ISSN 1674-1056
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Citation: Chin. Phys. B . 2020, 29(5): 050304 . **doi:** 10.1088/1674-1056/ab7da3
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Qubit movement-assisted entanglement swapping

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(Received 23 January 2020; revised manuscript received 19 February 2020; accepted manuscript online 9 March 2020)

We propose a scheme to generate entanglement between two distant qubits (two-level atom) which are separately trapped in their own (in general) non-Markovian dissipative cavities by utilizing entangling swapping, considering the case in which the qubits can move along their cavity axes rather than a static state of motion. We first examine the role of movement of the qubit by studying the entropy evolution for each subsystem. The average entropy over the initial states of the qubit is calculated. Then by performing a Bell state measurement on the fields leaving the cavities, we swap the entanglement between qubit-field in each cavity into qubit-qubit and field-field subsystems. The entangling power is used to measure the average amount of swapped entanglement over all possible pure initial states. Our results are presented in two weak and strong coupling regimes, illustrating the positive role of movement of the qubits on the swapped entanglement. It is revealed that by considering certain conditions for the initial state of qubits, it is possible to achieve a maximally long-leaving stationary entanglement (Bell state) which is entirely independent of the environmental variables as well as the velocity of qubits. This happens when the two qubits have the same velocities.

Keywords: dissipative systems, quantum entanglement, entanglement swapping

PACS: 03.65.Yz, 03.65.Ud, 03.67.Mn **DOI:** 10.1088/1674-1056/ab7da3

1. Introduction

In recent decades, there is a great deal of evidence that quantum phenomena play a central role in the development of information theory. Coherent superposition is one of these features which is usually referred as quantum coherence. The non-local quantum correlations among composite subsystems is called entanglement.[1] The importance of quantum entanglement arises from its various exciting applications such as quantum teleportation,[2,3] quantum cryptography,[4] sensitive measurements,[5] quantum telecloning,[6,7] and superdense coding.[8] Due to the rapid growth of the applications of these kinds of quantum states in quantum information processing implementations, a great deal of attention has been paid to the generation and detection of entangled states. Most of these proposals rely on the interaction of atoms (real or artificial) with optical cavities.[9] Other proposals include quantum dots,[10] atomic ensembles,[11] superconducting quantum interference devices,[12,13] photon pairs,[14] superconducting qubits,[15] and trapped ions.[16–18]

Approximately, all of the introduced schemes depend on the interactions (direct or indirect) between subsystems. For instance, it has been shown that the Jaynes–Cummings model (JCM), which describes the interaction of atoms (two- or multi-level) with cavity field,[19] could generate entanglement between an atom and a quantized field. This model has been extended to include the interaction of multiple atoms with a multi-mode electromagnetic field.[20] Thanks to the nonlocality of quantum correlations, it is possible to entangle two or more particles which are distributed over distances without any interactions and, or common history. This phenomenon is called entanglement swapping.[21,22] In this protocol, the basic recipe is to make a more general system. Then by projection of the quantum state of the whole system onto a maximally entangled Bell state, it is possible to swap the entanglement between these subsystems. There have been many works on this interesting topic. For instance, it has been generated for continuous-variable systems in Ref. [23]. Multiparticle entanglement swapping has been studied in Ref. [24]. The unconditional entanglement swapping has been experimentally demonstrated in Ref. [25]. In Ref. [26], the authors have discussed this phenomenon using a quantum-dot spin system. Researchers have already shown that entanglement swapping could be used for the optimization of entanglement purification.[27] One-cavity scheme enabling to implement delayed choice for entanglement swapping in cavity QED has been investigated in Ref. [28]. The effect of detuning and Kerr medium on the entanglement swapping has been studied in Ref. [29]. Recently, a high-fidelity unconditional entanglement swapping experiment in a superconducting circuit has been performed in Ref. [30]. Very recently, the swapping of entangled states between two pairs of photons emitted by a single quantum dot has been performed experimentally.[31]

On the other hand, contrary to the closed systems which are ideal, the real physical systems are open. This means that dissipation is always present in those systems. Actually, the inescapable interaction between the aim system and its sur-

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rounding environment makes the entanglement fragile.[32,33] Because a long-lasting entangled state is an essential resource for the quantum information theory, many strategies have been devoted to fight against the destructive environmental effects: the theory of open quantum systems.[34–42] However, it should be noted that the idea of interaction of a quantum system with the surrounding environment is not always bad. For instance, it has been shown that there exists a long-living entangled state due to the interaction of a two-qubit system with a common environment.[43–47] This idea has been generalized to an arbitrary number of qubits inside an environment.[48–50] Moreover, quantum reservoir engineering has been proven to be useful in stabilizing open quantum systems[51] and remote entanglement and concentration,[52] etc. Recently, it has been shown that an external classical field is a practical scheme to preserve the entanglement between two dissipative systems.[53] In this regard, many studies such as non-Markovianity,[54] quantum speedup,[55] quantum Fisher information[56] have been presented.

Recent experimental schemes in quantum information processing rely on the control of single qubits inside (optical) cavities. However, in practical implementations, achieving a static state of qubits in a cavity is a difficult task, although it is not impossible. In a pioneering work, the effect of the movement of two qubits inside non-Markovian environments on the protection of the initial entanglement has been studied in Ref. [57]. Moreover, other studies illustrated the effect of movement of qubits (both uniform and accelerated) on the interaction between such qubits and electromagnetic radiation.[58,59] This includes the relativistic velocities for qubits.[60,61] In a very recent paper, the authors showed the positive role of movement of qubits on the entanglement dynamics of an arbitrary number of qubits in a Markovian and/or non-Markovian environment.[62] It has also been shown that when all of the qubits have the same velocity, the stationary state of entanglement is independent of the velocity of qubits.[62]

All of the statements mentioned above motivate us to examine the effect of movement of qubits on the entanglement swapping between two separate subsystems. To end this, we consider two independent cavities, each contains a moving two-level system (qubit) in the presence of dissipation. We model the environment as a set of infinite quantized harmonic oscillators. We take the situation in which each qubit is allowed to move along the cavity axis. We also consider the non-relativistic velocities for qubits. In this situation, the exact dynamics for each subsystem is obtained for both weak and strong coupling regimes corresponding to bad and good cavity limits. We first explore the role of movement of the qubit on the entropy evolution for each subsystem. After that, we perform a Bell state measurement (BSM) on the cavity fields leaving the cavities. This swaps the entanglement between the qubit and the field in each cavity into qubit-qubit and field-field entanglements. We use the concurrence measure[63] to quantify the amount of swapped entanglement. Naturally, this depends on the initial state of the qubits. Our parametrization for the initial states of qubits allows us to establish an input-independent dynamics of entanglement by taking a statistical average over the initial states of two qubits. This is called entangling power originally introduced for unitary maps[64] and then generalized for dissipative channels.[65]

The rest of the paper is organized as follows. In Section 2 the model and the various related parameters are introduced, then the dynamics of each subsystem is obtained. Section 3 deals with the effect of the movement of the qubits on the entropy evolution of each subsystem. We study the entanglement swapping phenomena in detail in Section 4 Finally, in Section 5, we summarize the paper.

2. Dynamics of the single moving qubit system in dissipative regime

We consider two similar, but separate dissipative cavities, each containing a moving two-level atom (qubit) with an excited (ground) $|e\rangle$ ($|g\rangle$) state. These states are separated by transition frequency $\omega_{\rm qb}$. Each qubit is taken to move along the z-axis of the corresponding cavity with constant velocity v (see Fig. 1). The movement of qubits is characterized by a sine term due to the boundary conditions. The Hamiltonian for each system (with $\hbar = 1$) is given by

$$\hat{H}_{({\rm AF})_i} = \hat{H}^0_{({\rm AF})_i} + \hat{H}^{\rm Int}_{({\rm AF})_i}, \quad i = 1,2 \tag{1}$$

with

$$\hat{H}^0_{({\rm AF})_i} = \sum_k \omega_k \hat{a}^\dagger_k \hat{a}_k + \frac{\omega_{\rm qb}}{2}\hat{\sigma}_z, \tag{2a}$$

$$\hat{H}^{\rm Int}_{({\rm AF})_i} = \sum_k \alpha_i g_k f_k(z_i)\hat{a}_k \hat{\sigma}^+ + {\rm H.c.}, \tag{2b}$$

where $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli matrix, ω_k denotes the frequency of the cavity quantized modes, while $\hat{a}_k$ ($\hat{a}^\dagger_k$) are the annihilation (creation) operators of the cavity k-th mode; g_k denotes the coupling constant between the qubit and the k-th mode of environment. The interaction of the i-th qubit with the environment is measured by the dimensionless constant α_i; $\hat{\sigma}_+ = |e\rangle\langle g|$ ($\hat{\sigma}_- = |g\rangle\langle e|$) denotes the qubit raising (lowering) operator. Furthermore, the function $f_k(z_i)$ describes the shape function of the i-th qubit motion along z-axis.[66] In this regard, this parameter is given by[67]

$$f_k(z_i) = f_k(v_i t) = \sin[\omega_k(\beta_i t - \Gamma)], \quad i = 1,2, \tag{3}$$

where $\beta_i = v_i/c$ and $\Gamma = L/c$ with L being the size of cavity and c the velocity of light. The sine term in the above relation comes from the boundary conditions. It should be noted that the translational motion of the qubits has been treated

classically ($z = vt$). This is the situation in which the de Broglie wavelength λ_B of the qubits is much smaller than the wavelength λ_0 of the resonant transition (i.e., $\lambda_B/\lambda_0 \ll 1$),[57] which means $\beta_i \ll 1$.

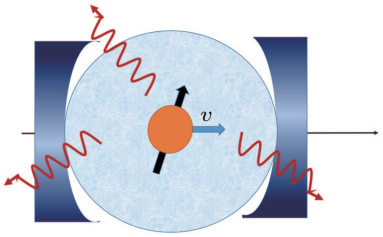


Fig. 1. Pictorial representation of a setup in which a moving qubit is interacting with a dissipative cavity.

Formally, it is more convenient to work in the interaction picture. The Hamiltonian (1) in the interaction picture is given by

$$\hat{\mathfrak{V}}_{(\mathrm{AF})_i} = \mathrm{e}^{\mathrm{i}\hat{H}^0_{(\mathrm{AF})_i}t}\,\hat{H}^{\mathrm{Int}}_{(\mathrm{AF})_i}\,\mathrm{e}^{-\mathrm{i}\hat{H}^0_{(\mathrm{AF})_i}t}. \tag{4}$$

After some manipulation, the explicit form of the Hamiltonian in the interaction picture may be obtained as

$$\hat{\mathfrak{V}}_{(\mathrm{AF})_i} = \sum_k \alpha_i g_k f_k(z_i)\hat{a}_k\hat{\sigma}^+\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_{\mathrm{qb}})t} + \mathrm{H.c.} \tag{5}$$

In the above relation, without loss of generality, we consider the case in which each qubit has the same speed, i.e., $v_1 = v_2 \equiv v$ ($\beta_1 = \beta_2 \equiv \beta$) and also the constants α_1 and α_2 are the same, i.e., $\alpha_1 = \alpha_2 \equiv \alpha$.

We suppose that there is no excitation in the cavities before the occurrence of interaction and each atom is in the coherent superposition of the exited $|e\rangle$ and ground state $|g\rangle$ as

$$|\psi(0)\rangle_i = \Big(\cos(\theta_i/2)|e\rangle + \sin(\theta_i/2)\,\mathrm{e}^{\mathrm{i}\phi_i}|g\rangle\Big)\,|\mathbf{0}\rangle_R, \tag{6}$$

in which $|0\rangle_R$ is the multi-mode vacuum state of the cavity. In the above relation, $\theta_i \in [0,\pi]$ and $\phi_i \in [0,2\pi]$ for $i = 1,2$. According to Hamiltonian (5), the quantum state of the i-th system at any time t can be written as

$$\begin{aligned}|\psi(t)\rangle_i \;=\; & \cos(\theta_i/2)\mathscr{E}(t)|e\rangle|\mathbf{0}\rangle_R + \sin(\theta_i/2)\,\mathrm{e}^{\mathrm{i}\phi_i}|g\rangle|\mathbf{0}\rangle_R \\ & + \cos(\theta_i/2)\sum_k \mathscr{G}_k(t)|g\rangle|\mathbf{1}_k\rangle,\end{aligned} \tag{7}$$

where $|\mathbf{1}_k\rangle$ describes the state of the environment with only one photon in the k-th mode.

Using the time-dependent Schrödinger equation in the interaction picture, we are ready to have the integro-differential equation for the amplitude $\mathscr{E}(t)$,

$$\dot{\mathscr{E}}(t) = -\alpha^2\int_0^t F(t,t')\mathscr{E}(t')\,\mathrm{d}t', \tag{8}$$

where the kernel $F(t,t')$ takes the form

$$F(t,t') = \sum_k |g_k|^2\,\mathrm{e}^{\mathrm{i}\delta_k(t-t')}f_k(vt)f_k(vt'), \tag{9}$$

with $\delta_k = \omega_{\mathrm{qb}} - \omega_k$.

As is seen, $\mathscr{E}(t)$ depends on the spectral density as well as the shape function of the qubit motion. In the continuum limit for the reservoir spectrum, the sum over the modes is replaced by the integral

$$\sum_k |g_k|^2 \to \int \mathrm{d}\omega J(\omega), \tag{10}$$

where $J(\omega)$ is the reservoir spectral density. The nonperfect reflectivity of the cavity mirrors implies a Lorentzian spectral density for the cavity as follows:[44]

$$J(\omega) = \frac{W^2}{\pi}\frac{\lambda}{(\omega-\omega_{\mathrm{qb}})^2+\lambda^2}, \tag{11}$$

where we have assumed that the qubit interacts resonantly with the central frequency of the cavity modes. The weight W is proportional to the vacuum Rabi frequency and λ is the width of the distribution which describes the cavity losses.

Substituting (11) into (9), we arrive at the following expression for $F(t,t')$:

$$\begin{aligned}F(t,t') \;=\; & \frac{W^2\lambda}{\pi}\int \mathrm{d}\omega \frac{\sin[\omega(\beta t-\Gamma)]\sin[\omega(\beta t'-\Gamma)]}{(\omega-\omega_{\mathrm{qb}})^2+\lambda^2} \\ & \times \mathrm{e}^{-\mathrm{i}(\omega-\omega_0)(t-t')}.\end{aligned} \tag{12}$$

In the continuum limit (i.e., $\Gamma \to \infty$),[68] the analytical solution of the above relation gives rise to

$$F(t,t') = \frac{W^2}{2}\,\mathrm{e}^{-\lambda(t-t')}\cosh[\beta\bar{\lambda}(t-t')], \tag{13}$$

in which $\bar{\lambda} \equiv \lambda + \mathrm{i}\omega_{\mathrm{qb}}$. According to the above relation, the argument of F is $t-t'$, i.e., $F = F(t-t')$, which motivates us to use the Laplace transformation technique. After some straightforward and long manipulations, we may obtain the analytical solution to (8) as follows:

$$\begin{aligned}\mathscr{E}(t) \;=\; & \frac{(q_1+y_+)(q_1+y_-)}{(q_1-q_2)(q_1-q_3)}\,\mathrm{e}^{q_1\lambda t} \\ & + \frac{(q_2+y_+)(q_2+y_-)}{(q_2-q_1)(q_2-q_3)}\,\mathrm{e}^{q_2\lambda t} \\ & + \frac{(q_3+y_+)(q_3+y_-)}{(q_3-q_1)(q_3-q_2)}\,\mathrm{e}^{q_3\lambda t},\end{aligned} \tag{14}$$

in which the quantities q_i ($i = 1,2,3$) are now the solutions to the cubic equation

$$q^3 + 2q^2 + \left(y_+y_- + \frac{R^2}{2}\right)q + \frac{R^2}{2} = 0 \tag{15}$$

with $y_\pm = 1 \pm \beta(1 + i\omega_{\mathrm{qb}}/\lambda)$ and $R = \mathscr{R}/\lambda$ with vacuum Rabi frequency $\mathscr{R} = \alpha W$.

3. Atom-field entanglement of subsystems

In the above section, we have solved the Schrödinger equation for the case of a moving qubit inside a cavity in both non-dissipative and dissipative regimes. Before we consider the entanglement swapping phenomena, we intend to illustrate the effect of the movement of the qubits on the entanglement dynamics. Among the various measures for computing the degree of entanglement between bipartite systems, we use the linear entropy defined as[69]

$$S_{\rm A}(\theta,\phi;t)=1-{\rm Tr}\left(\hat{\rho}_{\rm A}^2\right), \tag{16}$$

in which $\hat{\rho}_{\rm A}$ is the atomic reduced density matrix for each subsystem. We notice that we have omitted the subscript i ($i=1,2$) from parameters θ and ϕ, because we are dealing with only one subsystem. The linear entropy can range between zero, corresponding to a completely pure state, and $(1-1/d)$ corresponding to a completely mixed state, in which d is the dimension of the density matrix (here $d=2$).

Using Eqs. (7) and (16), the linear entropy at any time can be derived as

$$S_{\rm A}(\theta,\phi;t)=2\left(1-|\mathscr{E}(t)|^2\right)|\mathscr{E}(t)|^2\cos^4(\theta/2), \tag{17}$$

which does not depend on parameter ϕ. It is evident that the maximum amount of linear entropy is obtained for $\theta=2m\pi$ with $m=0,1,2,\ldots$, which corresponds to the situation in which the qubit is initially in the excited state. Also, one observes that the linear entropy is zero for the qubit which is initially in the ground state (i.e., $\theta=(2m+1)\pi$ with $m=0,1,2,\ldots$). For other values of θ, the behavior of the linear entropy is the same but with different amplitudes. This leads to the natural question: how much linear entropy on average is obtained over all initial states. This provides an input-independent dynamics. This can be carried out by computing the average of linear entropy with respect to all possible input states on the surface of the Bloch sphere as

$$S_{\rm A}^{\rm av}(t)=\int S_{\rm A}(\theta,\phi;t)\,{\rm d}\Omega, \tag{18}$$

in which ${\rm d}\Omega$ is the normalized $SU(2)$ Haar measure,

$${\rm d}\Omega=\frac{1}{4\pi}\sin\theta{\rm d}\theta{\rm d}\phi. \tag{19}$$

From Eqs. (17)–(19), it can be easily shown that

$$S_{\rm A}^{\rm av}(t)=\frac{2}{3}\left(1-|\mathscr{E}(t)|^2\right)|\mathscr{E}(t)|^2. \tag{20}$$

Figure 2 illustrates the linear entropy (17) for $\theta=0$ and the average of linear entropy (20) in the weak coupling regime ($R=0.1$) for different values of velocity of the qubit. Taking a glance at these plots reveals the positive role of the movement of the qubit on the survival of the linear entropy. As is seen, by increasing the velocity of the qubit, the linear entropy reaches its maximum value at longer times. The decaying behavior of linear entropy represents a Markovian process. The maximum amount of linear entropy is obtained for $\theta=0$.

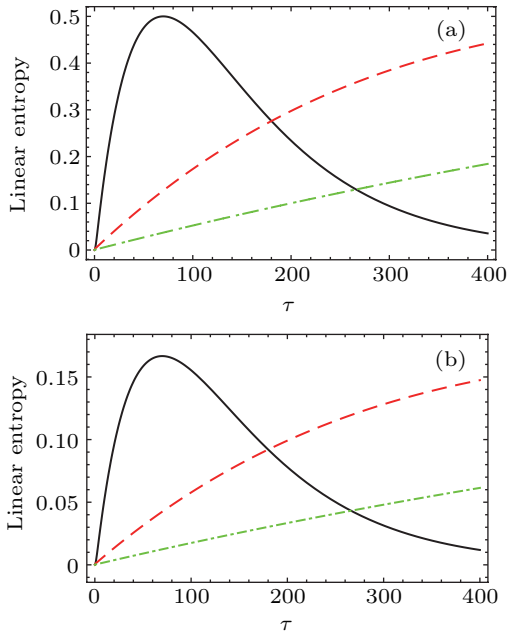


Fig. 2. Time evolution of (a) linear entropy for $\theta=0$ and (b) the average of linear entropy versus scaled time τ for weak coupling regime, i.e., $R=0.1$ with $\beta=0$ (solid black line), $\beta=2\times10^{-9}$ (dashed red line) and $\beta=4\times10^{-9}$ (dot-dashed green line). In these plots we have set $\omega_0/\lambda=1.5\times10^9$.

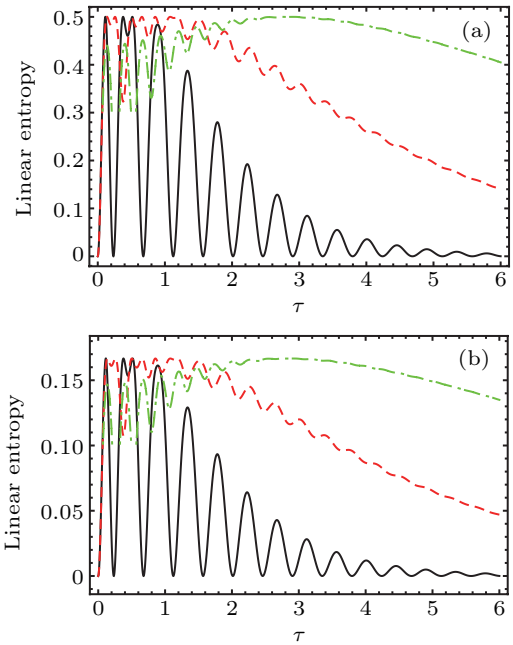


Fig. 3. Time evolution of (a) linear entropy for $\theta=0$ and (b) the average of linear entropy versus scaled time τ for strong coupling regime, i.e., $R=10$ with $\beta=0$ (solid black line), $\beta=10\times10^{-9}$ (dashed red line) and $\beta=15\times10^{-9}$ (dot-dashed green line). Again $\omega_0/\lambda=1.5\times10^9$.

In Fig. 3 we have plotted the linear entropy for $\theta=0$ and the average of linear entropy in the strong coupling regime ($R=10$) for several motion situations of the qubit. Again,

the positive role of the movement of qubit on the survival of the linear entropy is quite clear. Increasing the velocity of the qubit not only makes the linear entropy survive at longer times but also washes out the oscillatory behavior of the linear entropy. These oscillations are a sign of the non-Markovian process. Actually, in this coupling regime, the interaction between the qubit and its surrounding environment is so strong that part of the information that has been taken by the environment is fed back to the qubit. As is clear, in the presence of the movement of the qubit, the sudden death of linear entropy is no longer seen.

4. Entanglement swapping

In the above section, we examined the positive role of the movement of a single qubit on the entropy evolution of the qubit and its surrounding environment. In this section, we consider two similar but separable systems introduced in Section 2. The time evolution of each system is given in Eq. (7). As is expected, their states are separable, i.e.,

$$\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|, \tag{21}$$

in which $|\Psi(t)\rangle = |\psi(t)\rangle_1 \otimes |\psi(t)\rangle_2$. However, the qubit–qubit entangled states are more important due to their applications in quantum information processing. Therefore, it is quite logical to search a strategy to exchange the entanglement stored between qubit-field in each system into qubit-qubit and field-filed entanglement, as illustrated in Fig. 4. This can be performed by projection $|\Psi(t)\rangle$ onto one of the Bell states of the cavity fields.

Experimentally, there are many qubits inside each cavity. In this way, the qubits inside each dissipative cavity can be considered as a Bose–Einstein condensate in an optical cavity[70] in the presence of thermal noise which plays the role of the dissipation. Then, the output of the optical cavity of each node is sent to an intermediate site where a Bell-like detection is performed on these optical pairs.[70]

According to the wave function (7), one observes that there exist only the vacuum and first excited states of the field modes in each system. Therefore, it is possible to consider the following Bell-like states of the fields:[71]

$$|\psi^{\pm}\rangle_{\rm F} = \frac{1}{\sqrt{2}}\left(|\mathbf{0}\rangle_{R_1}|\mathbf{1}\rangle_{R_2} \pm |\mathbf{1}\rangle_{R_1}|\mathbf{0}\rangle_{R_2}\right), \tag{22a}$$

$$|\phi^{\pm}\rangle_{\rm F} = \frac{1}{\sqrt{2}}\left(|\mathbf{0}|_{R_1}|\mathbf{0}\rangle_{R_2} \pm |\mathbf{1}\rangle_{R_1}|\mathbf{1}\rangle_{R_2}\right), \tag{22b}$$

in which $|\mathbf{0}\rangle_{R_i}$ has been defined before and

$$|\mathbf{1}\rangle_{R_i} \equiv \sum_k \Theta_k |\mathbf{1}_k\rangle_i, \tag{23}$$

where $\sum_k |\Theta_k|^2 = 1$ with Θ_k related to the pulse shape associated with the incoming photons. The next step is to construct the projection operator $P_{\rm F} = |M\rangle_{\rm FF}\langle M|$ in which $M \in \{\psi^{\pm}, \phi^{\pm}\}$. Consequently, operating one of this projection operators onto $|\Psi(t)|$ leaves the field states in the corresponding Bell-type state and also establishes entangled atom-atom state.

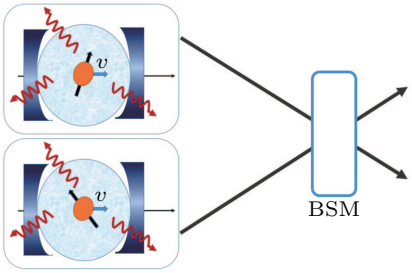


Fig. 4. Pictorial illustration of the entanglement swapping phenomenon. In each subsystem, the moving qubit is interacting with its own environment. Because of the nonperfect reflecting mirrors of the cavities, the photons can leak out of the cavities. Then a Bell state measurement is performed on these photons to establish the entanglement between the qubits.

In the continuation, we consider the following projection operator:

$$P_{\rm F}^{-} = |\Psi^{-}\rangle_{\rm FF}\langle\Psi^{-}|. \tag{24}$$

Then we have

$$P_{\rm F}^{-}|\Psi(t)\rangle = |\Psi^{-}\rangle_{\rm F} \otimes |\Psi_{\rm AA}(t)\rangle, \tag{25}$$

in which $|\Psi_{\rm AA}(t)\rangle$ is the qubit–qubit state (after normalization)

$$\begin{aligned}|\Psi_{\rm AA}(t)\rangle = \frac{1}{\sqrt{\mathscr{N}(t)}}\Big\{&X(\theta_1,\theta_2,t)\Big(|e\rangle|g\rangle - |g\rangle|e\rangle\Big)\\ &+ Y(\theta_1,\theta_2,\phi_1,\phi_2)|g\rangle|g\rangle\Big\},\end{aligned} \tag{26}$$

where the normalization coefficient reads

$$\mathscr{N}(t) = 2|X(\theta_1,\theta_2,t)|^2 + |Y(\theta_1,\theta_2,\phi_1,\phi_2)|^2. \tag{27}$$

In the above relations, we have defined

$$X(\theta_1,\theta_2,t) = \cos(\theta_1/2)\cos(\theta_2/2)\mathscr{E}(t), \tag{28a}$$

$$\begin{aligned}Y(\theta_1,\theta_2,\phi_1,\phi_2) = &\sin(\theta_1/2)\cos(\theta_2/2)\,{\rm e}^{{\rm i}\phi_1}\\ &-\sin(\theta_2/2)\cos(\theta_1/2)\,{\rm e}^{{\rm i}\phi_2}.\end{aligned} \tag{28b}$$

In what follows, we use concurrence as the figure of merit for the amount of entanglement between the two qubits. It has been defined as[63]

$$E(\hat{\rho}(t)) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{29}$$

where λ_i $(i = 1,2,3,4)$ are the eigenvalues (in decreasing order) of the matrix $\hat{\rho}_{\rm AA}\left(\sigma_1^y \otimes \sigma_2^y \hat{\rho}_{\rm AA}^{*} \sigma_1^y \otimes \sigma_2^y\right)$ with $\hat{\rho}_{\rm AA}^{*}$ the complex conjugate of $\hat{\rho}_{\rm AA}$ and $\sigma_k^y := {\rm i}(\sigma_k - \sigma_k^{\dagger})$. The concurrence varies between 0 (completely separable) and 1 (maximally entangled). For the state (26), this parameter reads

$$E(\hat{\rho}(t)) = \frac{2|X(\theta_1,\theta_2,t)|^2}{2|X(\theta_1,\theta_2,t)|^2 + |Y(\theta_1,\theta_2,\phi_1,\phi_2)|^2}. \tag{30}$$

A glance at the resulting concurrence reveals that the concurrence (30) does not depend on the shape of the incoming photons. Also, it is evident that whenever $|Y(\theta_1,\theta_2,\phi_1,\phi_2)|^2=0$, the concurrence would be independent of time and it always remains at its maximum value, i.e., 1. According to Eq. (30), it amounts to solve the following relation

$$\frac{1}{2}\big(1-\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)\big)=0. \quad (31)$$

The above relation is fulfilled with the following set of solutions:

$\theta_1=\theta_2=2n\pi$ and arbitrary values of ϕ_1 and ϕ_2 with $n=0,1,2,\ldots$;

$\theta_1=\theta_2$ and $\phi_1-\phi_2=2m\pi$, with $m=0,\pm1$.

These conditions lead to the maximally entangled Bell state (up to an irrelevant global phase)

$$|\Psi^-\rangle=\frac{1}{\sqrt{2}}(|e\rangle|g\rangle-|g\rangle|e\rangle). \quad (32)$$

On the other hand, for θ_1 or $\theta_2=(2n+1)\pi$ and arbitrary values of ϕ_1 and ϕ_2, the concurrence is always zero and we have the qubit-qubit state $|g\rangle|g\rangle$ as the stationary state.

Again, similar to the above section, we can establish an input-independent dynamics for the swapped entanglement, which is called entangling power. This is carried out by taking a statistical average over all initial states[65]

$$\mathfrak{E}(t):=\int E\left(\rho(t)\right)\mathrm{d}\mu(|\psi(0)\rangle), \quad (33)$$

where $\mathrm{d}\mu(|\psi(0)\rangle)$ is the probability measure over the submanifold of product states in $\mathbb{C}^2\otimes\mathbb{C}^2$. The latter is induced by the Haar measure of $SU(2)\otimes SU(2)$. Specifically, referring to the parametrization of (6), it reads

$$\mathrm{d}\mu(|\psi(0)\rangle)=\frac{1}{16\pi^2}\prod_{k=1}^{2}\sin\theta_k\mathrm{d}\theta_k\mathrm{d}\phi_k. \quad (34)$$

According to the above definition, the entangling power $\mathfrak{E}$ is normalized to 1. It is trivial that in this case, it lies in $[0,1]$.

Figure 5 illustrates the entangling power as a function of scaled time $\tau=\lambda t$ in the strong coupling regime (i.e., $R=10$) for different values of β. In this case, entangling power has an oscillating behavior which is a characteristic feature of non-Markovian regime. Entanglement sudden death phenomenon is clearly seen. As is observed, the entangling power exhibits an oscillatory decay behavior in the absence and presence of the movement of the qubits. However, the presence of movement of qubits makes the decay of entanglement becomes slow. This can be understood by paying attention to the fact that the entanglement between the two qubits depends directly on the entanglement between the qubit and its surrounding filed in each cavity. Therefore, the less entanglement between qubit and field (in each cavity), the less swapped entanglement between qubits. Thanks to the results presented in Section 3, we already know the positive role of the movement of qubit on the entanglement between the qubit and the cavity field. This means that, by choosing a suitable value of β, a long-living stationary entangled state between two qubits can be created.

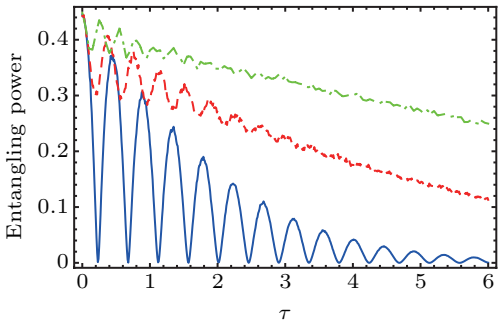


Fig. 5. Time evolution of the entangling power of the atom-atom state after BSM ($P_{\mathrm{F}}^{-}=|\Psi^-\rangle_{\mathrm{FF}}\langle\Psi^-|$) as a function of scaled time τ for strong coupling regime, i.e., $R=10$ with $\beta=0$ (solid blue line), $\beta=10\times10^{-9}$ (dashed red line) and $\beta=15\times10^{-9}$ (dot-dashed green line). We have set $\omega_0/\lambda=1.5\times10^9$.

In order to explore a deep insight into the role of movement of the qubits on the swapped entanglement, we have shown the density matrix of two qubits at the scaled time $\tau=1$ for two values of β (see Fig. 6). We have considered the initial state of the qubits to be $|\Psi_{\mathrm{AA}}(0)\rangle=|e\rangle\otimes\frac{1}{\sqrt{2}}\big(|e\rangle+|g\rangle\big)$. As is observed, the population of the state $|g,g\rangle$ (which is zero at the beginning of the interaction) is increased due to the dissipation sources in the cavities. It is evident that by increasing the value of β, the population of this state is decreased, which leads to the survival of swapped entanglement at more significant times.

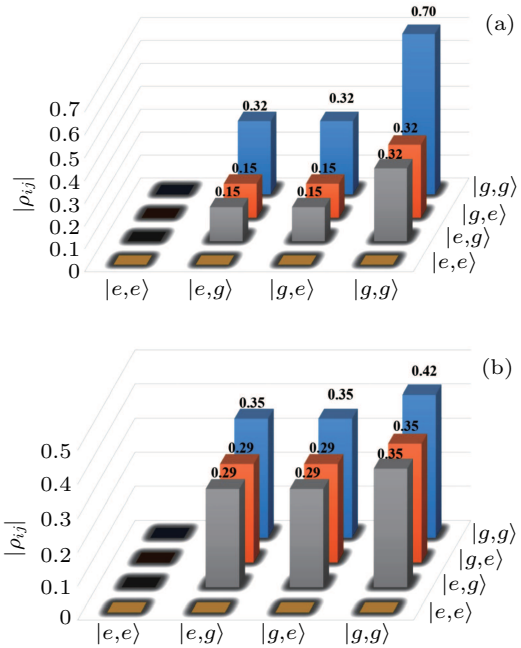


Fig. 6. Histogram of density matrix of two qubits at scaled time $\tau=1$ with initial state $|\Psi_{\mathrm{AA}}(0)\rangle=|e\rangle\otimes\frac{1}{\sqrt{2}}\big(|e\rangle+|g\rangle\big)$ for strong coupling regime, i.e., $R=10$ with (a) $\beta=0$ and (b) $\beta=15\times10^{-9}$. In these plots $\omega_0/\lambda=1.5\times10^9$.

In Fig. 7, we have illustrated the entangling power (33) as a function of scaled time $\tau = \lambda t$ in the weak coupling regime for various values of β. Again, the entangling power has decaying behavior. In this case, the oscillatory behavior is no longer observed. The positive role of the movement of the qubits is clearly apparent.

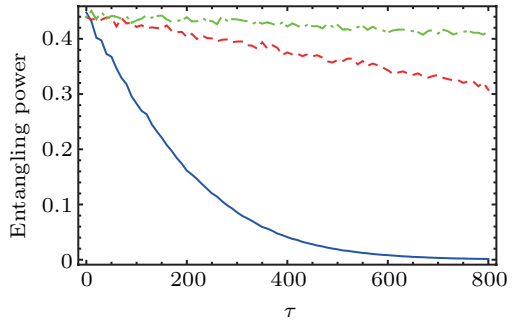


Fig. 7. Time evolution of the entangling power of the atom-atom state after BSM ($P_{\rm F}^{-} = |\Psi^{-}\rangle_{\rm FF}\langle\Psi^{-}|$) as a function of scaled time τ for weak coupling regime, i.e., $R = 0.1$ with $\beta = 0$ (solid blue line), $\beta = 2\times10^{-9}$ (dashed red line) and $\beta = 4\times10^{-9}$ (dot-dashed green line). We have set $\omega_0/\lambda = 1.5\times10^{9}$.

Finally, we take into account the effect of initial qubit state on the evolution of the swapped entanglement. In Fig. 8, we plot the time evolution of the concurrence for different initial states of the qubits. Again we consider both Markovian and non-Markovian environments. It is revealed that both coupling regimes lead to the same stationary value of entanglement. We observe the similar behavior of entanglement for different initial states. However, the amplitude of entanglement depends on the initial state of qubits.

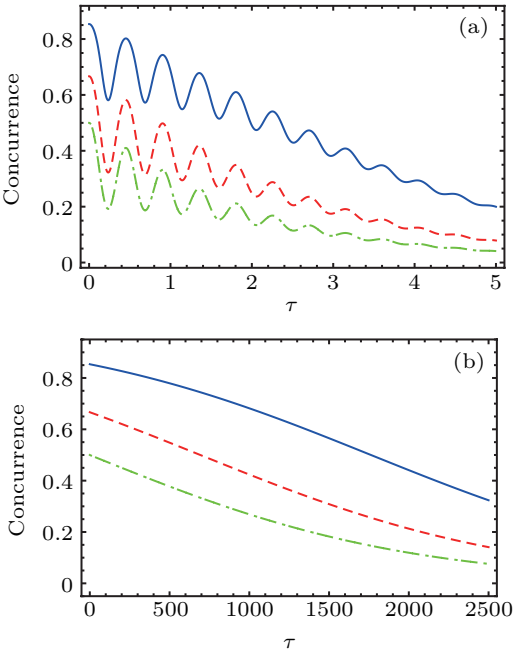


Fig. 8. Time evolution of the concurrence for different initial states of the qubits for (a) $R = 10$ and (b) $R = 0.1$. We have set $\beta = 2\times10^{-9}$, $\omega_0/\lambda = 1.5\times10^{9}$ and $\theta_1 = \frac{\pi}{2}$, $\phi_1 = 0$, $\theta_2 = \frac{\pi}{4}$, $\phi_2 = 0$ (solid blue line), $\theta_1 = \frac{\pi}{2}$, $\phi_1 = 0$, $\theta_2 = 0$, $\phi_2 = 0$ (dashed red line) and $\theta_1 = \frac{\pi}{2}$, $\phi_1 = \pi$, $\theta_2 = \frac{\pi}{4}$, $\phi_2 = 0$ (dot-dashed green line).

5. Concluding remarks

In this work, we have considered a model to study the possibility of entanglement swapping between two subsystems each contains a moving qubit inside an environment. Our model allows us to treat the environment in both weak and strong coupling regimes corresponding to bad and good cavity limits. By good cavity limit, we have meant an oscillation behavior of entanglement which is due to the memory depth of the environment. We treat the movement of qubits to be classical. Under certain conditions, we have solved the time-dependent Schrödinger equation for each subsystem.

Before considering the entanglement swapping phenomenon, we have examined the influence of the movement of qubits on the entropy evolution of subsystems. This gives us an insight into the possible role of the movement of qubits on the entanglement dynamics. Our parametrization for the initial state of the qubits in each subsystem allows us to construct an initial state independently for the entropy (and later on for the swapped entanglement). This has been carried out by taking a statistical average over all of the initial states of the qubits. The results show the entanglement between qubits and its surrounding environment due to the interaction among them. However, due to the environmental effects, the entropy has decaying behavior. This deterioration of entanglement is suppressed in the presence of movement of the qubit.

Then we turn to the problem of entanglement swapping between such two subsystems. Our goal is to swap the stored qubit-field entanglement in each subsystem into qubit-qubit and field-filed entanglements. Since the cavities are not perfect, the photons can leak out them. This allows us to perform a BSM on the photons leaving the cavities. We have obtained the analytical expression for the normalized state of qubit-qubit after BSM. We have used the concurrence parameter to quantify the amount of swapped entanglement. Several interesting and noticeable points are found. First of all, by considering a suitable Bell state for field modes, the concurrence would be independent of the shape of the incoming photons. Second, there is a set of the initial states which lead to a long-lived maximally entangled state corresponding to stationary state $|\Psi^{-}\rangle = \frac{1}{\sqrt{2}}(|e,g\rangle - |g,e\rangle)$. On the other hand, we have determined the initial states which lead to no entangled state at all. Therefore, the average of entanglement over all initial states of the qubits determines whether our model is a good entangler or not, the notion of entangling power. From the information supplied, the average of swapped entanglement is always greater than zero, which signifies that our proposed model is a good entangler. The positive influence of the movement of qubits on the swapped entanglement is clearly apparent for both weak and strong coupling regimes. Since

the velocities of the qubits are treated classically, it is possible to reach the velocities for qubits in which a nearly stationary swapped entanglement is obtained.

The present work can be considered as an extension of Ref. [72] where it has been shown that entanglement swapping in the presence of dissipation can be performed. First of all, by considering a static state of motion for qubits ($\beta = 0$), we recover the results presented in the mentioned work. Therefore, this guarantees the validity of our model. Second, it is interesting to notice that the long-lived maximally swapped entanglement (corresponding to a Bell state) (see Eq. (34)) is independent of the movement of qubits. This is exactly the result reported in Ref. [72]. In very recent work, the possibility of entanglement swapping in the presence of dissipation has been studied in Ref. [73] when the qubits are driven by an external classical laser field. Again, it is shown that the long-lived maximally swapped entanglement state can be achieved under certain conditions. Moreover, this state is independent of the effect of the classical field. Altogether, we can conclude the superiority of considering the movement of qubits, especially when the speed of qubits is reasonably large. Under this condition, the swapped entanglement remains static over sufficiently long times, a new result that was not reported in Ref. [72].

We should emphasize that our results could be used in quantum communication applications. The main idea behind these protocols is to transmit and exchange quantum information (entangled states) over long distances.[74] For instance, generating and swapping entanglement are at the heart of quantum repeater protocols. In this regard, since the environmental effects are always present, our results could boost the efficiency of such protocols. We also note that our results could be useful in the preparation of quantum states. For instance, in order to prepare an entangled state between two qubits that are located in their own distinct cavities, it is enough to detect a photon from a cavity. If we do not know that from which cavity the detected photon is coming, the qubits will be in an entangled state.

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