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# $M^2$-factor of high-power laser beams through a multi-apertured *ABCD* optical system*

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Based on the generalized truncated second-order moments, an approximate analytical formula of the beam propagation factor $M^2$ of high-power laser beams passing through the optical system with multiple hard-edged apertures is deduced. Numerical examples of the beams passing through an aperture-spatial filter are enclosed, and the influences of amplitude modulations (AMs) and phase fluctuations (PFs) on the beam propagation quality of high-power laser beams passing through the multi-apertured *ABCD* optical system are considered and discussed. It is shown that PFs are able to degrade the beam propagation quality of laser beams more than AMs when the high-power laser beams passing through the aperture-spatial filter, furthermore, one or two aperture-lens optical systems configured appropriate aperture parameters are both able to upgrade the beam propagation quality of high-power laser beams. The $M^2$ factor of Gaussian beam passing through the multi-aperture optical system is a special case in this paper.

**Keywords:** amplitude modulations (AMs), phase fluctuations (PFs), multiple hard-edged aperture, beam propagation factor

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## 1. Introduction

In the past decade, relying on the wide application from material processing and nonlinear optics to free-space optical communication and other fields, the high-power laser beams[1–4] have increasingly drawn attention, on the other hand, an increase in the laser output power inevitably results in the gain non-uniformity, gain saturation, nonlinear optical effects, and thermal effect further triggering beam distortions on both amplitude and phase which are able to impact the propagation properties of laser beams.[5–11] Therefore, it is practical important to study the propagation properties of laser beams with beam distortion, including amplitude modulations and phase fluctuations. In order to analyze propagation properties of laser beams with beam distortions theoretically, Manes and Simmons proposed statistical optics model of high-power laser beams for the first time.[12] Further studies[12,13] have shown that statistical optics methods can provide reliable estimates with experimental results and simulations. Besides, the beam propagation quality factor $M^2$ introduced by Siegman[14] is a very useful beam parameter to feature propagation properties of laser beams in various applications.[15–18]

Generally, the larger the value of the factor is, the poorer the beam quality is. Meanwhile, the practical optical systems mostly are equipped with limited apertures which usually cause the laser beams propagating through these systems truncated. Thus, based on the mathematics model proposed by Manes and Simmons, a series of works concerning the propagation properties have been accomplished,[19–25] including $M^2$ factor of truncated high-power laser beams passing through turbulent atmospheric and passing through optical system with an aperture. The results showed that the propagation properties of high-power laser beams are related to both amplitude modulations and phase fluctuations when passing through optical system with an aperture. In addition, it is also shown that when lasers propagate through turbulent atmosphere, the decrement of the turbulence effect on laser beams is caused by amplitude modulations and phase fluctuations. In 2004, by expanding the window function into complex Gaussian function sum, an approximate analytical recurrence formula specified for laser beams with amplitude modulations and phase fluctuations propagating through multi-apertured *ABCD* system was deduced by Lü *et al.*,[26] which provided a fast algorithm for simulating the beam propagation through complex optical trains with a series of apertures. This analytical expression is in satisfactory agreement with direct diffraction integration. To the best of our knowledge, as the valuable theoretical reference for the design and asses of high-power laser optical system, the generalized $M^2$ factor of high-power laser beams passing through multi-apertured *ABCD* system has not been reported yet. With the truncated second-order moments method and expanding the hard-edged aperture function into a finite sum of complex Gaussian functions, in this

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paper, we deduce an approximate analytical expression for the generalized $M^2$ factor of high-power laser beams passing through the multi-apertured *ABCD* optical system, and discuss the influence of beam distortions on $M^2$ factor of laser beams. The approximate analytical propagation expression deduced in Ref. [26] is applied and the generalized $M^2$ factor of high-power laser beams passing through an aperture-spatial filter system is calculated numerically in this paper.

## 2. Basic theory

In the space–time domain, the high-power laser beams with AMs and PFs introduced by Manes and Simmons[12] is featured with the mutual intensity $J_0(x_1', x_2', z=0)$ in two-dimensional case are expressed as

$$J_0(x_1',x_2',0) = I\exp\left[-\frac{(x_1'-x_2')^2\sigma_{\rm p}^2}{L_{\rm p}^2}\right] + \delta_{\rm A}^2\exp\left[-\left(\frac{1}{L_{\rm A}^2}+\frac{\sigma_{\rm p}^2}{L_{\rm p}^2}\right)(x_1'-x_2')^2\right], \quad (1)$$

where $L_{\rm A}$ and $L_{\rm p}$ are the scale of the AMs and PFs respectively, and $\sigma_{\rm A}^2$ and $\sigma_{\rm P}^2$ measure the strengths of the intensity modulations and the amplitude of the phase error respectively. $I$ presenting Gaussian profile is generally much larger than the noise intensity $\sigma_{\rm A}^2$, *i.e.*,

$$I = \exp\left(-\frac{x_1'^2+x_2'^2}{w_0^2}\right), \quad (2)$$

where $w_0$ is the waist width.

Letting

$$\sigma_0^2 = \frac{1}{2}\left(\frac{L_{\rm P}}{\sigma_{\rm P}}\right)^2, \quad (3)$$

$$\gamma^2 = \frac{1}{2}\frac{1}{(1/L_{\rm A}^2)+(\sigma_{\rm P}/L_{\rm p})^2}. \quad (4)$$

The high-power laser beams is assumed to propagate through the paraxial *ABCD* optical system consisting of $mA_iB_iC_iD_i$ subsystems with $m$ hard-edged apertures in the $b_i$ $(i=1,2,\ldots,m)$ width, as shown in Fig. 1, the $\begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix}$ herein refers to the ray transfer matrix.

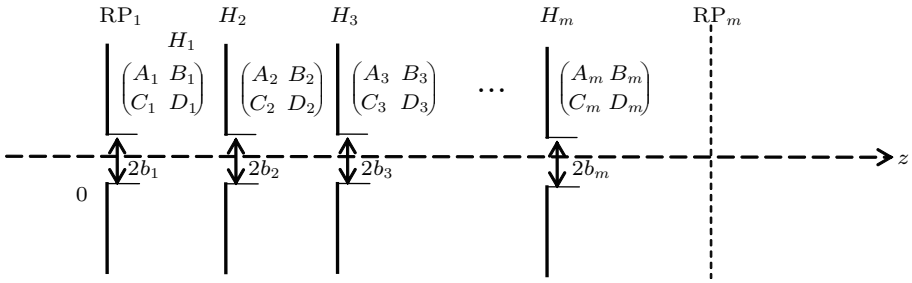



**Fig. 1.** A multi-apertured *ABCD* optical system.

The window function of a rectangular aperture is expressed as $T(x)$. According to Wen and Breazeale,[22] $T(x)$ can be expanded into a finite sum of complex-valued Gaussian functions

$$T(x) = \sum_{j=1}^{M} F_j \exp\left(-\frac{G_j x^2}{b^2}\right), \quad (5)$$

where $F_j$ and $G_j$ denote the expansion coefficients and Gaussian coefficients respectively, which are able to be yielded through optimization computation directly for $M=10$ in Table 2 in Ref. [27].

The relevant parameters and expressions in this paper are subject to the corresponding parameters and expressions in Ref. [26]. On the basis of the mutual intensity expression $J_m(x_1, x_2, z)$ at the output plane $\mathrm{RP}_m$ in Ref. [26], the total power of high-power laser beams, passing through the $m$-th hard-edged rectangular aperture with width of $b_m$ at the $m$-th aperture plane $z$ can be deduced as

$$I_0 = \int_{-b_m}^{b_m} J_{m-1}(x,x,z)\mathrm{d}x = \begin{cases} \sum\limits_{j_1=1}^{M} F_{j_1}\sqrt{\pi}\alpha_1^{-1} + 2b_1\delta_{\rm A}^2, & m=1, \\ \left(\dfrac{\pi}{\lambda}\right)^{m-1}\dfrac{1}{B_1B_2\cdots B_{m-1}}\sum\limits_{j_1=1}^{M}\sum\limits_{j_1'=1}^{M}\cdots\sum\limits_{j_{m-1}=1}^{M}\sum\limits_{j_{m-1}'=1}^{M}\sum\limits_{j_m=1}^{M}\sqrt{\pi}F_{j_1}F_{j_1'}^*\cdots F_{j_{m-1}}F_{j_{m-1}'}^*F_{j_m} & \\ \times\left\{\dfrac{1}{P_1P_2\cdots P_{2\,m-2}}\alpha_m^{-1} + \dfrac{\sigma_{\rm A}^2}{P_1'P_2'\cdots P_{2\,m-2}'}\alpha_m'^{-1}\right\}, & m\geqslant 2. \end{cases} \quad (6)$$

The truncated second-order moments in the spatial domain of high-power laser beams through the $m$-th hard-edged rectangular aperture at the $m$-th aperture plane $z$ is given by[28]

$$\begin{aligned}\langle x^2\rangle &= \frac{1}{I_0}\int_{-b_m}^{b_m} x^2 J_{m-1}(x,x,z)\\ &= \begin{cases} \sum\limits_{j_1=1}^{M} F_{j_1}\dfrac{\sqrt{\pi}}{2}\alpha_1^{-3}+\dfrac{2}{3}\delta_{\mathrm{A}}^2 b_1^3, & m=1,\\ \left(\dfrac{\pi}{\lambda}\right)^{m-1}\dfrac{1}{B_1B_2\cdots B_{m-1}}\sum\limits_{j_1=1}^{M}\sum\limits_{j_1'=1}^{M}\cdots\sum\limits_{j_{m-1}=1}^{M}\sum\limits_{j_{m-1}'=1}^{M}\sum\limits_{j_m=1}^{M} F_{j_1}F_{j_1'}^{*}\cdots F_{j_{m-1}}F_{j_{m-1}'}^{*}F_{j_m}\dfrac{\sqrt{\pi}}{2} & \\ \quad\left\{\dfrac{1}{P_1P_2\cdots P_{2\,m-2}}\alpha_m^{-3}+\dfrac{\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2\,m-2}'}\alpha_m'^{-3}\right\}, & m\geqslant 2, \end{cases}\end{aligned} \tag{7}$$

where

$$\alpha_1^2=\frac{2}{w_0^2}+\frac{G_{j_1}}{b_1^2},$$

$$\alpha_m^2=\frac{G_{j_m}}{b_m^2}-\left(\frac{\mathrm{i}\pi}{\lambda B_{m-1}P_{2m-3}}\right)^2-\frac{\left(S_{m-1}-\dfrac{\mathrm{i}2\pi}{\lambda B_{m-1}}\right)^2}{4P_{2m-2}^2},$$

$$\alpha_m'^2=\frac{G_{j_m}}{b_m^2}-\left(\frac{\mathrm{i}\pi}{\lambda B_{m-1}P_{2m-3}'}\right)^2-\frac{\left(S_{m-1}'-\dfrac{\mathrm{i}2\pi}{\lambda B_{m-1}}\right)^2}{4P_{2m-2}'^2}. \tag{8}$$

The truncated second-order moments in the spatial-frequency-domain of high-power laser beams through the $m$-th hard-edged rectangular aperture at the $m$-th aperture plane $z$ is found to be[28]

$$\begin{aligned}\langle p^2\rangle &= \frac{1}{k^2I_0}\int_{-b_m}^{b_m}\frac{\partial^2 J_{m-1}(x_1,x_2,z)}{\partial x_1\partial x_2}|_{x_1=x_2=x}\,\mathrm{d}x+\frac{4}{k^2I_0b_m}\left[J_{m-1}(b_m,b_m)+J_{m-1}(-b_m,-b_m)\right]\\ &= \frac{1}{k^2I_0}\begin{cases} \sum\limits_{j_1=1}^{M}\left(\dfrac{1}{\sigma_0^2}F_{j_1}\sqrt{\pi}\alpha_1^{-1}+\left(\dfrac{2}{w_0^2}\right)^2F_{j_1}\dfrac{\sqrt{\pi}}{2}\alpha_1^{-3}\right)+2\dfrac{\delta_{\mathrm{A}}^2}{\gamma^2}b_1+\dfrac{8}{b_1}\left(\exp\left(-\dfrac{2b_1^2}{w_0^2}\right)+\delta_{\mathrm{A}}^2\right), & m=1,\\ \left(\dfrac{\pi}{\lambda}\right)^{m-1}\dfrac{1}{B_1B_2\cdots B_{m-1}}\sum\limits_{j_1=1}^{M}\sum\limits_{j_1'=1}^{M}\cdots\sum\limits_{j_{m-1}=1}^{M}\sum\limits_{j_{m-1}'=1}^{M}\sum\limits_{j_m=1}^{M}F_{j_1}F_{j_1'}^{*}\cdots F_{j_{m-1}}F_{j_{m-1}'}^{*} & \\ \quad\left\{\dfrac{F_{j_m}}{P_1P_2\cdots P_{2\,m-2}}\left(\sqrt{\pi}\alpha_m^{-1}F_m+(F_m+2E_m)(F_m-2E_m)\dfrac{\sqrt{\pi}}{2}\alpha_m^{-3}\right)\right. & \\ \quad+\dfrac{F_{j_m}\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2\,m-2}'}\left(\sqrt{\pi}\alpha_m'^{-1}F_m'+(F_m'+2E_m')(F_m'-2E_m')\dfrac{\sqrt{\pi}}{2}\alpha_m'^{-3}\right) & \\ \quad\left.+\dfrac{8}{b_m}\left(\dfrac{1}{P_1P_2\cdots P_{2\,m-2}}\exp(E_m+F_m-G_m)b_m^2+\dfrac{\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2\,m-2}'}\exp(E_m'+F_m'-G_m')b_m^2\right)\right\}, & m\geqslant 2, \end{cases}\end{aligned} \tag{9}$$

where

$$\begin{aligned}E_m&=\frac{S_{m-1}^2}{4P_{2m-2}^2}-\frac{\mathrm{i}\pi D_{m-1}}{\lambda B_{m-1}}-\left(\frac{\pi}{\lambda B_{m-1}P_{2m-3}}\right)^2,\\ F_m&=-\frac{\mathrm{i}\pi S_{m-1}}{\lambda B_{m-1}P_{2m-2}^2},\\ G_m&=\frac{\pi^2}{\lambda^2B_{m-1}^2P_{2m-2}^2}-\frac{\mathrm{i}\pi D_{m-1}}{\lambda B_{m-1}},\end{aligned} \tag{10}$$

and $k=2\pi/\lambda$ is the wavenumber with incident wavelength $\lambda$. By replacing $P_{2m-3}$,$P_{2m-2}$,$S_{m-1}$ on the right-hand side of Eqs. (10) with $P_{2m-3}'$, $P_{2m-2}'$, $S_{m-1}'$ respectively, $E_m'$, $F_m'$, $G_m'$ can be obtained from Eq. (10).

The corresponding cross second-order moment can be obtained as

$$\begin{aligned}\langle xp\rangle &= \frac{1}{2\mathrm{i}kI_0}\int_{-b_m}^{b_m}\left[x_1\frac{\partial J_{m-1}(x_1,x_2,z)}{\partial x_2}\right.\\ &\quad\left.-x_2\frac{\partial J_{m-1}(x_1,x_2,z)}{\partial x_2}\right]_{x_1=x_2=x}\mathrm{d}x=0.\end{aligned} \tag{11}$$

By substituting Eqs. (6)–(11) into the definition of $M^2$ factor, we obtain

$$M^2=2k\left(\langle x^2\rangle\langle p^2\rangle-\langle xp\rangle\right)^{1/2}, \tag{12}$$

the generalized $M^2$ factor of high-power laser beams passing through a multi-apertured optical $ABCD$ system can be yielded below:

$$
M^2 = \begin{cases}
\dfrac{2}{\sum\limits_{j_1=1}^{M} F_{j_1}\sqrt{\pi}+2b_1\delta_{\mathrm{A}}^2}\left\{\left[\sum\limits_{j_1=1}^{M}\left(\dfrac{1}{\sigma_0^2}F_{j_1}\sqrt{\pi}\alpha_1^{-1}+\left(\dfrac{2}{w_0^2}\right)^2 F_{j_1}\dfrac{\sqrt{\pi}}{2}\alpha_1^{-3}\right)+2\dfrac{\delta_{\mathrm{A}}^2}{\gamma^2}b_1\right.\right. \\
\left.\left.+\dfrac{8}{b_1}\left(\exp\left(-\dfrac{2b_1^2}{w_0^2}\right)+\delta_{\mathrm{A}}^2\right)\right]\cdot\left[\sum\limits_{j_1=1}^{M}F_{j_1}\dfrac{\sqrt{\pi}}{2}+\dfrac{2}{3}b_1^3\delta_{\mathrm{A}}^2\right]\right\}^{1/2}, & m=1, \\
\dfrac{\left\{\sum\limits_{j_1=1}^{M}\sum\limits_{j_1'=1}^{M}\cdots\sum\limits_{j_{m-1}=1}^{M}\sum\limits_{j_{m-1}'=1}^{M}\sum\limits_{j_m=1}^{M}F_{j_1}F_{j_1'}^{*}\cdots F_{j_{m-1}}F_{j_{m-1}'}^{*}F_{j_m}\left\{\dfrac{1}{P_1P_2\cdots P_{2m-2}}\alpha_m^{-3}+\dfrac{\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2m-2}'}\alpha_m'^{-3}\right\}\right\}^{1/2}}{\sum\limits_{j_1=1}^{M}\sum\limits_{j_1'=1}^{M}\cdots\sum\limits_{j_{m-1}=1}^{M}\sum\limits_{j_{m-1}'=1}^{M}\sum\limits_{j_m=1}^{M}F_{j_1}F_{j_1'}^{*}\cdots F_{j_{m-1}}F_{j_{m-1}'}^{*}F_{j_m}\left\{\dfrac{1}{P_1P_2\cdots P_{2m-2}}\alpha_m^{-1}+\dfrac{\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2m-2}'}\alpha_m'^{-1}\right\}} \\
\times\left\{\sum\limits_{j_1=1}^{M}\sum\limits_{j_1'=1}^{M}\cdots\sum\limits_{j_{m-1}=1}^{M}\sum\limits_{j_{m-1}'=1}^{M}\sum\limits_{j_m=1}^{M}F_{j_1}F_{j_1'}^{*}\cdots F_{j_{m-1}}F_{j_{m-1}'}^{*}\right. \\
\times\left\{\dfrac{F_{j_m}}{P_1P_2\cdots P_{2m-2}}\left(\sqrt{\pi}\alpha_m^{-1}F_m+(F_m+2E_m)(F_m-2E_m)\alpha_m^{-3}\right)\right. \\
+\dfrac{F_{j_m}\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2m-2}'}\left(\sqrt{\pi}\alpha_m'^{-1}F_m'+(F_m'+2E_m')(F_m'-2E_m')\alpha_m'^{-3}\right) \\
\left.\left.+\dfrac{16}{b_m\sqrt{\pi}}\left(\dfrac{1}{P_1P_2\cdots P_{2m-2}}\exp(E_m+F_m-G_m)b_m^2+\dfrac{\sigma_{\mathrm{A}}^2}{P_1'P_2'\cdots P_{2m-2}'}\exp(E_m'+F_m'-G_m')b_m^2\right)\right\}\right\}^{1/2}, & m\geqslant 2.
\end{cases} \tag{13}
$$

Supposing that $\sigma_0\to\infty$, $\delta_{\mathrm{A}}^2=0$, $\gamma\to\infty$, the approximate analytical expression (13) will be reduced to the generalized $M^2$ factor of Gaussian beams passing through a multi-apertured optical *ABCD* system. Furthermore, supposing that $m=1$, the approximate analytical expression (13) will be reduced to the generalized $M^2$ factor of hard-edged apertured diffracted Gaussian beams, which is consistent with Eq. (25) in Ref. [29].

## 3. Numerical results and analyses

In this section, some numerical calculations have been involved and accomplished with the above deduced analytical expression.

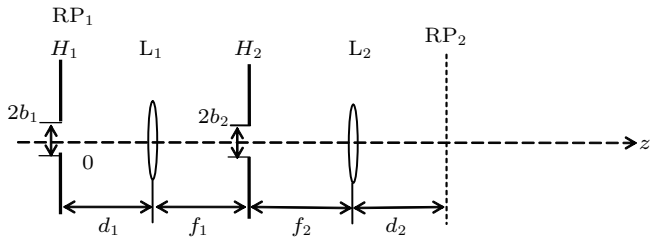



**Fig. 2.** An aperture-spatial filter system.

Figure 2 is a special case of Fig. 1, *i.e.*, the aperture-spatial filter system. Among calculation parameters, we take $w_0=1$ mm, $\lambda=1.06$ μm, $f_1=f_2=1$ m, and $d_1=d_2=1$ m, $f_1$ and $f_2$ refer to the focal lengths of the two lens and $d_1$ and $d_2$ refer to the separation of the first aperture to the first lens and the second aperture to the second lens. The generalized $M^2$ factor of different beams (Gaussian beam and high-power laser beams) passing through one aperture-lens optical system *versus* truncation parameter $b_1/w_0$ is shown in Fig. 3. It is worth noting that the width of the high-power laser beams is assumed to $2w_0$, in this circumstance, the diffraction effect can be negligible if the aperture width is larger than $2w_0$. From Figs. 3(a)–3(b), it is seen that the $M^2$ factor decreases with increasing $\sigma_0$ or decreasing $\delta_{\mathrm{A}}^2$ for a certain $b_1/w_0$. The $M^2$ factor of truncated Gaussian beam and truncated high-power laser beams with only PFs ($\delta_{\mathrm{A}}^2=0$) decreases with increasing $b_1/w_0$ till it respectively reaches the valve 1, 1.421 corresponding to the non-truncated case, whereas $M^2$ factor varies non-monotonically with $b_1/w_0$ for the truncated high-power laser beams with $\delta_{\mathrm{A}}^2\neq 0$. In the beginning, the $M^2$ factor decreases with increasing $b_1/w_0$ till it reaches minimum $M_{\min}^2=2.127$ ($\delta_{\mathrm{A}}^2=0.1$, $\sigma_0=0.5$ mm), 1.606 ($\delta_{\mathrm{A}}^2=0.1$, $\sigma_0=1$ mm), 1.316 ($\delta_{\mathrm{A}}^2=0.1$, $\sigma_0=\infty$), and 1.734 ($\delta_{\mathrm{A}}^2=0.2$, $\sigma_0=1$ mm) respectively, thereafter, it increases with increasing $b_1/w_0$. With $b_1/w_0$ further increasing, the $M^2$ factor reaches a certain value corresponding to the non-truncated case, for instance, $M^2=2.969$ ($\delta_{\mathrm{A}}^2=0.1$, $\sigma_0=0.5$ mm), 2.033 ($\delta_{\mathrm{A}}^2=0.1$, $\sigma_0=1$ mm), 1.605 ($\delta_{\mathrm{A}}^2=0.1$, $\sigma_0=\infty$), and 2.328 ($\delta_{\mathrm{A}}^2=0.2$, $\sigma_0=1$ mm) respectively. Furthermore, regarding to the strong truncated case, the $M^2$ factor converges to 2.313 of truncated plane waves which coincides with the conclusion in Ref. [29]. It demonstrates that when high-power laser beams are passing through an aperture-lens optical system, AMs will change the dependence of Gaussian beams on the truncation parameters $b_1/w_0$, moreover, both AMs and PFs will degrade the beam propagation quality significantly compared with Gaussian beams. It is also suggested in Fig. 3 that when the aperture width $b_1$ is within a suitable range, the optical system with an aperture-lens will result in the decreasing of $M^2$ factor of high-power laser beams with both AMs and

PFs.

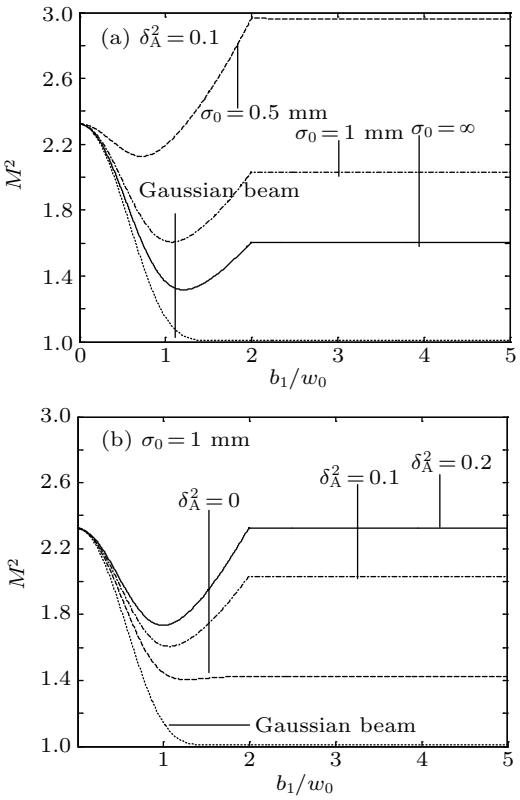



**Fig. 3.** $M^2$ factor *versus* $b_1/w_0$.

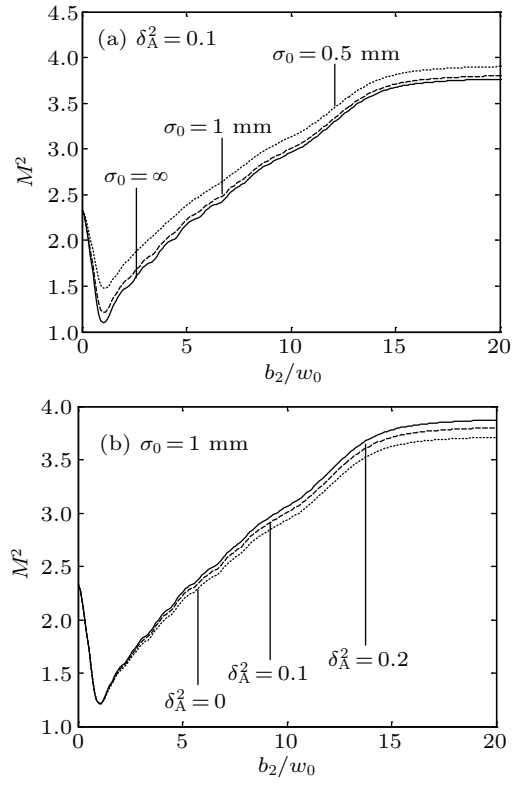



**Fig. 4.** $M^2$ factor *versus* $b_2/w_0$.

The generalized $M^2$ factor of high-power laser beams passing through the aperture-spatial filter system *versus* $b_2/w_0$ for different $\sigma_0$ and $\delta_A^2$ with $b_1/w_0 = 1$ is shown in Fig. 4. From Figs. 4(a) and 4(b), it turns out that the influence of $\sigma_0$ and $\delta_A^2$ on $M^2$ factor is negligible when $b_2/w_0$ is small, nevertheless, when $b_2/w_0$ is large, the $M^2$ factor will increase with the increase of $\delta_A^2$ or decrease of $\sigma_0$ for a certain $b_2/w_0$. Regarding to high-power laser beams passing through the aperture-spatial filter system and the aperture-lens optical system, the dependences of their $M^2$ factor on $b_2/w_0$ and on $b_1/w_0$ are parallel. In other words, there is an optimal aperture width $b_2$ resulting in $M^2$ minimal with preset beam parameters and $l$-th aperture-lens parameters. Respectively, the $M^2_{\min}$ is 1.87, 1.39, 1.1 corresponding to $\sigma_0 = 0.5$ mm, 1 mm, $\infty$ with $\delta_A^2 = 0.1$, as shown in Fig. 4(a). The $M^2_{\min}$ is 1.35, 1.39, 1.41 respectively corresponding to $\delta_A^2 = 0$,, 0.1, 0.2 with $\sigma_0 = 1$ mm, as shown in Fig. 4(b). Regarding the high-power laser beams passing through the aperture-spatial filter system, its $M^2$ will be reduced similarly by setting the appropriate aperture width $b_2$ with the preset aperture width $b_1$.

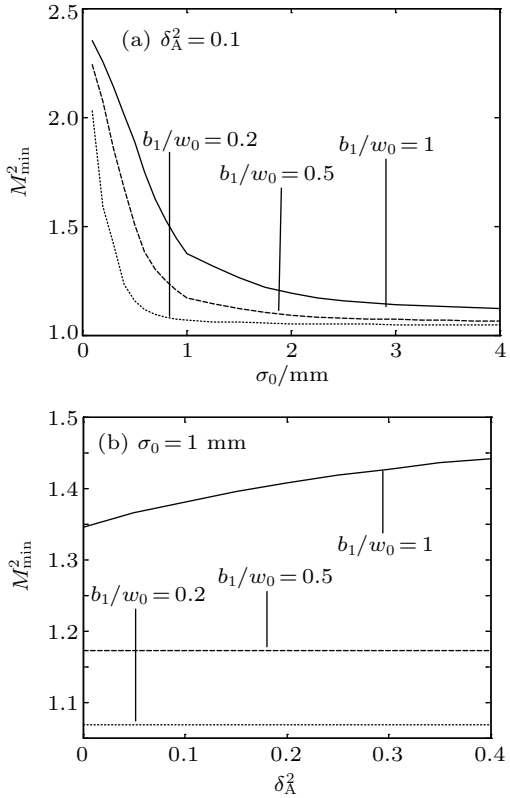



**Fig. 5.** $M^2_{\min}$ *versus* $\sigma_0$ or $\delta_A^2$.

Figures 5(a) and 5(b) show $M^2_{\min}$ *versus* $\sigma_0$ and $M^2_{\min}$ *versus* $\delta_A^2$ when high-power laser beams passing through the aperture-spatial filter system, which is suggested that $M^2_{\min}$ will decrease to an asymptotic value when $\sigma_0$ increases with preset $b_1/w_0$. Moreover, when $\sigma_0$ is determined, the larger the $b_1/w_0$ value is, the larger $M^2_{\min}$ will be. As shown in Fig. 5(b), it can be seen $\delta_A^2$ will not affect the value of $M^2_{\min}$ for $b_1/w_0 = 0.2$, 0.5, nevertheless when $b_1/w_0 = 1$, $M^2_{\min}$ will increase with the increase of $\delta_A^2$. In other words, the aperture-spatial filter system with smaller aperture width $b_1$ can be ap-

plied to upgrade the beam propagation quality of high-power laser beams passing through the aperture-spatial filter system.

Figure 6 shows $M^2$ factor *versus* $b_2/w_0$ for different wavelengths $\lambda$ with $b_1/w_0 = 1$, $\sigma_0 = 1$ mm, $\delta_A^2 = 0.1$ when high-power laser beams passing through the aperture-spatial filter system. It is clear that the variation rule of $M^2$ factor with $b_2/w_0$ will not be impacted by the variation of wavelengths. Nevertheless, when $b_2/w_0$ is small, the variation of the wavelengths will affect the value of $M^2$ factor, and when $b_2/w_0$ is large, corresponding to one aperture-lens case, $M^2$ factor will not vary with the variation of wavelengths. It is concluded that the $M^2$ factor is independent of the wavelengths $\lambda$ when high-power laser beams passing through one-apertured *ABCD* optical system.

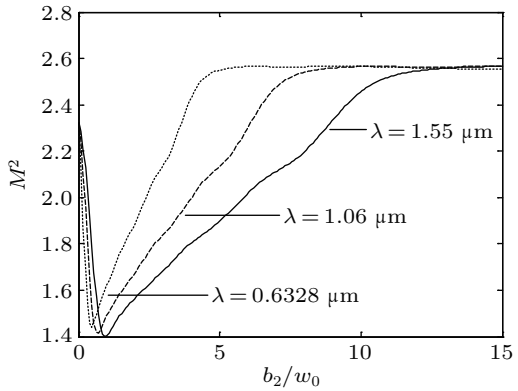



**Fig. 6.** $M^2$ factor *versus* $b_2/w_0$.

It is worth noting that further numerical calculations on the $M^2$ factor of high-power laser beams passing through three or more aperture-lens optical system have as well been fulfilled. The results show that when the number of aperture-lens is equal to or larger than three, $M^2$ factor with preset high-power laser beams parameters can be also minimized by optimal aperture parameters and the influence of AMs and PFs on the $M^2$ factor is negligible with small width of the front apertures, the relevant detailed statements is omitted.

## 4. Conclusion

In this paper, the approximate propagation expression of beam propagation factor $M^2$ of high-power laser beams passing through a multi-apertured optical *ABCD* system is derived with the truncated second-order moments method and by expanding the rectangular function into a finite sum of complex Gaussian functions, it can be applied to evaluate the beam propagation quality of high-power laser beams passing through a multi-apertured optical *ABCD* system. Numerical calculation results indicate the influence of the multi-apertures on the beam propagation quality of high-power laser beams. It is suggested that the $M^2$ factor of high-power laser beams will be decreased with optimal beam parameters and aperture parameters regardless passing through one or two apertures, and the phase fluctuations have greater influence on $M^2$ factor than amplitude modulations. The conclusions reported in this paper provides a reference for design and engineering applications of high-power laser spatial filter systems.

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