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# The effect of phase fluctuation and beam splitter fluctuation on two-photon quantum random walk*

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In the optical quantum random walk system, phase fluctuation and beam splitter fluctuation are two unavoidable decoherence factors. These two factors degrade the performance of quantum random walk by destroying coherence, and even degrade it into a classical one. We propose a scheme for the simulation of quantum random walk using phase shifters, tunable beam splitters, and photodetectors. This proposed scheme enables us to analyze the effect of phase fluctuation and beam splitter fluctuation on two-photon quantum random walk. Furthermore, it is helpful to guide the control of phase fluctuation and beam splitter fluctuation in the experiment.

**Keywords:** quantum random walk, decoherence, random walk

**PACS:** 05.40.Fb, 05.40.–a, 03.65.Yz **DOI:** 10.1088/1674-1056/ab6654

## 1. Introduction

Classical random walk is widely used in classical algorithms and plays an important role in many areas, such as polymer chemistry, biology, computer science, economics, and so on.[1–3] By endowing the walker with quantum properties, quantum interference leads to a new type of walk, named quantum random walk, which diffuses much faster than its corresponding classical counterpart.[4] Quantum random walk quickly becomes a research hotspot with two reasons: on the one hand, quantum random walk has richer dynamical properties than the classical one for an interest in basic science;[5–10] on the other hand, quantum random walk may provide a similar insight for quantum algorithms, which is equivalent to classical random walk for classical algorithms.[11,12] It has been proven that quantum random walk allows the realization of universal quantum computation[13] and the speed-up of search algorithms.[14] There have been several suggestions for a practical implementation of quantum random walk, using waveguide structures,[15] single photon systems,[16] two-photon systems,[17,18] trapped atoms,[19] trapped ions,[20–22] and nuclear magnetic resonance (NMR) systems.[23,24] More recent works on the implementation of quantum walk are reported, based on a photonic chip,[25] cross-Kerr nonlinearity,[26] dense coding of coin operators,[27] spin–orbital angular momentum space of photons,[28] and a lattice with twisted photons,[29] even quantum walk with one variable absorbing boundary.[30]

All practical implementations of various quantum random walk might be destroyed by decoherence in a real environment. Coherence is the essence of the difference between quantum random walk and classical random walk. With the increase of the degree of decoherence, the distribution of a totally quantum random walk can change into a classical Gaussian distribution.[31,32] Recently, decoherence in quantum random walk has been discussed by many researchers. Brun *et al.* studied quantum random walk with decoherent coins, through increasing the number of coins which drive the walk and using a new coin at each step.[33,34] Broome *et al.* controlled a relative angle between two beam displacers to adjust photon's temporal and spatial mode overlap, in order to discuss tunable decoherence in quantum random walk.[35] Romanelli *et al.* investigated the quantum walk when decoherence was introduced through random failures in the links between neighboring sites.[36] Quantum random walk is highly sensitive to decoherence with the increase of the number of steps.[37] This is because the effects of decoherence will be accumulated, and therefore decoherence research is more important for the application of large-scale quantum random walk system in the future. Photons are well known to be extremely powerful for carrying and manipulating quantum information. In this paper, phase shifters and tunable beam splitters are used to simulate two main kinds of decoherence factors, phase fluctuation and beam splitter fluctuation (erratic transmittivity, *i.e.*, coin fluctuation). We in detail discuss the effect of these two kinds of decoherence factors on two-photon quantum random walk. This is of great significance to both basic theory and experimental research. On the one hand, the role of two kinds of decoherence factors in two-photon quantum random walk is clarified theoretically; on the other hand, it is helpful to preserve quantum properties in the experiment of two-photon quantum random walk by controlling two main kinds of decoherence

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factors.

## 2. Theoretical analysis

Tunable decoherence two-photon quantum random walk system scheme is composed of a series of phase shifters, tunable beam splitters and detectors, as shown in Fig. 1. Two entangled photons $|\psi_{\text{in}}\rangle = (|H\rangle|H\rangle + |V\rangle|V\rangle)/\sqrt{2}$ are used as input signals and enter into this system from the above two ports. Beam splitter implements the quantum coin operation, deciding whether the walker moves left or right in each step. Each beam splitter is randomly adjustable with erratic transmittivity, in order to simulate beam splitter fluctuation $T$ that can be described as Gaussian distribution $(\mu_{\text{BS}}, \sigma_{\text{BS}})$. Besides, phase fluctuation is introduced by phase shifters placed at each beam splitter's input port, and phase fluctuation $\Phi$ can be described as Gaussian distribution $(\mu_{\text{phase}}, \sigma_{\text{phase}})$.

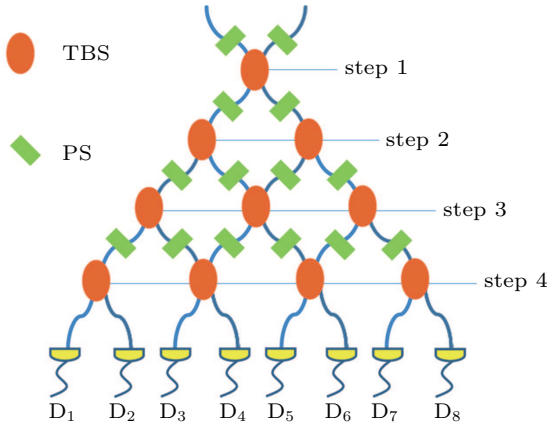



**Fig. 1.** Two-photon quantum random walk system scheme with tunable beam splitters and phase shifters. TBS: tunable beam splitter; PS: phase shifter; $D_1$–$D_8$: detectors.

As is well known, a perfect beam splitter (BS) can be represented as a $2\times 2$ matrix $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix}$. Here considering two main kinds of decoherence factors (beam splitter fluctuation $T$ and phase fluctuation $\Phi$), the imperfect BS $2\times 2$ matrix is

$$U=\begin{pmatrix} \sqrt{T}\exp(\Phi \mathrm{i}) & \sqrt{1-T}\exp[(\Phi+\pi/2)\mathrm{i}] \\ \sqrt{1-T}\exp[(\Phi+\pi/2)\mathrm{i}] & \sqrt{T}\exp(\Phi \mathrm{i}) \end{pmatrix}. \tag{1}$$

After the first beam splitter, the output results of the first step is obtained as $|\psi_1\rangle = U|\psi_{\text{in}}\rangle$. Then the output results of the $n$-th step can be got through the iterating method

$$|\psi_n\rangle = [U \otimes I(n)]|\psi_{n-1}\rangle. \tag{2}$$

The first term of this formula is the tensor product of imperfect BS $2\times 2$ matrix $U$ and $n$-order unit matrix. The imperfect BS $2\times 2$ matrix $U$ is expanded into a $2n\times 2n$ matrix, which represents the total effect of $n$ imperfect BS on the $n$-th step; The second term is the input states of the $n$-th step. When $n>1$, considering two outermost beam splitters of each step of two-photon quantum random walk system, only one port has input and the other port is idle. Two idle ports on either side can be treated as the vacuum state $|0\rangle$. Thus according to $|\psi_n\rangle$, the probability distribution of the $n$-th step is obtained $P(n) = \langle\psi_n|\psi_n\rangle$. Next, the effects of phase fluctuation and beam splitter fluctuation on two-photon quantum random walk are in detail analyzed.

## 3. Simulation results and analysis

First, we control each phase shifter to generate independent phase fluctuation $\Phi$ which satisfies Gaussian distribution $(\mu_{\text{phase}}, \sigma_{\text{phase}})$. The mean value is $\mu_{\text{phase}} = 0$, and the standard deviation $\sigma_{\text{phase}}$ are 0, $0.1\pi$, $0.2\pi$, $0.3\pi$, respectively. Their coincidence probability distribution results are shown in Figs. 2(a), 2(c), 2(e), 2(g). Here coincidence probability of any point $(x, y)$ is the probability of one photon at position $x$ and one photon at position y. Figures 2(b), 2(d), 2(f), 2(h) are photon distributions of panels (a), (c), (e), (g), respectively. When $\sigma_{\text{phase}} = 0$, two-photon quantum random walk shows typical quantum characteristics. At this time, photons diffuse rapidly, and two discrete peaks appear. With the increase of phase fluctuation, it can be seen that phase fluctuation gradually weakens quantum characteristics, and the coincidence probability distribution and photon distribution are gradually close to the center. When $\sigma_{\text{phase}} = 0.3\pi$, it is almost completely degenerated to Gaussian distribution of the classical random walk. This shows that the fundamental reason why quantum random walk is different from classical random walk is interference.[38] The interference requires a stable phase environment, and however random phase fluctuation will degrade the interference effect rapidly, making quantum random walk degenerate into classical random walk.

In order to quantitatively analyze the effect of phase fluctuation on two-photon quantum random walk, we calculate the photon distribution variance and the coincidence probability distribution similarity, as shown in Figs. 3(a) and 3(b), respectively.

Photon distribution variance $v = \sum_i P_i |x-\bar{x}|^2$,[33,34,39] where $x$ is the position of the triggered detector, and $\bar{x}$ is the mean position, which is $\bar{x} = \sum_i P_i \cdot x.P_i$ is photon distribution probability of the $i$-th position (detector). Photon distribution variance directly shows the diffusion rates of 'walker' photons. In Fig. 3(a), photon distribution variance decreases gradually with the increase of phase fluctuation, that is to say the diffusion rates of 'walker' photons decrease gradually. When $\sigma_{\text{phase}} \geqslant 0.3\pi$, photon distribution variance remains almost unchanged, and this is because it basically approaches classical random walk. Besides, figures 3(b) shows coincidence probability distribution similarity with different phase

fluctuation. Coincidence probability distribution similarity $S = \left(\sum_{j,k}\sqrt{P_{j,k}\cdot P'_{j,k}}\right)^2 / \left(\sum_{j,k}P_{j,k}\cdot\sum_{j,k}P'_{j,k}\right)$,[40,41] where $P_{j,k}$ is coincidence probability distribution with no phase fluctuation $\sigma_{\text{phase}} = 0$ and $P'_{j,k}$ is coincidence probability distribution with phase fluctuation $\sigma_{\text{phase}} \neq 0$. Coincidence probability distribution similarity directly shows the effect of phase fluctuation on coincidence probability distribution of quantum random walk. It can be seen that coincidence probability distribution similarity decreases gradually with the increase of phase fluctuation. When $\sigma_{\text{phase}} \geqslant 0.3\pi$, coincidence probability distribution similarity also remains unchanged. This change point $\sigma_{\text{phase}} = 0.3\pi$ is in accordance with the one of photon distribution variance, which shows it basically approaches classical random walk with $\sigma_{\text{phase}} = 0.3\pi$. In addition, from Figs. 3(a) and 3(b), it can be seen that the larger the step of the quantum random walk is, the greater the effect of the phase fluctuation will be.

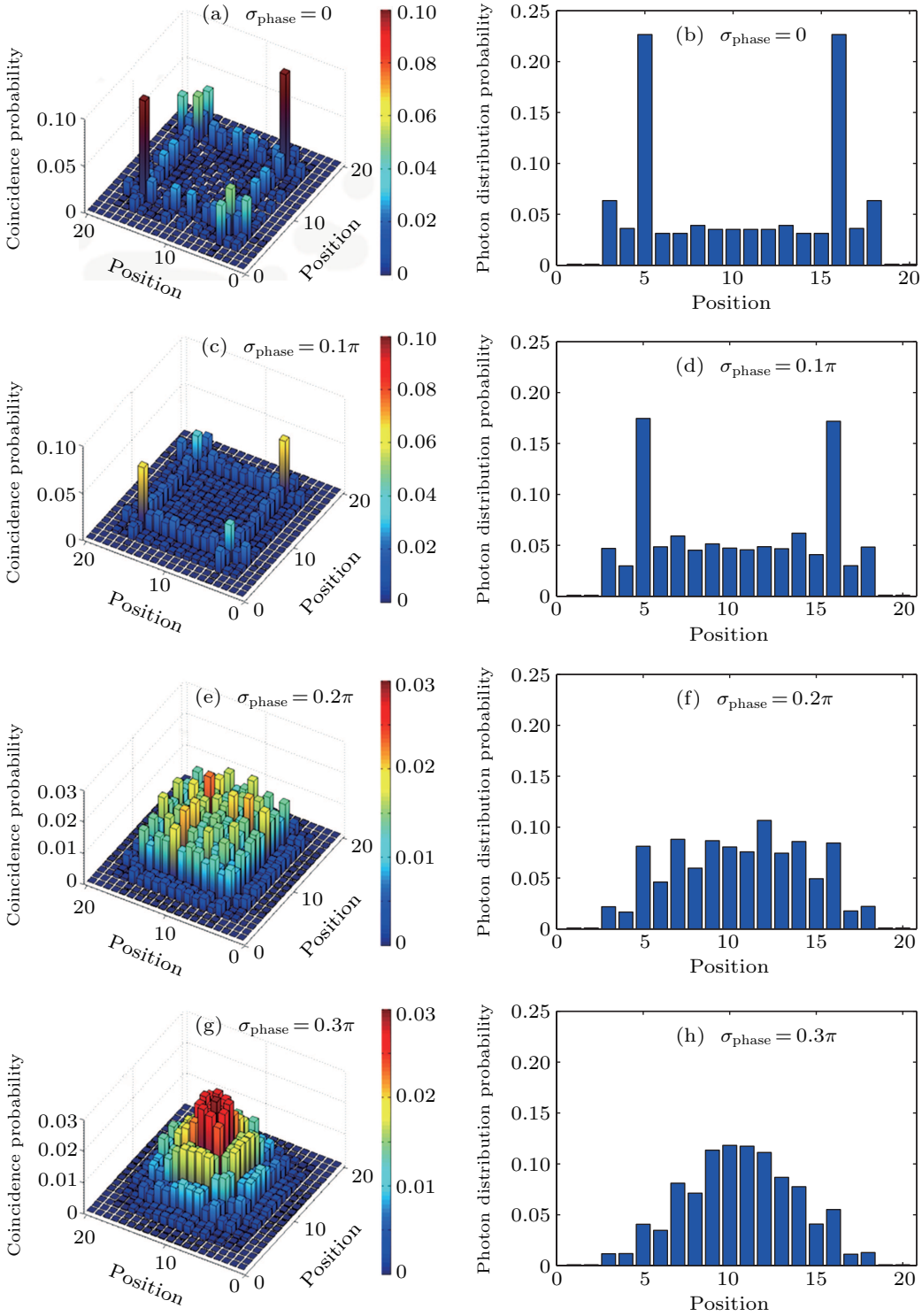



**Fig. 2.** Coincidence probability distribution of 10-step two-photon quantum random walk with phase fluctuation of (a) $\sigma_{\text{phase}} = 0$, (c) $\sigma_{\text{phase}} = 0.1\pi$, (e) $\sigma_{\text{phase}} = 0.2\pi$, (g) $\sigma_{\text{phase}} = 0.3\pi$. Panels (b), (d), (f), (h) are photon distributions of panels (a), (c), (e), (g), respectively.

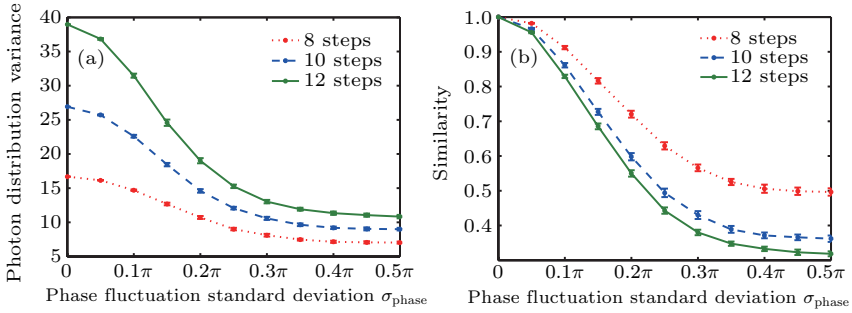



**Fig. 3.** (a) Photon distribution variance and (b) similarity of two-photon quantum random walk with different phase fluctuations.

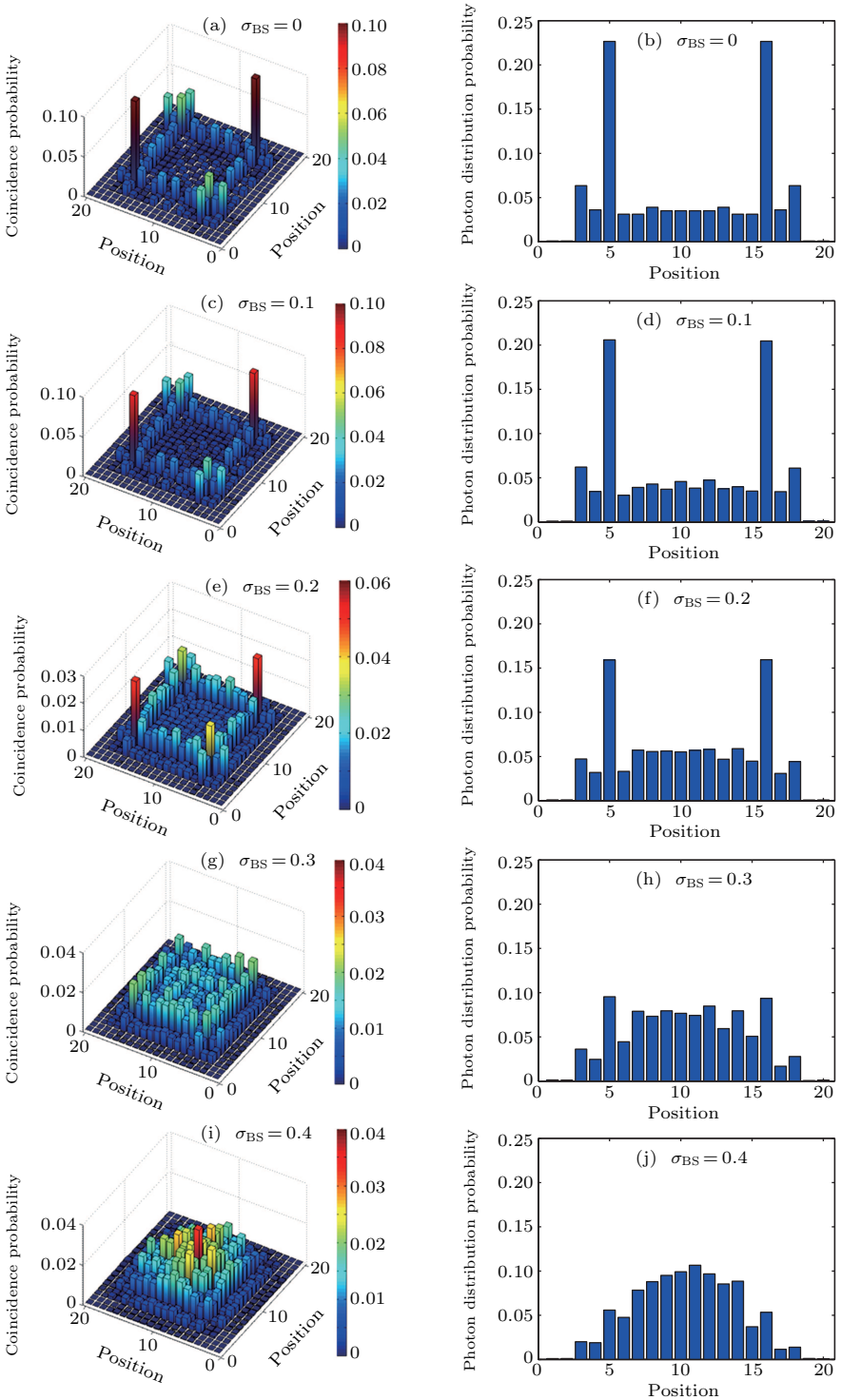



**Fig. 4.** Coincidence probability distribution of 10-step two-photon quantum random walk with tunable beam splitters of (a) $\sigma_{\rm BS}=0$, (c) $\sigma_{\rm BS}=0.1$, (e) $\sigma_{\rm BS}=0.2$, (g) $\sigma_{\rm BS}=0.3$, (i) $\sigma_{\rm BS}=0.4$. Panels (b), (d), (f), (h), (j) are photon distributions of panels (a), (c), (e), (g), (i), respectively.

Secondly, tunable beam splitters (TBS) are used to generate independent random beam splitter fluctuation $T$ which satisfies Gaussian distribution $(\mu_{\rm BS}, \sigma_{\rm BS})$. The mean value is $\mu_{\rm BS} = 0.5$, and the standard deviation $\sigma_{\rm BS}$ are 0, 0.1, 0.2, 0.3, 0.4, respectively. Their coincidence probability distributions are shown in Figs. 4(a), 4(c), 4(e), 4(g), 4(i). Figures 4(b), 4(d), 4(f), 4(h), 4(j) are photon distributions of panels (a), (c), (e), (g), (i), respectively. As the standard deviation of beam splitter fluctuation increases, coincidence probability distribution and photon distribution get close to the center gradually, and degenerate into a classical one. This indicates that beam splitter fluctuation is also able to affect the interference effect, so that quantum random walk gradually degenerates into the classical random walk.

We calculate the photon distribution variance and the coincidence probability distribution similarity under different beam splitter fluctuations, as shown in Figs. 5(a) and 5(b), respectively. In Fig. 5(a), photon distribution variance decreases gradually with the increase of beam splitter fluctuation, that is to say the diffusion rates of 'walker' photons decreases gradually. Figure 5(b) shows coincidence probability distribution similarity decreases gradually with the increase of beam splitter fluctuation. Besides, figures 5(a) and 5(b) show that the larger the step of quantum random walk is, the greater the effect of beam splitter fluctuation will be.

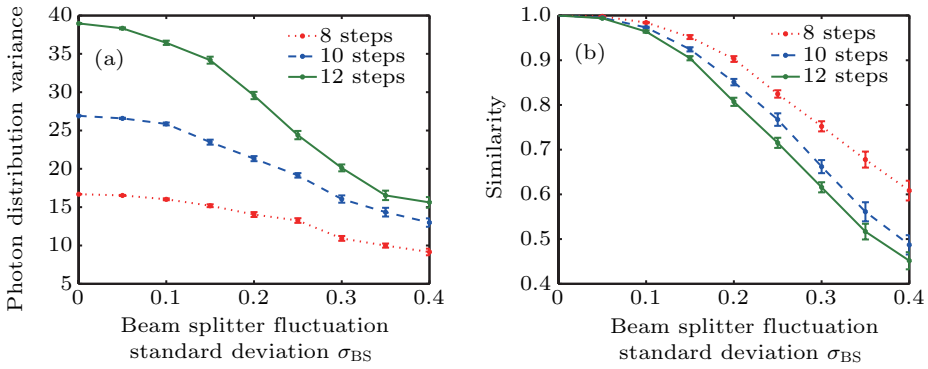



**Fig. 5.** (a) Photon distribution variance and (b) similarity of two-photon quantum random walk with beam splitter fluctuation.

Finally, we synthetically consider the effect of two factors (phase fluctuation and beam splitter fluctuation). Figures 6(a) and (b) show that photon distribution variance and coincidence probability distribution similarity of 10-step two-photon quantum random walk with phase fluctuation and beam splitter fluctuation. The results of Fig. 6 are helpful to guide us implement a better quantum random walk through the control of phase fluctuation and beam splitter fluctuation in the experiment. For example, coincidence probability distribution similarity of more than 90% requires that phase fluctuation standard deviation should be controlled within $\sigma_{\rm phase} \leqslant 0.05\pi$ and beam splitter fluctuation standard deviation should be controlled within $\sigma_{\rm BS} \leqslant 0.1$. Controlling $\sigma_{\rm BS} \leqslant 0.1$ is easy to achieve, while the main difficulty that needs our attention is to control phase fluctuation within $\sigma_{\rm phase} \leqslant 0.05\pi$ in the experiment.

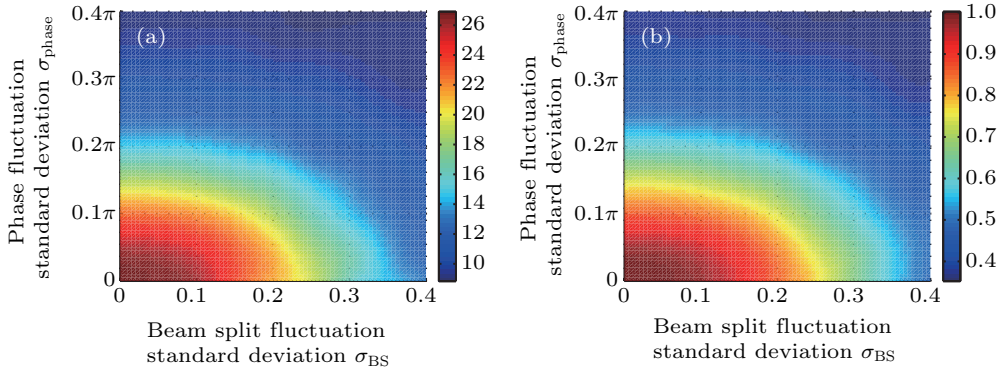



**Fig. 6.** (a) Photon distribution variance and (b) coincidence probability distribution similarity of 10-step two-photon quantum random walk with phase fluctuation and beam splitter fluctuation.

## 4. Conclusion

In this paper, we introduce phase shifters and tunable beam splitters into a two-photon quantum random walk system. Phase shifters and tunable beam splitters are used to simulate two unavoidable decoherence factors, phase fluctuation, and beam splitter fluctuation. We find that both coincidence probability distribution and photon distribution gradually tend to the center and degenerate into a classical Gaussian distri-

bution with the increase of phase fluctuation and beam splitter fluctuation. The photon distribution variance and coincidence probability distribution similarity are used to quantitatively analyze the effect of phase fluctuation and beam splitter fluctuation on the two-photon quantum random walk. Our work shows that the controlling phase fluctuation is the key point for realizing a better result of quantum random walk. When phase fluctuation $\sigma_{\text{phase}} \geqslant 0.3\pi$, the two-photon quantum random walk basically approaches classical random walk. If similarity of more than 90% for a typical quantum result is required, phase fluctuation within $\sigma_{\text{phase}} \leqslant 0.05\pi$ is a key point and must be satisfied.

## References


[1] Barber M N, Ninham B W and Ninham B W 1970 *Random and restricted walks: Theory and applications* (New York: Science Publishers) pp. 1–2

[2] Schoning T 1999 *Foundations of Computer Science, 40th Annual Symposium on IEEE*, October 17–19, 1999, New York, USA, pp. 410–414

[3] Dana R A and Jeanblanc M 2007 *Financial markets in continuous time* (Springer Science & Business Media) pp. 1–2

[4] Aharonov Y, Davidovich L and Zagury N 1993 *Phys. Rev. A* **48** 1687

[5] Roldán E and Soriano J C 2005 *J. Mod. Opt.* **52** 2649

[6] Dür W, Raussendorf R, Kendon V M and Briegel H J 2002 *Phys. Rev. A* **66** 052319

[7] Knight P L, Roldán E and Sipe J E 2003 *Phys. Rev. A* **68** 020301

[8] Do B, Stohler M L, Balasubramanian S, Elliott D S, Eash C, Fischbach E, Fischbach M A, Mills A and Zwickl B 2005 *J. Opt. Soc. Am. B* **22** 499

[9] Zou X, Dong Y and Guo G 2006 *New J. Phys.* **8** 81

[10] Ma Z Y, Burnett K, d'Arcy M B and Gardiner S A 2006 *Phys. Rev. A* **73** 013401

[11] Childs A M, Cleve R, Deotto E, Farhi E, Gutmann S and Spielman D A 2003 *Proceedings of the thirty-fifth annual ACM symposium on theory of computing. ACM*, June 9–11, 2003, San Diego, USA, pp. 59–68

[12] Shenvi N, Kempe J and Whaley K B 2003 *Phys. Rev. A* **67** 052307

[13] Lovett N B, Cooper S, Everitt M, Trevers M and Kendon V 2010 *Phys. Rev. A* **81** 042330

[14] Potoček V, Gábris A, Kiss T and Jex I 2009 *Phys. Rev. A* **79** 012325

[15] Perets H B, Lahini Y, Pozzi F, Sorel M, Morandotti R and Silberberg Y 2008 *Phys. Rev. Lett.* **100** 170506

[16] Schreiber A, Cassemiro K N, Potocek V, Gabris A, Mosley P, Andersson E, Jex I and Silberhorn C 2010 *Phys. Rev. Lett.* **104** 050502

[17] Peruzzo A, Lobino M, Matthews J C F, Matsuda N, Politi A, Poulios K, Zhou X Q, Lahini Y, Ismail N, Wrhoff K, Bromberg Y, Silberberg Y, Thompson M G and OBrien J L 2010 *Science* **329** 1500

[18] Owens J O, Broome M A, Biggerstaff D N, Goggin M E, Fedrizzi A, Linjordet T, Ams M, Marshall G D, Twamley J, Withford M J and White A G 2011 *New J. Phys.* **13** 075003

[19] Karski M, Förster L, Choi J M, Steffen A, Alt W, Meschede D and Widera A 2009 *Science* **325** 174

[20] Schmitz H, Matjeschk R, Schneider C, Glueckert J, Enderlein M, Huber T and Schaetz T 2009 *Phys. Rev. Lett.* **103** 090504

[21] Zahringer F, Kirchmair G, Gerritsma R, Solano E, Blatt R and Roos C F 2010 *Phys. Rev. Lett.* **104** 100503

[22] Travaglione B C and Milburn G J 2002 *Phys. Rev. A* **65** 032310

[23] Du J, Li H, Xu X, Shi M, Wu J, Zhou X and Han R 2003 *Phys. Rev. A* **67** 042316

[24] Ryan C A, Laforest M, Boileau J C and Laflamme R 2005 *Phys. Rev. A* **72** 062317

[25] Tang H, Lin X F, Feng Z, Chen J Y, Gao J, Sun K, Wang C Y, Lai P C, Xu X Y, Wang Y, F Qiao L, Yang A L and Jin X M 2018 *Sci. Adv.* **4** 3174

[26] Gao W C, Cao C, Liu X F, Wang T J and Wang C 2019 *OSA Continuum* **2** 1667

[27] Yang Y G, Zhang Y C, Xu G, Chen X B, Zhou Y H and Shi W M 2018 *Sci. China- Phys. Mech. Astron.* **61** 030312

[28] Zhang P, Liu B H, Liu R F, Li H R, Li F L, Guo G C 2010 *Phys. Rev. A* **81** 052322

[29] Cardano F, Massa F, Qassim H, Karimi E, Slussarenko S, Paparo D, Lisio C, Sciarrino F, Santamato E, Boyd R W and Marrucci L 2015 *Sci. Adv.* **1** 1500087

[30] Wang F R, Zhang P, Wang Y L, Liu R F, Gao H and Li F L 2017 *Phys. Lett. A* **381** 65

[31] Jeong H, Paternostro M and Kim M S 2004 *Phys. Rev. A* **69** 012310

[32] Kendon V 2007 *Math. Struct. Comp. Sci.* **17** 1169

[33] Brun T A, Carteret H A and Ambainis A 2003 *Phys. Rev. A* **67** 032304

[34] Brun T A, Carteret H A and Ambainis A 2003 *Phys. Rev. Lett.* **91** 130602

[35] Broome M A, Fedrizzi A, Lanyon B P, Kassal I, Aspuru-Guzik A and White A G 2010 *Phys. Rev. Lett.* **104** 153602

[36] Romanelli A, Siri R, Abal G, Auyuanet A and Donangelo R 2005 *Physica A* **347** 137

[37] Kendon V and Tregenna B 2003 *Phys. Rev. A* **67** 042315

[38] Kempe J 2003 *Contemp. Phys.* **44** 307

[39] Jeong Y C, Franco C D, Lim H T, Kim M S and Kim Y H 2013 *Nat. Commun.* **4** 2471

[40] Crespi A, Osellame R, Ramponi R, Giovannetti V, Fazio R, Sansoni L, Nicola F D, Sciarrino F and Mataloni P 2013 *Nat. Photon.* **7** 322

[41] Schreiber A, Gábris A, Rohde P P, Laiho K, Štefaňák M, Potoček V, Hamilton C, Jex I and Silberhorn C 2012 *Science* **336** 55