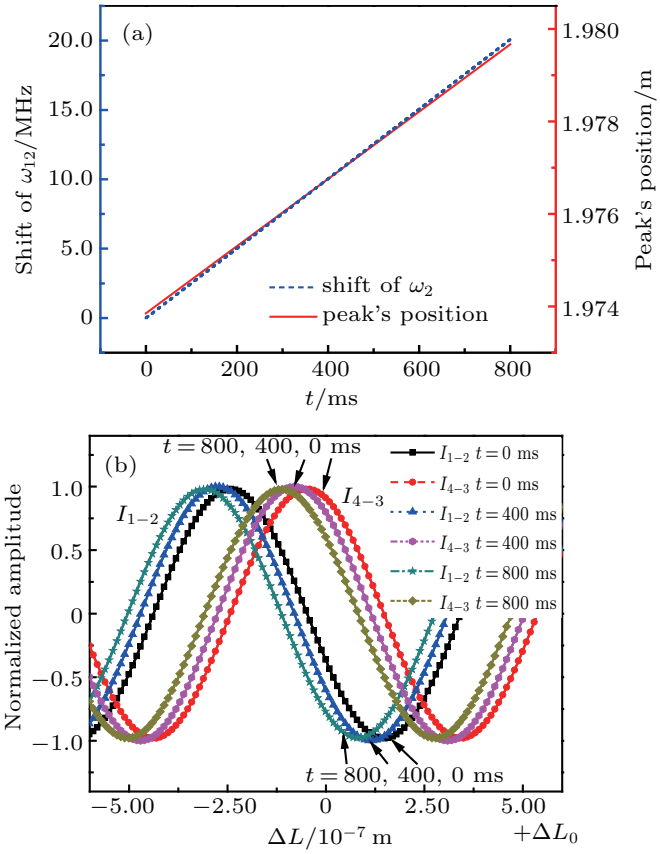


Fig. 6. The changes of (a) ω_{12} (the blue line) and envelope peak (the red line) as frequency chirp time; (b) the phase change of the fine interference fringes, the red line and circular symbols, pink line and hexagonal symbols, brown line and diamond symbols represent I_{4-3}, while the black line and square symbols, blue line and triangle symbols, blue line and pentagonal symbols represent I_{1-2} when $t = 0$, 400, and 800 ms, respectively.

The fringe movement due to frequency shift can be represented in phase as

$$\varphi(t) = \frac{A_{1,\omega_{12}}}{A_{1,\omega_{11}} + A_{1,\omega_{12}}} 2\pi \frac{\alpha t \Delta L_0}{c} = \frac{A_{1,\omega_{12}}}{A_{1,\omega_{11}} + A_{1,\omega_{12}}} 2\pi \frac{\alpha \Delta L_0}{fc}, \quad (12)$$

where $t = 0$ is the beginning of frequency scanning, f is the sampling rate. According to Eq. (12), the shift of the fringe phase is proportional to the frequency chirp rate thus can be canceled by linearly compensate the phase. Given a suitable parameter, we can obtain the maximum phase error induced by frequency chirping after compensation is about 10^{-4} mrad.

If no compensation measures are taken, the maximum phase error is about 10^{-1} mrad, as shown in Fig. 7.

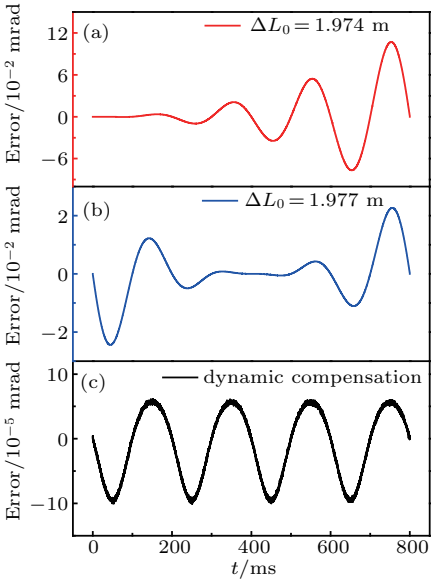


Fig. 7. The maximum phase measurement error is about (a) 1.2×10^{-1} mrad when ΔL_0 is set at the envelope peak point at $t = 0$ ms; (b) 2×10^{-2} mrad when ΔL_0 is set at the peak point of envelope at $t = 400$ ms; (c) 10^{-4} mrad when ΔL_0 is dynamically compensated. The interferometer time in the numerical simulations is 800 ms.

4.5. The effect of the air refractive index

Besides above characters of the MIRL, the air refractive index is also an important factor. Ignoring the variation of air composition, the air refractive index has a relative error of $\Delta n/n = 10^{-8}$.[23] The measurement error induced by the change of the air refractive index is expressed as $\Delta\varphi_{\text{air}} = 2\pi\Delta n L_{\text{air}}/(n\lambda_0)$, where λ_0 is the laser wavelength in vacuum, and L_{air} is the total arm length of MI in air. We evaluate the maximum measurement error caused by the change of air refractive index as 1.6 mrad from the maximum $L_{\text{air}} \sim 4.4$ cm of the beat period length.

4.6. The total precision

The total phase measurement error of the above factors is calculated by $\delta\varphi_{\text{tot}} = \sqrt{\sum \delta\varphi_i^2} < 4$ mrad and the corresponding mirror displacement measurement error (accuracy) is evaluated as 0.48 nm with 780-nm wavelength, as shown in Table 1. Therefore, the corresponding gravity measurement accuracy can be expressed as

$$\delta_{\text{g,vib}} = \frac{\delta\varphi_{\text{tot}}}{(k_{\text{eff_2}} - \alpha)\, gT^2}, \tag{13}$$

where $k_{\text{eff_1}} \approx k_{\text{eff_2}} \approx 1.6 \times 10^7$, T is in the order of 100 milliseconds. If we take $T = 400$ ms as an example, the accuracy of gravity measurement is 10^{-10} g in a single shot measurement. This result is one order of magnitude better than that of mechanical vibration compensation under the same conditions.[15]

Table 1. Systematic error budget of the MIRL vibration measurement method.

Systematic effects	Phase error/mrad	Displacement error/nm
Single-frequency approximate	$\sim 10^{-4}$	$\sim 2.5 \times 10^{-5}$
The chirp of the Raman laser	$\sim 10^{-4}$	$\sim 2.5 \times 10^{-5}$
Laser linewidth	< 3.54	< 0.44
The change in air refractive index	< 1.6	< 0.2
Total	< 4	< 0.5

5. Conclusion

We proposed an innovative method to measure the mirror vibration noise contribution of the atomic gravimeter based on a Michelson interferometer of Raman laser (MIRL). This MIRL is constructed with the atomic gravimeter intrinsic Raman beam combing a four-channel phase shift detector. Therefore, this vibration monitoring method has the advantages of simple, low cost, completely synchronized with the gravity measurement process, perfectly localized with the Raman beams and high measurement accuracy of 4 mrad or 10^{-10} g, *etc*. Considering this MIRL only can measure the relative motion between the beam splitter and bottom mirror, we propose a modified "AI–MI–AI" tri-interferometer system and "the triangle cap" method to measure the contribution of the mirror vibration on the instability of g measurement. Although this method cannot give the absolute vibration of the bottom mirror because the limit of the equivalent principle, its advantages of simple, low cost, effective and high-precision enable it an alternative method to evaluate the contribution of vibration noise to the instability of g measurement besides the commercial seismometer or accelerometer. Therefore, this paper is beneficial to the noise evaluation, miniaturization development, and field application of the atomic gravimeter.

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