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Generation of tripartite Einstein–Podolsky–Rosen steering by cascaded nonlinear process*

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A scheme is proposed to generate genuine tripartite Einstein–Podolsky–Rosen (EPR) steering in cascaded nonlinear process of the fourth-harmonic generation. The second-harmonic is generated by the first double-frequency process in an optical superlattice. Then, the fourth-harmonic is produced by the second cascaded double-frequency process through quasi-phase-matching technique in the same optical superlattice. The genuine tripartite EPR steering among the pump, the second-harmonic, and the fourth-harmonic beams can be obtained by this cascaded nonlinear process according to a criterion for genuine multipartite quantum steering. The quantum steering properties are discussed by adjusting the parameters related to the cascaded nonlinear system. The present research provides a reference scheme and data for obtaining good multipartite EPR steering in experiment and can advance the applications of quantum steering in the quantum information processing.

Keywords: quantum steering, harmonic generation, quasi-phase-matching, optical superlattice

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1. Introduction

The issue of quantum correlation has attracted a lot of interest. The interpretation of quantum correlation can be dated back to Einstein, Podolski, and Rosen's research of the completeness of quantum mechanics in 1935.[1] This research is the famous Einstein–Podolski–Rosen (EPR) paradox. In response to EPR paradox, Schrödinger proposed the concept of quantum steering (EPR steering)[2–4] which described by using different observables to detect one of the particles, causing the corresponding one to collapse to a different state. In the decades after this concept was proposed, scientists have been focusing on quantum entanglement[5–8] and Bell nonlocality.[9,10] Until 2007, Wiseman *et al.*[11,12] found that in the form of quantum information tasks, quantum steering can predict a new property that cannot be described by the local hidden state. Wiseman *et al.*[11] gave the rigorous definition of quantum steering which lies between quantum entanglement and Bell non-locality. Because of the feature of asymmetry, EPR steering has been deep researched in recent years which aims to a better understanding of the steerability.

The criterion of quantum steering is the theoretical basis for realizing EPR steering, it has obtained the most research results recently.[13–19] The simplicity, operability, and practicality of its criteria will greatly facilitate and stimulate the realization and development of EPR steering. In theory, various forms of EPR steering criteria have been developed and applied to EPR steering experiments of different systems and different ideas. In 1992, Ou *et al.*[20] achieved the violation of EPR steering inequalities for continuous variable by using a nondegenerate optical parametric oscillator (OPO). In 2012, Smith *et al.*[21] closed the detection of loophole in the optical system and observed the phenomenon of EPR steering. In the same year, Wittmann *et al.*[22] further closed the free loophole in the experiment. In addition, there are experimentally operable EPR steering criteria.[14,23] For continuous variables, the Reid criterion can be used to detect EPR steering.[15] In 2013, He and Reid put forward a criterion of genuine multipartite EPR steering.[24] Hereafter, by using a continuous variable optical system, a scheme of quantum security communication was proposed and finished.[25] Recently, it has been proved theoretically that EPR steering is an asymmetric quantum nonlocality[26,27] and has been demonstrated in experiments.[28,29] In 2017, Olsen theoretically studied the quantum correlation in the OPO with an injected signal and found the asymmetric steering is controllable.[30] Subsequently, bipartite entanglement and EPR steering cor-

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relation was shown by studying the cascaded $\chi^{(2)}$ system[31] and the cascaded third-harmonic generation process.[32] Three schemes were experimentally demonstrated to manipulate the direction of EPR steering.[33] EPR steering was proposed can be generated via atomic coherence,[34–36] optomechanical systems,[37] and cascaded nonlinear processes in optical superlattice.[38] These great achievements have made us understand deeper about the EPR steering. So far, the EPR steering is fast becoming a key instrument in secure quantum communication[39–41] and quantum key distribution.[42] Tripartite entanglement in cascaded nonlinear process of the fourth-harmonic generation was investigated both in an optical cavity[43] and without optical cavity.[44] However, multipartite quantum steering in this cascaded nonlinear process has not been investigated up to now.

In this paper, we present the genuine tripartite quantum steering in cascaded nonlinear process of quasi-phase-matching fourth-harmonic generation in an optical cavity according to the criterion of genuine multipartite EPR steering which put forward by He and Reid.[24] We also discuss the quantum steering properties by adjusting the parameters related to the system. Our research provides a reference scheme and data for obtaining good multipartite quantum steering in experiment and can advance the applications of quantum steering in quantum information processing. The rest of this paper is arranged as follows. In Section 2, the equations of motion are solved and the stationary solutions are obtained in the positive-P representation.[45] The genuine tripartite EPR steering is presented and the quantum characteristics is discussed in Section 3. Finally, a brief summary about this work is given in Section 4.

2. Equations of motion and the stationary solutions

In this work, A fundamental field with the frequency ω_0 is incident onto the optical cavity which an optical superlattice (OSLT) is placed inside which can be seen in Fig. 1(a). The second-harmonic field at frequency ω_1 is generated by the cascaded nonlinear sum-frequency process, where $\omega_1 = 2\omega_0$. Then, by a second cascaded process between the second-harmonic and second-harmonic fields, the fourth-harmonic field at frequency $\omega_2 = 2\omega_1 = 4\omega_0$ is generated. In the two cascaded nonlinear sum-frequency processes, the phase mismatching can be simultaneously compensated by the reciprocal vectors $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ provided by the OSLT through quasi-phase-matching technique[46] as $\boldsymbol{k}_1 = \boldsymbol{k}_0 + \boldsymbol{k}_0 + \boldsymbol{G}_1$ and $\boldsymbol{k}_2 = \boldsymbol{k}_1 + \boldsymbol{k}_1 + \boldsymbol{G}_2$, which is shown in Fig. 1(b).

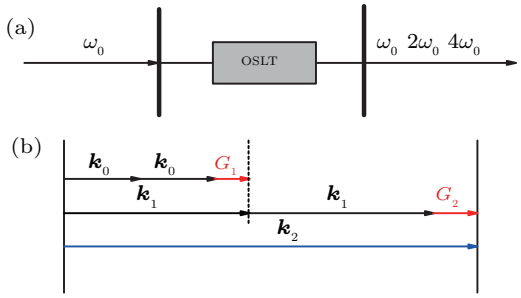


Fig. 1. (a) Sketch of the optical cavity, (b) quasi-phase-matching schematic diagram for the cascaded nonlinear interaction process.

The interaction Hamiltonian for this cascaded nonlinear process can be written as[43]

$$\mathcal{H}_{\mathrm{I}} = \mathrm{i}\hbar\kappa_0\hat{a}_0^2\hat{a}_1^\dagger + \mathrm{i}\hbar\kappa_1\hat{a}_1^2\hat{a}_2^\dagger + \mathrm{h.c.}, \tag{1}$$

where κ_0 and κ_1 represent the nonlinear coupling constants of optical superlattice, $\hat{a}_i$ $(i = 0, 1, 2)$ is the bosonic annihilation operator. Generally speaking, it is taken to be real[47] for the purpose of simplicity. The Hamiltonian of the cavity pumping reads as

$$\mathcal{H}_{\mathrm{pump}} = \mathrm{i}\hbar(\varepsilon\hat{a}_0^\dagger - \varepsilon^*\hat{a}_0), \tag{2}$$

where ε is the amplitude of pumping that it is also considered to be real. Following the description of Lindblad superoperator, the losses of the three fields can be written as

$$L_i\hat{\rho} = \gamma_i(2\hat{a}_i\hat{\rho}\hat{a}_i^\dagger - \hat{a}_i^\dagger\hat{a}_i\hat{\rho} - \hat{\rho}\hat{a}_i^\dagger\hat{a}_i), \tag{3}$$

where $\hat{\rho}$ is the system density matrix and γ_i $(i = 0, 1, 2)$ represent the cavity loss at ω_i. And the master equation of this system can be expressed as

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}_{\mathrm{I}} + \mathcal{H}_{\mathrm{pump}}, \hat{\rho}] + \sum_{i=0}^{2} L_i\hat{\rho}. \tag{4}$$

One can map the master equation onto Fokker–Planck equation (FPE) in the positive-P representation for the sake of being able to study quantum steering characteristics better. Here, α_i and α_i^+ are independent variables which correspond to α_i and $\alpha_i^\dagger$ when the averages of products converge to normally ordered operator expectation values.[30] Thus, we obtain the FPE of the system as

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}t} = \Big\{ &-(\varepsilon - \gamma_0\alpha_0 - 2\kappa_0\alpha_0^+\alpha_1)\frac{\partial}{\partial\alpha_0} - (\varepsilon^* - \gamma_0\alpha_0^+ - 2\kappa_0\alpha_0\alpha_1^+)\frac{\partial}{\partial\alpha_0^+} - (-\gamma_1\alpha_1 - 2\kappa_1\alpha_1^+\alpha_2 + \kappa_0\alpha_0^2)\frac{\partial}{\partial\alpha_1} \\ &- (\kappa_0\alpha_0^{+2} - \gamma_1\alpha_1^+ - 2\kappa_1\alpha_1\alpha_2^+)\frac{\partial}{\partial\alpha_1^+} - (-\gamma_2\alpha_2 + \kappa_1\alpha_1^2)\frac{\partial}{\partial\alpha_2} - (-\gamma_2\alpha_2^+ + \kappa_1\alpha_1^{+2})\frac{\partial}{\partial\alpha_2^+} + \frac{1}{2}\frac{\partial}{\partial^2\alpha_0}(-2\kappa_0\alpha_1) \\ &+ \frac{1}{2}\frac{\partial}{\partial^2\alpha_0^+}(-2\kappa_0\alpha_1^+) + \frac{1}{2}\frac{\partial}{\partial^2\alpha_1}(-2\kappa_1\alpha_2) + \frac{1}{2}\frac{\partial}{\partial^2\alpha_1^+}(-2\kappa_1\alpha_2^+)\Big\}P(\alpha).\end{aligned} \tag{5}$$

Following the standard processing, the equations of motion can be derived in the positive-P representation,

$$\begin{aligned}
\frac{\mathrm{d}\alpha_0}{\mathrm{d}t} &= \varepsilon - \gamma_0\alpha_0 - 2\kappa_0\alpha_0^+\alpha_1 + \sqrt{-2\kappa_0\alpha_1}\eta_1,\\
\frac{\mathrm{d}\alpha_0^+}{\mathrm{d}t} &= \varepsilon^* - \gamma_0\alpha_0^+ - 2\kappa_0\alpha_0\alpha_1^+ + \sqrt{-2\kappa_0\alpha_1}\eta_1^\dagger,\\
\frac{\mathrm{d}\alpha_1}{\mathrm{d}t} &= -\gamma_1\alpha_1 + \kappa_0\alpha_0^2 - 2\kappa_1\alpha_1^+\alpha_2 + \sqrt{-2\kappa_1\alpha_2}\eta_2,\\
\frac{\mathrm{d}\alpha_1^+}{\mathrm{d}t} &= -\gamma_1\alpha_1^+ + \kappa_0\alpha_0^{+2} - \kappa_1\alpha_1\alpha_2^+ + \sqrt{-2\kappa_1\alpha_2^+}\eta_2^\dagger,\\
\frac{\mathrm{d}\alpha_2}{\mathrm{d}t} &= -\gamma_2\alpha_2 + \kappa_1\alpha_1^2,\\
\frac{\mathrm{d}\alpha_2^+}{\mathrm{d}t} &= -\gamma_2\alpha_2^+ + \kappa_1\alpha_1^{+2},
\end{aligned} \tag{6}$$

where $\eta_i(t)$ $(i=0,1,2)$ are the complex Gaussian noise terms which are different from the case in Ref. [43]. They satisfy the relations $\langle\eta_i(t)\rangle = \langle\eta_i^\dagger(t)\rangle = 0$, $\langle\eta_i(t)\eta_j(t')\rangle = \langle\eta_i^\dagger(t)\eta_j^\dagger(t')\rangle = 0$, and $\langle\eta_i(t)\eta_j^\dagger(t')\rangle = \delta_{ij}\delta(t-t')$. In these equations, α_0, α_1, and α_2 correspond to the fundamental, second-harmonic, and fourth-harmonic field amplitudes in the positive-P representation, respectively. By setting $\mathrm{d}\alpha_i/\mathrm{d}t = 0$, we obtain the steady-state solutions when the noise terms are ignored and found the steady-state solution A_1 of α_1 satisfies the following relationship

$$J_1 J_0^2 - J_2 = 0, \tag{7}$$

where $J_0 = 2\kappa_0 A_1 + \gamma_0$, $J_1 = 2\kappa_1^2 A_1^3 + \gamma_1\gamma_2 A_1$, and $J_2 = \varepsilon\kappa_0\gamma_2$. And we take $\gamma_1 = \gamma_2 = \gamma$ for the purpose of simple calculations. However, equation (7) is difficult to solve analytically since it requires the solution of a fifth-order polynomial. For this reason, the other two steady-state solutions A_0 and A_2 can be represented with A_1 as follows:

$$A_0 = \frac{1}{J_0}, \qquad A_2 = \frac{\kappa_1}{\gamma_2}A_1^2. \tag{8}$$

In the following, we can expand the variables into their steady-state expectation values and small Gaussian fluctuations terms close to the steady-state values as $\alpha_i = A_i + \delta\alpha_i$ $(i=0,1,2)$. Based on that, by means of linear processing approach, equation (6) can be rewritten as

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\delta\alpha_0 &= -\gamma_0\delta\alpha_0 - 2\kappa_0 A_1\delta\alpha_0^+ - 2\kappa_0 A_0^*\delta\alpha_1 + \sqrt{-2\kappa_0\alpha_1}\eta_1,\\
\frac{\mathrm{d}}{\mathrm{d}t}\delta\alpha_0^+ &= -2\kappa_0 A_1^*\delta\alpha_0 - \gamma_0\delta\alpha_0^+ - 2\kappa_0 A_0\delta\alpha_1^+ + \sqrt{-2\kappa_0\alpha_1}\eta_1^\dagger,\\
\frac{\mathrm{d}}{\mathrm{d}t}\delta\alpha_1 &= 2\kappa_0 A_0\delta\alpha_0 - \gamma\delta\alpha_1 - 2\kappa_1 A_2\delta\alpha_1^+ - 2\kappa_1 A_1^*\delta\alpha_2 + \sqrt{-2\kappa_1\alpha_2}\eta_2,\\
\frac{\mathrm{d}}{\mathrm{d}t}\delta\alpha_1^+ &= 2\kappa_0 A_0^*\delta\alpha_0^+ - \kappa_1 A_2^*\delta\alpha_1 - \gamma\delta\alpha_1^+ - \kappa_1 A_1\delta\alpha_2^+ + \sqrt{-2\kappa_1\alpha_2}\eta_2^\dagger,\\
\frac{\mathrm{d}}{\mathrm{d}t}\delta\alpha_2 &= 2\kappa_1 A_1\delta\alpha_1 - \gamma\delta\alpha_2, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\delta\alpha_2^+ = 2\kappa_1 A_1^*\delta\alpha_1^+ - \gamma\delta\alpha_2^+.
\end{aligned} \tag{9}$$

We can rewrite it in a matrix form as

$$\mathrm{d}\delta\tilde{\alpha} = -\boldsymbol{A}\delta\tilde{\alpha}\,\mathrm{d}t + \boldsymbol{B}\,\mathrm{d}W \tag{10}$$

with

$$\delta\tilde{\alpha} = [\delta\alpha_0, \delta\alpha_0^\dagger, \delta\alpha_1, \delta\alpha_1^\dagger, \delta\alpha_2, \delta\alpha_2^\dagger]^{\mathrm{T}}, \tag{11}$$

where $\boldsymbol{B}$ is the matrix of noise terms contains the steady-state solutions which found from the original FPE and $\mathrm{d}W$ is a vector of Wiener increments.[45] The drift matrix $\boldsymbol{A}$ can be written in the following form:

$$\boldsymbol{A} = \begin{pmatrix}
\gamma_0 & 2\kappa_0 A_1 & 2\kappa_0 A_0^* & 0 & 0 & 0\\
2\kappa_0 A_1^* & \gamma_0 & 0 & 2\kappa_0 A_0 & 0 & 0\\
-2\kappa_0 A_0 & 0 & \gamma_1 & 2\kappa_1 A_2 & 2\kappa_1 A_1^* & 0\\
0 & -2\kappa_0 A_0^* & 2\kappa_1 A_2^* & \gamma_1 & 0 & 2\kappa_1 A_1\\
0 & 0 & -2\kappa_1 A_1 & 0 & \gamma_2 & 0\\
0 & 0 & 0 & -2\kappa_1 A_1^* & 0 & \gamma_2
\end{pmatrix}. \tag{12}$$

Only when the above drift matrix $\boldsymbol{A}$ has no negative eigenvalues, can the system be in a steady state. In Fig. 2, we show that the real parts of eigenvalues of the drift matrix $\boldsymbol{A}$ (RPEA) *versus* (a) ε, (b) γ_0, (c) γ, and (d) κ_1/κ_0, respectively. As figure 2 shows, the eigenvalues of the drift matrix $\boldsymbol{A}$ are all non-negative in parameter ranges. The satisfaction of this condition, *i.e.*, the drift matrix $\boldsymbol{A}$ has no negative eigenvalues, guarantees that we can discuss quantum steering characteristics in the steady state. The difference in the selected parameters in the present work in order to obtain better quantum steering causes the result to be different from our previous work[43] for obtaining better quantum entanglement state. Then, the satisfaction of the condition of matrix $\boldsymbol{A}$ allows us to calculate the intracavity spectra in the foundation of Fourier transformation via

$$\boldsymbol{S}(\omega) = (\boldsymbol{A} + \mathrm{i}\omega\boldsymbol{I})^{-1}\boldsymbol{D}(\boldsymbol{A}^{\mathrm{T}} - \mathrm{i}\omega\boldsymbol{I})^{-1}, \tag{13}$$

where ω is the Fourier analysis frequency, $\boldsymbol{I}$ is the identity matrix, and $\boldsymbol{D} = \boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}$ is a 6×6 matrix. According to the standard input–output relationship,[49] the output spectra can

be calculated from Eq. (13).

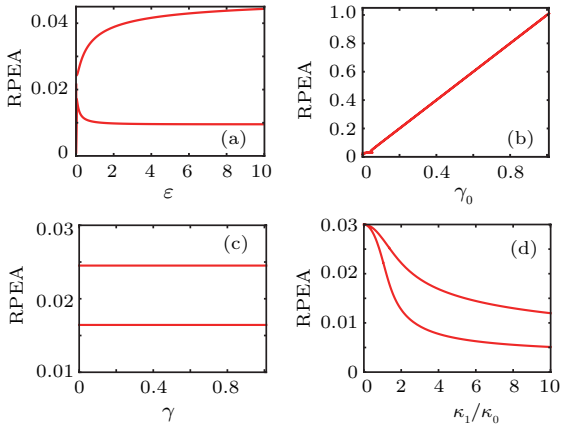


Fig. 2. Real parts of eigenvalues of drift matrix $\boldsymbol{A}$ (RPEA) *versus* (a) ε, (b) γ_0, (c) γ, and (d) κ_1/κ_0, respectively.

3. Genuine tripartite EPR steering

There are many criteria for EPR steering, such as the criteria of Reid[15] in continuous variable, the criteria of multipartite system,[24] and experimentally operable EPR steering criteria.[14,50] Bipartite asymmetric quantum steering in the cascaded $\chi^{(2)}$ system[30,31] and the third-harmonic quantum steering[32] are studied, there is almost no genuine EPR steering involved. In this paper, the properties of quantum steering among three fields in the cascaded sum-frequency process will be investigated according to the criterion of multipartite system.[24] We give the quadrature definitions as $\alpha_i = X_i + \mathrm{i}Y_i$, and $\alpha_i^{\dagger} = X_i - \mathrm{i}Y_i$, all the relationships we present involved X_i and Y_i. Thus, based on the criterion of multipartite system, we write the three equalities in the form as

$$\begin{aligned} S_0 &= \Delta(X_0 - X_1)\Delta(Y_0 + Y_1 + Y_2), \\ S_1 &= \Delta(X_1 - X_2)\Delta(Y_0 + Y_1 + Y_2), \\ S_2 &= \Delta(X_2 - X_0)\Delta(Y_0 + Y_1 + Y_2). \end{aligned} \tag{14}$$

By measuring the values of S_i $(i = 0, 1, 2)$ and comparing with 1 to confirm whether the system i is steerable or not. When the condition $S_i < 1$, it means the success of EPR steering of system i and also indicates that the three fields are bipartite quantum steering each other. There is no doubts that when the measurement of

$$S_{\mathrm{tot}} = S_0 + S_1 + S_2 < 1 \tag{15}$$

will be sufficient to demonstrate genuine tripartite steering.[24]

Parameters will affect the results that whether the system is steerable or not in the cascaded sum-frequency process, thence we take the normalized analysis frequency $\Omega = \omega/\gamma_0$ into account first. The values of S_i and S_{tot} *versus* the normalized analysis frequency Ω with $\gamma_0 = 0.001$, $\gamma_1 = \gamma_2 = 0.03$, $\kappa_0 = 0.05$, $\kappa_1 = 1.5\kappa_0$, and $\varepsilon = 0.06\kappa_0$, respectively, is shown in Fig. 3. The choice of parameters is partially identical to our previous work[43] in order to obtain better tripartite quantum steering. As shown in Fig. 3, S_i $(i = 0, 1, 2)$ are all below 1 in a wide range of the analysis frequency which indicates that bipartite quantum steering exists in the three fields. And it can also be found that the value of $S_{\mathrm{tot}} < 1$ across the whole range of analysis frequency Ω from Fig. 3. That is to say, the genuine tripartite steering condition is satisfied, which further confirms the genuine tripartite steering.

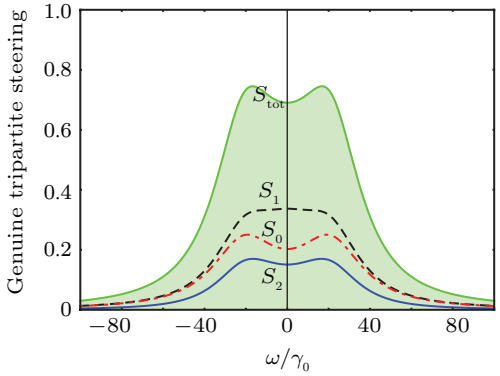


Fig. 3. The values of S_0 (red dashed line), S_1 (black dot-dashed line), S_2 (blue thin line), and S_{tot} (green thick line) *versus* the the normalized analysis frequency Ω with $\gamma_0 = 0.001$, $\gamma_1 = \gamma_2 = 0.03$, $\kappa_0 = 0.05$, $\kappa_1 = 1.5\kappa_0$, and $\varepsilon = 0.06\kappa_0$, respectively.

In quasi-phase-matching fourth-harmonic generation process, we can adjust nonlinear coupling parameters. Figure 4 plots the values of S_i and S_{tot} *versus* the nonlinear coupling parameter κ_1/κ_0 with $\gamma_0 = 0.001$, $\kappa_0 = 0.03$, $\gamma_1 = \gamma_2 = 0.05$, and $\omega = 4\gamma_0$, respectively. We can clearly see that the values of the S_i and the S_{tot} increase as the increase of the ratio κ_1/κ_0 in this range. The value of the S_{tot} will exceed 1 when the ratio of κ_1/κ_0 is larger than 12 which proves that it is not a genuine tripartite steering. Only when the condition $S_{\mathrm{tot}} < 1$, will the genuine tripartite steering be demonstrated. We found that the value of the nonlinear coupling parameter κ_0 directly affects the value of S_i and S_{tot} which determined whether the system is steerable and genuine tripartite steering or not in the cascaded sum-frequency processes.

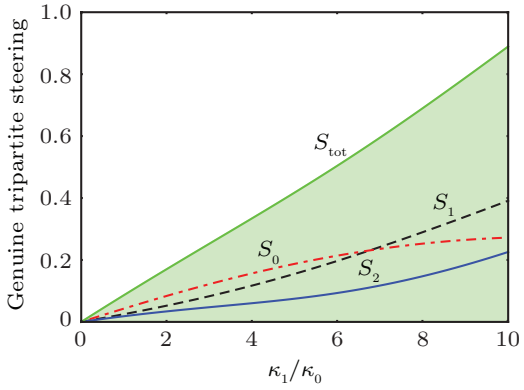


Fig. 4. The values of S_0 (red dashed line), S_1 (black dot-dashed line), S_2 (blue thin line), and S_{tot} (green thick line) *versus* the nonlinear coupling parameter κ_1/κ_0 with $\gamma_0 = 0.001$, $\kappa_0 = 0.03$, $\gamma_1 = \gamma_2 = 0.05$, and $\omega = 4\gamma_0$, respectively.

Figure 5 depicts the values of S_i and $S_{\rm tot}$ *versus* the damping rates γ_1/γ_0 with $\gamma_0 = 0.001$, $\kappa_0 = 0.01$, $\kappa_1 = 2\kappa_0$, $\varepsilon = 0.03\kappa_0$, and $\omega = 5\gamma_0$, respectively. One can see that the value of S_i and $S_{\rm tot}$ are far less than 1 comprehensively. It is below 1, although the values of S_i and $S_{\rm tot}$ reache the peak at a rate of about 1.5, which indicates that bipartite EPR steering and genuine tripartite steering can be generated in the cascaded nonlinear process.

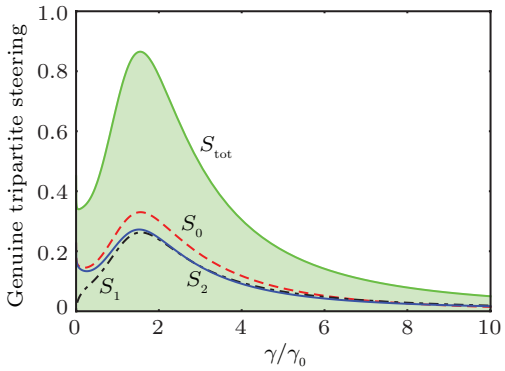


Fig. 5. The values of S_0 (red dashed line), S_1 (black dot-dashed line), S_2 (blue thin line), and $S_{\rm tot}$ (green thick line) *versus* the damping rate of γ_1/γ_0 with $\gamma_0 = 0.001$, $\kappa_0 = 0.01$, $\kappa_1 = 2\kappa_0$, $\varepsilon = 0.03\kappa_0$, and $\omega = 5\gamma_0$, respectively.

4. Conclusion

In this paper, the cascaded nonlinear process of the fourth-harmonic generation is analyzed in the positive-P representation. We demonstrate that the genuine tripartite quantum steering can be obtained in the cascaded nonlinear process based on the criterion for the genuine multipartite quantum steering.[24] The parameters that affect the results are also discussed and we found the better quantum steering can be obtained when the cavity loss is lower. The value of the nonlinear coupling parameter directly determined whether the system is steerable or not. In general, quantum steering, as an emerging quantum information processing sub-direction, has some important and valuable results, but there are still many unknowns waiting for further study. The tripartite steering can also be generated by four-wave mixing (FWM) in rubidium atoms combined with a linear beamsplitter or cascaded a second FWM and their results, showing that the cascaded FWM scheme is a promising candidate to generate stronger multipartite EPR steering.[37] Compared with their scheme, our scheme only needs one pump and one optical superlattice, which is more simple in experiment. However, their scheme has more flexibility to manipulate the monogamy relation. In addition, in their scheme, the correlation level of three-mode is weaker than the case of twin beams since the three-mode correlation condition is more stringent,[51,52] which is consistent with our results as shown in Figs. 2–5. In previous single-pass cascaded nonlinear experiments, the frequency conversion efficiencies are up to 20% for the second-harmonic generation and close to 23% for the cascaded third-harmonic generation by quasi-phase-matching technique.[46,53,54] Theoretically, the frequency conversion efficiency for the cascaded fourth-harmonic generation is up to 4%. In our scheme, when taking it into optical cavity, the conversion efficiency will be further increased. Therefore, we think that our scheme is feasible in experiment. The difficulty is how to design an optical resonator so that three light fields of different frequencies can resonate at the same time. We hope the results reported in this paper could enable researchers to understand the EPR steering deeper, and give some help for potential applications in quantum information processing.

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