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Citation: Chin. Phys. B . 2020, 29(6): 064214 . **doi:** 10.1088/1674-1056/ab8622
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Chin. Phys. B . 2015, 24(4): 044205 . **doi:** 10.1088/1674-1056/24/4/044205

Extended validity of weak measurement*

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(Received 8 November 2019; revised manuscript received 5 March 2020; accepted manuscript online 3 April 2020)

We introduce a modified weak value that is related to the mean value of input meter variable. With the help of the modified weak value, the validity conditions for various modified versions of weak value formalism are investigated, in terms of the dependence of the pointer shift on the mean value of the input meter. The weak value formalism, often used to represent the pointer shift, with the modified weak value is of great use in simplifying calculations and giving guidance of practical experiments whenever the mean value of the input meter variable is nonzero. The simulation in a qubit system is presented and coincident well with our theoretical result.

Keywords: weak measurement, quantum measurement, modified weak value

PACS: 42.50.Tx, 03.65.Ta, 42.50.Xa **DOI:** 10.1088/1674-1056/ab8622

1. Introduction

The concept of weak measurement was first proposed by Aharonov, Albert, and Vaidman (AAV),[1] which involves weak coupling of the system and the meter. The weak values, which could be complex and exceed the eigenvalue spectrum of observable, have been demonstrated to have important applications in exploring fundamental problems of quantum physics in Refs. [2–10]. As an important extension, the weak value amplification technique acts an essential role in precision metrology[11–13] and has been exploited to amplify minute physical quantities.[14–23,23] Recently it was also used to realize highly sensitive optical shifts in position or time domains by weak measurement with a small weak value.[24,25]

Among the practical applications of the weak measurement, it is necessary to establish an exact relationship, which was often expressed by the standard weak value formalism proposed by AAV, among the pointer shift, weak value and the coupling constant. For the standard weak value formalism (AAV formalism), which is the first order approximation in coupling strength, Jozsa gave a physical interpretation of the weak values in terms of the pointer shift.[26] The mean position shift of the pointer is divided two parts: one is proportional to the real part of the weak value and the other is proportional to the pointer spreading rate in space. Obviously, the influence of the latter is less intuitive than that of the former. It was also used to describe numerous physical phenomenons, which has resembled the optical beam propagation enhancement.[14,27] The AAV formalism related to the imaginary part of weak value was also expressed as a shift induced by varying wave function phase[28] or by covariance between the meter's input and output variables.[12,29]

Furthermore, some other investigations beyond the AAV formalism were discussed and analyzed, such as higher-order expansions for nearly orthogonal pre- and post-selected states,[30,31] nonperturbative pre- and post-selected measurement,[29] weak measurement with the orbital angular momentum state,[32–34] the fast oscillations of large pointer shifts[35,36] and the amplification limitations of pointer shifts.[37–40]

In fact, the pointer shift is related not only to the weak value of system and the coupling constant, but to the meter's characteristics. The previous discussions were mainly concentrated on uncertainty of meter's input variable,[1,30,41] varying phase of wave function[12,14,26–29] and more in Refs. [32–36]. However, in some occasions the pointer shift cannot be simply expressed by the AAV formalism as shown in Refs. [2,3,18,20,23,42–48] for the dependence of the pointer shift on the central frequency. Furthermore, a series of validity conditions of the weak measurement were explored, which showed that the mean value of meter's input variable has the same influence as the meter's uncertainty on validity conditions of the weak measurement.[29] Hence, the effect of the mean value of meter's input variable cannot be simply neglected because it not

*Project supported by the National Key R&D Program of China (Grant No. 2017YFA0305200), the National Natural Science Foundation of China (Grant Nos. 11674234 and 11605205), the Natural Science Foundation of Chongqing, China (Grant Nos. cstc2015jcyjA00021 and cstc2018jcyjAX0656), Innovation Project of Sichuan University, China (Grant No. 2018SCUH0021), the Youth Innovation Promotion Association, Chinese Academy of Sciences (Grant No. 2015317), the Entrepreneurship and Innovation Support Program for Chongqing Overseas Returnees, China (Grant Nos. cx2017134 and cx2018040), the Fund of CAS Key Laboratory of Microscale Magnetic Resonance, and the Fund of CAS Key Laboratory of Quantum Information, China.

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only affects the pointer shift but also constrains the validity of weak measurement.

Several treatments in two level systems were presented, such as the redefined system states with a rotation of the polarization of the input field according to frequency,[2,3,44,46] directly integrating the probability distributions of the final state to calculate the central frequency shifts without using the weak value formalisms.[45,48,49] Moreover, the inverse of central frequency was regarded as the interaction strength to realize phase estimation.[43] Unfortunately, a general theoretical solution has not been provided so far.

In this paper, we analyze the role of the mean value of the meter's input variable in such a process. In order to settle this doubt, we present a modified formulas for the pointer shift, coming from the modified "standard" procedure, and a modified weak value is meanwhile introduced, related to the mean value of meter's input variable and interaction constant, as there are also other theory about the modular value[50–52] and general approaches.[53,54] By the modified formulas, the relationship among the pointer shift, the mean value of the meter's input variable and the validity conditions of weak measurement is clarified. Such a kind of treatment is helpful for extending the validity of weak measurement, in particular, making it capable to go on exploiting the weak value formalism with the modified weak value to precisely represent the pointer shift when the mean value of the meter's input variable is large and necessary to be taken into account for the pointer shift. Therefore, it is of great benefit to give guidance of practical experiments and to simplify calculations whenever the mean value of input meter variable is nonzero.

2. Weak measurement: a brief review

The system and meter are initially prepared in the states $|\psi\rangle$ and $|\psi_{\rm m}\rangle$, respectively. In the Von-Neumann measurement scheme, the interaction between the system and meter is described by the coupling Hamiltonian $\hat{H}=\gamma\hat{A}\otimes\hat{F}$, where γ is the coupling strength, $\hat{A}$ is the operator corresponding to the measured system observable A, and $\hat{F}$ is the operator corresponding to the meter's input variable F. The unitary operator is expressed as $\hat{U}=\exp(-{\rm i}\gamma\hat{A}\otimes\hat{F})$ with the free Hamiltonian of the system and meter being neglected. The total initial state undergoes unitary transformation and becomes joint $|\psi_{\rm int}\rangle=\hat{U}|\psi\rangle|\psi_{\rm m}\rangle$, through which the information about the parameter to measure is encoded in the meter. After postselection on the state of the system $|\phi\rangle$, the final state of the meter is $|\Psi_{\rm s}\rangle=\langle\phi|\hat{U}|\psi\rangle|\psi_{\rm m}\rangle$ without normalization. Through measurement on the meter's output observable $\hat{R}=\sum R|R\rangle\langle R|$ (then called the pointer variable), in which for continuous variables the sum becomes integral, the pointer shift is given by

$$\delta R=\overline{R_{\rm s}}-\overline{R}, \tag{1}$$

where $\overline{R_{\rm s}}=\langle\Psi_{\rm s}|\hat{R}|\Psi_{\rm s}\rangle/\langle\Psi_{\rm s}|\Psi_{\rm s}\rangle$ and $\overline{R}=\langle\psi_{\rm m}|\hat{R}|\psi_{\rm m}\rangle$ are the average values of the meter observable $\hat{R}$ in the final state and the initial state, respectively.

In general, the pointer shift can be rewritten as[29,30]

$$\delta R=\frac{{\rm Tr}[(\Pi_\phi\otimes\hat{R}_{\rm c})\hat{U}(\rho\otimes\rho_{\rm m})\hat{U}^\dagger]}{{\rm Tr}[(\Pi_\phi\otimes I_{\rm M})\hat{U}(\rho\otimes\rho_{\rm m})\hat{U}^\dagger]}, \tag{2}$$

where $\Pi_\phi=|\phi\rangle\langle\phi|$ is the postselected state of system, $\rho=|\psi\rangle\langle\psi|$, $\rho_{\rm m}=|\psi_{\rm m}\rangle\langle\psi_{\rm m}|$, $\hat{R}_{\rm c}=\hat{R}-\overline{R}$, and $I_{\rm M}$ is the identity operator of the pointer variable.

Under the weak validity condition[29]

$$|\gamma A_{\phi\psi}|(|\overline{F}|+\Delta F)\ll 1, \tag{3}$$

where $A_{\phi\psi}=\langle\phi|\hat{A}|\psi\rangle$, ΔF is the uncertainty of F in the state $\rho_{\rm m}$, and $\overline{F}={\rm Tr}[\rho_{\rm m}\hat{F}]$ is the mean value of the meter's input variable in the initial state. The nonlinear formula for the pointer shift is obtained as follows:

$$\delta R=\frac{2\gamma{\rm Im}(\overline{R_{\rm c}F}A_{\rm w})+\gamma^2\overline{FR_{\rm c}F}|A_{\rm w}|^2}{1+2\gamma\overline{F}{\rm Im}A_{\rm w}+\gamma^2\overline{F^2}|A_{\rm w}|^2}, \tag{4}$$

where the overline represents the average over $\rho_{\rm m}$, so that $\overline{O}={\rm Tr}(\hat{O}\rho_{\rm m})$ for a meter operator $\hat{O}$, and $A_{\rm w}$ is the weak value $A_{\rm w}=\langle\phi|\hat{A}|\psi\rangle/\langle\phi|\psi\rangle$ (then below called the normal weak value) first proposed by AAV.[1]

In the linear regime,[29] i.e. the first order approximation in γ,

$$|\gamma A_{\rm w}|(|\overline{F}|+\Delta F)\ll 1, \tag{5}$$

with canonically conjugate variable $[\hat{R},\hat{F}]={\rm i}$ and the real Gaussian initial state in the variable F space, the pointer shifts are obtained as follows:

$$\delta R = \gamma{\rm Re}A_{\rm w}, \tag{6}$$

$$\delta F = 2\gamma\Delta F^2{\rm Im}A_{\rm w}. \tag{7}$$

The result of Eqs. (6) and (7) shows that the pointer shift is linear to the coupling constant, which is of great benefit in the practical amplification techniques.[14–19]

3. Weak value formalism with extended validity

According to the weak condition (3) and linear regime condition (5), it is found that the average value $\overline{F}$ and uncertainty ΔF have the same influence on the validity condition for the nonlinear formula and linear formula of the pointer shift. The occasions, $\gamma|A_{\phi\psi}\overline{F}|\gg 1$ or $\gamma|A_{\rm w}\overline{F}|\gg 1$ corresponding to $\gamma|A_{\phi\psi}|\Delta F\ll 1$ or $\gamma|A_{\rm w}|\Delta F\ll 1$ respectively, may sometimes occur, showing the corresponding total validity Eq. (3) or Eq. (5) is violated but the conditions related to ΔF are satisfied. Hence, the normal weak value cannot be used to represent the pointer shift in Eq. (4) or Eq. (6).

In order to settle this doubt, the modified weak value is introduced,

$$A'_{\rm w}=\frac{\langle\phi|\hat{A}\exp(-{\rm i}\gamma\overline{F}\hat{A}))|\psi\rangle}{\langle\phi|\exp(-{\rm i}\gamma\overline{F}\hat{A}))|\psi\rangle}, \tag{8}$$

where $\overline{F}$ is the mean value of meter's input variable. By the modified weak value, the normal formula of Eq. (4) or Eq. (6) can still represent the pointer shift. In the following, we present the process of how to get it and analyze its validity.

Based on the pointer shift form shown in Eq. (2), we perform the following transformations:[29]

$$\hat{F}\to\hat{F}'=\hat{F}-F_0, \tag{9}$$

$$\rho\to\rho'=\exp(-{\rm i}\gamma' F_0\hat{A})\rho\exp({\rm i}\gamma' F_0\hat{A}), \tag{10}$$

$$\Pi_\phi\to\Pi'_\phi=\exp({\rm i}\gamma'' F_0\hat{A})\Pi_\phi\exp(-{\rm i}\gamma'' F_0\hat{A}), \tag{11}$$

where F_0, γ', γ'' are arbitrary real numbers and $\gamma=\gamma'+\gamma''$. Taking the transformations of Eqs. (9)–(11) into Eq. (2), we can obtain a modified formula with the whole pointer shift form unchanged. Then the expression (2) can be rewritten as

$$\delta R=\frac{{\rm Tr}[(\Pi'_\phi\otimes R_{\rm c})U'(\rho'\otimes\rho_{\rm m})(U')^\dagger]}{{\rm Tr}[(\Pi'_\phi\otimes I_{\rm M})U'(\rho'\otimes\rho_{\rm m})(U')^\dagger]}, \tag{12}$$

where the unitary operator is replaced by $U'=\exp(-{\rm i}\gamma\hat{A}\otimes\hat{F}')$, which is derived from the transformation of the meter's input variable F in Eq. (9).

By the transformations, the initial state and the postselected state of the system become ρ' and Π'_ϕ, respectively. The meter's input variable becomes $\hat{F}'$ with its mean value $\overline{F'}=\overline{F}-F_0$. For the purpose of eliminating the influence of mean value $\overline{F'}$ on the validity conditions (3) an (5), the arbitrary number is replaced by $F_0=\overline{F}$ with a new modified operator

$$\hat{F}'=\hat{F}_{\rm c}=\hat{F}-\overline{F}, \tag{13}$$

resulting in $\overline{F'}=0$ but the uncertainty of the variable F unchanged, i.e., $\Delta F'=\Delta F$.

For deducing the general nonlinear formula (4), three replacements need be made, of which the $\hat{F}$ with $\hat{F}-\overline{F}$, the initial state of system $|\psi\rangle$ with $|\psi'\rangle=\exp(-{\rm i}\gamma'\overline{F}\hat{A})|\psi\rangle$ and the final state of system $|\phi\rangle$ with $|\phi'\rangle=\exp({\rm i}\gamma'\overline{F}\hat{A})|\phi\rangle$. The corresponding modified weak value $A'_{\rm w}$ expressed by Eq. (8) is obtained, depending on the coupling constant γ and the meter's parameter $\overline{F}$.

By the modified procedure, the weak validity condition becomes

$$|\gamma A'_{\phi\psi}|\Delta F\ll 1, \tag{14}$$

where the $A'_{\phi\psi}=\langle\phi|\hat{A}\exp(-{\rm i}\gamma\overline{F}\hat{A})|\psi\rangle$. Under this condition the pointer shift is obtained as follows:

$$\delta R'=\frac{2\gamma{\rm Im}(\overline{R_{\rm c}F}A'_{\rm w})+\gamma^2\overline{F_{\rm c}R_{\rm c}F_{\rm c}}|A'_{\rm w}|^2}{1+\gamma^2\Delta F^2|A'_{\rm w}|^2}. \tag{15}$$

Comparing this formula with Eq. (4), we can see the differences that the first order part in γ of the denominator of the nonlinear formula (4) vanishes, the normal weak value $A_{\rm w}$ and the meter's input variable $\hat{F}$ are replaced by the modified weak value $A'_{\rm w}$ and $\hat{F}-\overline{F}$, respectively.

The linear regime condition becomes

$$\gamma|A'_{\rm w}|\Delta F\ll 1, \tag{16}$$

with canonically conjugate variable $[\hat{R},\hat{F}]={\rm i}$ and the real Gaussian initial state in the variable F space, the formulas for the pointer shift are obtained as follows:

$$\delta R' = \gamma{\rm Re}A'_{\rm w}, \tag{17}$$

$$\delta F' = 2\gamma\Delta F^2{\rm Im}A'_{\rm w}. \tag{18}$$

The weak value in the linear formula is just replaced by the modified one.

The weak validity condition and the linear regime condition become $\gamma|A'_{\phi\psi}|\Delta F\ll 1$ and $|\gamma A'_{\rm w}|\Delta F\ll 1$, respectively, showing a possibility of going on the using weak value formulas to represent the pointer shift with the modified weak value, particularly when the value $\overline{F}$ makes the validity condition (3) or (5) unsatisfied but the corresponding condition $|\gamma A'_{\phi\psi}|\Delta F\ll 1$ or $|\gamma A'_{\rm w}|\Delta F\ll 1$ satisfied respectively.

Moreover, the roles of value $\overline{F}$ in the formulas and the corresponding validity conditions make a change that being as a multiplier factor becomes being as a phase factor. Meanwhile, it reveals a deep truth that the uncertainty of the meter acts more essential roles in the validity of weak measurement.

4. The extended validity by numerical analysis

We take an example to show the superiority of the weak value formalisms with the modified weak value, to represent pointer shift. Here we choose a qubit system with the operator of the system observable $\hat{A}=|0\rangle\langle 0|-|1\rangle\langle 1|$, the initial state of the system $|\psi\rangle=\frac{\sqrt{2}}{2}|0\rangle+\frac{\sqrt{2}}{2}|1\rangle$ and postselected state $|\phi\rangle=\frac{\sqrt{2}}{2}|0\rangle-\frac{\sqrt{2}}{2}\exp({\rm i}\varphi)|1\rangle$. The modified weak value by Eq. (8) becomes

$$A'_{\rm w}={\rm i}\cot(\omega_0\tau-\varphi/2). \tag{19}$$

The meter's input variable F is chosen as the frequency ω. The initial state of the meter's input variable is supposed as $|\psi_{\rm m}\rangle=|f(\omega)\rangle$ with

$$f(\omega)=C\exp(-\sigma^2(\omega-\omega_0)^2), \tag{20}$$

where $C=(\sqrt{\pi}/\sqrt{2}\sigma)^{-1/2}$ is the normalization constant, $1/\sigma$ is the width of frequency spectrum, and the ω_0 is the central frequency. The system and meter are correlated by unitary operator $U=\exp(-{\rm i}\tau\hat{A}\otimes\hat{\omega})$, with the coupling strength $\gamma=\tau$.

After postselection on the state of system $|\phi\rangle$, the final state of the meter is given by $|\psi_f\rangle = \langle\phi|\exp(-i\tau\hat{A}\otimes\hat{\omega})|\psi\rangle|f(\omega)\rangle$. The frequency spectrum of the meter final state, without normalization, is obtained

$$\Psi(\omega) = |\langle\omega|\psi_f\rangle|^2 = \sin(\tau\omega - \varphi/2)^2 f(\omega)^2. \tag{21}$$

In the following, three treatments are made to obtain the average frequency shift. First the frequency shift is calculated by direct integral $\int\Psi(\omega)(\omega-\omega_0)\mathrm{d}\omega/\int\Psi(\omega)\mathrm{d}\omega$, a treatment used as comparison. The exact expression is given by

$$\delta\omega_{\mathrm{I}} = \frac{2\tau\Delta\omega^2\exp(-\frac{\tau^2}{2\sigma^2})\sin(2\omega_0\tau-\varphi)}{1-\exp(-\frac{\tau^2}{2\sigma^2})\cos(2\omega_0\tau-\varphi)}, \tag{22}$$

where $\Delta\omega = 1/2\sigma$ is standard deviation of the initial frequency spectrum. The second treatment is to represent the pointer shift by the modified formulas. The modified formulas (15) and (18) with our parameters are given by

$$\delta\omega'_{\mathrm{N}} = \frac{2\tau\Delta\omega^2\mathrm{Im}A'_{\mathrm{w}}}{1+\tau^2\Delta\omega^2|A'_{\mathrm{w}}|^2}, \tag{23}$$

$$\delta\omega'_{\mathrm{L}} = 2\tau\Delta\omega^2\mathrm{Im}A'_{\mathrm{w}}. \tag{24}$$

The third treatment is to represent the pointer shift with the normal weak value. The nonlinear formula (4) is given by

$$\delta\omega_{\mathrm{N}} = \frac{2\tau\Delta\omega^2\mathrm{Im}A_{\mathrm{w}} + 2\tau^2\omega_0\Delta\omega^2|A_{\mathrm{w}}|^2}{1+2\tau\omega_0\mathrm{Im}A_{\mathrm{w}} + \tau^2(\omega_0^2+\Delta\omega^2)|A_{\mathrm{w}}|^2}. \tag{25}$$

The linear formula (7) is $\delta\omega_{\mathrm{L}} = 2\tau\Delta\omega^2\mathrm{Im}A_{\mathrm{w}}$, where the weak value A_{w} is a fixed value and is not related to the central frequency.

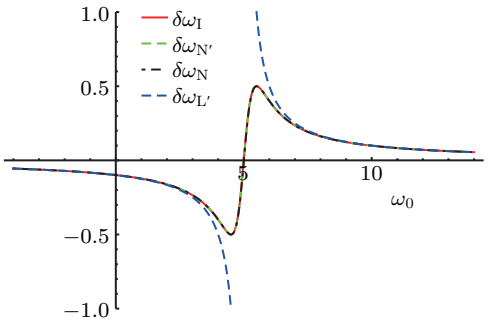


Fig. 1. The average frequency shift versus ω_0 for the four formulas. Parameters $\sigma = 1$, $\tau = 0.01$, $\varphi = 0.1$. The red curve represented by direct integral shown in Eq. (22), the green dashed curve represented by nonlinear formula with the modified weak value shown in Eq. (23), the black dot-dashed curve represented by the nonlinear formula with the normal weak value shown in Eq. (25) and the blue curve represented by linear formula shown in Eq. (24).

In order to analyze the validity of the latter two treatments, the frequency shift calculated by four formulas are shown in Fig. 1 with the specific parameters $\sigma = 1$, $\tau = 0.01$, $\varphi = 0.1$. As the dependence of the pointer shifts on the central frequency is shown in Fig. 1, it is not enough to take the linear formula $2\tau\Delta\omega^2\mathrm{Im}A_{\mathrm{w}}$ to represent the pointer shift in more general occasion, showing its necessity of considering the effect of $\overline{F}$ on the pointer shift. Then the blue dashed curve represented by the linear formula (24) is partially, just on the two sides of the curve, fitted well with the exact red curve (Eq. (22)), while the middle part is not. The reason is that the linear validity condition $\tau|A'_{\mathrm{w}}|\Delta\omega \ll 1$ shown in Fig. 2 on the two sides of the curve is no longer satisfied.

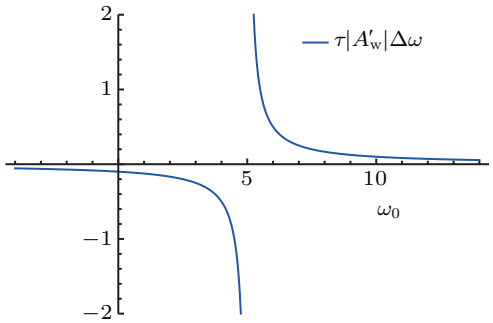


Fig. 2. The value $\tau|A'_{\mathrm{w}}|\Delta\omega$ for the linear validity condition with the modified value versus ω_0. Parameters $\sigma = 1$, $\tau = 0.01$, $\varphi = 0.1$.

The green dashed curve represented by nonlinear formula, with the modified weak value, shown in Eq. (23) and the black dot-dashed curve represented by the nonlinear formula, with the normal weak value, shown in Eq. (25) are fitted well with the exact red curve represented by direct integral Eq. (22). This is because the nonlinear conditions $\tau\Delta\omega\cos(\omega_0\tau - \varphi/2) \leq \tau\Delta\omega \ll 1$ and $\tau(\Delta\omega+\omega_0)\cos(-\varphi/2) \leq \tau(\Delta\omega + \omega_0^{\max}) \ll 1$ for Eqs. (23) and (25), where $\omega_0^{\max}$ is the maximum coordinate value of ω_0 in Fig. 1, are satisfied under their given parameters.

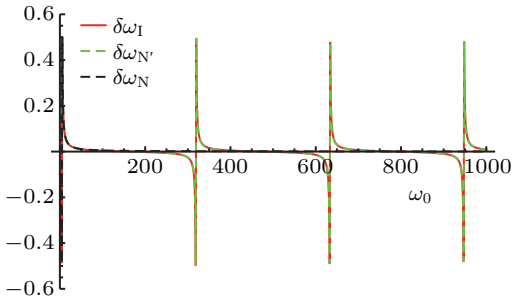


Fig. 3. The average frequency shift versus the larger central frequency ω_0 for the three formulas.

For larger values ω_0, the curves for the two kinds of nonlinear formulas are shown in Fig. 3. It is shown that the green dashed curve representing the nonlinear formula (23) is still fitted well with the exact red curve, while the black dashed curve representing the nonlinear formula (25) is not. This is because the nonlinear condition $\tau\Delta\omega\cos(\omega_0\tau - \varphi/2) \ll 1$ for the formula (23) is still satisfied but the condition $\tau(\Delta\omega + \omega_0)\cos(-\varphi/2) \ll 1$ for the formula (25) is obviously violated.

Moreover, the periodicity of variation of the pointer shift with the value $\bar{F}$ can be interestingly observed, whereas this phenomenon would not occur through the formula (4). This is because the value $\bar{F}$ in the modified formula (15) is indeed a phase factor rather than a multiplier factor.

By the modified "standard" procedure of the weak measurement, the extended validity of weak measurement has been verified in our simulation. Since the frequency shift would be influenced by central frequency, the former nonlinear formula (4) and nonlinear formula (7) with normal weak value cannot be simply used to represent the pointer shift, particularly as the central frequency makes the validity of the two formulas violated. Under this occasion, the two formulas (15) and (18) with the modified weak value works better. Through the above analysis, the simulation is coincident well with the theoretical treatment.

5. Conclusion

By the modified "standard" procedure for the pointer shift, general nonlinear formulas with the modified weak value related to the mean value of the meter's input variable and the coupling constant have been presented. The mean value $\overline{F}$, treated as a multiplier factor under the validity conditions and the former formulas, becomes a phase influence on the modified formulas. The relationship between the mean value of meter's input variable, validity conditions and the pointer shift has been clarified. Such a kind of treatment helps extend the validity of weak measurement, especially for the occasions with a large value $\overline{F}$. It is of great benefit in giving guidance of practical experiments and simplifying calculations whenever the mean value of the meter's input variable is nonzero.

As the importance of the mean value of the meter's input variable in weak measurements had been shown in the earlier work,[29] we give a way how to deal with it. For the role of the mean value $\overline{F}$ as a multiplier factor in the validity conditions becomes the one as phase factor, it reveals a deep truth that the uncertainty of the meter's input variable may play more essential role than the meter's input variable in the validity of weak measurement.

Our work unveils the deep connection between weak measurement and meter's input variable, thereby making progress toward understanding and solving the problems when the mean value of meter's input variable need be taken into account. Furthermore, we may leave a talk about if the system phase can influence the mean value of the meter's input variable, or if the system phase can influence the meter input variable.

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