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Quantum teleportation of particles in an environment*

Lu Yang(杨璐)[1,2], Yu-Chen Liu(刘雨辰)[3], and Yan-Song Li(李岩松)[1,2,†]

[1] State Key Laboratory of Low-Dimensional Quantum Physics and Department of Physics, Tsinghua University, Beijing 100084, China
[2] Frontiers Science Center of Quantum Information, Beijing 100084, China
[3] School of Physics, Jilin University, Changchun 130012, China

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We discuss the teleportation of particles in an environment of an N-body system. In this case, we can change a many-body system into an arbitrary shape in space by teleporting some or all the constituent particles, and thus we call the quantum teleportation under this circumstance as quantum tele-transformation (QTT). The particular feature of QTT is that the wave function of the internal degrees of freedom remains the same, while the spatial wave function experiences a drastic change. The notion of QTT provides conceptual and pedagogical convenience for quantum information processing. In view of QTT, teleportation is the change of a single particle in space, while entanglement swapping is the change of one particle of an entangled pair.

Keywords: quantum tele-transformation, teleportation, entanglement swapping

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1. Introduction

Quantum entanglement is one of the interesting physical properties of quantum mechanics and has been widely studied.[1–14] By using prior shared entanglement and classical communication, Alice can send Bob who locates remotely an unknown quantum state without actually sending the particle using quantum teleportation.[15] Quantum teleportation has wide applications in quantum communication and quantum computing. It is an essential ingredient in quantum repeater,[16–19] measurement-device-independent quantum secure direct communication (QSDC),[20,21] device-independent QSDC,[22] three-party QSDC,[23] measurement-device-independent quantum key distribution,[24] device-independent quantum key distribution,[25] quantum communication,[26,27] and certifying entanglement.[28,29] Quantum teleportation of multiple degrees of freedom of a single photon[30,31] and the logic qubit teleportation[32,33] are also studied. Concurrently with quantum teleportation, entanglement swapping was also proposed, in which two particles in two EPR pairs who never meet each other could be entangled by performing Bell-state measurement of their partner particles.[34] In this work, we will extend the quantum teleportation to the case where the particle to be teleported is within a general many-body system. We call this kind of teleportation as quantum tele-transformation (QTT), because it can transform a many-body system into an arbitrary shape in space, while keeping the wave function in other degrees of freedom the same. In the original quantum teleportation scheme, an isolated single particle with the wave function $\alpha|0\rangle+\beta|1\rangle$ is teleported to a distant place by using pre-shared entanglement, classical communication, and single particle unitary operation. What happens if the single particle to be teleported is in a complicated environment? We will discuss this problem in this short paper. This problem seems simple to some physicists, but is not apparent to the general researchers in the field.

2. Quantum tele-transformation

Suppose a general N-body quantum system is located at Alice's site with the following wave function in the internal degrees of freedom, for instance, the spins of electrons:

$$|\Psi\rangle=\sum_{i_1=0}^{d_1-1}\sum_{i_2=0}^{d_2-1}\cdots\sum_{i_N=0}^{d_N-1}c_{i_1i_2\cdots i_N}|i_1\rangle|i_2\rangle\cdots|i_N\rangle, \tag{1}$$

where d_i is the dimension of the i-th particle's Hilbert space. The spatial wave functions are just the products of the individual particles, and we will not write them out explicitly. The important fact is that the N particles can be anywhere in space. The dimension of the Hilbert space of each particle may be different, as it is usually the case in nature. Suppose we want to teleport the k-th particle which is coupled to the rest $(N-1)$ particles in a very complicated manner. Then we can rewrite Eq. (1) as follows:

$$\begin{aligned}|\Psi\rangle&=\sum_{i_1=0}^{d_1-1}\sum_{i_2=0}^{d_2-1}\cdots\sum_{i_N=0}^{d_N-1}c_{i_1i_2\ldots i_N}|i_1\rangle|i_2\rangle\ldots|i_N\rangle\\&=\sum_{i_k=0}^{d_k-1}\left(\sum_{i_k,j\neq k}^{d_j-1}c_{i_1\ldots i_k\ldots i_N}\left|i_1\cdots i_{j\neq k}\cdots i_N\right\rangle\right)|i_k\rangle\end{aligned}$$

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†Corresponding author. E-mail: ysli@tsinghua.edu.cn

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$$= \sum_{i_k=0}^{d_k-1} \alpha_k |i_k\rangle, \tag{2}$$

where we have written all the coefficients, including the quantum states of other particles, as a coefficient α_k. The quantum states of the N particles commute with each other, therefore they can be treated as numerical factors. It is equivalent to say that we can treat all other particles as bystanders. Then the teleportation process is the same as an isolated particle. Here we discuss the general case of teleportation in arbitrary high dimension, which was first proposed in Ref. [15] and has been experimentally demonstrated recently.[35,36] The Bell states in d_k dimension are

$$\left|\psi_{nm}^k\right\rangle = \sum_{j=0}^{d_k-1} \mathrm{e}^{2\pi \mathrm{i}\, jn/d_k} |j\rangle\, |(j+m)\mathrm{Mod}\ d_k\rangle / \sqrt{d_k}, \tag{3}$$

where $j,n,m = 0,1,\ldots,d_k-1$ and $k = 1,2,\ldots,N$. Explicitly, suppose particles k_A and k_B at Bob's site are entangled in one of the Bell states[15]

$$\left|\psi_{00}^k\right\rangle = \sum_{j=0}^{d_k-1} |j\rangle\, |j\rangle / \sqrt{d_k}, \tag{4}$$

and particle k_A is distributed to Alice. Then Alice performs generalized Bell-state measurement on particle k_A from the shared Bell state and particle k at her site, and the result will be one of the Bell states $\left|\psi_{nm}^k\right\rangle$ with probability $1/d_k$. After Alice tells Bob her measurement result, Bob performs a unitary operation

$$U_{nm}^k = \sum_{j=0}^{d_k-1} \mathrm{e}^{2\pi \mathrm{i}\, jn/d_k} |j\rangle\langle (j+m)\mathrm{Mod}\ d_k|, \tag{5}$$

on particle k_B. By now particle k at Alice's site has been teleported to Bob's site. However the wave function of the internal degrees of freedom is still the same, namely,

$$\left|\Psi'\right\rangle = \sum_{i_1=0}^{d_1-1} \sum_{i_2=0}^{d_2-1} \cdots \sum_{i'_k}^{d_k-1} \cdots \sum_{i_N=0}^{d_N-1} c_{i_1 i_2 \ldots i'_k \cdots i_N} |i_1\rangle\, |i_2\rangle \ldots \left|i'_k\right\rangle \cdots |i_N\rangle, \tag{6}$$

where $\left|i'_k\right\rangle$ means particle k is now at Bob's site as shown in Fig. 1. All the other particles are just bystanders that have not been influenced by the process of the teleportation of their companying particle k.

QTT is just the generalization of the idea of teleportation of an isolated particle into the teleportation of a particle in an environment. We find that such a teleportation, QTT, changes the shape of the many-body system. QTT simplifies the conceptual understanding of the nonlocal processes in quantum information, and provides pedagogical convenience in application. Here are some examples.

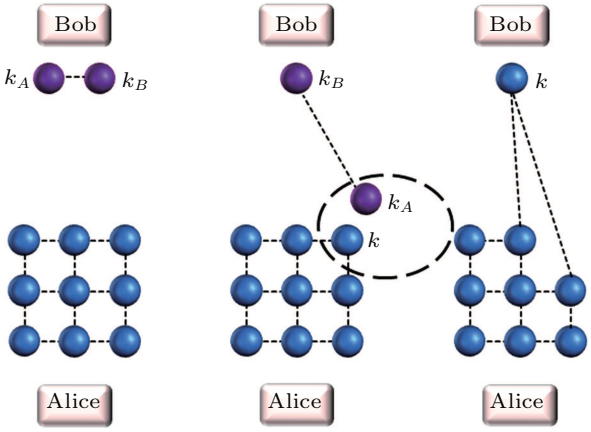


Fig. 1. Illustration of quantum tele-transformation.

When there is only one particle in QTT, it is simply the original quantum teleportation.

When there are two particles that are in entangled states

$$|\psi_{nm}\rangle = \sum_{j=0}^{1} \mathrm{e}^{2\pi \mathrm{i}\, jn/2} |j\rangle\, |(j+m)\mathrm{Mod}\ 2\rangle / \sqrt{2}, \tag{7}$$

a QTT on the second particle will teleport it to Bob's site, thus the entanglement at Alice's site is stretched to cover the distance from Alice's to Bob's sites. This is just entanglement swapping.[34] In entanglement swapping, the counter intuitive feature is that the two particles in the newly formed entanglement have never met before.

Quantum repeater is the repeated use of QTT to move a qubit further and further to increase the communication distance.

QTT can make an N-body quantum system into an arbitrary shape by teleporting each particle at Alice's site to many Bobs vastly locating in remote places. Of course, Alice can teleport all her particles to Bob.

Using QTT, an N-body system can be divided into arbitrary partitions and then sent to different locations. For instance, in distributed quantum computing, a big task can be divided into several smaller ones, and each can be completed with a smaller number of qubits. QTT can distribute these smaller tasks to some small quantum computers. After the completions of the smaller tasks, the small quantum computers return their homework to the central quantum computing workstation using QTT.

This picture can also be used to understand the nonlocal gate operations. For instance, the optimal nonlocal CNOT gate between qubit A in state $\alpha|0\rangle_A + \beta|1\rangle_A$ and qubit B in state $\gamma|0\rangle_B + \delta|1\rangle_B$ was proposed in Ref. [37], and its recent experimental realization in an ion trap system was reported in Ref. [38]. It has been generalized to logic qubits.[39] How to perform the nonlocal CNOT gate if particles A and B are mingled with other particles? By rewriting the wave functions of the two mingled systems with the components of particles

A and B explicitly written out as in Eq. (2), the wave functions of particles A and B can still be written as $\alpha|0\rangle_A + \beta|1\rangle_A$ and $\gamma|0\rangle_B + \delta|1\rangle_B$, respectively, with the understanding that the coefficients now contain the wave functions of the remaining particles they couple with. Then we can perform the nonlocal gate in the same way as if we are performing it on two isolated particles.

3. Conclusion and perspectives

Quantum tele-transformation can transform a many-body system into an arbitrary shape in space. It simplifies the conceptual understanding of the nonlocal processes in quantum information and provides pedagogical convenience in application. Its implementation is relatively simple. It requires only long coherence quantum memories, efficient measurement devices, and single particle manipulations. Finally, it is worth noting that in this study, we concentrate on the influence of the environment of the teleported particle using the standard quantum teleportation protocol,[15] which is different from the studies of quantum teleportation using different entangled states, for instance, Ref. [40].

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