ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理 Chinese Physics B

Quantum adiabatic algorithms using unitary interpolation

Shuo Zhang(张硕)[1], Qian-Heng Duan(段乾恒)[1], Tan Li(李坦)[1], Xiang-Qun Fu(付向群)[1], He-Liang Huang(黄合良)[1], Xiang Wang(汪翔)[1], Wan-Su Bao(鲍皖苏)[1,2,3]

Citation: Chin. Phys. B . 2020, 29(1): 010308 . **doi:** 10.1088/1674-1056/ab5f02
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Nb-based Josephson parametric amplifier for superconducting qubit measurement

Fei-Fan Su(宿非凡)[1,2], Zi-Ting Wang(王子婷)[1,2], Hui-Kai Xu(徐晖凯)[1,2], Shou-Kuan Zhao(赵寿宽)[1,2], Hai-Sheng Yan(严海生)[1,2], Zhao-Hua Yang(杨钊华)[1,2], Ye Tian(田野)[1,2], Shi-Ping Zhao(赵士平)[1,2,3]

Chin. Phys. B . 2019, 28(11): 110303 . **doi:** 10.1088/1674-1056/ab4578

Experimental implementation of a continuous-time quantum random walk on a solid-state quantum information processor

Maimaitiyiming Tusun(麦麦提依明·吐孙)[1,2,3,4], Yang Wu(伍旸)[1,2,3], Wenquan Liu(刘文权)[1,2,3], Xing Rong(荣星)[1,2,3], Jiangfeng Du(杜江峰)[1,2,3]

Chin. Phys. B . 2019, 28(11): 110302 . **doi:** 10.1088/1674-1056/ab44ae

Tunable coupling between Xmon qubit and coplanar waveguide resonator

He-Kang Li(李贺康), Ke-Min Li(李科敏), Hang Dong(董航), Qiu-Jiang Guo(郭秋江), Wu-Xin Liu(刘武新), Zhan Wang(王战), Hao-Hua Wang(王浩华), Dong-Ning Zheng(郑东宁)

Chin. Phys. B . 2019, 28(8): 080305 . **doi:** 10.1088/1674-1056/28/8/080305

Realization of *t*-bit semiclassical quantum Fourier transform on IBM's quantum cloud computer

Xiang-Qun Fu(付向群), Wan-Su Bao(鲍皖苏), He-Liang Huang(黄合良), Tan Li(李坦), Jian-Hong Shi(史建红), Xiang Wang(汪翔), Shuo Zhang(张硕), Feng-Guang Li(李风光)

Chin. Phys. B . 2019, 28(2): 020302 . **doi:** 10.1088/1674-1056/28/2/020302

MatCloud, a high-throughput computational materials infrastructure: Present, future visions, and challenges

Xiaoyu Yang(杨小渝), Zongguo Wang(王宗国), Xushan Zhao(赵旭山), Jianlong Song(宋健龙), Chao Yu(虞超), Jiaxin Zhou(周嘉欣), Kai Li(李凯)

Chin. Phys. B . 2018, 27(11): 110301 . **doi:** 10.1088/1674-1056/27/11/110301

Quantum adiabatic algorithms using unitary interpolation*

Shuo Zhang(张硕)[1], Qian-Heng Duan(段乾恒)[1], Tan Li(李坦)[1], Xiang-Qun Fu(付向群)[1], He-Liang Huang(黄合良)[1], Xiang Wang(汪翔)[1], and Wan-Su Bao(鲍皖苏)[1,2,3,†]

[1] *Henan Key Laboratory of Quantum Information and Cryptography, SSE IEU, Zhengzhou* 450001, *China*

[2] *Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei* 230026, *China*

(Received 30 June 2019; revised manuscript received 19 November 2019; accepted manuscript online 5 December 2019)

We present two efficient quantum adiabatic algorithms for Bernstein–Vazirani problem and Simon's problem. We show that the time complexities of the algorithms for Bernstein–Vazirani problem and Simon's problem are $O(1)$ and $O(n)$, respectively, which are the same complexities as the corresponding algorithms in quantum circuit model. In these two algorithms, the adiabatic Hamiltonians are realized by unitary interpolation instead of standard linear interpolation. Comparing with the adiabatic algorithms using linear interpolation, the energy gaps of our algorithms keep constant. Therefore, the complexities are much easier to analyze using this method.

Keywords: adiabatic quantum computation, quantum adiabatic algorithms

PACS: 03.67.Lx, 03.67.Ac **DOI:** 10.1088/1674-1056/ab5f02

1. Introduction

In 2001, Farhi *et al.* proposed a novel quantum computing model, the adiabatic quantum computation (AQC).[1] The adiabatic quantum computation has proved to be computational equivalent to the quantum circuit model[2,3] and robustness against thermal fluctuations.[4–6] Therefore, it has received extensive attention, and several quantum adiabatic algorithms (QAAs) have been proposed and demonstrated, *e.g.*, adiabatic algorithms for unstructured search,[7] Deutsch–Jozsa problem,[8–12] Bernstein–Vazirani problem,[13] Simon's problem,[13–15] and integer number factorization problem.[16–18]

A general QAA is achieved as follows. One first designs the target Hamiltonian H_p, whose ground state encodes the solution of the problem, and then prepare the system in the ground state of the initial Hamiltonian H_0. After adiabatically evolving from H_0 to H_p, which is achieved by means of a linearly interpolating Hamiltonian $H(s(t)) = (1-s)H_0 + sH_p$, with $s(0) = 0$ and $s(T) = 1$, the system will finally evolve to the ground state of H_p, and the solution is therefore obtained.

The quantum adiabatic condition[19]

$$\max_{0\le t\le T} \frac{\left|\langle E_1(t)|\frac{\partial}{\partial t}H(t)|E_0(t)\rangle\right|}{|\Delta(t)|^2} \sim \varepsilon \ll 1 \tag{1}$$

guarantees the adiabaticity during the evolution, where $|E_0(t)\rangle$ and $|E_1(t)\rangle$ are the instantaneous ground state and first excited state of $H(t)$, $\Delta(t)$ denotes the energy gap between these two energy levels. The computational complexity of a QAA, regarding to the run time T, can be estimated using the adiabatic condition (1) as well. However, the estimation of $\Delta(t)$ in expression (1) is a hard problem. Hence the complexity of some QAAs might be difficult to analyze.

In 2005, Siu introduced the unitary interpolation method[20] to achieve the time-dependent Hamiltonian $H(t)$, in which the time dependent Hamiltonian has the form

$$H(t) = U(t)H_0U^\dagger(t), \tag{2}$$

where $U(0) = I$, and $U(T)H_0U^\dagger(T) = H_p$. Then Sarandy and Lidar proposed an adiabatic implementation of the Deutsch–Jozsa algorithm using unitary interpolation.[10] It can be proved that the energy gap keep invariant in such method. Therefore, it is much easier to obtain the time complexity using unitary interpolation.

In this paper, we present two alternative QAAs for Bernstein–Vazirani problem and Simon's problem, where the time-dependent Hamiltonians $H(t)$ are achieved by means of unitary interpolation. We show that the Bernstein–Vazirani problem and Simon's problem can be solved by using quantum adiabatic algorithms in $O(1)$ and $O(n)$ runtime, respectively, which is the same as the quantum circuit model.

2. The QAA for Bernstein–Vazirani problem

In Bernsterin–Vazirani problem, we are given an n-bit function

$$f_{\boldsymbol{a}}(\boldsymbol{x}) = \boldsymbol{a}\cdot\boldsymbol{x} := a_0x_0 \oplus a_1x_1 \oplus \cdots \oplus a_{n-1}x_{n-1}. \tag{3}$$

The task is to determine the n-bit $\boldsymbol{a}$. Classically, the problem can be solved within n queries of $f(\cdot)$, while the

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11504430, 11805279, 61501514, and 61502526).

†Corresponding author. E-mail: bws@qiclab.cn

© 2020 Chinese Physical Society and IOP Publishing Ltd

Bernsterin–Vazirani algorithm[21] in quantum circuit model only requires a single query of $f(\cdot)$ to achieve the same task.

The initial Hamiltonian for solving the Bernstein–Vazirani problem is

$$H_0 = I - |+\rangle\langle+|, \tag{4}$$

where we have normalized the energy scale to unity.

The ground state of H_0 is $|+\rangle$, which is an equal superposition over all n-bit string

$$|+\rangle = \frac{1}{\sqrt{2^n}} \sum_{\boldsymbol{x}\in\{0,1\}^n} |\boldsymbol{x}\rangle. \tag{5}$$

The target Hamiltonian is

$$H_p = UH_0U^\dagger = I - U|+\rangle\langle+|U^\dagger, \tag{6}$$

with the unitary operator

$$U = \sum_{\boldsymbol{x}\in\{0,1\}^n} \mathrm{e}^{\mathrm{i}\pi(\boldsymbol{a}\cdot\boldsymbol{x})} |\boldsymbol{x}\rangle\langle\boldsymbol{x}|, \tag{7}$$

and the ground state of H_p is

$$U|+\rangle = \frac{1}{\sqrt{2^n}} \sum_{\boldsymbol{x}\in\{0,1\}^n} (-1)^{(\boldsymbol{a}\cdot\boldsymbol{x})} |\boldsymbol{x}\rangle. \tag{8}$$

The QAA for Bernsterin–Vazirani problem is achieved as follows.

Step 1 Prepare $|+\rangle$, which is the ground state of H_0, as the initial state of the quantum system

$$|\psi(t=0)\rangle = |+\rangle. \tag{9}$$

Step 2 Adiabatically evolve the system from H_0 to the target Hamiltonian H_P, under the following time-dependent Hamiltonian

$$H(t) = U(t) H_0 U^\dagger(t), \tag{10}$$

where the unitary operator

$$U(t) = \sum_{\boldsymbol{x}\in\{0,1\}^n} \mathrm{e}^{\mathrm{i}\pi(\boldsymbol{a}\cdot\boldsymbol{x})\frac{t}{T}} |\boldsymbol{x}\rangle\langle\boldsymbol{x}|. \tag{11}$$

The unitary operator can be realized by $U(t) = \exp(-\mathrm{i}H_a t)$ with Hamiltonian

$$H_a = -\frac{\pi}{T} \sum_{\boldsymbol{x}\in\{0,1\}^n} (\boldsymbol{a}\cdot\boldsymbol{x}) |\boldsymbol{x}\rangle\langle\boldsymbol{x}|.$$

Under the adiabatic approximation, the system finally evolves to the ground state of H_p, which is

$$|\psi(t=T)\rangle \approx U|+\rangle = \frac{1}{\sqrt{2^n}} \sum_{\boldsymbol{x}\in\{0,1\}^n} \mathrm{e}^{\mathrm{i}\pi(\boldsymbol{a}\cdot\boldsymbol{x})} |\boldsymbol{x}\rangle.$$

The final state can be rewritten in x-basis. Define $|0'\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ and $|1'\rangle = (1/\sqrt{2})(|0\rangle - |1\rangle)$, then

$$\begin{aligned} |0\rangle &= \frac{1}{\sqrt{2}} \left(|0'\rangle + |1'\rangle\right), \\ |1\rangle &= \frac{1}{\sqrt{2}} \left(|0'\rangle - |1'\rangle\right). \end{aligned} \tag{12}$$

The above equations can be summarized in more concise form as

$$|x\rangle = \frac{1}{\sqrt{2}} \sum_{y=0,1} (-1)^{xy} |y'\rangle. \tag{13}$$

For the n-bit case, the state $|\boldsymbol{x}\rangle$ is

$$|\boldsymbol{x}\rangle = \frac{1}{\sqrt{2^n}} \sum_{\boldsymbol{y}\in\{0,1\}^n} (-1)^{\boldsymbol{x}\cdot\boldsymbol{y}} |\boldsymbol{y}'\rangle. \tag{14}$$

Then the final state can be rewritten as

$$\begin{aligned} &|\psi(t=T)\rangle \approx U|+\rangle \\ &= \frac{1}{2^n} \sum_{\boldsymbol{x}\in\{0,1\}^n} \sum_{\boldsymbol{y}\in\{0,1\}^n} (-1)^{\boldsymbol{x}\cdot\boldsymbol{y}+\boldsymbol{x}\cdot\boldsymbol{a}} |\boldsymbol{y}'\rangle. \end{aligned} \tag{15}$$

Notice that

$$\frac{1}{2^n} \sum_{\boldsymbol{x}\in\{0,1\}^n} \sum_{\boldsymbol{y}\in\{0,1\}^n} (-1)^{\boldsymbol{x}\cdot\boldsymbol{y}+\boldsymbol{x}\cdot\boldsymbol{a}} = \delta_{\boldsymbol{a}\boldsymbol{y}},$$

then $|\psi(t=T)\rangle \approx U|+\rangle = |\boldsymbol{a}'\rangle$.

Step 3 At the end of the evolution, measure each qubit of the system in the $\{|0'\rangle, |1'\rangle\}$ basis, and $\boldsymbol{a}$ can be obtained.

The run time of the algorithm is determined by the adiabatic condition

$$\max_{0\le t\le T} \frac{\left|\langle E_1(t)| \frac{\partial}{\partial t} H(t) |E_0(t)\rangle\right|}{|\Delta(t)|^2} \sim \varepsilon \ll 1. \tag{16}$$

Obviously, the energy gap is invariant during the adiabatic evolution, *i.e.*, $\Delta(t) = \Delta(0) = 1$. On the other hand,

$$\left|\langle E_1(t)| \frac{\partial}{\partial t} H(t) |E_0(t)\rangle\right| = \left|\langle E_1(t)| \left(\frac{\partial}{\partial t} U(t)\right) H_0 U^\dagger(t) |E_0(t)\rangle + \langle E_1(t)| U(t) H_0 \left(\frac{\partial}{\partial t} U^\dagger(t)\right) |E_0(t)\rangle\right|. \tag{17}$$

As $\frac{\partial}{\partial t} U(t) = -\mathrm{i}U(t) H_a$, and H_a is commutative with $U(t)$ and $U^\dagger(t)$, therefore

$$\left|\langle E_1(t)| \frac{\partial}{\partial t} H(t) |E_0(t)\rangle\right| = |\langle E_1| [H_0, H_a] |E_0\rangle| = \Delta(0) |\langle E_1| H_a |E_0\rangle|. \tag{18}$$

Note that any eigenvalue λ of H_a satisfies $|\lambda| \leq \pi/T$, therefore $|\langle E_1|H_a|E_0\rangle| \leq \pi/T$. Thus

$$\max_{0\leq t\leq T} \frac{\left|\langle E_1(t)|\frac{\partial}{\partial t}H(t)|E_0(t)\rangle\right|}{|\Delta(t)|^2} \leq \frac{\pi}{T} \sim \varepsilon, \tag{19}$$

and the total evolution time

$$T \sim \frac{\pi}{\varepsilon}, \tag{20}$$

that means the algorithm can be achieved in $O(1)$ run time, which is the same order as the algorithm using quantum circuit mode.

3. The QAA for Simon's problem

In Simon's problem, a function from n-bit strings to n-bit strings is given by

$$f:\{0,1\}^n \longrightarrow \{0,1\}^n,\ \boldsymbol{x} \longrightarrow \boldsymbol{f}(\boldsymbol{x}), \tag{21}$$

where we are given the promise that there is one unknown string $\boldsymbol{a} = a_1a_2\cdots a_n$ so that $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{y})$ $(\boldsymbol{x} \neq \boldsymbol{y})$ if and only if $\boldsymbol{y} = \boldsymbol{x} \oplus \boldsymbol{a}$.

In order to find $\boldsymbol{a}$, namely the period of the function, classical algorithm requires exponentially queries of $\boldsymbol{f}(\cdot)$, while Simon's algorithm in quantum circuit model requires only $O(n)$ queries.[22]

Here, we propose a QAA for Simon's problem. We choose the initial Hamiltonian

$$H_0 = I_{2n} - |+\rangle\langle+| \otimes |\boldsymbol{0}\rangle\langle\boldsymbol{0}|. \tag{22}$$

The algorithm requires two n-bit registers with the first one encodes the inputs and the second one records the output of $\boldsymbol{f}(\cdot)$.

Meanwhile, the target Hamiltonian is designed as

$$H_p = UH_0U^\dagger = I_{2n} - U|+\rangle\langle+| \otimes |\boldsymbol{0}\rangle\langle\boldsymbol{0}|U^\dagger,$$

The unitary operator is

$$U = \sum_{\boldsymbol{x}\in\{0,1\}^n} |\boldsymbol{x}\rangle\langle\boldsymbol{x}| \otimes \mathrm{e}\left[-\mathrm{i}(\pi/2)f_1(\boldsymbol{x})\sigma_{x,1}\right]\mathrm{e}\left[-\mathrm{i}(\pi/2)f_2(\boldsymbol{x})\sigma_{x,2}\right]\cdots\mathrm{e}\left[-\mathrm{i}(\pi/2)f_n(\boldsymbol{x})\sigma_{x,n}\right],$$

where $\sigma_{x,i}$ is the σ_x operator acting on the i-th bit of the second register, and $f_i(\boldsymbol{x})$ denotes the i-th bit of $\boldsymbol{f}(\boldsymbol{x})$.

The ground state of H_p is

$$\begin{aligned} U|+\rangle|\boldsymbol{0}\rangle &= \frac{1}{\sqrt{2^n}}U\sum_{\boldsymbol{x}\in\{0,1\}^n}|\boldsymbol{x}\rangle|\boldsymbol{0}\rangle = \frac{1}{\sqrt{2^n}}\sum_{\boldsymbol{x}\in\{0,1\}^n}|\boldsymbol{x}\rangle \otimes |f_1(\boldsymbol{x})\rangle|f_2(\boldsymbol{x})\rangle\cdots|f_n(\boldsymbol{x})\rangle \\ &= \frac{1}{\sqrt{2^n}}\sum_{\boldsymbol{x}\in\{0,1\}^n}|\boldsymbol{x}\rangle \otimes |\boldsymbol{f}(\boldsymbol{x})\rangle. \end{aligned} \tag{23}$$

The quantum adiabatic algorithm for Simon's problem proceeds as the following steps.

Step 1 Prepare the ground state of H_0, as the initial state of the quantum system.

$$|\psi(t=0)\rangle = |+\rangle|\boldsymbol{0}\rangle.$$

Step 2 Adiabatically evolve the system from H_0 to the target Hamiltonian H_p, by means of the time-dependent Hamiltonian

$$H(t) = U(t)H_0U^\dagger(t),$$

where the unitary operator

$$U(t) = \sum_{\boldsymbol{x}\in\{0,1\}^n}|\boldsymbol{x}\rangle\langle\boldsymbol{x}| \otimes \exp\left[-\mathrm{i}\frac{\pi}{2}\frac{t}{T}\sum_{i=1}^n f_i(\boldsymbol{x})\sigma_{x,i}\right].$$

The unitary operator can be realized by $U(t) = \exp(-\mathrm{i}H_at)$ with Hamiltonian

$$H_a = \frac{\pi}{2T}\sum_{\boldsymbol{x}\in\{0,1\}^n}\left(|\boldsymbol{x}\rangle\langle\boldsymbol{x}| \otimes \sum_{i=1}^n f_i(\boldsymbol{x})\sigma_{x,i}\right).$$

At the end of the evolution, we approximately obtain the ground state of H_p

$$|\psi(t=T)\rangle \approx U|+\rangle = \frac{1}{\sqrt{2^n}}\sum_{x\in\{0,1\}^n}|\boldsymbol{x}\rangle \otimes |\boldsymbol{f}(\boldsymbol{x})\rangle.$$

Step 3 Measure the second register, and obtain some values $|\boldsymbol{f}(\boldsymbol{y})\rangle$, then the first register will be collapsed to the state $(1/\sqrt{2})(|\boldsymbol{y}\rangle + |\boldsymbol{y}\oplus\boldsymbol{a}\rangle)$, which can be rewritten in the $\{|0'\rangle, |1'\rangle\}$ basis as

$$\begin{aligned} \frac{1}{\sqrt{2}}(|\boldsymbol{y}\rangle + |\boldsymbol{y}\oplus\boldsymbol{a}\rangle) &= \frac{1}{\sqrt{2^n}}\left(\sum_{\boldsymbol{z}\in\{0,1\}^n}(-1)^{\boldsymbol{y}\cdot\boldsymbol{z}}|\boldsymbol{z}'\rangle + \sum_{\boldsymbol{z}\in\{0,1\}^n}(-1)^{(\boldsymbol{y}\oplus\boldsymbol{a})\cdot\boldsymbol{z}}|\boldsymbol{z}'\rangle\right) \\ &= \frac{1}{\sqrt{2^{n-1}}}\sum_{\boldsymbol{a}\cdot\boldsymbol{z}=\boldsymbol{0}}(-1)^{\boldsymbol{y}\cdot\boldsymbol{z}}|\boldsymbol{z}'\rangle. \end{aligned}$$

Step 4 Measure the first register in the $\{|0'\rangle, |1'\rangle\}$ basis, some values $\boldsymbol{z}$ which satisfy $\boldsymbol{a}\cdot\boldsymbol{z}=\boldsymbol{0}$ can be obtained randomly.

Repeating from Step 1 to Step 4 m times, it yields the following linear equation

$$\begin{cases} \boldsymbol{a}\cdot\boldsymbol{z}_1=\boldsymbol{0}, \\ \boldsymbol{a}\cdot\boldsymbol{z}_2=\boldsymbol{0}, \\ \cdots \\ \boldsymbol{a}\cdot\boldsymbol{z}_m=\boldsymbol{0}. \end{cases} \tag{24}$$

It can be proved that for $m=n+O(1)$, $\boldsymbol{a}$ can be uniquely determined from Eq. (24) with probability at least $2/3$,[23], so the QAA for Simon problem requires $O(n)$ evaluations of f.

Same as in Section 2, energy gap $\Delta(t)=\Delta(0)$. From the adiabatic condition,

$$\max_{0\le t\le T}\frac{\left|\langle E_1(t)|\frac{\partial}{\partial t}H(t)|E_0(t)\rangle\right|}{|\Delta(t)|^2}=\max_{0\le t\le T}\frac{|\langle E_1|H_a|E_0\rangle|}{|\Delta(t)|} \le \frac{\pi}{2T}\le\varepsilon,$$

the run time of each cycle is $T\ge\pi/2\varepsilon$. Therefore, the complexity of the algorithm is $O(n)$.

4. Discussion and conclusion

We first compare our scheme with Ref. [13]. In Ref. [13], I. Hen has proposed the QAAs for the same two problems as well, but his algorithms are significantly different from ours. On the one hand, in our work, the algorithms are devised using unitary interpolation, while in Ref. [13] the algorithms are achieved in linear interpolation. On the other hand, in Ref. [13], the ground states of the Hamiltonian is highly degenerate, and higher levels of coherence are needed,[13] in addition, the adiabatic evolution is achieved in each invariant subspaces, which is different from the traditional AQC model. While, in our work the ground state is unique, which indicates that the two alternative algorithms we proposed can be more robust against noises.

We now discuss the physical realization. The initial and target Hamiltonians in our schemes are one-dimensional projectors, *i.e.*, $H=I-|\alpha\rangle\langle\alpha|$. It is n-local Hamiltonian, which cannot be physically realized. In the QAA for Bernstein–Vazirani problem, we can take alternative 1-local initial Hamiltonian

$$H_0=-\sigma_{x,1}-\sigma_{x,1}-\cdots-\sigma_{x,n}$$

with energy gap $\Delta(0)=1$, and the time-dependent Hamiltonian is

$$H(t)=U(t)H_0U^{\dagger}(t) =-\sum_k\left[\sigma_{x,k}\cos\left(\frac{\pi}{T}a_kt\right)+\sigma_{y,k}\sin\left(\frac{\pi}{T}a_kt\right)\right],$$

which keeps 1-local during the evolution. From the adiabatic condition (1), the algorithm can also be achieved in $O(1)$ running time.

However, in the QAA for Simon's problem, the two registers are correlated after adiabatic evolution, and might be entangled. It means that the target Hamiltonian might be n-local and not physically realizable, even the initial Hamiltonian is chosen to be 1-local. Nevertheless, like other oracle-based adiabatic algorithms with n-local Hamiltonians, our algorithm provides a meaningful example to investigate some aspects of QAC, such as the role of coherence,[24] and performances in open system.[10,25]

In summary, we have proposed two quantum adiabatic algorithms for Bernstein–Vazirani problem and Simon's problem. The algorithm for Bernstein–Vazirani problem has an $O(1)$ complexity, and the complexity of the algorithm for Simon's problem is $O(n)$. Both of the two algorithms have the same complexities as the algorithms in quantum circuit model. In further studies, we would like to find more realizable algorithms using local Hamiltonian, and extend the unitary interpolation method to devise other quantum adiabatic algorithms, such as unstructured search problem and prime factorization problem.

References

[1] Farhi E, Goldstone J, Gutmann S, Lapan J, Lundgren A and Preda D 2001 *Science* **292** 472
[2] Mizel A, Lidar D A and Mitchell M 2007 *Phys. Rev. Lett.* **99** 070502
[3] Yu H, Huang Y and Wu B 2018 *Chin. Phys. Lett.* **35** 110303
[4] Aharonov D, van Dam W, Kempe J, Landau Z, Lloyd S and Regev O 2007 *SIAM J. Comput.* **37** 166
[5] Childs A M and Farhi E 2001 *Phys. Rev. A* **65** 012322
[6] Roland J and Nicolas J C 2005 *Phys. Rev. A* **71** 032330
[7] Roland J and Cerf N J 2002 *Phys. Rev. A* **65** 042308
[8] Das S, Kobes R and Kunstatter G 2002 *Phys. Rev. A* **65** 062310
[9] Wei Z and Ying M 2006 *Phys. Lett. A* **354** 271
[10] Sarandy M S and Lidar D A 2005 *Phys. Rev. Lett.* **95** 250503
[11] Sun J, Lu S, Liu F and Gao C 2014 *Quantum Inf. Process.* **13** 731
[12] Sun J, Lu S, Liu F, Gao C, Zhou Q and Zhang Z 2014 *Chin. Phys. Lett.* **31** 070304
[13] Hen I 2014 *Europhys. Lett.* **105** 50005
[14] Rao M V P 2003 *Phys. Rev. A* **67** 052306
[15] Long Y, Feng G, Tang Y, Qin W and Long G 2013 *Phys. Rev. A* **88** 012306
[16] Du J, Xu N, Peng X, Wang P, Wu S and Lu D 2010 *Phys. Rev. Lett.* **104** 030502
[17] Li Z, Dattani N S, Chen X, Liu X, Wang H, Tanburn R, Chen H, Peng X and Du J 2017 *rXiv: 1706.08061*
[18] Kieu T D 2018 arXiv: 1808.02781.
[19] Messiah A 1962 *Quantum Mechanics*, Vol. II (Amsterdam: North Holland Publishing Company)
[20] Siu M S 2005 *Phys. Rev. A* **71** 062314
[21] Bernsterin E and Vazirani U 1997 *SIAM J. Comput.* **26** 1411
[22] Simon D R 1997 *SIAM J. Comput.* **26** 1474
[23] Kaye P, Laflamme R and Mosca M 2007 *An Introduction to Quantum Computing* (New York: Oxford University Press Inc.)
[24] Li F, Bao W, Zhang S, Huang H, Li T, Wang X and Fu X 2018 *Phys. Lett. A* **382** 2709
[25] Wild D S, Gopalakrishnan S, Knap M, Yao N Y and Lukin M D 2016 *Phys. Rev. Lett.* **117** 150501