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Phase-modulated quadrature squeezing in two coupled cavities containing a two-level system*

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The phase-modulated quadrature squeezing in the system that composed of two coupled cavities interacting with a two-level atom is investigated. The variances of the amplitude and phase quadrature of the output field are calculated. It turns out that the squeezing behaviors of the output field can be obviously modified due to the phase difference of the coupling strengths between the atom and the two cavities. The squeezing in one quadrature (*i.e.*, phase quadrature) can be transferred into another (*i.e.*, amplitude quadrature), or the quadrature squeezing located at the low-frequency region can be transferred into the high-frequency region by modulating the relative phase of the coupling strengths. Furthermore, the effects of the decay mismatch between the two cavities and the coupling mismatch between the atom and the cavities on the quadrature squeezing have been discussed. The results show that both the decay mismatch and the coupling mismatch play a positive role in generating better quadrature squeezing.

Keywords: quadrature squeezing, cavity QED, phase modulation

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1. Introduction

Nonclassical light source is an essential element for implementation quantum information processing,[1] which is extensively applied to quantum simulations,[2] quantum networks,[3] quantum communications,[4] and linear optical quantum computations.[5] In recent years, the nonclassical properties of light, such as antibunching,[6] entanglement,[7] and squeezing,[8] have been widely studied in quantum optical systems. Squeezing,[9] which possesses variance of a quadrature amplitude or phase below that of a vacuum or coherent state, is regarded as one of the most peculiar nonclassical phenomena. Such a squeezing plays a crucial role in noiseless communications,[10] gravitational wave detectors,[11] and other highly precision measurements,[12,13] such as precision interferometry and atomic spectroscopy.

The generation of squeezed light was first realized in an atomic vapor based on the four-wave mixing process.[14] After then, a variety of schemes for generating squeezed state have been proposed,[15–28] including the optical parametric process,[15,16] optomechanics systems,[17–19] time-delayed coherent feedback,[20] periodically modulation,[21] and cavity quantum electrodynamics (QED).[6,22–24] The physics behind them is mainly related to the nonlinear optical process. For cavity QED system, a single atom in a suitable setup can work as an effective nonlinear-optical medium operating in the quantum regime. Quadrature squeezing can be observed by trapping a single atom into a weakly driven high-Q cavity. It is shown that the squeezing comes from a cavity-enhanced coherent interaction and an optimal squeezing can be obtained by adjusting the coupling strength.[29,30] However, in these proposals, only the amplitude of the coupling strength is considered, while the influence of the phase effect of the coupling strength on the squeezing properties has been ignored.

It is well known that the phase effect plays a crucial role in quantum mechanics. The physical observation of a quantum superposition state in measurement statistics is much different due to the phase difference between the amplitudes. The physical mechanisms of the phase-controlled quantum feature in an atom–photon coupling system have been investigated.[31–36] For example, the single-photon transmission and reflection spectra can be significantly influenced by modulating the phase differences of the coupling strengths between the atom and photons,[35] or between the external driving fields.[36] A single-photon router or switch can be realized based on such phase modulation, which can be used as a building block for various quantum information processing.

Recently, phase-dependent quantum statistical properties in a driven cavity QED system have also been reported.[37–39]

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They indicated that the phase difference of the coupling strengths between the atom and the cavity modes or the relative phase between the external driving fields plays an important role in the generation of quantum light. For instance, the photonic statistical properties of the light field from a cavity that contains two atoms are closely related to the atom's relative positions (which leads to the phase difference of the coupling strengths between the cavity mode and different atoms). The light field can be tuned from antibunched to bunched or superbunched by modifying the phase difference between the coupling strengths.[37] Similar results have also been shown by coupling two cavity modes to a two-level system[38] or a microresonator optomechanical system.[39] They found that photon antibunching can be optimally modulated by properly engineering the relative phase between the coupling strengths or between the driving fields, which is useful for generating single photon sources or nonclassical fields.

Motivated by these advances, in this paper, the phase-modulated quadrature squeezing of the output field in a system contains two coupled cavities interacting with a two-level atom is investigated. The results show that the squeezing properties depend strongly on the phase and an optimal quadrature squeezing can be obtained by carefully engineering the relative phase of the coupling strengths between the atom and the two cavity modes. The squeezing behaviors in the amplitude and phase quadrature, or located at the low-frequency region and high-frequency region can be transferred into each other by modulating the relative phase between the coupling strengths. Furthermore, this phase-modulated quadrature squeezing also related to the decay mismatch and the coupling mismatch. These mismatch can be applied to generate better quadrature squeezing.

This paper is organized as follows. In Section 2, we introduce the model and the basic theory to describe the squeezing properties of the system. In Section 3, we analyze how the phase difference of the coupling strengths between the atom and the two cavities affects the squeezing properties of the output field. Besides, the influences of the decay mismatch between the two cavities and the coupling mismatch between the atom and the two cavity modes on the phase-dependent quadrature squeezing are explored in detail. Finally, a conclusion is drawn in Section 4.

2. Model and basic theory

The model under consideration is constituted by a two-level atom and two coupled single mode cavities. An external coherent probe field is applied to drive the mode of cavity 1, as shown in Fig. 1. Under the rotation wave approximation, the Hamiltonian of the driven system can be written as

$$H = \frac{1}{2}\hbar\omega_{\rm a}\sigma_z + \sum_{j=1,2}\hbar\omega_{{\rm c},j}a_j^\dagger a_j + \hbar\left(g_1 a_1^\dagger\sigma_- + g_2 a_2^\dagger\sigma_- + Ja_1^\dagger a_2 + \varepsilon a_1^\dagger {\rm e}^{-{\rm i}\omega t} + {\rm H.c.}\right). \quad (1)$$

Here, $\omega_{\rm a}$ is the transition frequency of the two-level atom, $\omega_{{\rm c},j}$ is the resonance frequency of the j-th cavity, a_j and $a_j^\dagger$ are the annihilation and creation operators of the j-th cavity mode, respectively. $\sigma_- = |g\rangle\langle e|$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ are the atomic lowering operator and the z-component Pauli matrix, respectively. g_j is the coupling strength between the j-th cavity and the atom. In order to show the influence of phase difference between the coupling strengths on the squeezing properties, g_j is assumed be a complex number. J is the coupling strength between cavity 1 and cavity 2, which can be tuned by adjusting the distance of the two cavities. ε and ω are the amplitude and frequency of the input field $S_{\rm in}$, respectively.

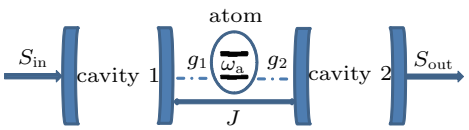


Fig. 1. Schematic diagram of a two-level atom coupled to two single mode cavities with coupling strengths g_1 and g_2, respectively. J denotes the coupling strength between cavity 1 and cavity 2, $\omega_{\rm a}$ denotes the transition frequency of the two-level atom, $S_{\rm in}$ and $S_{\rm out}$ represent the input and output of an external probe field, respectively.

In a frame rotating at the input frequency ω, the Hamiltonian in Eq. (1) takes the form

$$\tilde{H} = \frac{1}{2}\hbar\Delta_{\rm a}\sigma_z + \sum_{j=1,2}\hbar\Delta_{{\rm c},j}a_j^\dagger a_j + \hbar\left(g_1 a_1^\dagger\sigma_- + g_2 a_2^\dagger\sigma_- + Ja_1^\dagger a_2 + \varepsilon a_1^\dagger + {\rm H.c.}\right), \quad (2)$$

where $\Delta_{\rm a} = \omega_{\rm a} - \omega$ ($\Delta_{{\rm c},j} = \omega_{{\rm c},j} - \omega$) is the detuning between the atom (cavity j) and the input field. The dynamics of the system in the presence of dissipation can be described by Heisenberg equation. The equations of relevant operators can be written as

$$\dot{a}_1 = -(\kappa_1/2 + {\rm i}\Delta_{{\rm c},1})a_1 - {\rm i}g_1\sigma_- - {\rm i}Ja_2 - {\rm i}\varepsilon, \quad (3)$$

$$\dot{a}_2 = -(\kappa_2/2 + {\rm i}\Delta_{{\rm c},2})a_2 - {\rm i}g_2\sigma_- - {\rm i}Ja_1, \quad (4)$$

$$\dot{\sigma}_- = -(\gamma_{\rm a}/2 + {\rm i}\Delta_{\rm a})\sigma_- + {\rm i}g_1^*\sigma_z a_1 + {\rm i}g_2^*\sigma_z a_2, \quad (5)$$

$$\dot{\sigma}_z = -\gamma_{\rm a}(1+\sigma_z) + 2{\rm i}\sum_{j=1,2}\left(g_j a_j^\dagger\sigma_- - g_j^*\sigma_+ a_j\right). \quad (6)$$

Here, $\gamma_{\rm a}$ is the atomic decay rate, and κ_j is the leakage rate of the photons from the j-th cavity to the outside. The cavity operators a_j can be obtained directly from Eqs. (3) and (4) in the steady state ($\dot{a}_j = 0$),

$$a_1 = -\frac{{\rm i}\varepsilon(\kappa_2/2 + {\rm i}\Delta_{{\rm c},2}) + [{\rm i}(\kappa_2/2 + {\rm i}\Delta_{{\rm c},2})g_1 + Jg_2]\sigma_-}{(\kappa_1/2 + {\rm i}\Delta_{{\rm c},1})(\kappa_2/2 + {\rm i}\Delta_{{\rm c},2}) + J^2}, \quad (7)$$

$$a_2 = -\frac{J\varepsilon + [{\rm i}(\kappa_1/2 + {\rm i}\Delta_{{\rm c},1})g_2 + Jg_1]\sigma_-}{(\kappa_1/2 + {\rm i}\Delta_{{\rm c},1})(\kappa_2/2 + {\rm i}\Delta_{{\rm c},2}) + J^2}. \quad (8)$$

Using the standard input-output theory,[40,41] the output $S_{\rm out}$ from cavity 2 can be related to the cavity mode a_2 as

$$S_{\rm out} = \sqrt{\kappa_2}a_2. \quad (9)$$

By substituting Eqs. (7) and (8) into Eqs. (5) and (6), we can obtain the expected value of the atomic operators, which takes the form

$$\langle\sigma_-\rangle = -\frac{\varepsilon\left[P^* + \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right]\left(K_2 g_1^* - \mathrm{i}J g_2^*\right)}{\left|P - \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right|^2 + 2\varepsilon^2\left|K_2^* g_1 + \mathrm{i}J g_2\right|^2}, \quad (10)$$

$$\langle\sigma_z\rangle = -\frac{\left|P - \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right|^2}{\left|P - \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right|^2 + 2\varepsilon^2\left|K_2^* g_1 + \mathrm{i}J g_2\right|^2}. \quad (11)$$

Here, we denote $g_1 \equiv |g_1|\,\mathrm{e}^{\mathrm{i}\phi_1}$ and $g_2 \equiv |g_2|\,\mathrm{e}^{\mathrm{i}\phi_2}$, $K_j \equiv \kappa_j/2 + \mathrm{i}\Delta_{\mathrm{c},j}$ $(j = 1,2)$, $K_\mathrm{a} \equiv \gamma_\mathrm{a}/2 + \mathrm{i}\Delta_\mathrm{a}$, and $P \equiv K_\mathrm{a}\left(K_1 K_2 + J^2\right) + K_2\left|g_1\right|^2 + K_1\left|g_2\right|^2$.

In order to measure the squeezing, the field operator must be expressed in terms of quadrature components. For example, the output operator S_out should be divided into two quadrature components as[42]

$$S_\mathrm{out} = \mathrm{e}^{\mathrm{i}\theta/2}\left(X_\mathrm{Amp} + \mathrm{i}X_\mathrm{Ph}\right). \quad (12)$$

Here θ is an adjustable phase, and we set $\theta = 0$ below. X_Amp and X_Ph are the amplitude and phase quadrature of the output field, respectively, which satisfy the commutation relation $\left[X_\mathrm{Amp}, X_\mathrm{Ph}\right] = \mathrm{i}/2$. According to the definition of the variance $\langle M,N\rangle = \langle MN\rangle - \langle M\rangle\langle N\rangle$, *i.e.*, $\langle S_\mathrm{out}, S_\mathrm{out}\rangle = \langle S_\mathrm{out}S_\mathrm{out}\rangle - \langle S_\mathrm{out}\rangle\langle S_\mathrm{out}\rangle$, the corresponding quadrature variance of the output field S_out read as

$$\begin{aligned}\left\langle :X_\mathrm{Amp}, X_\mathrm{Amp}: \right\rangle &= \frac{1}{4}\langle S_\mathrm{out}, S_\mathrm{out}\rangle + \frac{1}{2}\left\langle S_\mathrm{out}^\dagger, S_\mathrm{out}\right\rangle \\ &\quad + \frac{1}{4}\left\langle S_\mathrm{out}^\dagger, S_\mathrm{out}^\dagger\right\rangle, \quad (13)\\ \left\langle :X_\mathrm{Ph}, X_\mathrm{Ph}: \right\rangle &= -\frac{1}{4}\langle S_\mathrm{out}, S_\mathrm{out}\rangle + \frac{1}{2}\left\langle S_\mathrm{out}^\dagger, S_\mathrm{out}\right\rangle \\ &\quad - \frac{1}{4}\left\langle S_\mathrm{out}^\dagger, S_\mathrm{out}^\dagger\right\rangle. \quad (14)\end{aligned}$$

The notation $\langle :\,, :\rangle$ represents the normally ordered variance, which originate from the definition $\langle :X_i, X_i:\rangle = \langle X_i, X_i\rangle - 1/4$, $i = \mathrm{Amp}, \mathrm{Ph}$. If $\langle :X_i, X_i:\rangle$ is negative, the output field exhibits quadrature squeezing.[42] In detail, $\left\langle :X_\mathrm{Amp}, X_\mathrm{Amp}:\right\rangle$ measures the squeezing of the amplitude quadrature, while $\left\langle :X_\mathrm{Ph}, X_\mathrm{Ph}:\right\rangle$ measures the squeezing of the phase quadrature.

The variances of the output operator S_out, *i.e.*, $\langle S_\mathrm{out}, S_\mathrm{out}\rangle$ and $\left\langle S_\mathrm{out}^\dagger, S_\mathrm{out}\right\rangle$, can be obtained from Eqs. (7)–(11),

$$\langle S_\mathrm{out}, S_\mathrm{out}\rangle = -\frac{\varepsilon^2\kappa_2\left[P^* + \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right]^2\left(K_2 g_1^* - \mathrm{i}J g_2^*\right)^2\left(J g_1 + \mathrm{i}K_1 g_2\right)^2}{\left(K_1 K_2 + J^2\right)^2\left[\left|P - \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right|^2 + 2\varepsilon^2\left|K_2^* g_1 + \mathrm{i}J g_2\right|^2\right]^2}, \quad (15)$$

$$\left\langle S_\mathrm{out}^\dagger, S_\mathrm{out}\right\rangle = \frac{2\varepsilon^4\kappa_2\left|K_2^* g_1 + \mathrm{i}J g_2\right|^4\left|J g_1 + \mathrm{i}K_1 g_2\right|^2}{\left|K_1 K_2 + J^2\right|^2\left|\left|P - \mathrm{i}J\left(g_1^* g_2 + g_1 g_2^*\right)\right|^2 + 2\varepsilon^2\left|K_2^* g_1 + \mathrm{i}J g_2\right|^2\right|^2}. \quad (16)$$

It is clear from Eqs. (13)–(16) that the normally ordered variance of the output field is related to both the amplitude and phase of the coupling strengths g_1 and g_2. As a result, we can explore the phase-controlled squeezing effect in this system. Furthermore, in this work, we are mainly interested in the influence of the phase effect on the squeezing behaviors of the output field. Thus, in the following discussion, for simplicity but without loss of generality, we assume that the two cavities have the same frequency $\omega_{\mathrm{c},1} = \omega_{\mathrm{c},2} \equiv \omega_\mathrm{c}$ and interact resonantly with the two-level atom $\omega_\mathrm{a} = \omega_\mathrm{c}$, *i.e.*, $\Delta_{\mathrm{c},1} = \Delta_{\mathrm{c},2} = \Delta_\mathrm{a}$.

3. Results and discussion

In this section, we study the squeezing properties of the transmitted field and focus on discussing how the quadrature squeezing can be affected by the phase difference of the coupling strengths between the two-level atom and the two cavities. First, we discuss the case where the two cavities have the same decay rates $\kappa_1 = \kappa_2 = \kappa$ and couple to the atom with the same coupling strength $|g_1| = |g_2| = |g|$. Then, the mismatch between the coupling strengths $|g_1|$ and $|g_2|$, and the mismatch between the decay rates κ_1 and κ_2 will be considered later.

In Figs. 2(a)–2(c), the variances of the transmitted field *versus* the detuning $\Delta/\kappa = (\omega - \omega_\mathrm{c})/\kappa$ between the probe field and the cavity modes with different phase differences $\phi_2 - \phi_1$ are plotted. Here, we study how the variances of the amplitude and phase quadrature vary with the detuning, and show how the squeezing properties are modulated by the relative phase of the coupling strengths. It can be seen from Figs. 2(a)–2(c) that there are several dips with variance below zero in the squeezing spectra. The number and location of the dips are strongly related to phase difference. If the atom couples to the two cavities in phase ($\phi_2 - \phi_1 = 0$), there is only a dip in the phase quadrature located at $\Delta = J$, which means that only the phase quadrature can be squeezed, as shown in Fig. 2(a). If the atom couples to the two cavities out of phase ($\phi_2 - \phi_1 = \pi$), a similar squeezing property in the phase quadrature can be obtained, except that the location of the dip changes from $\Delta = J$ to $\Delta = -J$, as shown in Fig. 2(b). However, If the atom couples to the two cavities with a relative phase $\pi/2(\phi_2 - \phi_1 = \pi/2)$, the squeezing behaviors of the phase and amplitude quadrature are much different from that of the cases in phase or out of phase. There are two dips with equal variance in the phase quadrature located at $\Delta = -J$ and $\Delta = J$, respectively. Furthermore, there is also a dip in the

amplitude quadrature located at $\Delta = 0$, which means that, in the present case, an obviously squeezing behavior also can be observed in the amplitude quadrature, as shown in Fig. 2(c).

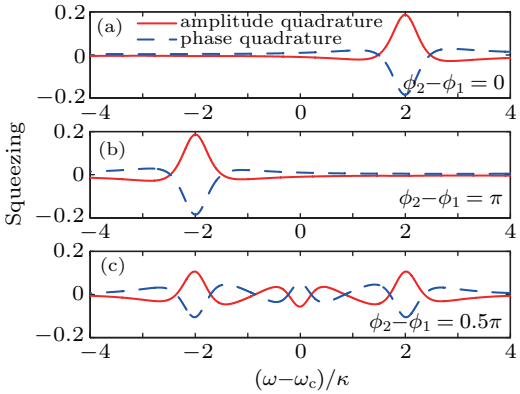


Fig. 2. The variances of the amplitude and phase quadrature as a function of the detuning for different phase difference: (a) $\phi_2-\phi_1=0$; (b) $\phi_2-\phi_1=\pi$; (c) $\phi_2-\phi_1=0.5\pi$. All the red solid and blue dashed curves represent the amplitude and phase quadrature, respectively. The common parameters for this situation are $|g_1|=|g_2|=5\kappa$, $J=2\kappa$, $\varepsilon=0.5\kappa$, $\gamma_a=0.1\kappa$, and $\kappa_1=\kappa_2=\kappa$.

The physical mechanism of this phase-dependent quadrature squeezing can be explained in the normal mode picture.[43,44] The system contains two cavities interacting with an atom supports a pair of standing wave modes A and B, which are the superposition of the two cavity modes, *i.e.*, $A=(a_1+a_2)/\sqrt{2}$ with frequency ω_c+J, and $B=(a_1-a_2)/\sqrt{2}$ with frequency ω_c-J. For convenience, we use the words high-frequency mode and low-frequency mode to describe the standing wave modes A and B, respectively. In this new normal mode representation, the Hamiltonian in Eq. (2) can be rewritten as

$$\begin{aligned}\tilde{H}_{\rm eff} =& \frac{1}{2}\hbar\Delta_a\sigma_z+\hbar(\Delta_a+J)A^\dagger A+\hbar(\Delta_a-J)B^\dagger B\\&+\hbar\left(\frac{|g_1|+|g_2|\,e^{i\Delta\phi}}{\sqrt{2}}A^\dagger\sigma_-+\frac{|g_1|-|g_2|\,e^{i\Delta\phi}}{\sqrt{2}}B^\dagger\sigma_-\right.\\&\left.+\varepsilon\frac{A+B}{\sqrt{2}}+\text{H.c.}\right).\end{aligned}\tag{17}$$

Here, $\Delta\phi=\phi_2-\phi_1$. From Eq. (17), we can find that the effective coupling strengths between the atom and the standing wave modes A and B are given by $g_{\text{eff},A}=(|g_1|+|g_2|\,e^{i\Delta\phi})/\sqrt{2}$ and $g_{\text{eff},B}=(|g_1|-|g_2|\,e^{i\Delta\phi})/\sqrt{2}$, respectively, and these effective coupling strengths can be modified by tuning the relative phase ($\Delta\phi=\phi_2-\phi_1$). When $|g_1|=|g_2|=|g|$, the effective coupling strengths $g_{\text{eff},A}$ and $g_{\text{eff},B}$ become $g_{\text{eff},A}=|g|(1+e^{i\Delta\phi})/\sqrt{2}$ and $g_{\text{eff},B}=|g|(1-e^{i\Delta\phi})/\sqrt{2}$, which can be as large as $\sqrt{2}|g|$ or as small as zero when the relative phase changes from zero to π. For the case $\phi_2-\phi_1=0$, we can obtain $g_{\text{eff},A}=\sqrt{2}|g|$ and $g_{\text{eff},B}=0$, which means that the atom couples to the high-frequency mode A with a maximal effective coupling strength, while decouples completely from the low-frequency mode B. The coherent interaction between the atom and the normal mode A induces a strong nonlinearity in the system, which leads to the phase quadrature squeezing around the high-frequency ω_c+J region (*i.e.*, $\Delta=J$). *Vice versa*, for the case $\phi_2-\phi_1=\pi$, the atom only couples to the normal mode B ($g_{\text{eff},A}=0$ and $g_{\text{eff},B}=\sqrt{2}|g|$), which result in a similar squeezing behavior around the low-frequency ω_c-J region (*i.e.*, $\Delta=-J$). However, for the situation $\phi_2-\phi_1=\pi/2$, the atom couples to both the normal modes A and B. As a consequence, the phase quadrature squeezing can be observed simultaneously around the low-frequency and high-frequency region. Furthermore, the squeezing behavior in the amplitude quadrature can also be observed around the bare cavity resonant region $\Delta=0$ due to the joint contribution of the two normal modes A and B.

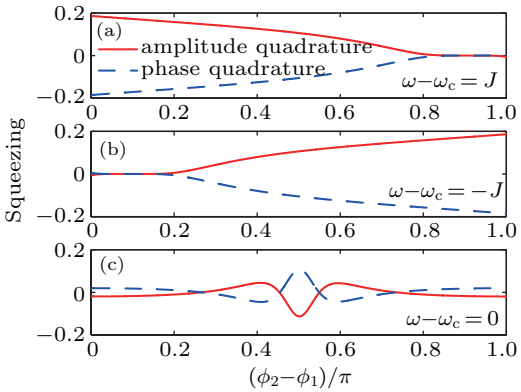


Fig. 3. The variances of the amplitude and phase quadrature as a function of the phase difference for different detunings: (a) $\Delta=\omega-\omega_c=J$; (b) $\Delta=\omega-\omega_c=-J$; (c) $\Delta=\omega-\omega_c=0$. All the red solid and blue dashed curves represent the amplitude and phase quadrature, respectively. The common parameters are the same as those in Fig. 2.

A remarkable feature of the quadrature squeezing of the transmitted field is that the squeezing behaviors can be transferred from the low-frequency mode to the high-frequency mode, and can be transferred from the phase quadrature to the amplitude quadrature by modulating the relative phase of the coupling strengths. To clearly show this, we first plot the variances of the phase and amplitude quadrature *versus* the phase difference with three special detuning $\Delta=\pm J$ and $\Delta=0$ in Figs. 3(a)–3(c), respectively. As the input field resonant with the high-frequency mode A (*i.e.*, $\Delta=J$), the squeezing behavior can be observed in the phase quadrature when the atom couples to the two cavities in phase $\phi_2-\phi_1=0$. However, the degree of squeezing decreases with increasing $\phi_2-\phi_1$. The squeezing behavior around the high-frequency mode A even disappear as the phase difference increases to π, as shown in Fig. 3(a). On the contrary, as the input field resonant with the low-frequency mode B (*i.e.*, $\Delta=-J$), no obvious squeezing behaviors around the low-frequency mode B can be obtained in both the amplitude and phase quadrature at first ($\phi_2-\phi_1=0$). However, by increasing the phase difference to π, the phase quadrature can be squeezed and the degree of squeezing increases to the same value (about -0.2) as that obtained around the high-frequency mode A, as

shown in Fig. 3(b). It is clear from Figs. 3(a) and 3(b) that the squeezing behaviors can be transferred completely from the high-frequency region to the low-frequency region with the phase difference $\phi_2-\phi_1$ ranges from zero to π. Furthermore, figure 3(c) shows that, as the input field resonant with the bare cavity mode (*i.e.*, $\Delta=0$), the squeezing behaviors can be transferred between the amplitude and phase quadrature by increasing the phase difference, and the squeezing behavior in the amplitude quadrature can only be obtained obviously with the phase difference around $\pi/2$.

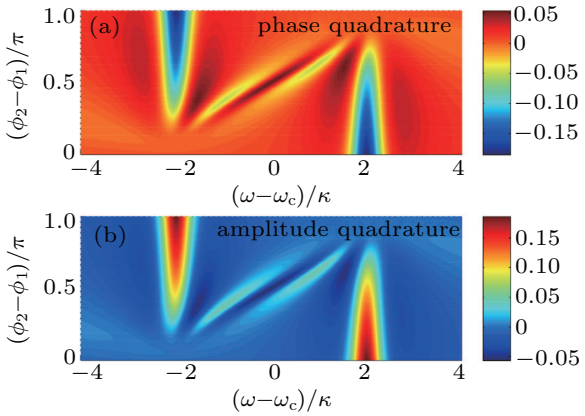


Fig. 4. The variances of the phase quadrature (a) and the amplitude quadrature (b) as a function of both the phase difference and the detuning. The common parameters are the same as those in Fig. 2.

In order to further illuminate these features and study the effect of the relative phase on the quadrature squeezing in a more general case, the variances of the phase and amplitude quadrature *versus* both the detuning $\Delta/\kappa=(\omega-\omega_c)/\kappa$ and the phase difference $\phi_2-\phi_1$ are plotted in Figs. 4(a) and 4(b), respectively. We can find that, for a given detuning, the squeezing behaviors in the high and low-frequency regions are much different. The squeezing effect in the low-frequency region becomes more and more significant while that in the high-frequency region disappears gradually as the relative phase increasing, and *vice versa*. This shows the transition of the squeezing between the high and low-frequency regions, which is consistent with the results shown in Figs. 3(a) and 3(b). Furthermore, comparing Figs. 4(a) and 4(b), it is found that the squeezing behavior in the phase quadrature is also much different from that in the amplitude quadrature around the bare cavity resonant region ($\Delta/\kappa=(\omega-\omega_c)/\kappa=0$). The variance of the phase or amplitude quadrature increases above zero or decreases blew zero alternatively with the change of the phase difference around $\pi/2$, which means that the squeezing between the phase quadrature and the amplitude quadrature can be transferred back and forth. Therefore, the squeezing transfer between the phase and amplitude quadrature or between the low and high-frequency regions can be achieved by simply modulating the relative phase of the coupling strengths between the atom and the two cavities.

In the above discussion, we have assumed that the two cavities with the same decay rates $\kappa_1=\kappa_2$, and coupled symmetrically to the two-level atom, *i.e.*, $|g_1|=|g_2|$. We find that, under this conditions, the squeezing properties of the transmitted field depend strongly on the phase difference of the coupling strengths between the atom and the two cavities. However, it is clear from Eqs. (15) and (16) that the phase-dependent quadrature squeezing is also related to the intensity of the decay rates and the coupling constants. Below, we focus on analyzing the effect of decay mismatch $\kappa_1\neq\kappa_2$ and coupling mismatch $|g_1|\neq|g_2|$ on the phase-dependent quadrature squeezing properties.

First, we study how the decay mismatch between the two cavities affects the quadrature squeezing by setting $|g_1|=|g_2|$. In Figs. 5(a)–5(c), the variances of both the amplitude and phase quadrature are plotted as a function of the decay mismatch κ_1/κ_2 for three detunings, *i.e.*, $\Delta=0$ and $\Delta=\pm J$. Here, we take the phase difference $\phi_2-\phi_1=\pi/2$ for example. Figure 5 shows clearly that the degree of squeezing depends strongly on the decay mismatch. As the decay rate ratio κ_1/κ_2 increases, both the variance of the phase quadrature located at $\Delta=\pm J$, and the variance of the amplitude quadrature located at $\Delta=0$ increase monotonically, as shown by the blue dashed lines in Fig. 5(a) and Fig. 5(c), and the red solid line in Fig. 5(b), respectively. Compare with Fig. 2(c), where $\kappa_1/\kappa_2=1$ ($\kappa_1=\kappa_2$), the degree of squeezing increases when $\kappa_1/\kappa_2<1$ ($\kappa_1<\kappa_2$). The reason is that, as the probe field is injected into cavity 1, an unexpected noise will be injected into the cavities at the same time, which weaken the quadrature squeezing of the transmitted field. However, as the decay rate of cavity 1 κ_1 decreases, partial fluctuation in the input field can be effectively suppressed, which leads to the enhancement of the squeezing of the output field.[45] As a result, a better quadrature squeezing of the output field can be obtained with a proper choice of the decay mismatch κ_1/κ_2.

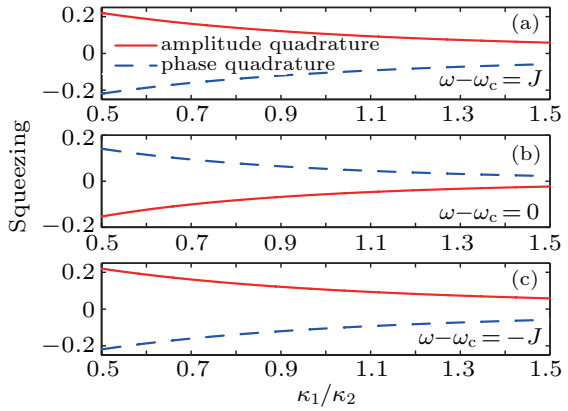


Fig. 5. The variances of the amplitude and phase quadrature as a function of the decay mismatch for different detunings: (a) $\Delta=\omega-\omega_c=J$; (b) $\Delta=\omega-\omega_c=0$; (c) $\Delta=\omega-\omega_c=-J$. All the red solid and blue dashed curves represent the amplitude and phase quadrature, respectively. The common parameters for this situation are $|g_1|=|g_2|=5\kappa_2$, $J=2\kappa_2$, $\varepsilon=0.5\kappa_2$, and $\gamma_a=0.1\kappa_2$.

Then, we proceed to study how the coupling mismatch between the two cavities and the atom affects the quadrature squeezing by setting $\kappa_1 = \kappa_2$. In Figs. 6(a) and 6(b), the quadrature squeezing is plotted as a function of the coupling mismatch $|g_2|/|g_1|$ with the phase difference $\phi_2 - \phi_1 = 0$ for two detuning, *i.e.*, $\Delta = \pm J$. For the case $\omega - \omega_c = -J$, the variances of the phase and amplitude quadrature are approach to zero when $|g_2|/|g_1| = 1$ ($|g_1| = |g_2|$), which means that no obvious squeezing effect can be observed around $\Delta = -J$. This is consistent with the result shown in Fig. 2(a). However, when $|g_2|/|g_1| \neq 1$ ($|g_1| \neq |g_2|$), the variances of both the phase and amplitude quadrature can become much smaller than zero, which means that a better quadrature squeezing can be obtained due to the asymmetric interaction between the atom and the two cavities. Furthermore, the squeezing behaviors of the phase and amplitude quadrature can be transferred into each other by modulating the coupling mismatch, *i.e.*, the squeezing effect of the phase quadrature can be transferred into the amplitude quadrature by increasing $|g_2|/|g_1|$ at first, and then transferred back to the phase quadrature by further increasing the coupling mismatch, as shown in Fig. 6(a). While for the case $\omega - \omega_c = J$, the squeezing behaviors can only be observed in the phase quadrature, and the degree of squeezing is not very sensitive to the coupling mismatch $|g_2|/|g_1|$, as shown in Fig. 6(b).

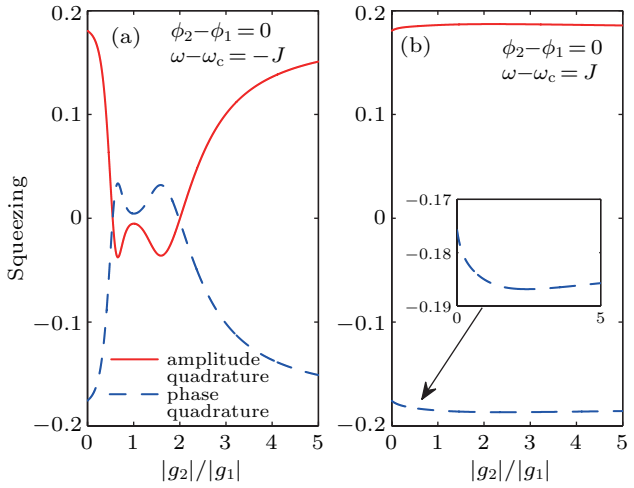


Fig. 6. The variances of the amplitude and phase quadrature as a function of the coupling mismatch for two detunings: (a) $\Delta = \omega - \omega_c = -J$; (b) $\Delta = \omega - \omega_c = J$. All the red solid and blue dashed curves represent the amplitude and phase quadrature, respectively. The common parameters for this situation are $J = 2\kappa$, $\varepsilon = 0.5\kappa$, $\gamma_a = 0.1\kappa$, and $\kappa_1 = \kappa_2 = \kappa$.

Compared to Fig. 2(a), we can find that the squeezing behaviors of the transmitted field become much different due to the coupling mismatch. This difference can be further explaining in the normal mode picture. As mentioned above, for the case $|g_1| = |g_2| = |g|$, the effective coupling strengths between the atom and the standing wave modes A and B, *i.e.*, $g_{\text{eff},A} = |g|(1 + e^{i\Delta\phi})/\sqrt{2}$ and $g_{\text{eff},B} = |g|(1 - e^{i\Delta\phi})/\sqrt{2}$, can reach to their maximum value $g_{\text{eff},A} = \sqrt{2}|g|$ and minimum value $g_{\text{eff},B} = 0$, respectively, as $\Delta\phi = \phi_2 - \phi_1 = 0$. This result in an obvious quadrature squeezing around the high-frequency mode A ($\Delta = J$), but no quadrature squeezing can be observed around the low-frequency mode B ($\Delta = -J$), as shown in Fig. 2(a). However, for the situation $|g_1| \neq |g_2|$, the expressions of the effective coupling strengths should be rewritten as $g_{\text{eff},A} = (|g_1| + |g_2|\, e^{i\Delta\phi})/\sqrt{2}$ and $g_{\text{eff},B} = (|g_1| - |g_2|\, e^{i\Delta\phi})/\sqrt{2}$, respectively. It is clear that, in the present case, the effective coupling strength $g_{\text{eff},B}$ becomes nonzero even though the atom couples to the two cavities in phase ($\Delta\phi = 0$), which leads to the quadrature squeezing around the low-frequency mode B ($\Delta = -J$), as shown in Fig. 6(a).

Furthermore, it is worth noting that the effect of the coupling mismatch on squeezing with other phase difference can be studied in the same way as those for $\Delta\phi = 0$. Actually, the quadrature squeezing vary with $|g_2|/|g_1|$ for the phase difference $\Delta\phi = \pi$ also have been investigated. It is confirmed that a similar squeezing behavior can be obtained and the results are the mirror images of those in Fig. 6. Namely, as the two cavities couple to the atom out of phase $\Delta\phi = \pi$, the squeezing behaviors and the degree of squeezing located at the high-frequency mode A ($\Delta = J$) are very sensitive to $|g_2|/|g_1|$. Obviously, a better quadrature squeezing effect can also be achieved by adjusting the coupling mismatch between $|g_1|$ and $|g_2|$.

Before summary, let us provide a brief discussion on the experimental feasibility of our scheme. The interaction between a two-level atom and two directly coupled cavities can be realized by coupling a quantum dot to a semiconductor bimodal microcavity,[46,47] which supports two degenerate counterpropagating cavity modes (a clockwise propagating mode and a counterclockwise propagating mode). The coupling between these two counterpropagating modes can be realized due to the surface scattering of a nanoparticle, and the mode coupling strength can be controlled by tuning the radius of the nanoparticle or the number of the nanoparticles.[48] Furthermore, in our model, a key element is that the atom couples to the two cavity modes with different relative phase. It has been demonstrated that, in the bimodal microcavity, the field distributions of the clockwise and counterclockwise propagating modes have an azimuthal dependence.[43,49] Thus, the coupling strengths and their relative phase between the two-level atom and the two cavities can be modulated experimentally by varying the position of the atom relative to the azimuthal distribution of the cavity modes.

4. Conclusion

In this paper, we have investigated the quadrature squeezing of the output field from a cavity QED system, which contains two coupled cavities and a two-level atom. We mainly

focus on studying the influence of the phase effect of the coupling strengths on the squeezing. It is found that the squeezing behaviors can be transferred between the phase quadrature and the amplitude quadrature, and can be transferred between the low-frequency region and high-frequency region by tuning the phase difference of the coupling strengths between the atom and the two cavities. Besides, the decay mismatch between the two cavities and the coupling mismatch between the atom and the two cavities can be used to generate better quadrature squeezing. The degree of squeezing of the output field can be remarkably enhanced with the help of these mismatch, which is useful for the application in highly precision measurements and atomic spectroscopy.

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