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Four-soliton solution and soliton interactions of the generalized coupled nonlinear Schrödinger equation*

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Based on the generalized coupled nonlinear Schrödinger equation, we obtain the analytic four-bright–bright soliton solution by using the Hirota bilinear method. The interactions among four solitons are also studied in detail. The results show that the interaction among four solitons mainly depends on the values of solution parameters; k_1 and k_2 mainly affect the two inboard solitons while k_3 and k_4 mainly affect the two outboard solitons; the pulse velocity and width mainly depend on the imaginary part of k_i ($i = 1, 2, 3, 4$), while the pulse amplitude mainly depends on the real part of k_i ($i = 1, 2, 3, 4$).

Keywords: coupled nonlinear Schrödinger equation, four-soliton solution, soliton interaction

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1. Introduction

Optical soliton, generated from the balance between nonlinearity and dispersion (or diffraction), has received much attention in research due to its potential applications in long distance optical communication, optical devices, and optical computers. The propagation of optical soliton pulses through a fiber medium is governed by the nonlinear Schrödinger (NLS) equation and its variants.[1,2] These NLS equations have two types of soliton solutions, namely bright and dark solitons.[3,4] The bright soliton is generated in the anomalous dispersion region of the fiber, and the dark soliton, which is more stable than bright soliton, exists in the normal dispersion region. In general, due to birefringence effect,[5,6] single mode propagation of light pulses is governed by the coupled NLS (CNLS) equations.[7–9] In recent years, the CNLS equations with higher-order effects have attracted much attention, such as self-steepening,[10] Raman scatting,[11] and four-wave mixing.[12] The four wave mixing effect is a basic nonlinear phenomenon having fundamental relevance and practical applications particularly in nonlinear optics, optical processing,[13] real time holography,[14] measurement of atomic energy structures and decay rates.[15] The generalized coupled nonlinear Schrödinger (GCNLS)[16,17] equation with four-wave mixing effect can be expressed as

$$\begin{aligned} &\mathrm{i}p_t + p_{xx} + 2(a|p|^2 + c|q|^2 + bpq^* + b^*p^*q)p = 0,\\ &\mathrm{i}q_t + q_{xx} + 2(a|p|^2 + c|q|^2 + bpq^* + b^*p^*q)q = 0, \end{aligned} \quad (1)$$

where p and q are slowly varying pulse envelopes; a and c are real constants and correspond to self-phase and cross-phase modulation coefficients, respectively; b is a complex constant corresponding to four-wave mixing effect; symbol * denotes the complex conjugation. It has been shown that system (1) is completely integrable for arbitrary values of the system parameters, a, b and c, through Weiss–Tabor–Carnevale (WTC) algorithm.[18] When $a = c$ and $b = 0$, system (1) reduces to the Manakov system.[19] When $a = -c$ and $b = 0$, it reduces to the mixed coupled nonlinear Schrödinger equation.[20] In recent years, different soliton solutions of GCNLS equation have been extensively studied.[21–32] In 2010, two-soliton solution of GCNLS equation and the collision dynamics between solitons were studied in Ref. [16], and a new soliton phenomenon (soliton reflection) was found. Later, the dark–dark soliton, general breather solution (GB), Akhmediev breather solution (AB), Ma soliton solution (MS), and rouge wave solution (RW) of GCNLS equation were obtained by use of Hirota bilinear method.[23–31] The N-soliton solutions of the GCNLS equation with four-wave mixing effect were presented and the collision properties of one-soliton, two-soliton, and three-soliton solutions have also been studied in detail in Ref. [17]. Among them, the quasi-breather-dark (breather-like) soliton has novel properties that one component has oscillatory behavior and another component's amplitude remains unchanged.[24] But there are few reports about breather-like soliton and four-soliton solution. Therefore the four-bright–bright soliton solution of GCNLS equation is derived by using Hirota bilinear method in this paper. And a breather-like four-soliton solution is also obtained and the collision dynamics between solitons is studied in detail. The rest of this paper is organized as follows. In Section 2, the four-bight–bright soliton solution is deduced by the Hirota bilinear method. In Section 3, the collision dynamics between solitons is discussed in detail. Finally, some conclusions are drawn from the present study in Section 4.

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2. Four-soliton solution

We construct a four-soliton solution of Eq. (1) through the Hirota bilinear method when $a, c > 0$. We consider the bilinearizing transformations that are in the following form:[17]

$$p = \frac{g^{(1)}}{f}, \quad q = \frac{g^{(2)}}{f}, \tag{2}$$

where $g^{(1)}$ and $g^{(2)}$ are complex functions while f is a real function. Substituting Eq. (2) into the GCNLS Eq. (1) and rewriting the resulting equation appropriately, we obtain the following bilinear equations:[17]

$$\begin{aligned}&(\mathrm{i}D_t + D_x^2)(g^{(s)} \cdot f) = 0,\\ &D_x^2 (f \cdot f) = 2(a\left|g^{(1)}\right|^2 + c\left|g^{(2)}\right|^2\\ &\qquad + bg^{(1)}g^{(2)*} + b^* g^{(2)} g^{(1)*}).\end{aligned} \tag{3}$$

In the above expressions, D_t and D_x^2 are the Hirota bilinear operators. We expand the functions $g^{(s)}$, $s = 1,2$, and f formally as power series expansions in terms of a small arbitrary real parameter ε, that is

$$\begin{aligned}&g^{(s)} = \varepsilon g_1^{(s)} + \varepsilon^3 g_3^{(s)} + \varepsilon^5 g_5^{(s)} + \varepsilon^7 g_7^{(s)},\\ &f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \varepsilon^6 f_6 + \varepsilon^8 f_8,\end{aligned} \tag{4}$$

where supposing $\varepsilon = 1$ and substituting Eq. (4) into Eq. (3), we can obtain the four-soliton solution of GCNLS Eq. (1). In order to obtain the explicit form of four-soliton solution, we define the following (1×4) row matrices Δ_s, (2×1) column matrices ψ_j, $j = 1, 2, 3, 4$, column matrix ϕ, and the (4×4) identity matrix I as follows:

$$\Delta_{\mathrm{s}} = -\left(\varsigma_1^{(s)}, \varsigma_2^{(s)}, \varsigma_3^{(s)}, \varsigma_4^{(s)}\right), \quad \psi_{\mathrm{j}} = \begin{pmatrix} \varsigma_j^{(1)} \\ \varsigma_j^{(2)} \end{pmatrix}, \tag{5}$$

$$\varphi = \begin{pmatrix} \exp(\eta_1) \\ \exp(\eta_2) \\ \exp(\eta_3) \\ \exp(\eta_4) \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

Here, $\eta_j = k_j x + \mathrm{i}k_j^2 t$, ς_j^s, and k_j are the complex parameters. We can write the four-soliton solution of GCNLS equation in the form of

$$p = \frac{g^{(1)}}{f}, \quad q = \frac{g^{(2)}}{f}, \tag{7}$$

where

$$g^{(s)} = \begin{vmatrix} \Xi & I & \phi \\ -I & \Omega & \mathbf{0}^{\mathrm{T}} \\ \mathbf{0} & \Delta_s & 0 \end{vmatrix}, \quad f = \begin{vmatrix} \Xi & I \\ -I & \Omega \end{vmatrix}, \quad s = 1,2. \tag{8}$$

The matrices Ξ_{mn} and Ω_{mn} are defined by

$$\begin{aligned}&\Xi_{mn} = \frac{\exp(\eta_m + \eta_n^*)}{k_m + k_n^*}, \quad \Omega_{mn} = \frac{\psi_m^\dagger \psi_n'}{k_m^* + k_n},\\ &\psi_n' = \begin{pmatrix} a & b^* \\ b & c \end{pmatrix} \psi_n, \quad m,n = 1,2,3,4,\end{aligned} \tag{9}$$

where symbol † represents the transpose conjugate. It has been proved that the Gram determinant forms $g^{(s)}$ and f satisfy the bilinear Eq. (3) in Ref. [17], and the expressions are as follows:

$$g^{(s)} = \begin{vmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 1 & 0 & 0 & 0 & \exp(\eta_1) \\ \Xi_{21} & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 1 & 0 & 0 & \exp(\eta_2) \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & \Xi_{34} & 0 & 0 & 1 & 0 & \exp(\eta_3) \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & 0 & 0 & 0 & 1 & \exp(\eta_4) \\ -1 & 0 & 0 & 0 & \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 \\ 0 & -1 & 0 & 0 & \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 \\ 0 & 0 & -1 & 0 & \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} & 0 \\ 0 & 0 & 0 & -1 & \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} & 0 \\ 0 & 0 & 0 & 0 & -\varsigma_1^{(s)} & -\varsigma_2^{(s)} & -\varsigma_3^{(s)} & -\varsigma_4^{(s)} & 0 \end{vmatrix}, \tag{10}$$

$$f = \begin{vmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 1 & 0 & 0 & 0 \\ \Xi_{21} & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 1 & 0 & 0 \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & \Xi_{34} & 0 & 0 & 1 & 0 \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ 0 & -1 & 0 & 0 & \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ 0 & 0 & -1 & 0 & \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ 0 & 0 & 0 & -1 & \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{vmatrix}, \tag{11}$$

where

$$\Omega_{mn} = \frac{a\varsigma_n^{(1)}\varsigma_m^{(1)*} + c\varsigma_n^{(2)}\varsigma_m^{(2)*} + b\varsigma_n^{(1)}\varsigma_m^{(2)*} + b^*\varsigma_n^{(1)*}\varsigma_m^{(2)}}{(k_n + k_m^*)}.$$

Substituting expressions (10) and (11) into Eq. (7) and simplifying the resulting expression, we can obtain the four-soliton solution of Eq. (1) as shown in Appendix A.

Because of the large number and complexity of parameters in the expression p, q, D, they are not presented here. The pulsed collisions of four-soliton solutions are numerically studied in the following section.

3. Collision behavior between solitons

Lots of computed results show that the collision behavior between solitons of four-soliton solution is mainly related to the values of parameters k_1, k_2, k_3, k_4, ς_1^s, ς_2^s, ς_3^s, ς_4^s ($s = 1, 2$). And here we only present two major cases.

3.1. k_i ($i = 1, 2, 3, 4$) being all complex

When k_1, k_2, k_3, and k_4 are all complex, the other parameters are $a = 2$, $b = 1 + 2\mathrm{i}$, $c = 2$ and the following condition is satisfied:

$$\frac{\varsigma_1^1}{\varsigma_1^2} = \frac{\varsigma_2^1}{\varsigma_2^2} = \frac{\varsigma_3^1}{\varsigma_3^2} = \frac{\varsigma_4^1}{\varsigma_4^2} = 1. \tag{12}$$

The collision dynamics between solitons are elastic and shown in Figs. 1 and 2. We can see that the pulse amplitudes do not change during the collision in components $|p|$ and $|q|$. But it is clear that the outboard two solitons' widths are smaller than those of the inboard solitons. Lots of numerical simulation results show that k_1 and k_2 mainly affect the two inboard solitons and k_3, k_4 mainly affect the two outboard solitons, while the pulses' velocity and width mainly depend on the imaginary part of k_i ($i = 1, 2, 3, 4$) and the pulse amplitude mainly depends on the real part of k_i. The pulse amplitudes are all equal to 0.4 in Fig. 1. But in Fig. 2, the inboard pulse amplitudes are 0.8 and the outboard pulse amplitudes are 0.2. With decreasing of the imaginary part of k_1 and k_2 and the increasing of the imaginary part of k_3 and k_4, the velocities of inboard two solitons decrease and the widths of outboard solitons become smaller as shown in Fig. 2.

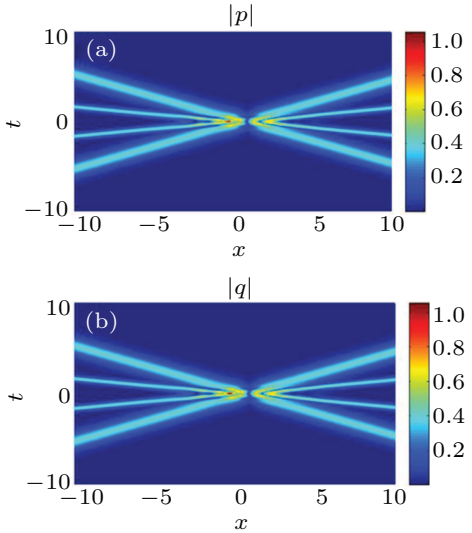


Fig. 1. Contour plots of elastic interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1 + \mathrm{i}$, $k_2 = 1 - \mathrm{i}$, $k_3 = 1 + 3\mathrm{i}$, and $k_4 = 1 - 3\mathrm{i}$.

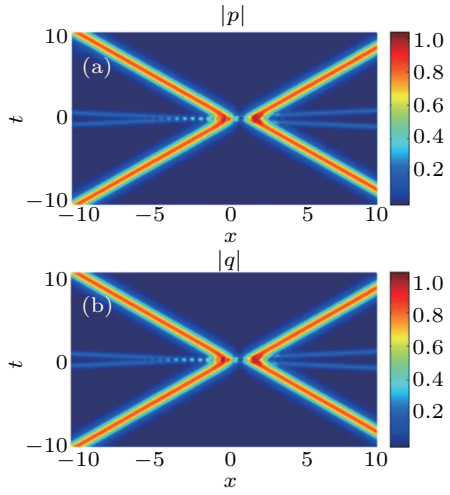


Fig. 2. Contour plots of elastic interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 2 + 0.5\mathrm{i}$, $k_2 = 2 - 0.5\mathrm{i}$, $k_3 = 0.5 + 6\mathrm{i}$, and $k_4 = 0.5 - 6\mathrm{i}$.

When the condition (12) is not satisfied, and

$$\varsigma_1^1 = \varsigma_2^1 = \varsigma_1^2 = \varsigma_2^2 = 0, \quad \varsigma_3^1 = \varsigma_4^1 = \varsigma_3^2 = \varsigma_4^2, \tag{13}$$

$$\varsigma_3^1 = \varsigma_4^1 = \varsigma_3^2 = \varsigma_4^2 = 0, \quad \varsigma_1^1 = \varsigma_2^1 = \varsigma_1^2 = \varsigma_2^2, \tag{14}$$

the values of k_i ($i = 1, 2, 3, 4$) are the same as those in Fig. 1, ς_3^1, ς_4^1, ς_3^2, ς_4^2 can also be real or complex. Under the condition (13) or (14), the interactions among the solitons are presented as two disappearing solitons and two quasi-elastic collision solitons as shown in Fig. 3. We can see that the two inboard solitons disappear and the two outboard solitons proceed quasi-elastic collision under condition (13). However the interaction among solitons under condition (14) is just opposite to that under condition (13), the two outboard solitons disappear and the two inboard solitons proceed quasi-elastic collision.

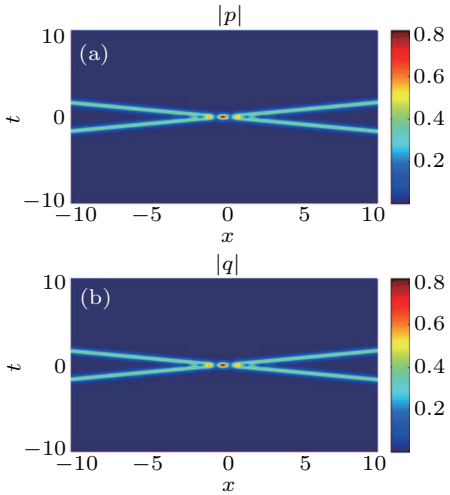


Fig. 3. Contour plots of quasi-elastic interactions among four solitons in components (a) $|p|$ and (b) $|q|$ for $\varsigma_1^1 = \varsigma_2^1 = \varsigma_1^2 = \varsigma_2^2 = 0$, $\varsigma_3^1 = \varsigma_4^1 = \varsigma_3^2 = \varsigma_4^2$.

Under the following condition:

$$\varsigma_3^1 = \varsigma_4^1 = \varsigma_1^2 = \varsigma_2^2 = 0, \quad \varsigma_1^1 = \varsigma_2^1 = \varsigma_3^2 = \varsigma_4^2 = 1, \tag{15}$$

$$\varsigma_1^1 = \varsigma_2^1 = \varsigma_3^2 = \varsigma_4^2 = 0, \quad \varsigma_3^1 = \varsigma_4^1 = \varsigma_1^2 = \varsigma_2^2 = 1, \tag{16}$$

soliton interactions are displayed in Fig. 4. In the $|p|$ component, the two outboard solitons disappear and two inboard solitons present quasi-elastic collision behavior. Unlike the $|p|$ component, the four solitons in the component $|q|$ present inelastic collision. The interaction between solitons under condition (16) is similar to that under condition (15).

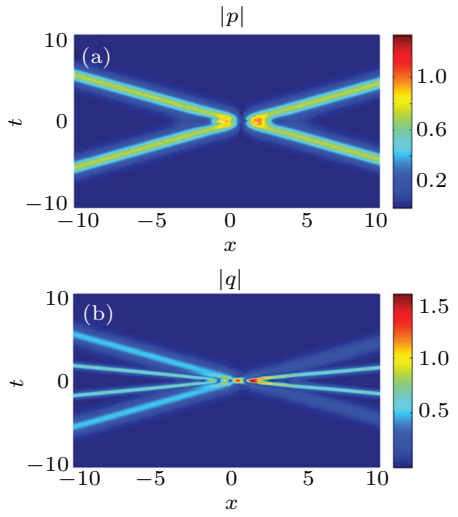


Fig. 4. Contour plots of interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for where $\varsigma_3^1 = \varsigma_4^1 = \varsigma_1^2 = \varsigma_2^2 = 0$, $\varsigma_1^1 = \varsigma_2^1 = \varsigma_3^2 = \varsigma_4^2 = 1$.

When the conditions (12)–(16) are all not satisfied and one or more of the parameters ς_i^s are complex, figure 5 illustrates the inelastic collision behavior of four solitons in component $|p|$ and $|q|$.

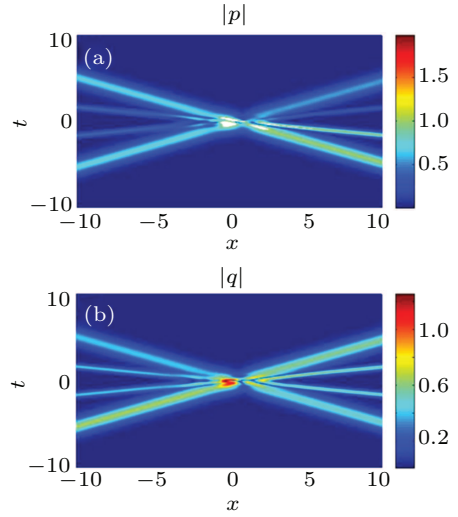


Fig. 5. Contour plots of inelastic interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $\varsigma_1^1 = 1+\mathrm{i}$, $\varsigma_2^1 = 1+2\mathrm{i}$, $\varsigma_3^1 = \varsigma_4^1 = \varsigma_1^2 = \varsigma_2^2 = \varsigma_3^2 = \varsigma_4^2 = 1$.

3.2. k_1, k_2 are real and k_3, k_4 are complex

In this situation, the two inboard solitons' velocities are zero. When condition (12) is satisfied, we can obtain that the two inboard zero velocity solitons integrated into one and the two outboard solitons present quasi-elastic collision as shown in Fig. 6. When k_1 and k_2 are not equal, an interesting quasi-breathing soliton can be obtained as shown in Fig. 7, two inboard solitons attract and repel each other periodically. And the breathing period becomes smaller with the increasing of $|k_1 - k_2|$ as shown in Fig. 8.

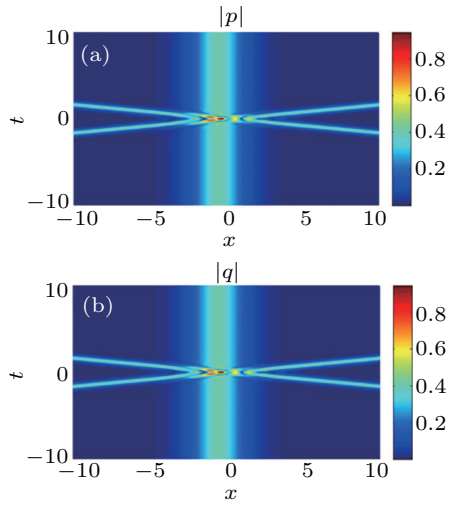


Fig. 6. Contour plots of interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 1$, $k_3 = 1+3\mathrm{i}$, and $k_4 = 1-3\mathrm{i}$.

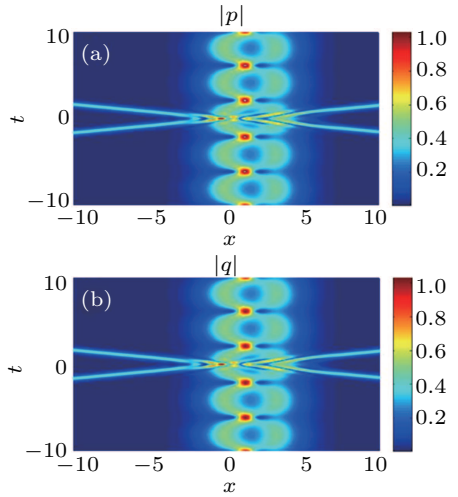


Fig. 7. Contour plots of interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 1.6$, $k_3 = 1+3\mathrm{i}$, and $k_4 = 1-3\mathrm{i}$.

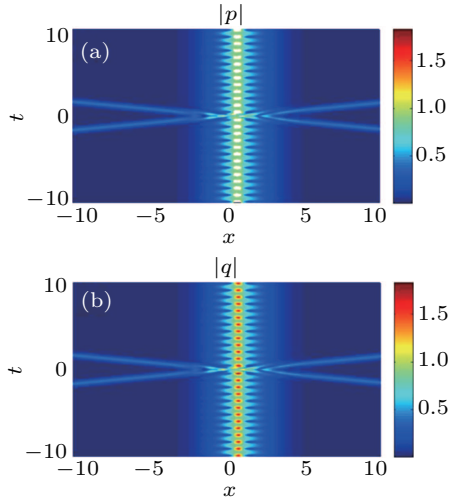


Fig. 8. Contour plots of interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 3$, $k_3 = 1+3\mathrm{i}$, and $k_4 = 1-3\mathrm{i}$.

When the condition (13) is satisfied and ς_i^s are all real, the interactions among solitons are exactly the same as those in Fig. 3. But under the condition (14), the outboard solitons disappear and the inboard solitons are similar to those in Fig. 7 as show in Fig. 9. The breathing period is also inversely proportional to $|k_1 - k_2|$, which is obvious by comparing Fig. 9 with Fig. 10. Under the conditions (15) and (16), the soliton collisions are different in component $|p|$ and $|q|$ as shown in Figs. 11 and 12. In Fig. 11, the outboard solitons disappear in component $|p|$ and the inboard solitons interact with eah other periodically. In component $|q|$, both the outboard and inboard solitons present quasi-elastic collision and the inboard solitons' amplitudes decrease obviously. In Fig. 12, the condition (16) is satisfied, the two inboard solitons disappear in the component $|p|$ and the two outboard solitons present quasi-elastic collision, while completely inelastic collision in component $|q|$.

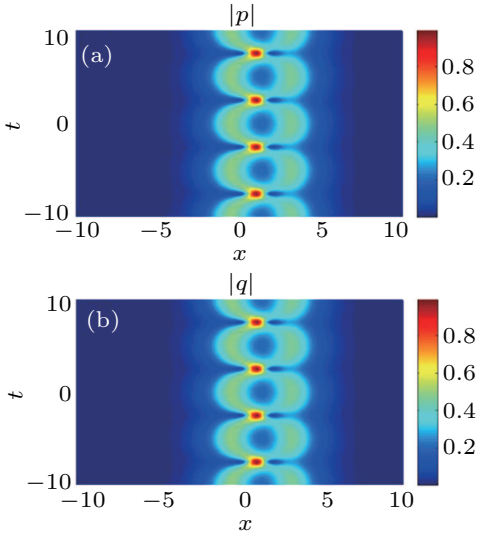


Fig. 9. Contour plots of quasi-elastic interactions among four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 1.5$, $k_3 = 1 + 3\mathrm{i}$, and $k_4 = 1 - 3\mathrm{i}$.

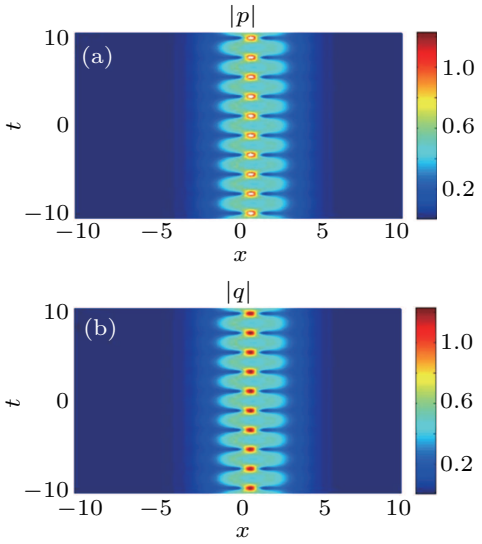


Fig. 10. Contour plots of quasi-elastic interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 2$, $k_3 = 1 + 3\mathrm{i}$, and $k_4 = 1 - 3\mathrm{i}$.

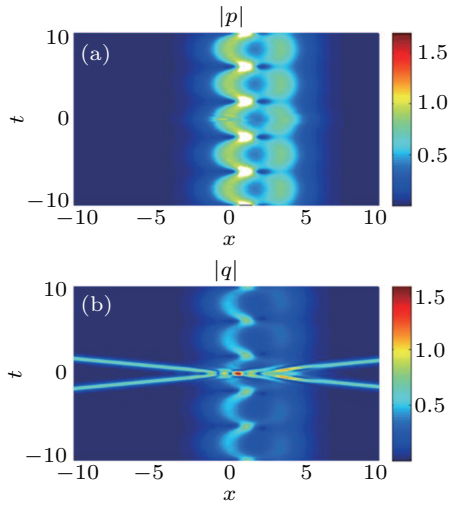


Fig. 11. Contour plots of interactions of four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 1.6$, $k_3 = 1 + 3\mathrm{i}$, and $k_4 = 1 - 3\mathrm{i}$.

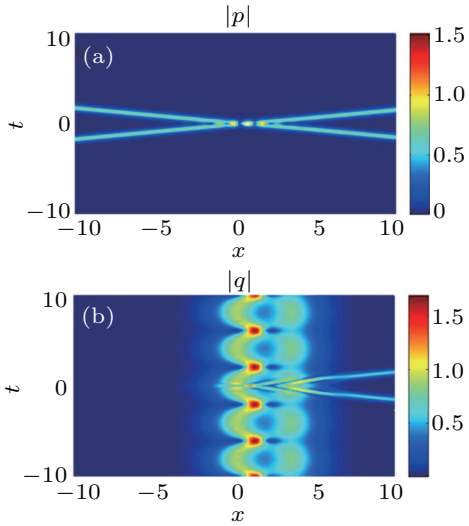


Fig. 12. Contour plots of interactions among four solitons in components (a) $|p|$ and (b) $|q|$ for $k_1 = 1$, $k_2 = 1.6$, $k_3 = 1 + 3\mathrm{i}$, and $k_4 = 1 - 3\mathrm{i}$.

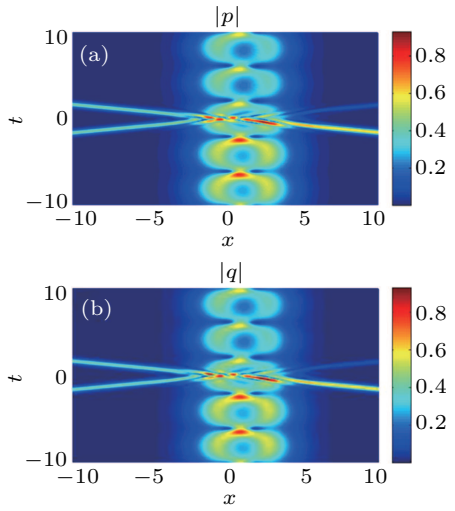


Fig. 13. Contour plots of interactions among four solitons in components (a) $|p|$ and (b) $|q|$ for $\varsigma_1^1 = \varsigma_2^1 = \varsigma_1^2 = \varsigma_2^2 = 1 + \mathrm{i}$, $\varsigma_3^1 = \varsigma_4^1 = \varsigma_3^2 = \varsigma_4^2 = 1$.

When conditions (12)–(16) are all not satisfied and one or more of the parameters ς_i^s ($s = 1, 2$, $i = 1, 2, 3, 4$)

are complex, we can obtain completely inelastic collisions in components $|p|$ and $|q|$. Especially, when the parameters are $\varsigma_1^1=\varsigma_2^1=\varsigma_1^2=\varsigma_2^2=1+\mathrm{i}$, $\varsigma_3^1=\varsigma_4^1=\varsigma_3^2=\varsigma_4^2=1$ or $\varsigma_1^1=\varsigma_2^1=\varsigma_1^2=\varsigma_2^2=1$, $\varsigma_3^1=\varsigma_4^1=\varsigma_3^2=\varsigma_4^2=1+\mathrm{i}$, the soliton interactions in two components are similar as shown in Fig. 13. Otherwise, the inelastic collision behaviors of the two components are different.

4. Conclusions

In this work, we obtain the exact four-soliton solution of the GCNLS Eq. (1) by the Hirota bilinear method and the interactions between solitons are discussed in detail. The numerical simulation results show that the interactions among four solitons depend on the solution parameters' values and we discuss only two major cases. The first case is that k_i ($i=1$, 2, 3, 4) are all complex. In this case, if ς_i^s ($s=1$, 2, $i=1$, 2, 3, 4) are all real, the elastic collisions occur among four solitons both in components $|p|$ and $|q|$ when the condition (12) is satisfied. While ς_i^s ($s=1$, 2, $i=1$, 2, 3, 4) are all real and meet conditions (13)–(16) or they are all complex, soliton disappears and inelastic collision occurs. The second case is that k_1 and k_2 are real while k_3 and k_4 are complex. There are two zero velocity solitons in this case and the interaction behaviors are similar to those in the first case when parameters ς_i^s ($s=1$, 2, $i=1$, 2, 3, 4) satisfy different conditions. Lots of numerical simulation results show that k_1 and k_2 mainly affect the two inboard solitons, and k_3 and k_4 the two outboard solitons, while the pulses' velocity and width mainly depend on the imaginary part of k_i ($i=1$, 2, 3, 4), the pulse amplitude the real part of k_i. Our results will be helpful in the understanding of design of all-optical switch, soliton control, fiber amplifier, *etc*., at the same time, they provide theoretical support for the experimental study of soliton excitation and control. These results have certain applications in all-optical communication system, nonlinear optics, optical conversion equipment, and other fields.

Appendix A

The four-soliton solution expressions can be expressed as follows:

$$
\begin{aligned}
p\times D=&\,\varsigma_1^{(1)}\exp(\eta_1)+\varsigma_2^{(1)}\exp(\eta_2)+\varsigma_3^{(1)}\exp(\eta_3)+\varsigma_4^{(1)}\exp(\eta_4)+\exp(\eta_1+\eta_1^*+\eta_2+\delta_{11})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\delta_{21})+\exp(\eta_1+\eta_1^*+\eta_4+\delta_{31})+\exp(\eta_2+\eta_2^*+\eta_1+\delta_{41})+\exp(\eta_2+\eta_2^*+\eta_3+\delta_{51})\\
&+\exp(\eta_2+\eta_2^*+\eta_4+\delta_{61})+\exp(\eta_3+\eta_3^*+\eta_1+\delta_{71})+\exp(\eta_3+\eta_3^*+\eta_2+\delta_{81})+\exp(\eta_3+\eta_3^*+\eta_4+\delta_{91})\\
&+\exp(\eta_4+\eta_4^*+\eta_1+\delta_{101})+\exp(\eta_4+\eta_4^*+\eta_2+\delta_{111})+\exp(\eta_4+\eta_4^*+\eta_3+\delta_{121})+\exp(\eta_1^*+\eta_2+\eta_3+\delta_{131})\\
&+\exp(\eta_1^*+\eta_2+\eta_4+\delta_{141})+\exp(\eta_1^*+\eta_3+\eta_4+\delta_{151})+\exp(\eta_2^*+\eta_1+\eta_3+\delta_{161})+\exp(\eta_2^*+\eta_1+\eta_4+\delta_{171})\\
&+\exp(\eta_2^*+\eta_3+\eta_4+\delta_{181})+\exp(\eta_3^*+\eta_1+\eta_2+\delta_{191})+\exp(\eta_3^*+\eta_1+\eta_4+\delta_{201})+\exp(\eta_3^*+\eta_2+\eta_4+\delta_{211})\\
&+\exp(\eta_4^*+\eta_1+\eta_2+\delta_{221})+\exp(\eta_4^*+\eta_1+\eta_3+\delta_{231})+\exp(\eta_4^*+\eta_2+\eta_3+\delta_{241})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\tau_{11})+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_4+\tau_{21})+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_2+\tau_{31})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\tau_{41})+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+\eta_2+\tau_{51})+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+\eta_3+\tau_{61})\\
&+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_1+\tau_{71})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\tau_{81})+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_1+\tau_{91})\\
&+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_3+\tau_{101})+\exp(\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_1+\tau_{111})+\exp(\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_2+\tau_{121})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_3^*+\eta_4+\tau_{131})+\exp(\eta_1+\eta_1^*+\eta_2+\eta_4^*+\eta_3+\tau_{141})+\exp(\eta_1+\eta_1^*+\eta_3+\eta_2^*+\eta_4+\tau_{151})\\
&+\exp(\eta_2+\eta_2^*+\eta_1+\eta_3^*+\eta_4+\tau_{161})+\exp(\eta_2+\eta_2^*+\eta_1+\eta_4^*+\eta_3+\tau_{171})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_4^*+\eta_1+\tau_{181})\\
&+\exp(\eta_3+\eta_3^*+\eta_1+\eta_2^*+\eta_4+\tau_{191})+\exp(\eta_3+\eta_3^*+\eta_1+\eta_4^*+\eta_2+\tau_{201})+\exp(\eta_3+\eta_3^*+\eta_2+\eta_1^*+\eta_4+\tau_{211})\\
&+\exp(\eta_4+\eta_4^*+\eta_2+\eta_1^*+\eta_3+\tau_{221})+\exp(\eta_4+\eta_4^*+\eta_1+\eta_2^*+\eta_3+\tau_{231})+\exp(\eta_4+\eta_4^*+\eta_1+\eta_3^*+\eta_2+\tau_{241})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\mu_{11})+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_3+\mu_{21})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_2+\mu_{31})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_1+\mu_{41}),
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
q\times D=&\,\varsigma_1^{(2)}\exp(\eta_1)+\varsigma_2^{(2)}\exp(\eta_2)+\varsigma_3^{(2)}\exp(\eta_3)+\varsigma_4^{(2)}\exp(\eta_4)+\exp(\eta_1+\eta_1^*+\eta_2+\delta_{12})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\delta_{22})+\exp(\eta_1+\eta_1^*+\eta_4+\delta_{32})+\exp(\eta_2+\eta_2^*+\eta_1+\delta_{42})+\exp(\eta_2+\eta_2^*+\eta_3+\delta_{52})\\
&+\exp(\eta_2+\eta_2^*+\eta_4+\delta_{62})+\exp(\eta_3+\eta_3^*+\eta_1+\delta_{72})+\exp(\eta_3+\eta_3^*+\eta_2+\delta_{82})+\exp(\eta_3+\eta_3^*+\eta_4+\delta_{92})\\
&+\exp(\eta_4+\eta_4^*+\eta_1+\delta_{102})+\exp(\eta_4+\eta_4^*+\eta_2+\delta_{112})+\exp(\eta_4+\eta_4^*+\eta_3+\delta_{122})+\exp(\eta_1^*+\eta_2+\eta_3+\delta_{132})\\
&+\exp(\eta_1^*+\eta_2+\eta_4+\delta_{142})+\exp(\eta_1^*+\eta_3+\eta_4+\delta_{152})+\exp(\eta_2^*+\eta_1+\eta_3+\delta_{162})+\exp(\eta_2^*+\eta_1+\eta_4+\delta_{172})\\
&+\exp(\eta_2^*+\eta_3+\eta_4+\delta_{182})+\exp(\eta_3^*+\eta_1+\eta_2+\delta_{192})+\exp(\eta_3^*+\eta_1+\eta_4+\delta_{202})+\exp(\eta_3^*+\eta_2+\eta_4+\delta_{212})
\end{aligned}
$$

$$
\begin{aligned}
&+\exp(\eta_4^*+\eta_1+\eta_2+\delta_{222})+\exp(\eta_4^*+\eta_1+\eta_3+\delta_{232})+\exp(\eta_4^*+\eta_2+\eta_3+\delta_{242})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\tau_{12})+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_4+\tau_{22})+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_2+\tau_{32})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\tau_{42})+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+\eta_2+\tau_{52})+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+\eta_3+\tau_{62})\\
&+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_1+\tau_{72})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\tau_{82})+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_1+\tau_{92})\\
&+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_3+\tau_{102})+\exp(\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_1+\tau_{112})+\exp(\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_2+\tau_{122})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_3^*+\eta_4+\tau_{132})+\exp(\eta_1+\eta_1^*+\eta_2+\eta_4^*+\eta_3+\tau_{142})+\exp(\eta_1+\eta_1^*+\eta_3+\eta_2^*+\eta_4+\tau_{152})\\
&+\exp(\eta_2+\eta_2^*+\eta_1+\eta_3^*+\eta_4+\tau_{162})+\exp(\eta_2+\eta_2^*+\eta_1+\eta_4^*+\eta_3+\tau_{172})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_4^*+\eta_1+\tau_{182})\\
&+\exp(\eta_3+\eta_3^*+\eta_1+\eta_2^*+\eta_4+\tau_{192})+\exp(\eta_3+\eta_3^*+\eta_1+\eta_4^*+\eta_2+\tau_{202})+\exp(\eta_3+\eta_3^*+\eta_2+\eta_1^*+\eta_4+\tau_{212})\\
&+\exp(\eta_4+\eta_4^*+\eta_2+\eta_1^*+\eta_3+\tau_{222})+\exp(\eta_4+\eta_4^*+\eta_1+\eta_2^*+\eta_3+\tau_{232})+\exp(\eta_4+\eta_4^*+\eta_1+\eta_3^*+\eta_2+\tau_{242})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\mu_{12})+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_3+\mu_{22})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_2+\mu_{32})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+\eta_1+\mu_{42}),
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
D=\;&1+\exp(\eta_1+\eta_1^*+R_1)+\exp(\eta_2+\eta_2^*+R_2)+\exp(\eta_3+\eta_3^*+R_3)+\exp(\eta_4+\eta_4^*+R_4)\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+R_5)+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+R_6)+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+R_7)\\
&+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+R_8)+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+R_9)+\exp(\eta_3+\eta_3^*+\eta_4+\eta_4^*+R_{10})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\eta_3^*+R_{11})+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+R_{12})\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+R_{13})+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+R_{14})\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\eta_4^*+R_{15})+\exp(\eta_1+\eta_2^*+\delta_{10})+\exp(\eta_1+\eta_3^*+\delta_{20})\\
&+\exp(\eta_1+\eta_4^*+\delta_{30})+\exp(\eta_2+\eta_1^*+\delta_{10}^*)+\exp(\eta_2+\eta_3^*+\delta_{40})+\exp(\eta_2+\eta_4^*+\delta_{50})\\
&+\exp(\eta_3+\eta_1^*+\delta_{20}^*)+\exp(\eta_3+\eta_2^*+\delta_{40}^*)+\exp(\eta_3+\eta_4^*+\delta_{60})+\exp(\eta_4+\eta_1^*+\delta_{30}^*)\\
&+\exp(\eta_4+\eta_2^*+\delta_{50}^*)+\exp(\eta_4+\eta_3^*+\delta_{60}^*)+\exp(\eta_1+\eta_1^*+\eta_2+\eta_3^*+\tau_1)+\exp(\eta_1+\eta_1^*+\eta_3+\eta_2^*+\tau_1^*)\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_4^*+\tau_2)+\exp(\eta_1+\eta_1^*+\eta_4+\eta_2^*+\tau_2^*)+\exp(\eta_1+\eta_1^*+\eta_3+\eta_4^*+\tau_3)\\
&+\exp(\eta_1+\eta_1^*+\eta_4+\eta_3^*+\tau_3^*)+\exp(\eta_2+\eta_2^*+\eta_1+\eta_3^*+\tau_4)+\exp(\eta_2+\eta_2^*+\eta_3+\eta_1^*+\tau_4^*)\\
&+\exp(\eta_2+\eta_2^*+\eta_1+\eta_4^*+\tau_5)+\exp(\eta_2+\eta_2^*+\eta_4+\eta_1^*+\tau_5^*)+\exp(\eta_2+\eta_2^*+\eta_3+\eta_4^*+\tau_6)\\
&+\exp(\eta_2+\eta_2^*+\eta_4+\eta_3^*+\tau_6^*)+\exp(\eta_3+\eta_3^*+\eta_1+\eta_2^*+\tau_7)+\exp(\eta_3+\eta_3^*+\eta_2+\eta_1^*+\tau_7^*)\\
&+\exp(\eta_3+\eta_3^*+\eta_1+\eta_4^*+\tau_8)+\exp(\eta_3+\eta_3^*+\eta_4+\eta_1^*+\tau_8^*)+\exp(\eta_3+\eta_3^*+\eta_2+\eta_4^*+\tau_9)\\
&+\exp(\eta_3+\eta_3^*+\eta_4+\eta_2^*+\tau_9^*)+\exp(\eta_4+\eta_4^*+\eta_1+\eta_2^*+\tau_{10})+\exp(\eta_4+\eta_4^*+\eta_2+\eta_1^*+\tau_{10}^*)\\
&+\exp(\eta_4+\eta_4^*+\eta_1+\eta_3^*+\tau_{11})+\exp(\eta_4+\eta_4^*+\eta_3+\eta_1^*+\tau_{11}^*)+\exp(\eta_4+\eta_4^*+\eta_2+\eta_3^*+\tau_{12})\\
&+\exp(\eta_4+\eta_4^*+\eta_3+\eta_2^*+\tau_{12}^*)+\exp(\eta_1+\eta_2^*+\eta_3+\eta_4^*+\tau_{13})+\exp(\eta_2+\eta_1^*+\eta_4+\eta_3^*+\tau_{13}^*)\\
&+\exp(\eta_1+\eta_2^*+\eta_4+\eta_3^*+\tau_{14})+\exp(\eta_2+\eta_1^*+\eta_3+\eta_4^*+\tau_{14}^*)+\exp(\eta_1+\eta_3^*+\eta_2+\eta_4^*+\tau_{15})\\
&+\exp(\eta_3+\eta_1^*+\eta_4+\eta_2^*+\tau_{15}^*)+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\eta_4^*+\mu_1)\\
&+\exp(\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_4+\eta_3^*+\mu_1^*)+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_2+\eta_4^*+\mu_2)\\
&+\exp(\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_4+\eta_2^*+\mu_2^*)+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+\eta_2+\eta_3^*+\mu_3)\\
&+\exp(\eta_1+\eta_1^*+\eta_4+\eta_4^*+\eta_3+\eta_2^*+\mu_3^*)+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_1+\eta_4^*+\mu_4)\\
&+\exp(\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_4+\eta_1^*+\mu_4^*)+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_1+\eta_3^*+\mu_5)\\
&+\exp(\eta_2+\eta_2^*+\eta_4+\eta_4^*+\eta_3+\eta_1^*+\mu_5^*)+\exp(\eta_4+\eta_4^*+\eta_3+\eta_3^*+\eta_1+\eta_2^*+\mu_6)\\
&+\exp(\eta_4+\eta_4^*+\eta_3+\eta_3^*+\eta_2+\eta_1^*+\mu_6^*).
\end{aligned}
\tag{A3}
$$

References

[1] Wright O C, Gregory F M and Mclaughlin K T R 1999 *Phys. Lett. A* **257** 170

[2] Sun J J and Ma S W 2015 *J. Math. Phys.* **56** 022110

[3] Belanger N and Belanger P A 1996 *Opt. Commun.* **124** 301

[4] Mofiz U A 2007 *Phys. Plasmas.* **14** 112906

[5] Chen Y J 1989 *J. Appl. Phys.* **66** 43

[6] Sun W R, Liu D Y and Xie X Y 2017 *Chaos* **27** 043114

[7] Tang X Y, Guo Y and Shukla P K 2011 *Eur. Phys. J. D* **61** 677

[8] Wang Z and Cui S B 2015 *J. Math. Phys.* **56** 021503

[9] Qi F H, Ju H M and Meng X H 2014 *Nonlinear Dyn.* **77** 1331

[10] Zhong X Q, Tang T T, Xiang A P and Cheng K 2011 *Opt. Commun.* **284** 4727

[11] Calegar F, Vozzi C and Gasilov S 2008 *Phys. Rev. Lett.* **100** 123006

[12] Chen D, Sun B and Wei Y 2010 *Laser Phys.* **20** 1733

[13] Song S S, Howard S S and Liu Z J 2006 *Appl. Phys. Lett.* **89** 041115

[14] Fischer B, Cronin-Golomb M, White J and Yariv A 1981 *Opt. Lett.* **6** 519
[15] Yajima T and Souma H 1978 *Phys. Rev. A* **17** 309
[16] Wang D S, Zhang D J and Yang J K 2010 *J. Math. Phys.* **51** 023510
[17] Vishnu P N and Senthilvelan M 2016 *Commun. Nonlinear Sci. Numer. Simulat.* **36** 366
[18] Lu X and Peng M S 2013 *Nonlinear Dyn.* **73** 405
[19] Hua S H and Mihalache D 2015 *J. Phys. A: Math. Theor.* **48** 215202
[20] Vijayajayanthi M, Kanna T and Lakshmanan M 2008 *Phys. Rev. A* **77** 013820
[21] Wang Y F, Tian B, Li M, Wang P and Wang M 2014 *Commun. Nonlinear Sci. Numer. Simulat.* **19** 1783
[22] Musammil N M, Porsezian K, Subha P A and Nithyanandan K 2017 *Chaos* **27** 023113
[23] Zhang H Q and Ma W X 2014 *Appl. Math. Comput.* **230** 509
[24] Agalarov A, Zhulego V and Gadzhimuradov T 2015 *Phys. Rev. E* **91** 042909
[25] Vishnu P N, Senthilvelan M and Lakshmanan M 2014 *Phys. Rev. E* **89** 062901
[26] Khare A and Saxena A 2015 *J. Math. Phys.* **56** 032104
[27] Du Z, Tian B, Chai H B, Sun Y and Zhao X H 2018 *Chaos* **109** 90
[28] Esen A, Sulaiman T A, Bulut H and Baskonus H M 2018 *Optik* **167** 150
[29] Song J Y, Hao H Q and Zhang X M 2018 *Appl. Math. Lett.* **78** 126
[30] Wang H and Li B 2011 *Chin. Phys. B* **20** 040203
[31] Zuo J M and Zhang Y M 2011 *Chin. Phys. B* **20** 010205
[32] Li M M, Hu C L, Wu J, Lai X J and Wang Y Y 2019 *Chin. Phys. B* **28** 120502