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Quantum fluctuation of entanglement for accelerated two-level detectors*

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It is well known that the quantum fluctuation of entanglement (QFE) between Unruh–De Witt detector (modeled by a two-level atom) is always investigated in a relativistic setting. However, both of the Unruh radiation and quantum fluctuation effects play an important role in precise measurements of quantum entanglement. In this paper, we have quantitatively analyzed how the relativistic motion affects the QFE for two entangled Unruh–De Witt detectors, one of which is accelerated and interacting with the neighbor external scalar field. Our results show that the QFE, which initially increases by the Unruh thermal noise, will suddenly decay when the acceleration reaches to a considerably large value. Therefore, the relativistic effect will lead to non-negligible QFE effect. We also find that the initial QFE (without acceleration effect) reaches its minimum value at the maximally entangled state and the separable state. More importantly, our analysis demonstrates that although the QFE has a huge decay when the acceleration is greater than ~ 0.96, the ratio of $\Delta E/C$ is still very large, due to the simultaneous decay of concurrence to a very low value. Finally, enlightened by the well-known equivalence principle, we discuss the possibility of applying the above findings to the dynamics of QFE under the influence of gravitation field.

Keywords: relativistic quantum information, quantum fluctuation, Unruh effect

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1. Introduction

The difference between classical and quantum theory lies in the fact that physical quantities in quantum mechanics are corresponded to Hermitian operator, which could directly determine the state of the quantum system.[1] Thus, the eigenvalues are the possible values of the Hermitian operator acting on the quantum state via measurements. However, one should note that the outcome of each measurement of a physical quantity should not be identical. For a physical quantity in quantum mechanics, the expected value is usually used to represent the eigenvalue of the quantity, *i.e.*, the expected value stands for the statistical mean of the measurement outcome by repeated several times measurements. As is known, the fluctuations are inevitable in practical measurement processes. More specifically, for a physical quantities A in the quantum state ψ, the fluctuation is defined as $\overline{\Delta A^2} = \overline{(\hat{A}-\overline{A})^2} = \langle\psi|(\hat{A}-\overline{A})^2|\psi\rangle$. Focusing on one of the most general quantum resources, quantum entanglement, it can be quantified by one statistical quantity or operator, von Neumann entropy.[2,3] Recently, the relations between the fluctuation of quantified entropy and entanglement has been widely studied in Refs. [4–7].

On the other hand, as one of the most important developments in modern physics, quantum entanglement has been extensively investigated in most recent years.[8–10] It is interesting to note that, the importance of quantum entanglement not only embodies the fundamental perspective of quantum information task, but also attributes to its advantages in practical aspects.[11–13] Although quantum entanglement has been achieved in many experiments, however, most of these measurements were carried out without considering the effect of acceleration. Actually, in realistic situation, the preparation of quantum system and the procession of quantum information tasks are always accompanied by accelerated effects.[14–19] In the framework of such accelerated quantum system, the Unruh effect will be generated, which indicates that quantum properties of fields are observer-dependent.[20,21] From theoretical point of view, the Unruh effect will reveal thermal radiation detected by a uniformly accelerated detector in the Minkowski vacuum and that associated with the proper acceleration of the detector. Following this direction, a number of combined analyses involving the dynamics of quantum entanglement and steering between two correlated Unruh–De Witt detectors have been performed in the literature,[10,22,23] which indicated that the type of quantum resource will be reduced by the Unruh effect, while the acceleration effect on quantum systems is non-negligible when using quantum resources to perform quantum information task.[24–28]

Inspire by the above works, in this paper we will investi-

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gate the quantum fluctuation of entanglement (QFE) for a two-level atom accelerator, which is modeled by Unruh–De Witt detectors in the relativistic setting. Compared with the global free models extensively used in many papers,[29,30] the Unruh–De Witt detector model applied in this analysis to study the behavior of QFE in a non-inertial system[31] has more advantages. On the one hand, the problems related to single-mode approximation and physically unfeasible detection of quantum correlations in the full space-time can be effectively avoided,[34] which could provide us a better understanding of quantum entanglement. On the other hand, precise measurements of quantum entanglement are highly dependent on the Unruh radiation and quantum fluctuation effects,[35] which supports a quantitative analysis of QFE in relativistic setting. In this paper, we will carry out a quantitative analysis and explore how the Unruh radiation affects the QFE.

This paper is organized as follows. In Section 2, we briefly describe the Unruh–De Witt detectors and the evolution of the prepared state under the Unruh thermal bath. In Section 3 we introduce the quantum fluctuation of entanglement. The behaviors of QFE with the detectors model in a relativistic setting are are presented in Section 4. Finally, we summarize our conclusions in Section 5.

2. Evolution of the detectors' state under relativistic motion

In this section, from the view point of quantum information, we will give a brief description of the Unruh–De Witt detectors[22] and furthermore discuss the dynamics of a pair of detectors (considering the relativistic motion of one detector). Based on the well known two-level atom system, the simplest model for one quantum system, our Unruh–De Witt detectors are modeled by a point-like two-level atoms (each atom represents a detector). However, in our analysis we also try to extend the interaction term and investigate the interaction between this two-level system and its nearby fields. Note that the detector is semiclassical, because it possesses a classical world line but its internal degree of freedom is treated quantum mechanically. In order to investigate the behavior of quantum properties, one usually assume that the detectors are initially sharing some quantum correlations between the Minkowski spacetime and observed by two observers called Alice and Rob, respectively. Alice and Rob's detectors initially are prepared for the inertial frame, then we let that Alice still keeps inertial and always be switched off, while Rob's detector interacts with the scalar field, and moves with uniform acceleration for a time duration Δ. The Rob's detector will move with its world line as

$$t(\tau) = a^{-1}\sinh a\tau,$$
$$x(\tau) = a^{-1}\cosh a\tau, \quad y(\tau) = z(\tau) = 0, \tag{1}$$

where a is the detector's proper acceleration, τ represents the detector's proper time, and (t,x,y,z) are the usual Cartesian coordinates in the Minkowski spacetime. For convenience, we employ the natural units $c = \hbar = \kappa_{\rm B} = 1$ throughout this paper.

Considering the interaction between Rob's detector and the field, the initial state of the total system can be expressed as

$$|\Psi_{t_0}^{AR\phi}\rangle = |\Psi_{AR}\rangle \otimes |0_M\rangle, \tag{2}$$

where $|\Psi_{AR}\rangle = \sin\theta|0_A\rangle|1_R\rangle + \cos\theta|1_A\rangle|0_R\rangle$ represents the initial state shared by Alice's (A) and Rob's (R) detectors, θ denotes the initial state parameter, and $|0_M\rangle$ represents the contribution of the external scalar field in Minkowski vacuum. Now the total Hamiltonian of the system with the scalar field can be written as

$$H_{AR\phi} = H_A + H_R + H_{KG} + H_{\rm int}^{R\phi}, \tag{3}$$

where KG (Klein–Gordon) denotes the scalar field, while $H_A = \Omega A^\dagger A$ and $H_R = \Omega R^\dagger R$ are Hamiltonians for the creation and annihilation operators (Ω is the energy gap of the detectors). The interaction term, $H_{\rm int}^{R\phi}(t)$, which describes how Rob's detector is coupled with the external massless scalar field ϕ, satisfies the Klein–Gordon (KG) equation[36]

$$H_{\rm int}^{R\phi}(t) = \varepsilon(t)\int_{\Sigma_t} {\rm d}^3\boldsymbol{x}\sqrt{-g}\phi(x)\left[\chi(\boldsymbol{x})R + \overline{\chi}(\boldsymbol{x})R^\dagger\right], \tag{4}$$

where $g \equiv \det(g_{ab})$, g_{ab} is the Minkowski metric, and $\boldsymbol{x}$ is the coordinates defined on the Cauchy surface $\sum_t = {\rm const}$, which is associated with the time-like isometries followed by the qubits. Meanwhile, $\varepsilon(t)$ represents a real-valued switching function, which keeps the detector switched on smoothly for a finite amount of proper time Δ. A point-like coupling function, $\chi(\boldsymbol{x}) = (\kappa\sqrt{2\pi})^{-3}\exp(-\boldsymbol{x}^2/2\kappa^2)$, guarantees that the space-localized detector interacts only with the field in a neighborhood of its world line. For simplicity, we take the detector-field free Hamiltonian as $H_0 = H_A + H_R + H_{KG}$, based on which the total Hamiltonian can be rewritten as

$$H_{AR\phi} = H_0 + H_{\rm int}. \tag{5}$$

Now working with the interaction term, we turn the state into the interaction representation labeled by I and rewrite the final state $|\Psi_t^{AR\phi}\rangle$ at the time of $t = t_0 + \Delta$ as

$$|\Psi_t^{AR\phi}\rangle = T\exp\left[-{\rm i}\int_{t_0}^{t}{\rm d}tH_{\rm int}^I(t)\right]|\Psi_{t_0}^{AR\phi}\rangle, \tag{6}$$

where T is the time-ordering operator, the term $H_{\rm int}^I(t) = U_0^\dagger(t)H_{\rm int}(t)U_0(t)$, and $U_0(t)$ is the unitary evolution operator associated with the Hamiltonian H_0. The dynamics of the atom–field system at $t = t_0 + \Delta$ can be calculated by the first order of perturbation over the coupling constant ε.[22] Based on the dynamic evolution described by the Hamiltonian given by Eq. (3), the final state $|\Psi_t^{AR\phi}\rangle$ could be expressed as[23,31]

$$|\Psi_t^{AR\phi}\rangle = [I - i(\phi(f)R + \phi(f)^\dagger R^\dagger)]|\Psi_{t_0}^{AR\phi}\rangle, \tag{7}$$

where I is the identity matrix with the same dimension of $|\Psi_{t_0}^{AR\phi}\rangle$, and the operator

$$\begin{aligned}\phi(f) &\equiv \int \mathrm{d}^4x\sqrt{-g}\chi(x)f \\ &= \mathrm{i}\left[a_{RI}(\overline{uEf}) - a_{RI}^{\dagger}(uEf)\right]\end{aligned} \tag{8}$$

is the distribution function corresponding to the external scalar field. Here Ef is approximately a positive-frequency solution of the scalar field,[23,31] while the operator u is an positive-frequency solution of the Klein–Gordon equation, with respect to the time-like isometry. Therefore, combined with the initial states [Eq. (2) and Eq. (7)], the final state of the total system can be expressed in terms of the Rindler operators ($a_{RI}^{\dagger}$ and a_{RI}), *i.e.*,

$$\begin{aligned}|\Psi_t^{AR\phi}\rangle = |\Psi_{t_0}^{AR\phi}\rangle &+ \sin\theta|0_A\rangle|0_R\rangle\otimes(a_{RI}^{\dagger}(\lambda)|0_M\rangle) \\ &+ \cos\theta|1_A\rangle|1_R\rangle\otimes(a_{RI}(\overline{\lambda})|0_M\rangle),\end{aligned} \tag{9}$$

where $\lambda = -uEf$, while the creation operator $a_{RI}^{\dagger}(\lambda)$ and annihilation operator $a_{RI}(\overline{\lambda})$ are defined in the Rindler region I. It should be pointed out that the relation between these two sets of operators are[23,31]

$$a_{RI}(\overline{\lambda}) = \frac{a_M(\overline{F_{1\Omega}}) + \mathrm{e}^{-\pi\Omega/a}a_M^{\dagger}(F_{2\Omega})}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}, \tag{10}$$

$$a_{RI}^{\dagger}(\lambda) = \frac{a_M^{\dagger}(F_{1\Omega}) + \mathrm{e}^{-\pi\Omega/a}a_M(\overline{F_{2\Omega}})}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}, \tag{11}$$

with

$$F_{1\Omega} = \frac{\lambda + \mathrm{e}^{-\pi\Omega/a}\lambda\circ w}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}, \quad F_{2\Omega} = \frac{\overline{\lambda\circ w} + \mathrm{e}^{-\pi\Omega/a}\overline{\lambda}}{(1-\mathrm{e}^{-2\pi\Omega/a})^{1/2}}.$$

Note that $w(t,x) = (-t,-x)$ is a wedge reflection isometry that reflects λ (defined in the Rindler region I) into $\lambda\circ w$ (in the other region II).[31–33]

With the aim of investigating the evolution of the detectors' states after interacting with the field, the part of the external field $\phi(f)$ should be traced. Then we obtain final matrix between Alice and Rob detectors

$$\rho_t^{AR} = \begin{pmatrix} \eta & 0 & 0 & 0 \\ 0 & 2\mu\sin^2\theta & \mu\sin 2\theta & 0 \\ 0 & \mu\sin 2\theta & 2\mu\cos^2\theta & 0 \\ 0 & 0 & 0 & \upsilon \end{pmatrix}, \tag{12}$$

where the parameters μ, υ, and η respectively take the form of

$$\begin{aligned}\mu &= \frac{1-q}{2(1-q)+2\nu^2(\sin^2\theta+q\cos^2\theta)}, \\ \upsilon &= \frac{\nu^2 q\cos^2\theta}{(1-q)+\nu^2(\sin^2\theta+q\cos^2\theta)}, \\ \eta &= \frac{\nu^2\sin^2\theta}{(1-q)+\nu^2(\sin^2\theta+q\cos^2\theta)}.\end{aligned} \tag{13}$$

In addition, the expression of $2\mu+\upsilon+\eta = 1$ and the basis of $|0_A\rangle\otimes|0_R\rangle, |0_A\rangle\otimes|1_R\rangle, |1_A\rangle\otimes|0_R\rangle$, and $|1_A\rangle\otimes|1_R\rangle$ are applied in this analysis. Here the acceleration parameter a is parameterized as $q \equiv \mathrm{e}^{-2\pi\Omega/a}$, and the combined coupling parameter ν satisfies $\nu^2 \equiv ||\lambda||^2 = (\varepsilon^2\Omega\Delta)/(2\pi)\,\mathrm{e}^{-\Omega^2\kappa^2}$.[23,31] We remark here that, the conditions of $\Omega^{-1} \ll \Delta$ and $\nu^2 \ll 1$ should be satisfied in the detector model, for the validity of the perturbation method.

3. Quantum fluctuations of entanglement

It is worth noting that, quantum fluctuation determined in terms of the von Neumann entropy operator is a stochastic quantity, the fluctuation of which will be taken int account and discussed in this section. In general, one can use entropy operator $\hat{S}$, or the so-called entanglement entropy operator $\hat{E}$ (see Refs. [37,38] for more details) to quantify the quantum entanglement. In such a case, the entanglement entropy operator $\hat{E}$ is equivalent to the entropy operator $\hat{S}$, *i.e.*,

$$\hat{E} = \hat{S} = -\log_2\rho_A, \tag{14}$$

where $\rho_A = \mathrm{Tr}_B(\rho_{AB})$ is the reduced density operator, with ρ_{AB} denoting the density matrix operator for an arbitrary pure bipartite system. Hereafter one can define the quantum fluctuation of entanglement through the entropy operator

$$\Delta E^2 = \left(\langle\hat{E}^2\rangle - (\langle\hat{E}\rangle)^2\right). \tag{15}$$

Following the detailed QFE expression for arbitrary bipartite systems in both pure and mixed state,[38] we employ the von Neumann entropy as a entanglement measurement of a pure bipartite state

$$E = -\mathrm{Tr}(\rho_A\log_2\rho_A) = -\mathrm{Tr}(\rho_B\log_2\rho_B), \tag{16}$$

where ρ_B represents the reduce density matrix by trace out subsystem A. Actually, the mean value of entropy operator $\hat{E}$ is equivalent to the von Neumann entropy, *i.e.*, $\langle\hat{E}\rangle = E$. For the mixed state with the density matrix in a diagonal form $\xi = \sum_i p_i|\psi_i\rangle\langle\psi_i|$, the definition of the von Neumann entropy changes as follows:

$$E = -\sum_i p_i\log_2 p_i, \tag{17}$$

where p_i is the eigenvalue of the density matrix ξ.

Moreover, in the following analysis we also focus on the concurrence (C), which has been widely applied in entanglement measurements in arbitrary bipartite systems,[39] to quantify quantum entanglement without statistical fluctuation (since concurrence is not dependent on entropy). Thus, the calculation of the entanglement fluctuation will reduce to the determination of the concurrence. Now the fluctuation of quantum entanglement with the arbitrary bipartite system is quantified as[40,41]

$$\Delta E = C\log_2\left[\frac{1}{C}\left(1+\sqrt{1-C^2}\right)\right]. \tag{18}$$

It is worthwhile to note that, different from the case for a pure state in the bipartite system (with the concurrence of $C = 2\sqrt{\det \rho_A} = 2\sqrt{\det \rho_B}$), the concurrence for mixed states takes slightly complicated form. More specifically, based on the spin flip operation defined by[39,42]

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{19}$$

the concurrence of a bipartite system can be written as

$$C = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}, \quad \lambda_i \geq \lambda_{i+1} \geq 0. \tag{20}$$

where λ_i is the eigenvalues with the matrix of $\rho\tilde{\rho}$.

4. Behaviors of QFE with the detectors model in a relativistic setting

In this section we will study the behaviors of QFE under the influence of the Unruh radiation. With the initial state of the system given in Eq. (2), for any nonzero θ, the detector A shares initial quantum correlation between detector R. Then Rob's detector is accelerated for a time duration Δ with constant acceleration and influenced by the Unruh thermal bath. We are interested in how the quantum fluctuations of entanglement is affected by the relativistic motion of the detectors, specially, the final mixed state between a pair of Unruh–De Witt detectors (with the acceleration of one detector given by Eq. (12)). With the expression of the QFE [Eqs. (18) and (20)], one could derive the eigenvalues of the R density matrix, *i.e.*,

$$\lambda_1 = \lambda_2 = \eta\upsilon, \quad \lambda_3 = 4\mu^2 \sin^2\theta, \quad \lambda_4 = 0, \tag{21}$$

where μ, υ, and η are expressed in Eq. (13).

It is interesting to understand whether the presence of the detector's acceleration will change the quantum fluctuation of entanglement (QFE). In Fig. 1, we plot the QFE betweens Alice and Rob as a function of the detector acceleration parameter q, for three initial state entanglement parameters (θ). The effective coupling parameter is fixed at $\nu = 0.05$. On the one hand, it is clearly shown that the QFE, which first increases with the Unruh thermal radiation, will suddenly decays when the acceleration parameter approaches 0.96. On the other hand, we also notice here that, as is illustrated in Fig. 1, the initial QFE ($q = 0$) is highly dependent on the initial parameter. More importantly, when the effective coupling parameter is fixed, the initial QFE of the system (without acceleration effect) will reach its minimum value at the maximally entangled state and the separable state. For the maximally entangled or the separable state, QFE seems difficult to generate. From the point of the definition of QFE, concurrence has one and zero value corresponding to the separable or the maximally entangled state, respectively, which causes QFE to be zero. Such tendency, *i.e.*, maximally entangled state will generate less QFE, strongly indicates the possibility of making use of quantum entanglement to achieve quantum information tasks. As a final comment, it is interesting to note that the QFE cannot be ignored with the relativistic motion, because the QFE is very large with extremely large acceleration, as can be seen from Fig. 1.

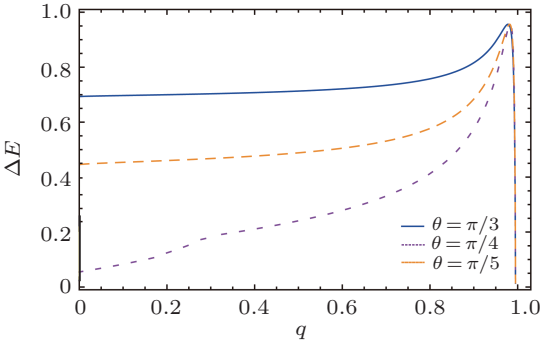


Fig. 1. The QFE ΔE as functions of the acceleration parameter q. The initial state entanglement parameter θ is fixed at $\theta = \pi/3$ (blue solid line), $\theta = \pi/4$ (orange dotted line), and $\theta = \pi/5$ (violet dotted line) with the effective coupling parameter $\nu = 0.05$.

Here, we are also interested in the dynamics of QFE for varying initial parameters, which determines the degree of quantum entanglement in our analysis ($\theta = \pi/4$ corresponds to the maximally entangled state). In Fig. 2, we plot the change of the QFE with the initial state parameter (for fixed effective coupling parameter $\nu = 0.05$) in the initial state ($q = 0$), final state I ($q = 0.5$) and final state II ($q = 0.9$). Our results show that the QFE reaches its maximum at the initial parameter $\theta = \pi/8, 3\pi/8$, while its minimum value is respectively determined at $\theta = 0$, $\pi/4$, and $\pi/2$. More interestingly, the oscillatory behavior of QFE (with a period of $\pi/2$) for different initial parameters is revealed in this paper. Such tendency can also be seen from the behavior of ΔE as a function of θ, in term of different q.

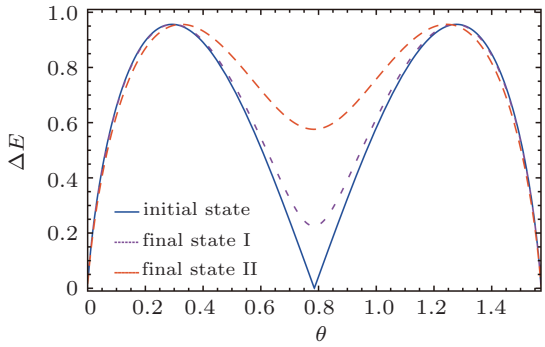


Fig. 2. The QFE ΔE of the initial and final states as a function of the initial entanglement parameter θ. The effective coupling parameter is fixed at $\nu = 0.05$ for final state I with $q = 0.5$ (violet dotted line) and for final state II with an extremely large acceleration $q = 0.8$ (orange dotted line). The initial state is denoted as blue solid line.

In order to obtain a better understanding of the effect of statistical fluctuation in entanglement, we illustrate in Fig. 3

the ratio between the QFE and the concurrence $\Delta E/C$ in terms of the Rob's acceleration parameter q. We emphasize that, the concurrence can be effectively used to quantify the quantum entanglement without statistical fluctuation, given the independence between the concurrence and entropy. By analyzing the behavior of the concurrence (C), the QFE (ΔE), one could clearly see the effect of the acceleration parameter on the ratio between concurrence and entanglement. Notice that the ratio of $\Delta E/C$ gradually increases with increasing Rob's acceleration, which suggests that the Unruh thermal radiation will inevitably makes quantum entanglement degeneration and concurrently induces quantum fluctuation of the entanglement. In addition, our analysis demonstrates that although QFE has a huge decay when the acceleration q is larger than ~ 0.96, the ratio of $\Delta E/C$ is still very large, due to the simultaneous decay of concurrence to a very low value. Therefore, concerning the realization of out quantum information task in relativistic setting, one should try to understand the background mechanism of the loss of quantum entanglement, and furthermore control the Unruh effect in the case of lower acceleration.

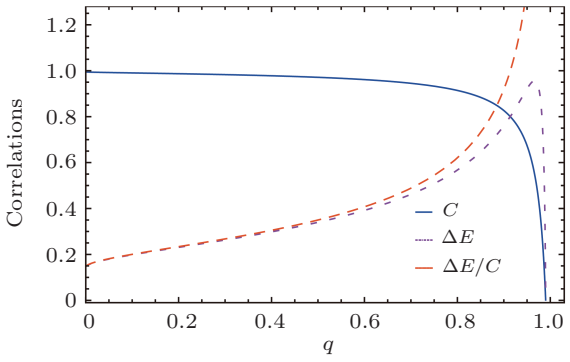


Fig. 3. The concurrence (C), the QFE (ΔE), and the ratio between concurrence and entanglement ($\Delta E/C$) as a function of the detector's acceleration parameter q. The effective coupling parameter is fixed at $\nu = 0.05$ for the initial parameter $\theta = \pi/4$.

5. Conclusions

The subject of quantum entanglement continues to be one of great importance in modern physics. Over the past decades, many of the studies in this field have concentrated on the realization of quantum entanglement without considering the effect of acceleration. However, in realistic situation, the preparation of quantum system and the procession of quantum information tasks are always accompanied by accelerated effects. In the framework of such accelerated quantum system, the Unruh effect will be generated, which indicates that quantum properties of fields are observer-dependent. Focusing on one of the most general quantum resources, quantum entanglement, it can be quantified by one statistical quantity or operator, von Neumann entropy. However, quantum entanglement has fluctuation under the description of von Neumann entropy. In this paper, we have investigated the dynamic of quantum fluctuation of entanglement (QFE) with two entangled Unruh-De Witt detectors(modeled by a two-level atom), one of which is accelerated and interacting with the neighbor external scalar field. Here we summarize our main conclusions in more details.

(i) Firstly, we find that the QFE initially increases with the Unruh thermal radiation and then suddenly decays when the acceleration reaches to $q \sim 0.96$, which indicates that QFE cannot be ignored when the relativistic motion is taken into consideration. It is found that the initial QFE ($q = 0$) is highly dependent on the initial parameter. More specifically, the initial QFE of the system (without acceleration effect) will reach its minimum value at the maximally entangled state and the separable state, which means that maximally entangled state will bring less QFE. Such findings strongly indicate the possibility of making use of quantum entanglement to achieve quantum information tasks.

(ii) Focusing on the dynamics of QFE for varying initial parameters, our results show that the QFE reaches its maximum at the initial parameter $\theta = \pi/8$, $3\pi/8$, while its minimum value is respectively determined at $\theta = 0$, $\pi/4$, $\pi/2$. The investigation of ΔE as a function of θ indicates that, the QFE between Alice and Rob's detectors will exhibit apparent oscillatory behavior with a period of $\pi/2$, in terms of different value for the initial parameters,

(iii) In order to quantify the the effect of statistical fluctuation on entanglement, we employ the ratio between QFE and quantum entanglement to describe this effect, in which quantum entanglement is described here in terms of concurrence. Our findings indicate that the ratio of $\Delta E/C$ gradually increases with increasing Rob's acceleration, which suggests that the Unruh thermal radiation will inevitably makes quantum entanglement degeneration and concurrently induces quantum fluctuation of entanglement. In addition, our analysis demonstrates that although QFE has a huge decay when the acceleration q is larger than ~ 0.96, the ratio of $\Delta E/C$ is still very large, due to the simultaneous decay of concurrence to a very low value.

(iv) Finally, with the rapid developments in both quantum technology and quantum communication, it is possible to achieve quantum entanglement by implementing quantum tasks with relativistic motion in the near future. Considering the fact that realistic quantum systems always exhibit gravitational and relativistic features, our analyses in this paper can be extended to the investigation of the dynamics of QFE, under the influence of gravitation field. This is supported by General Relativity, due to the equivalence principle that states all accelerated reference frames possess a gravitational field.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press) pp. 1–33

[2] Amico L, Fazio R, Osterloh A and Vedral V 2008 *Rev. Mod. Phys.* **80** 517
[3] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[4] Song H F, Rachel S and Hur K Le 2010 *Phys. Rev. B* **82** 012405
[5] Klich I and Levitov L 2009 *Phys. Rev. Lett.* **102** 100502
[6] Song H F, Rachel S, Flindt C, Klich I, Laflorencie N and Hur K Le 2012 *Phys. Rev. B* **85** 035409
[7] Calabrese P, Mintchev M and Vicari E 2012 *Europhys. Lett.* **98** 20003
[8] Wootters W K 2001 *Quantum Inf. Comput.* **11** 27
[9] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[10] Landulfo A G S and Matsas G E A 2009 *Phys. Rev. A* **80** 032315
[11] Dik B, Pan J W, Klaus M, Manfred E, Harald W and Anton Z 1997 *Nature* **390** 575
[12] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221
[13] Pan J W, Dik B, Harald W and Anton Z 1998 *Phys. Rev. Lett.* **80** 3891
[14] Alsing P M and Milburn G J 2003 *Phys. Rev. Lett.* **91** 180404
[15] Fuentes-Schuller I and Mann R B 2005 *Phys. Rev. Lett.* **95** 120404
[16] Downes T G, Fuentes I and Ralph T C 2011 *Phys. Rev. Lett.* **106** 210502
[17] Friis N 2013 *Phys. Rev. Lett.* **110** 113602
[18] Ostapchuk D C M, Lin S Y, Mann R B and Hu B L 2012 *JHEP* **07** 72
[19] Landulfo A G S and Matsas G E A 2009 *Phys. Rev. A* **80** 032315
[20] Unruh W G 1976 *Phys. Rev. D* **14** 870
[21] Crispino L 2008 *Rev. Mod. Phys.* **80** 787
[22] Unruh W G and Wald R M 1984 *Phys. Rev. D* **29** 1047
[23] Liu T H, Wang J C, Jing J L and Fan H 2018 *Annals of Physics* **390** 334
[24] Yin J, *et al.* 2017 *Science* **356** 1140
[25] Vallon G, Bacco D, Dequal D, Gaiarin S, Luceri V, Bianco G and Villoresi P 2015 *Phys. Rev. Lett.* **115** 040502
[26] Jofre M, *et al.* 2011 *Opt. Express* **19** 3825
[27] Liu T H, Jing J L and Wang J C 2018 *Adv. Quantum Technol.* **1** 1800072
[28] Liu T H, Wu S M and Cao S 2019 *Laser Phys. Lett.* **16** 095201
[29] Alsing P M, McMahon D and Milburn G J 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 8
[30] Schuller I F and Mann R B 2005 *Phys. Rev. Lett.* **95** 120404
[31] Céleri L C, Landulfo A G S, Serra R M and Matsas G E A 2010 *Phys. Rev. A* **81** 062130
[32] Wald R M 1990 *Vacuum States in Space-Times with Killing Horizons in Quantum Mechanics in Curved Space-Time* (NATO Advanced Study Institutes Series) Vol. B230
[33] Wald R M 1994 *Quantum Field Theory in Curved Spacetimes and Black Hole Thermodynamics* (Chicago: The University of Chicago Press) pp. 1–336
[34] Bruschi D E, Louko J, Martín-Martínez E, Dragan A and Fuentes I 2010 *Phys. Rev. A* **82** 042332
[35] Hao X and Wu Y Z 2016 *Annals of Physic.* **372** 110
[36] Penrose R and Rindler W 1987 *Spinors and Space-Time: Two-Spinor Calculus and Relativistic Fields* (Cambridge: Cambridge University Press) pp. 31–55
[37] Gadelha A L, Marchioro D Z and Nedel D L 2006 *Phys. Lett. B* **639** 383
[38] Feldman E B and Yurishchev M A 2009 *JETP Lett.* **90** 70
[39] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[40] Villaruel A M B and Paraan F N C 2016 *Phys. Rev. A* **94** 022323
[41] Furman G, Goren S D, Meerovich V and Sokolovsky V 2015 *Phys. Scr.* **90** 105301
[42] Coffman V, Kundu J and Wootters W K 2000 *Phys. Rev. A* **61** 052306