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Existence of spontaneous symmetry breaking in two-lane totally asymmetric simple exclusion processes with an intersection*

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We study two-lane totally asymmetric simple exclusion processes (TASEPs) with an intersection. Monte Carlo simulations show that only symmetric phases exist in the system. To verify the existence of asymmetric phases, we carry out a cluster mean-field analysis. Analytical results show that the densities of the two upstream segments of the intersection site are always equal, which indicates that the system is not in asymmetric phases. It demonstrates that the spontaneous symmetry breaking does not exist in the system. The density profiles and the boundaries of the symmetric phases are also investigated. We find that the cluster mean-field analysis shows better agreement with simulations than the simple mean-field analysis where the correlation of sites is ignored.

Keywords: spontaneous symmetry breaking, cluster mean-field analysis, simple mean-field analysis, totally asymmetric simple exclusion process (TASEP)

PACS: 05.70.Fh, 02.50.Ey, 05.60.Cd **DOI:** 10.1088/1674-1056/ab820e

1. Introduction

Asymmetric simple exclusion process (ASEP) has become a very prominent model in studies of non-equilibrium systems, because it plays the same role as Ising model in equilibrium systems.[1] The ASEP was first introduced in the description of kinetics of biopolymerization.[2] Despite its simplicity, the model has been extensively applied in descriptions of protein synthesis,[3] dense media polymer dynamics,[4] gel electrophoresis,[5] membrane channels diffusion,[6] traffic flow,[7–11] information flow,[12,13] and so on.

The totally asymmetric simple exclusion process (TASEP) is the simplest limit of ASEP. In one-dimensional TASEP, particles move in one direction and obey the hard-core exclusion principle. Each site of the model has two states, occupied by one particle or empty. However, many realistic situations are not in one dimension, for example, transport along parallel or intersected lanes is very common in vehicle traffic or intracellular transport. Therefore, studies of TASEP have been extended from one-lane to multi-lane.[14–18]

Spontaneous symmetry breaking (SSB) is one phenomenon observed in TASEP, and it has attracted many scientific researchers' interests in recent decades. This phenomenon was first observed in the 'bridge model'.[19] In this model, there are two species of particles hopping in the single lane and the hopping directions are opposite. It has been found that the asymmetric stationary states could exist under the symmetrical conditions for the two species of particles. However, it is unclear how the system can be transformed to the symmetry-breaking state.[20] Later, Pronina *et al*. investigated two-parallel-lane TASEPs.[21] There are also two species of particles in the model. The particles hop in opposite directions without changing lanes and obey the narrow entrance principle. Monte Carlo (MC) simulations showed that one asymmetric phase does not exist. However, the theoretical analysis indicated that the asymmetric phase exists. In the theoretical analysis, the simple mean-field (SMF) method was adopted, while the correlation of the two lanes was ignored. Tian *et al*. studied the system with the cluster mean-field (CMF) method. In the analysis, the correlation of two sites was firstly considered, then more sites of the two lanes were considered. Those theoretical analyses all indicated that the asymmetric phase does not exist.[22,23] However, the mechanism of the occurrence of SSB is still not well understood.[15]

Yuan *et al*.[18] investigated another two-lane TASEP. In Ref. [18], two lanes interact at a crossing site, and there are three different update rules of particles, which correspond to three models: A, B, and C. Model A can be used in simulation of motion of molecular motor, and models B and C can be used in simulation of vehicle traffic. In the models B and C, the asymmetric HD/HL phase exists, which has been verified by MC simulations and CMF analysis.[24,25] It means that the SSB does exist in both models. While in the model A, simulations show that there are only symmetric phases. It is still not known exactly whether the SSB (i.e., the asymmetric HD/HL phase) exists in the model. To clarify its existence, more careful investigations are needed.

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In Ref. [18], the models were analyzed by SMF. The asymmetric phases can not be investigated due to the ignorance of correlation. In the present paper, we carry out CMF analysis to study the asymmetric phase. In the CMF method, the correlation of sites is considered. When the correlation exists, the CMF analysis works better than the SMF analysis. We have considered the correlation of three and five sites in Refs. [24,25], respectively. It has been found that one boundary of asymmetric phase is determined by the difference of the upstream segment densities of the two lanes. In this paper, five sites are considered in the CMF analysis due to the update rules of particles. It is found that the densities of the two lanes are equal, which indicates that SSB does not exist in the system. The rest of this paper is organized as follows. In Section 2, the model is described in detail. In Section 3, the results of CMF analysis and MC simulations are discussed. In Section 4, conclusions are presented.

2. Model

Figure 1 shows a schematic view of the model. In the system, there are two intersected lanes. Lane 1 and lane 2 are in horizontal and vertical directions, respectively. Both lanes have L sites. Lane 1 is numbered from 1 to L, and lane 2 from $L+1$ to $2L$. Each site can be vacant or occupied by one particle. The updates of particles are random. The particles can enter lane 1 or lane 2 with a probability α, and they can leave lane 1 or lane 2 with a probability β. In the bulk (except site C), the particles can hop forward if the next site is empty.

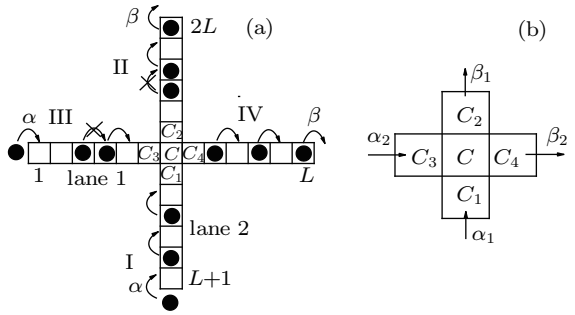


Fig. 1. (a) Schematic view of the model. The allowed moves are indicated by arrows and the prohibited hoppings are shown by crossed arrows. α and β correspond to the entrance and exit probabilities. (b) The illustration of cluster mean-field analysis. α_1 and α_2 (β_1 and β_2) denote effective injection (removal) probabilities.

In the model, if a particle of site C is chosen, the particle hops to site C_2 (C_4) if the site C_2 (C_4) is vacant and site C_4 (C_2) is occupied. When the sites C_2 and C_4 are both vacant, the particle hops to site C_2 or C_4 with an equal probability 0.5.

3. Analytical and simulation results

In Ref. [18], it has been found that there are three symmetric phases in the system, see Fig. 2. In these symmetric phases, the two lanes are in the same states and their densities are the same. When $\alpha < \beta$ and $\alpha < \lambda_1$, the two lanes are in low densities (LD) and the system is in a symmetric LL phase. The densities of the two lanes are equal to α. When $\beta < \alpha$ and $\beta < \lambda_1$, the two lanes are in high densities (HD) and the system is in a symmetric HH phase. The densities of the two lanes are equal to $1-\beta$. When $\alpha > \lambda_1$ and $\beta > \lambda_1$, the system is in symmetric HL phase. The downstream segments of the intersection of the lanes are in low densities and the upstream segments are in high densities. It is found that $\lambda_1 \approx 0.428$. Figure 3 shows the density profiles of the three phases.

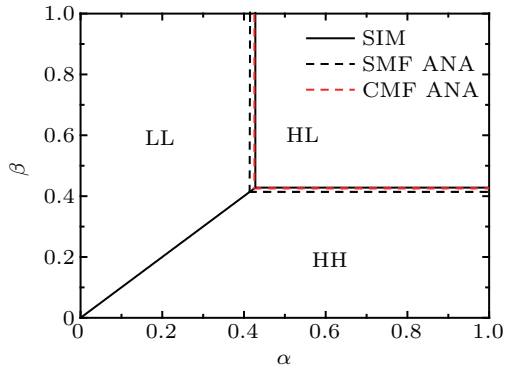


Fig. 2. Phase diagram of two-lane TASEPs with an intersection. Solid lines are obtained from MC simulations. Black dashed lines represent the results from SMF analysis. Red dashed lines correspond to the results from CMF analysis.

The boundaries of these phases have been obtained through analysis of the symmetric HL phase. In the analysis, the SMF method was adopted. The system was divided into four segments (i.e., I, II, III, and IV), and each segment was regarded as single-lane TASEP, see Fig. 1(a). It has been found that when

$$\alpha > \sqrt{2}-1, \quad \beta > \sqrt{2}-1 \tag{1}$$

are satisfied, the system is in a symmetric HL phase.

The SMF analytical results are shown in Fig. 2. We can see that the deviation between the analytical and simulation results exists. This is because of the ignorance of the correlation of sites. In other two models of Ref. [18], both MC simulations and theoretical analysis indicated that the SSB phenomenon does exist. When SSB occurs, the two lanes are in HD and HL phases, respectively, and their densities are not equal. However, the existence of the asymmetric phase of the present model can not be verified by the SMF analysis. Motivated by this, the CMF analysis is adopted to investigate the existence.

In the analysis, five sites (i.e., sites C, C_1, C_2, C_3, and C_4) are considered due to the update rule of the model, see Fig. 1(b). $Q_{\tau_1\tau_2\tau_3\tau_4\tau_5}$ is defined as the probability of site C_3 in state τ_1, site C in state τ_2, site C_4 in state τ_3, site C_1 in state τ_4, and site C_2 in state τ_5. Sites C_1, C_2, C_3, and C_4 have two states: empty or occupied, and τ_1, τ_3, τ_4, τ_5 can be 1 and 0 (1 represents occupation, 0 represents empty). Site C has three states,

and τ_2 can be 1, 2, and 0 (1 represents the state occupied by a particle moving in lane 1, 2 represents the state occupied by a particle moving in lane 2, and 0 represents the state of empty). The effective entrance rates into sites C_1 and C_3 are denoted by α_1 and α_2, and the effective exiting rates of particles from sites C_2 and C_4 are denoted by β_1 and β_2.

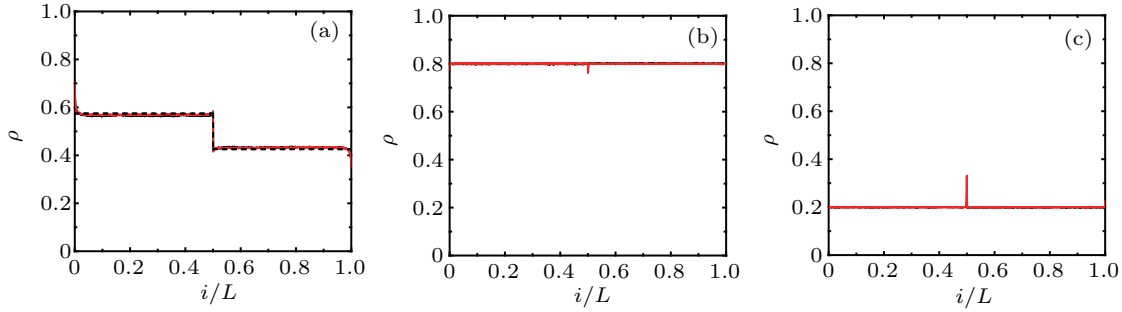


Fig. 3. Density profiles of the three symmetric phases. The results from MC simulations are shown by solid lines and those from CMF analysis are shown by dashed lines. (a) Symmetric HL phase with $\alpha = 0.8$, $\beta = 0.7$, (b) Symmetric HH phase with $\alpha = 0.7$, $\beta = 0.2$. (c) Symmetric LL phase with $\alpha = 0.2$, $\beta = 0.7$.

We first assume that the asymmetric phase exists. Then equations of $Q_{\tau_1\tau_2\tau_3\tau_4\tau_5}$ can be obtained. Take Q_{00000} for example,

$$\frac{\mathrm{d}Q_{00000}}{\mathrm{d}t} = -(\alpha_1+\alpha_2)Q_{00000} + \beta_1 Q_{00001} + \beta_2 Q_{00100}. \quad (2)$$

When the system reaches a stationary state, $\mathrm{d}Q_{\tau_1\tau_2\tau_3\tau_4\tau_5}/\mathrm{d}t = 0$ is satisfied. Then equation (2) can be simplified into

$$-(\alpha_1+\alpha_2)Q_{00000} + \beta_1 Q_{00001} + \beta_2 Q_{00100} = 0. \quad (3)$$

Other equations are shown in Appendix A. We can obtain 48 equations, but 47 of them are independent. In addition, for the conversation of these 48 probabilities, we have

$$\begin{aligned}
&Q_{12111}+Q_{12110}+Q_{12101}+Q_{12100}+Q_{12011}\\
&+Q_{12010}+Q_{12001}+Q_{12000}+Q_{11111}+Q_{11110}\\
&+Q_{11101}+Q_{11100}+Q_{11011}+Q_{11010}+Q_{11001}\\
&+Q_{11000}+Q_{10111}+Q_{10110}+Q_{10101}+Q_{10100}\\
&+Q_{10011}+Q_{10010}+Q_{10001}+Q_{10000}+Q_{02111}\\
&+Q_{02110}+Q_{02101}+Q_{02100}+Q_{02011}+Q_{02010}\\
&+Q_{02001}+Q_{02000}+Q_{01111}+Q_{01110}+Q_{01101}\\
&+Q_{01100}+Q_{01011}+Q_{01010}+Q_{01001}+Q_{01000}\\
&+Q_{00111}+Q_{00110}+Q_{00101}+Q_{00100}+Q_{00011}\\
&+Q_{00010}+Q_{00001}+Q_{00000} = 1. \quad (4)
\end{aligned}$$

The densities of the upstream segments of the intersection of lanes 1 and 2 can be expressed as

$$\begin{aligned}
\rho_1 &= Q_{12111}+Q_{12110}+Q_{12101}+Q_{12100}+Q_{12011}\\
&\quad +Q_{12010}+Q_{12001}+Q_{12000}+Q_{11111}+Q_{11110}\\
&\quad +Q_{11101}+Q_{11100}+Q_{11011}+Q_{11010}+Q_{11001}\\
&\quad +Q_{11000}+Q_{10111}+Q_{10110}+Q_{10101}+Q_{10100}\\
&\quad +Q_{10011}+Q_{10010}+Q_{10001}Q_{10000}, \quad (5)\\
\rho_2 &= Q_{00010}+Q_{00011}+Q_{00110}+Q_{00111}+Q_{01010}\\
&\quad +Q_{01011}+Q_{01110}+Q_{01111}+Q_{02010}+Q_{02011}\\
&\quad +Q_{02110}+Q_{02111}+Q_{10010}+Q_{10011}+Q_{10110}\\
&\quad +Q_{10111}+Q_{11010}+Q_{11011}+Q_{11110}+Q_{11111}\\
&\quad +Q_{12010}+Q_{12011}+Q_{12110}+Q_{12111}. \quad (6)
\end{aligned}$$

Furthermore, by ignoring the correlations, the flow rates of the upstream segments of lanes 1 and 2 can be given by

$$J_1 = \rho_1(1-\rho_1), \quad (7)$$
$$J_2 = \rho_2(1-\rho_2), \quad (8)$$

which can also be expressed as

$$J_1 = \alpha_2(1-\rho_1), \quad (9)$$
$$J_2 = \alpha_1(1-\rho_2). \quad (10)$$

On the other hand, the flow rates of the downstream segments of lanes 1 and 2 can be calculated by

$$\begin{aligned}
J_3 &= \rho_1(1-\rho_1) - 0.5(Q_{01100}+Q_{01110}+Q_{11100}+Q_{11110})\\
&\quad +0.5(Q_{02001}+Q_{02011}+Q_{12001}+Q_{12011}), \quad (11)\\
J_4 &= \rho_2(1-\rho_2) - 0.5(Q_{02001}+Q_{02011}+Q_{12001}+Q_{12011})\\
&\quad +0.5(Q_{01100}+Q_{01110}+Q_{11100}+Q_{11110}). \quad (12)
\end{aligned}$$

Without loss of generality, when the system is asymmetric, it is assumed that lanes 1 and 2 are in the HD and HL phases, respectively. Then J_3 satisfies

$$J_3 = \beta(1-\beta). \quad (13)$$

In lane 2, the upstream and downstream segments are in high and low densities. J_4 can be expressed by

$$\begin{aligned}
J_4 &= \beta_1(Q_{00001}+Q_{00011}+Q_{00101}+Q_{00111}+Q_{01001}\\
&\quad +Q_{01011}+Q_{01101}+Q_{01111}+Q_{02001}+Q_{02011}+Q_{02101}\\
&\quad +Q_{02111}+Q_{10001}+Q_{10011}+Q_{10101}+Q_{10111}+Q_{11001}\\
&\quad +Q_{11011}+Q_{11101}+Q_{11111}+Q_{12001}+Q_{12011}\\
&\quad +Q_{12101}+Q_{12111}). \quad (14)
\end{aligned}$$

In the analysis, there are 58 unknown quantities, including α_1, β_1, α_2, β_2, J_1, J_2, J_3, J_4, ρ_1, ρ_2 and the 48 probabilities $Q_{\tau_1\tau_2\tau_3\tau_4\tau_5}$. There are also 58 equations, including Eqs. (3)–(14) and (A1)–(A46). These equations can be solved to obtain ρ_1 and ρ_2. The relationship between ρ_1 (ρ_2) and β is shown in Fig. 4.

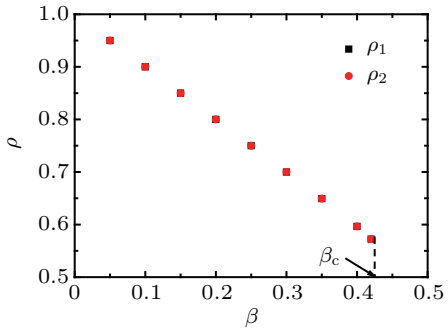


Fig. 4. The relationship between ρ_1 (ρ_2) and β. The scattered points are from the CMF analysis, β_c represents the value of the top boundary of the symmetric HH phase.

It can be seen that ρ_1 is always equal to ρ_2 with the change of β, which violates the assumed asymmetric phase. In addition, the results of MC simulations do not change with the system size, which means that the phases of the present model are also symmetric. From both CMF analysis and MC simulations, we demonstrate that the asymmetric phase does not exist in the system, which indicates that the SSB does not exist in the system.

The boundaries between these symmetric phases can also be analyzed. In the symmetric phases, due to the symmetry, $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $J_1 = J_2$, $J_3 = J_4$, $\rho_1 = \rho_2$. We have 53 unknown quantities including α_1 (α_2), β_1 (β_2), J_1 (J_2), J_3 (J_4), ρ_1 (ρ_2) and the 48 probabilities $Q_{\tau_1\tau_2\tau_3\tau_4\tau_5}$. We also have 53 equations including Eqs. (3)–(5), (7), (9), (12), (14), and (A1)–(A46). These equations can be solved, and the boundary between the symmetric HL and HH pahses can be determined by

$$\beta_c(1-\beta_c) = J_4. \tag{15}$$

The boundary of the symmetric HL and LL phases can also be obtained due to the symmetry in the phase diagram, see Fig. 2. It can be seen that the results from CMF analysis and simulations are in excellent agreement, and the CMF analysis shows better agreement with the MC simulations than the SMF analysis. In addition, the density profiles of the symmetric HL phase can also be analyzed. From Fig. 3(a), we can see that the CMF analytical results are also in excellent agreement with the simulations.

4. Conclusion

In this work, we have investigated two-lane TASEPs with an intersection under open boundaries. The random update is adopted. In the model, particles can change moving lanes with a probability 0.5 at the intersection site. The MC simulations show that the phases of the system are symmetric. To verify the existence of asymmetric phase, theoretical analysis is needed.

The system has been investigated by the SMF analysis. However, due to the ignorance of the correlation of sites, the asymmetric phase cannot be analyzed. Motivated by this, the CMF analysis is adopted to investigate the existence of the asymmetric phase, and five sites including the intersection site and four sites next nearest to it are considered. It is firstly assumed that the asymmetric phase exists. Through theoretical analysis, it is found that the densities of the two upstream segments of the intersection are always equal. It means that the two segments are symmetric, which violates the assumption that the two lanes are asymmetric. The MC simulations also indicate that the phases of the present model are symmetric. Therefore, it demonstrates that the asymmetric phase (spontaneous symmetry breaking) does not exist in the system.

The boundaries of the symmetric phases can also be analyzed. It has been found that the CMF analytical results are in excellent agreement with the simulations. By considering the correlation, the CMF analysis shows better agreement with the MC simulations than the SMF analysis where the correlation is ignored. We also investigate the density profiles when the system is in a symmetric HL phase, and it has been shown that both the CMF analytical and simulation results are also in excellent agreement.

Appendix A

$$\frac{\mathrm{d}Q_{00001}}{\mathrm{d}t} = -(\alpha_1+\alpha_2+\beta_1)Q_{00001} + \beta_2 Q_{00101} + 0.5Q_{01000} + 0.5Q_{02000} = 0, \tag{A1}$$

$$\frac{\mathrm{d}Q_{00010}}{\mathrm{d}t} = \alpha_1 Q_{00000} - (1+\alpha_2)Q_{00010} + \beta_1 Q_{00011} + \beta_2 Q_{00110} = 0, \tag{A2}$$

$$\frac{\mathrm{d}Q_{00011}}{\mathrm{d}t} = \alpha_1 Q_{00001} - (1+\alpha_2+\beta_1)Q_{00011} + \beta_2 Q_{00111} + 0.5Q_{01010} + 0.5Q_{02010} = 0, \tag{A3}$$

$$\frac{\mathrm{d}Q_{00100}}{\mathrm{d}t} = -(\alpha_1+\alpha_2+\beta_2)Q_{00100} + \beta_1 Q_{00101} + 0.5Q_{01000} + 0.5Q_{02000} = 0, \tag{A4}$$

$$\frac{\mathrm{d}Q_{00101}}{\mathrm{d}t} = -(\alpha_1+\alpha_2+\beta_1+\beta_2)Q_{00101} + Q_{01001} + Q_{01100} + Q_{02001} + Q_{02100} = 0, \tag{A5}$$

$$\frac{\mathrm{d}Q_{00110}}{\mathrm{d}t} = \alpha_1 Q_{00100} - (1+\alpha_2+\beta_2)Q_{00110} + \beta_1 Q_{00111} + 0.5Q_{01010} + 0.5Q_{02010} = 0, \tag{A6}$$

$$\frac{\mathrm{d}Q_{00111}}{\mathrm{d}t} = \alpha_1 Q_{00101} - (1+\alpha_2+\beta_1+\beta_2)Q_{00111} + Q_{01011}$$

$$+Q_{01110}+Q_{02011}+Q_{02110}=0, \tag{A7}$$

$$\frac{\mathrm{d}Q_{01000}}{\mathrm{d}t}=-(1+\alpha_1+\alpha_2)Q_{01000}+\beta_1Q_{01001}+\beta_2Q_{01100}+Q_{10000}=0, \tag{A8}$$

$$\frac{\mathrm{d}Q_{01001}}{\mathrm{d}t}=-(1+\alpha_1+\alpha_2+\beta_1)Q_{01001}+\beta_2Q_{01101}+Q_{10001}=0, \tag{A9}$$

$$\frac{\mathrm{d}Q_{01010}}{\mathrm{d}t}=\alpha_1Q_{01000}-(1+\alpha_2)Q_{01010}+\beta_1Q_{01011}+\beta_2Q_{01110}+Q_{10010}=0, \tag{A10}$$

$$\frac{\mathrm{d}Q_{01011}}{\mathrm{d}t}=\alpha_1Q_{01001}-(1+\alpha_2+\beta_1)Q_{01011}+\beta_2Q_{01111}+Q_{10011}=0, \tag{A11}$$

$$\frac{\mathrm{d}Q_{01100}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_2+1)Q_{01100}+\beta_1Q_{01101}+Q_{10100}=0, \tag{A12}$$

$$\frac{\mathrm{d}Q_{01101}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_1+\beta_2)Q_{01101}+Q_{10101}=0, \tag{A13}$$

$$\frac{\mathrm{d}Q_{01110}}{\mathrm{d}t}=\alpha_1Q_{01100}-(\alpha_2+\beta_2+1)Q_{01110}+\beta_1Q_{01111}+Q_{10110}=0, \tag{A14}$$

$$\frac{\mathrm{d}Q_{01111}}{\mathrm{d}t}=\alpha_1Q_{01101}-(\alpha_2+\beta_1+\beta_2)Q_{01111}+Q_{10111}=0, \tag{A15}$$

$$\frac{\mathrm{d}Q_{02000}}{\mathrm{d}t}=Q_{00010}-(1+\alpha_1+\alpha_2)Q_{02000}+\beta_1Q_{02001}+\beta_2Q_{02100}=0, \tag{A16}$$

$$\frac{\mathrm{d}Q_{02001}}{\mathrm{d}t}=Q_{00011}-(\alpha_1+\alpha_2+\beta_1+1)Q_{02001}+\beta_2Q_{02101}=0, \tag{A17}$$

$$\frac{\mathrm{d}Q_{02010}}{\mathrm{d}t}=\alpha_1Q_{02000}-(1+\alpha_2)Q_{02010}+\beta_1Q_{02011}+\beta_2Q_{02110}=0, \tag{A18}$$

$$\frac{\mathrm{d}Q_{02011}}{\mathrm{d}t}=\alpha_1Q_{02001}-(\alpha_2+\beta_1+1)Q_{02011}+\beta_2Q_{02111}=0, \tag{A19}$$

$$\frac{\mathrm{d}Q_{02100}}{\mathrm{d}t}=Q_{00110}-(1+\alpha_1+\alpha_2+\beta_2)Q_{02100}+\beta_1Q_{02101}=0, \tag{A20}$$

$$\frac{\mathrm{d}Q_{02101}}{\mathrm{d}t}=Q_{00111}-(\alpha_1+\alpha_2+\beta_1+\beta_2)Q_{02101}=0, \tag{A21}$$

$$\frac{\mathrm{d}Q_{02110}}{\mathrm{d}t}=\alpha_1Q_{02100}-(1+\alpha_2+\beta_2)Q_{02110}+\beta_1Q_{02111}=0, \tag{A22}$$

$$\frac{\mathrm{d}Q_{02111}}{\mathrm{d}t}=\alpha_1Q_{02101}-(\alpha_2+\beta_1+\beta_2)Q_{02111}=0, \tag{A23}$$

$$\frac{\mathrm{d}Q_{10000}}{\mathrm{d}t}=\alpha_2Q_{00000}-(1+\alpha_1)Q_{10000}+\beta_1Q_{10001}+\beta_2Q_{10100}=0, \tag{A24}$$

$$\frac{\mathrm{d}Q_{10001}}{\mathrm{d}t}=\alpha_2Q_{00001}-(1+\alpha_1+\beta_1)Q_{10001}+\beta_2Q_{10101}+0.5Q_{11000}+0.5Q_{12000}=0, \tag{A25}$$

$$\frac{\mathrm{d}Q_{10010}}{\mathrm{d}t}=\alpha_2Q_{00010}+\alpha_1Q_{10000}-2Q_{10010}+\beta_1Q_{10011}+\beta_2Q_{10110}=0, \tag{A26}$$

$$\frac{\mathrm{d}Q_{10011}}{\mathrm{d}t}=\alpha_2Q_{00011}+\alpha_1Q_{10001}-(2+\beta_1)Q_{10011}+\beta_2Q_{10111}+0.5Q_{11010}+0.5Q_{12010}=0, \tag{A27}$$

$$\frac{\mathrm{d}Q_{10100}}{\mathrm{d}t}=\alpha_2Q_{00100}-(1+\alpha_1+\beta_2)Q_{10100}+\beta_1Q_{10101}+0.5Q_{11000}+0.5Q_{12000}=0, \tag{A28}$$

$$\frac{\mathrm{d}Q_{10101}}{\mathrm{d}t}=\alpha_2Q_{00101}-(1+\alpha_1+\beta_1+\beta_2)Q_{10101}+Q_{11001}+Q_{11100}+Q_{12001}+Q_{12100}=0, \tag{A29}$$

$$\frac{\mathrm{d}Q_{10110}}{\mathrm{d}t}=\alpha_2Q_{00110}+\alpha_1Q_{10100}-(2+\beta_2)Q_{10110}+\beta_1Q_{10111}+0.5Q_{11010}+0.5Q_{12010}=0, \tag{A30}$$

$$\frac{\mathrm{d}Q_{10111}}{\mathrm{d}t}=\alpha_2Q_{00111}+\alpha_1Q_{10101}-(2+\beta_2+\beta_1)Q_{10111}+Q_{11011}+Q_{11110}+Q_{12011}+Q_{12110}=0, \tag{A31}$$

$$\frac{\mathrm{d}Q_{11000}}{\mathrm{d}t}=\alpha_2Q_{01000}-(1+\alpha_1)Q_{11000}+\beta_1Q_{11001}+\beta_2Q_{11100}=0, \tag{A32}$$

$$\frac{\mathrm{d}Q_{11001}}{\mathrm{d}t}=\alpha_2Q_{01001}-(1+\alpha_1+\beta_1)Q_{11001}+\beta_2Q_{11101}=0, \tag{A33}$$

$$\frac{\mathrm{d}Q_{11010}}{\mathrm{d}t}=\alpha_2Q_{01010}+\alpha_1Q_{11000}-Q_{11010}+\beta_1Q_{11011}+\beta_2Q_{11110}=0, \tag{A34}$$

$$\frac{\mathrm{d}Q_{11011}}{\mathrm{d}t}=\alpha_2Q_{01011}+\alpha_1Q_{11001}-(1+\beta_1)Q_{11011}+\beta_2Q_{11111}=0, \tag{A35}$$

$$\frac{\mathrm{d}Q_{11100}}{\mathrm{d}t}=\alpha_2Q_{01100}-(\alpha_1+\beta_2+1)Q_{11100}+\beta_1Q_{11101}=0, \tag{A36}$$

$$\frac{\mathrm{d}Q_{11101}}{\mathrm{d}t}=\alpha_2Q_{01101}-(\alpha_1+\beta_1+\beta_2)Q_{11101}=0, \tag{A37}$$

$$\frac{\mathrm{d}Q_{11110}}{\mathrm{d}t}=\alpha_2Q_{01110}+\alpha_1Q_{11100}-(\beta_2+1)Q_{11110}+\beta_1Q_{11111}=0, \tag{A38}$$

$$\frac{\mathrm{d}Q_{11111}}{\mathrm{d}t}=\alpha_2Q_{01111}+\alpha_1Q_{11101}-(\beta_1+\beta_2)Q_{11111}=0, \tag{A39}$$

$$\frac{\mathrm{d}Q_{12000}}{\mathrm{d}t}=\alpha_2Q_{02000}+Q_{10010}-(1+\alpha_1)Q_{12000}+\beta_1Q_{12001}+\beta_2Q_{12100}=0, \tag{A40}$$

$$\frac{\mathrm{d}Q_{12001}}{\mathrm{d}t}=\alpha_2Q_{02001}+Q_{10011}-(\alpha_1+\beta_1+1)Q_{12001}+\beta_2Q_{12101}=0, \tag{A41}$$

$$\frac{\mathrm{d}Q_{12010}}{\mathrm{d}t}=\alpha_2Q_{02010}+\alpha_1Q_{12000}-Q_{12010}+\beta_1Q_{12011}+\beta_2Q_{12110}=0, \tag{A42}$$

$$\frac{\mathrm{d}Q_{12011}}{\mathrm{d}t}=\alpha_2Q_{02011}+\alpha_1Q_{12001}-(\beta_1+1)Q_{12011}+\beta_2Q_{12111}=0, \tag{A43}$$

$$\frac{\mathrm{d}Q_{12100}}{\mathrm{d}t} = \alpha_2 Q_{02100} + Q_{10110} - (\alpha_1+\beta_2+1)Q_{12100} + \beta_1 Q_{12101} = 0, \tag{A44}$$

$$\frac{\mathrm{d}Q_{12101}}{\mathrm{d}t} = \alpha_2 Q_{02101} + Q_{10111} - (\alpha_1+\beta_1+\beta_2)Q_{12101} = 0, \tag{A45}$$

$$\frac{\mathrm{d}Q_{12110}}{\mathrm{d}t} = \alpha_2 Q_{02110} + \alpha_1 Q_{12100} - (1+\beta_2)Q_{12110} + \beta_1 Q_{12111} = 0, \tag{A46}$$

$$\frac{\mathrm{d}Q_{12111}}{\mathrm{d}t} = \alpha_2 Q_{02111} + \alpha_1 Q_{12101} - (\beta_1+\beta_2)Q_{12111} = 0. \tag{A47}$$

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