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Cyclotron dynamics of neutral atoms in optical lattices with additional magnetic field and harmonic trap potential*

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We analytically and numerically discuss the stability and dynamics of neutral atoms in a two-dimensional optical lattice subjected to an additional harmonic trap potential and artificial magnetic field. The harmonic trap potential plays a key role in modifying the equilibrium state properties of the system and stabilizing the cyclotron orbits of the condensate. Meanwhile, the presence of the harmonic trap potential and lattice potential results in rich cyclotron dynamics of the condensate. The coupling effects of lattice potential, artificial magnetic field, and harmonic trap potential lead to single periodic, multi-periodic or quasi-periodic cyclotron orbits of the condensate. So we can control the cyclotron dynamics of neutral atoms in optical lattice by manipulating the strength of harmonic confinement, artificial magnetic field, and initial conditions. Our results provide a direct theoretical evidence for the cyclotron dynamics of neutral atoms in optical lattices exposed to the artificial gauge magnetic field and harmonic trap potential.

Keywords: Bose–Einstein condensate, artificial magnetic field, optical lattice

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1. Introduction

Ultracold atoms in optical lattices provide an ideal platform for studying a lot of fundamental phenomena of many-body physics.[1,2] The technology of laser cooling and neutral atoms trapping[3] has been greatly developed, which provides the conditions for the realization of Bose–Einstein condensation (BEC). The dynamical properties of ultracold atoms trapped in optical lattices[4] subjected to an artificial gauge field have been attracted much attention in experimental and theoretical research.[5–9] Recently, the ways to synthesize optically magnetic field for neutral atoms resulting from the Berry's phase[10] have attracted great interests. When a neutral atom moves in a predesigned laser field, its center-of-mass motion is similar to that of an electron subjected to a Lorentz force in a magnetic field.[9] There are many ways to generate an artificial gauge field, such as laser-assisted hopping,[5,11] Raman coupling,[10,12] and lattice shaking.[13] The realization of the artificial magnetic field in optical lattices[14–18] leads to the observation of many new phenomena, for instance, the creation of vortices,[10,19,20] the coherent transportation of atoms,[21–25] the nonlinear waves,[26,27] the phase transitions,[28,29] and the oscillations of atomic clouds.[30,31] The creation of vortices without rotating the atomic cloud indicates the existence of the artificial gauge magnetic field. The significant effects of artificial magnetic field on the dynamics of neutral atoms make this research very popular. Particularly, the artificial gauge magnetic field induces cyclotron dynamics of neutral atoms in optical lattices, which is well observed experimentally.[5–8] The cyclotron dynamics of the neutral atom is another direct evidence for the generation of the artificial magnetic field and plays a key role in manipulating the coherent transportation of the atoms in optical lattices. Thus it would be significant to study how to control the cyclotron dynamics of neutral atoms in optical lattices theoretically.[32]

Ultracold atoms trapped in optical lattices subjected to an artificial gauge magnetic field provide an ideal platform for manipulating cyclotron dynamics of neutral atoms in a controllable way.[7,8] However, neutral atoms in pure optical lattices under the action of the artificial magnetic field will experience the instability problems.[33] In order to improve the stability, it is important to study the effect of the additional external confinement potential on the dynamics of the neutral atoms in optical lattices. The modifications of the topological states caused by the smooth confinement potential in optical lattice were discussed.[34–39] However, the dynamics of neutral atoms in optical lattices under the influence of an artificial gauge field and a confinement potential is not clear. Therefore, it would be important to consider the dynamics and stability of neutral atoms in optical lattice with additional artificial magnetic field and confinement potential.

The aim of the present work is to study the stability and dynamics of ultracold atoms in two-dimensional (2D) square

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optical lattices under the action of an artificial gauge magnetic field and harmonic trap potential theoretically and numerically. The coupling effects of lattice potential, artificial magnetic field, and harmonic trap potential result in rich and complex cyclotron dynamics phenomena of neutral atoms. Without the harmonic trap potential, there are multiple equilibrium points in the system and the cyclotron center of the condensate is also multiple. The distribution of the (un)stable equilibrium points in lattice plane changes periodically with the lattice site. However, after the addition of the harmonic trap potential, only one equilibrium point exists in the system. The harmonic trap potential plays an important role in modifying the equilibrium state properties and stabilizing the cyclotron orbit of the condensate in magnetized optical lattice. In addition, the existence of the lattice potential complicates the cyclotron orbits of the condensate. Due to the coupling effects of lattice potential, artificial magnetic field, and harmonic trap potential, the cyclotron orbit of the condensate can be single periodic, multiply periodic or quasi-periodic. Hence, we can manipulate the cyclotron dynamics of neutral atoms in optical lattice by controlling the strength of artificial gauge magnetic field, harmonic trap potential, and initial conditions.

2. Model and variational approach

We focus on the influence of an artificial gauge field and an additional external potential on the dynamics of neutral atoms in 2D square optical lattices in (x,y) plane. The optical lattice is so deep that we can work in the tight-binding limit.[40] The vector potential corresponding to the effect of the uniform artificial gauge field under the Landau gauge is $\boldsymbol{A}(\boldsymbol{r}) = (0, Bx, 0)$, which induces a magnetic field $\boldsymbol{B} = B\boldsymbol{e}_z$ applied perpendicular to the optical lattice plane. The presence of the magnetic field results in a phase shift ϕ for the hopping term along the y direction,[24,33] *i.e.*, $\phi = (1/\phi_0)\int_i^j \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{r} = Bd^2/\phi_0$, where $i \equiv (x = nd, y = md)$ is the lattice site, n and m are integers, d is the lattice period, Bd^2 is the magnetic flux per plaquette, ϕ_0 is the flux quantum. Here, only the nearest neighbors hopping is considered. The phase accumulated along one lattice plaquette is gauge-invariant. The corresponding Hamiltonian of the system can be described as[33,34,38]

$$\begin{aligned} H &= -J\sum_{n,m}(\hat{a}^{\dagger}_{n,m}\hat{a}_{n+1,m} + \mathrm{e}^{\mathrm{i}\phi n}\hat{a}^{\dagger}_{n,m}\hat{a}_{n,m+1} + \mathrm{H.c.}) \\ &\quad + \frac{g}{2}\sum_{n,m}(\hat{a}^{\dagger}_{n,m}\hat{a}_{n,m} - 1)\hat{a}^{\dagger}_{n,m}\hat{a}_{n,m} + \sum_{n,m} V_{n,m}\hat{a}^{\dagger}_{n,m}\hat{a}_{n,m}, \end{aligned} \quad (1)$$

where $\hat{a}^{\dagger}_{n,m}$ and $\hat{a}_{n,m}$ are the creation and annihilation operators for a boson at the sites of the square optical lattice (n,m), H.c. denotes the conjugate term, g is the strength of atomic interaction, ϕ represents the magnetic flux per plaquette in units of the magnetic flux quantum, J is hopping matrix element. A spatially dependent scalar potential $V_{n,m}$ caused by the finite width of the laser beams creates confinement potential such as a harmonic trap or an artificial hard-wall boundary. The situation we considered is a harmonically confined system which is circularly symmetric. Then the confinement potential can be written as $V_{n,m} = \gamma(n^2 + m^2)$, where γ expresses the strength of the harmonic confinement. Under the condition of weak atomic interaction and using the mean-field approximation $a_{n,m} = \langle \hat{a}_{n,m}\rangle = \psi_{n,m}$, the discrete mean-field Hamiltonian is generated as follows:

$$\begin{aligned} H^{\mathrm{MF}} &= \sum_{n,m} -(\psi^*_{n,m}\psi_{n+1,m} + \mathrm{e}^{\mathrm{i}\phi n}\psi^*_{n,m}\psi_{n,m+1} + \mathrm{H.c.}) \\ &\quad + \frac{g}{2J}\sum_{n,m}|\psi_{n,m}|^4 + \frac{\gamma}{J}\sum_{n,m}(n^2+m^2)|\psi_{n,m}|^2. \end{aligned} \quad (2)$$

Here, the asterisk denotes the conjugation. The first term of Eq. (2) is the kinetic energy, the second term represents the interaction energy between atoms, and the last term is the harmonic trap potential energy. The dynamics of ultracold atoms given by Eq. (2) can be depicted by $\dot{\psi}_{n,m} = \partial H^{\mathrm{MF}}/\partial(\mathrm{i}\psi^*_{n,m})$, with the following discrete nonlinear Gross–Pitaevskii equation

$$\begin{aligned} \mathrm{i}\dot{\psi}_{n,m} &= -(\psi_{n+1,m} + \psi_{n-1,m} + \mathrm{e}^{\mathrm{i}\phi n}\psi_{n,m+1} + \mathrm{e}^{-\mathrm{i}\phi n}\psi_{n,m+1}) \\ &\quad + \frac{g}{J}|\psi_{n,m}|^2\psi_{n,m} + \frac{\gamma}{J}(n^2+m^2)\psi_{n,m}. \end{aligned} \quad (3)$$

Obviously, the artificial magnetic field applied in the optical lattice is asymmetric in the x and y directions.

We focus on discussing the coupling effects of the optical lattice, artificial magnetic field, and harmonic trap potential on the cyclotron dynamics of the center of the mass of the condensate by using the variational method. Here we consider the case of noninteracting atoms (*i.e.*, with $g/J = 0$, which can be tuned by the Feshbach technique).[41] Thus, the Gaussian ansatz is appropriate and valid

$$\begin{aligned} \psi_{n,m}(t) &= \sqrt{\frac{2}{\pi R_x(t)R_y(t)}}\exp\left\{-\frac{(n-\xi_x)^2}{R_x^2(t)} - \frac{(m-\xi_y)^2}{R_y^2(t)}\right. \\ &\quad \left. + \mathrm{i}[p_x(n-\xi_x) + p_y(m-\xi_y)]\right\}, \end{aligned}$$

which defines a Gaussian distribution of the atoms centered at the position $(\xi_x(t),\ \xi_y(t))$ with width $(R_x(t), R_y(t))$ and momenta $(p_x(t),\ p_y(t))$ at a given time t. This Gaussian trial wave function is simple and convenient for obtaining the cyclotron dynamics of the center of the mass of the condensate with the variational analysis. For simplicity, we assume the widths of the condensate unchanged with time, *i.e.*, $R_x(t) \equiv R_x(0)$ and $R_y(t) \equiv R_y(0)$. Taking the advantage of Euler–Lagrangian equations, where Lagrangian $L = \sum_{n,m}\frac{\mathrm{i}}{2}(\dot{\psi}_{n,m}\psi^*_{n,m} - \psi_{n,m}\dot{\psi}^*_{n,m}) - H^{\mathrm{MF}}$ and $\mathrm{d}(\partial L/\partial\dot{q}_i)/\mathrm{d}t = \partial L/\partial q_i$,[40] $q_i = \xi_x, \xi_y, p_x, p_y$, we can get the effective Hamiltonian

$$H^{\mathrm{MF}} = -2\left[\cos p_x + \cos(p_y + \phi\xi_x)\exp\left(-\frac{\phi^2 R_x^2(0)}{8}\right)\right]$$

$$\times \exp\left[-\frac{1}{2R_x^2(0)}\right] + \frac{\gamma}{J}\left(\frac{R_x^2(0)+R_y^2(0)}{4}+\xi_x^2+\xi_y^2\right), \quad (4)$$

and the variational equations

$$\dot{p}_x = -2\phi\sin(p_y+\phi\xi_x) \times \exp\left[-\left(\frac{1}{2R_y^2(0)}+\frac{\phi^2R_x^2(0)}{8}\right)\right]-\frac{2\gamma}{J}\xi_x, \quad (5a)$$

$$\dot{p}_y = -\frac{2\gamma}{J}\xi_y, \quad (5b)$$

$$\dot{\xi}_x = 2\sin p_x \exp\left[-\frac{1}{2R_x^2(0)}\right], \quad (5c)$$

$$\dot{\xi}_y = 2\sin(p_y+\phi\xi_x)\exp\left[-(\frac{1}{2R_y^2(0)}+\frac{\phi^2R_x^2(0)}{8})\right]. \quad (5d)$$

Equation (5) gives the evolution of all free (time-dependent) parameters of the trial wave function. Once we know the behavior of both mass center and momentum of the condensate, we can completely describes the evolution of the Gaussian-like atomic cloud. Equations (5a)–(5d) indicate that the artificial magnetic field ϕ and harmonic trapping γ result in nonuniform x- and y-component momenta. Especially, the x- and y-component momenta are coupled by the harmonic trapping γ. Then, equations (5c) and (5d) show that the dynamics of the wave packets in the x and y directions are coupled by the artificial magnetic field ϕ and the harmonic trapping γ. This coupling suppresses the diffusion of the wave packets and therefore may coherently traps the atoms in a closed orbit. However, the orbital instability phenomena exist if the harmonic trapping is absent. The existence of the harmonic confinement γ modifies the stability of the system and enriches the dynamics of the system. The coupling effects of the lattice potential, artificial magnetic field, and harmonic trap potential have significant influence on the dynamics of the system. The results given by Eqs. (5a)–(5d) are confirmed by direct numerical simulation of Eq. (3).

3. Asymptotical cyclotron orbit and its stability

3.1. The case of $\gamma = 0$

When the harmonic trap potential is absent, *i.e.*, $\gamma = 0$, we can get the following equilibrium points from Eqs. (5a)–(5d) by assuming $R_x(0) = R_y(0) \equiv R_0$, in which (ξ_{x_0}, ξ_{y_0}) and (p_{x_0}, p_{y_0}) are the initial position and initial momenta of the condensate:

$$\bar{\xi}_x = (l\pi - p_{y_0})/\phi, \quad \bar{\xi}_y = \xi_{y_0} + (k\pi + p_{x_0})/\phi, \quad (6)$$

where $l, k = 0, \pm1, \pm2, \ldots$ When p_{x_0}, p_{y_0}, and ϕ are fixed, equation (6) gives multiple equilibrium points in (ξ_x, ξ_y) plane spaced by π/ϕ one by one in both ξ_x and ξ_y directions, which implies that there are multiply cyclotron center of the system. The more interesting is that the stability of the equilibrium points depends on l and k, which further determines the cyclotron dynamics of the condensate. Linear stability analysis shows that when $l+k$ is even, the equilibrium points are stable, otherwise, when $l+k$ is odd, the equilibrium points are unstable.[33] Therefore, the distribution of the (un)stable equilibrium points in lattice plane changes periodically with the lattice site. So, the stability of the condensate will also change.

At this case, the analytical results of cyclotron orbit of the condensate can also be obtained:

$$\cos\phi(\xi_x + p_{y_0}/\phi) + \exp\left(\frac{\phi^2{R_0}^2}{8}\right)\cos\phi[\xi_y - (\xi_{y_0} + p_{x_0}/\phi)] = -\frac{1}{2}\exp\left[\left(\frac{1}{2R_0^2}+\frac{\phi^2R_0^2}{8}\right)\right]{H_0}^{\mathrm{MF}}, \quad (7)$$

where ${H_0}^{\mathrm{MF}}$ is the initial Hamiltonian of the system. We can find that the cyclotron orbit of the condensate given by Eq. (7) is a periodic function of ϕ. When the condensate is initially located at a stable equilibrium point, the condensate will be localized (see the first row in Fig. 1). When the condensate is initially located at an unstable equilibrium point, the condensate will diffuse disorderly with time (see the first row in Fig. 2). At this time, the stable orbit circling around the stable equilibrium point is only the single periodic orbit which is given by Eq. (7).

3.2. The case of $\gamma \neq 0$

When the harmonic trap potential is present, *i.e.*, $\gamma \neq 0$, the system (2) only has one stable equilibrium point, *i.e.*, $(\bar{\xi}_x, \bar{\xi}_y) = (0,0)$, and all the cyclotron orbits around this equilibrium point are stable. Assuming $J = 1$, then the following equations can be obtained from Eqs. (5c) and (5d):

$$\dot{p}_x = \frac{\ddot{\xi}_x}{\sqrt{4\mathrm{e}^{-1/R_0^2}-\dot{\xi}_x^2}}, \quad (8)$$

$$\dot{p}_y = \frac{\ddot{\xi}_y}{\sqrt{4\exp\left[-\left(\frac{\phi^2R_0^2}{4}+\frac{1}{R_0^2}\right)\right]-\dot{\xi}_y^2}} - \phi\dot{\xi}_x. \quad (9)$$

Combining Eqs. (5a) and (8) and Eqs. (5b) and (9), respectively, we have

$$\ddot{\xi}_x = \sqrt{4\mathrm{e}^{-1/R_0^2}-\dot{\xi}_x^2}[-\phi\dot{\xi}_y - 2\gamma\xi_x], \quad (10)$$

$$\ddot{\xi}_y = \sqrt{4\exp\left[-\left(\frac{\phi^2R_0^2}{4}+\frac{1}{R_0^2}\right)\right]-\dot{\xi}_y^2}[\phi\dot{\xi}_x - 2\gamma\xi_y]. \quad (11)$$

Obviously, both the artificial magnetic field and harmonic confinement affect the cyclotron motion of the condensate. The coupling effects of the lattice potential, artificial magnetic

field, and harmonic trap potential make the cyclotron dynamics of the condensate quite complexity. Although we cannot find the analytic result, we can obtain the asymptotical cyclotron orbit of the condensate under the condition of small cyclotron radius. By linearizing Eqs. (10) and (11) at the equilibrium point (0,0), we have

$$\ddot{\tilde{\xi}}_x + 4\mathrm{e}^{-1/2R_0^2}\gamma\tilde{\xi}_x + 2\mathrm{e}^{-1/2R_0^2}\phi\dot{\tilde{\xi}}_y = 0, \tag{12}$$

$$\ddot{\tilde{\xi}}_y + 4\exp\left[-\left(\frac{1}{2R_0^2}+\frac{\phi^2R_0^2}{8}\right)\right]\gamma\tilde{\xi}_y - 2\exp\left[-\left(\frac{1}{2R_0^2}+\frac{\phi^2R_0^2}{8}\right)\right]\phi\dot{\tilde{\xi}}_x = 0. \tag{13}$$

Setting $D_1 = \mathrm{e}^{-1/2R_0^2}, D_2 = \mathrm{e}^{-(1/2R_0^2+\phi^2R_0^2/8)}$, the asymptotical orbit of the condensate center can be described by the following equations

$$\begin{aligned}\tilde{\xi}_x &= \sqrt{c_1^2+c_2^2}\cos(\omega_1 t-\varphi_1) \\ &\quad +\sqrt{c_3^2+c_4^2}\cos(\omega_2 t-\varphi_2),\end{aligned} \tag{14}$$

$$\begin{aligned}\tilde{\xi}_y &= \left(\frac{\omega_1^2}{8\gamma\phi D_1D_2}-\frac{\phi}{2\gamma}-\frac{1}{2\phi D_2}\right) \\ &\quad \times\omega_1\sqrt{(c_1^2+c_2^2)}\sin(\omega_1 t-\varphi_1) \\ &\quad +\left(\frac{\omega_2^2}{8\gamma\phi D_1D_2}-\frac{\phi}{2\gamma}-\frac{1}{2\phi D_2}\right) \\ &\quad \times\omega_2\sqrt{(c_3^2+c_4^2)}\sin(\omega_2 t-\varphi_2),\end{aligned} \tag{15}$$

where

$$\begin{aligned}\omega_{1,2} &= \{2[(D_1+D_2)\gamma+D_1D_2\phi^2] \\ &\quad \mp 2\sqrt{[(D_1+D_2)\gamma+D_1D_2\phi^2]^2-4D_1D_2\gamma^2}\}^{1/2},\end{aligned} \tag{16}$$

$$\varphi_1 = \arctan\left(\frac{c_2}{c_1}\right), \quad \varphi_2 = \arctan\left(\frac{c_4}{c_3}\right),$$

$$c_1 = \frac{16D_1D_2^2\gamma\phi\sin(p_{y_0}+\phi\xi_{x_0})+\omega_2^2\xi_{x_0}[4D_1(D_2\phi^2+\gamma)-\omega_2^2]}{(\omega_2^2-\omega_1^2)[4D_1(D_2\phi^2+\gamma)-\omega_1^2-\omega_2^2]},$$

$$c_2 = \frac{2\{4D_1D_2\gamma\phi\xi_{y_0}-D_1\sin p_{x_0}[4D_1(D_2\phi^2+\gamma)-\omega_2^2]\}}{\omega_1(\omega_2^2-\omega_1^2)},$$

$$c_3 = \xi_{x_0}-\frac{16D_1D_2^2\gamma\phi\sin(p_{y_0}+\phi\xi_{x_0})+\omega_2^2\xi_{x_0}[4D_1(D_2\phi^2+\gamma)-\omega_2^2]}{(\omega_2^2-\omega_1^2)[4D_1(D_2\phi^2+\gamma)-\omega_1^2-\omega_2^2]},$$

$$c_4 = \frac{2\{4D_1D_2\gamma\phi\xi_{y_0}-D_1\sin p_{x_0}[4D_1(D_2\phi^2+\gamma)-\omega_1^2]\}}{\omega_2(\omega_1^2-\omega_2^2)}.$$

Here, parameters D_1 and D_2 express the effect of the optical lattice. When $D_1 = D_2 = 1$, the lattice is absent. We can see that the cyclotron dynamics of the condensate is affected not only by the artificial magnetic field ϕ and harmonic trapping γ but also by the lattice potential and initial conditions. On the other hand, the presence of the additional harmonic trap potential changes the equilibrium state properties and modifies the cyclotron dynamics of the system.

To understand the modification of the harmonic trap potential on equilibrium state and its stability of the system, figures 1–2 show the wave packet dynamics obtained by direct numerical simulations of Eq. (3) with $(\xi_{x_0},\xi_{y_0}) = (50,50)$, $(p_{x_0},p_{y_0}) = (\pi,\pi)$, $R_0 = 5$ for different ϕ. When $\phi = 0.126$ and $\gamma = 0$, $(\xi_{x_0},\xi_{y_0}) = (50,50)$ is a stable equilibrium point. The condensate is localized at its initial position (see the first row in Fig. 1). At the same time, the amplitude of the condensate oscillates periodically with time. However, $(\xi_{x_0},\xi_{y_0}) = (50,50)$ with $\phi = 0.126$ is not the equilibrium point when the harmonic trap potential is present. Due to the existence of the harmonic trap potential, *i.e.*, $\gamma = 0.001$, there is only one equilibrium point of the system under the action of the optical lattices, artificial magnetic field, and harmonic trap potential. In this way, the condensate does cyclotron motion around the equilibrium point $(\bar{\xi}_x,\bar{\xi}_y) = (0,0)$ as time varies (see the second row in Fig. 1). The red lines in m–n plane are our asymptotic orbits given by Eqs. (14) and (15) and it is in good agreement with the direct numerical simulation of Eq. (3).

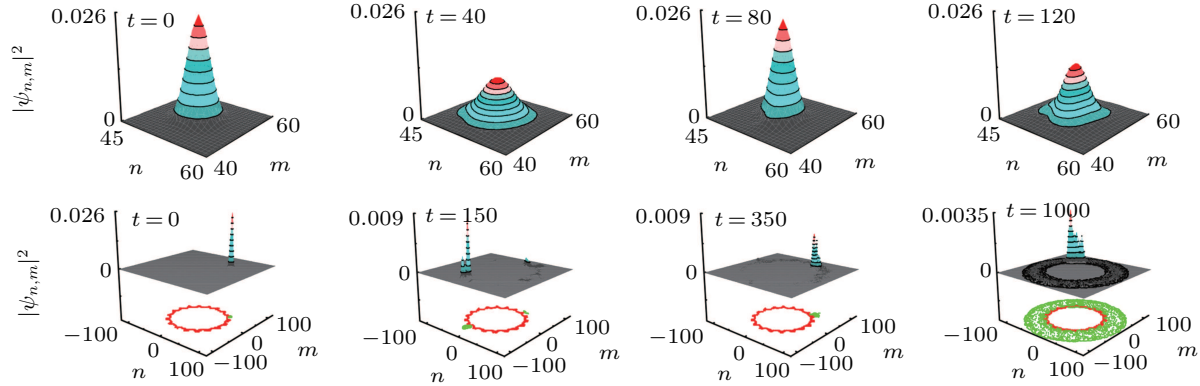


Fig. 1. The dynamics of the condensate with $\phi = 0.126$. From the first row to the second row: $\gamma = 0, 0.001$, respectively. The cyclotron orbit (red lines) of the second row in the m–n plane is given by Eqs. (14) and (15). The initial conditions are $(\xi_{x_0},\xi_{y_0}) = (50,50), (p_{x_0},p_{y_0}) = (\pi,\pi)$.

When $\phi = 0.189$ and $\gamma = 0$, $(\xi_{x_0},\xi_{y_0}) = (50,50)$ is an unstable equilibrium point under the action of lattice potential. At this case, the cyclotron orbit is unstable and the condensate expand disorderly with time (see the first row in Fig. 2). But

$(\xi_{x_0}, \xi_{y_0}) = (50, 50)$ with $\phi = 0.189$ is not the equilibrium point when the harmonic trap potential is present. In this case, as shown in the second row in Fig. 2, the condensate does a stable cyclotron motion along the asymptotic orbit predicted by Eqs. (14) and (15). Therefore we can gain a conclusion that, the harmonic trap potential plays role in modifying the equilibrium state properties and stabilizing the cyclotron orbits of the condensate in optical lattice under the artificial magnetic field. In addition, the presence of the harmonic trap potential makes the orbit quite complexity, and the complexity of the orbits depends on the coupling of the lattice potential, artificial magnetic field, and harmonic trap potential.

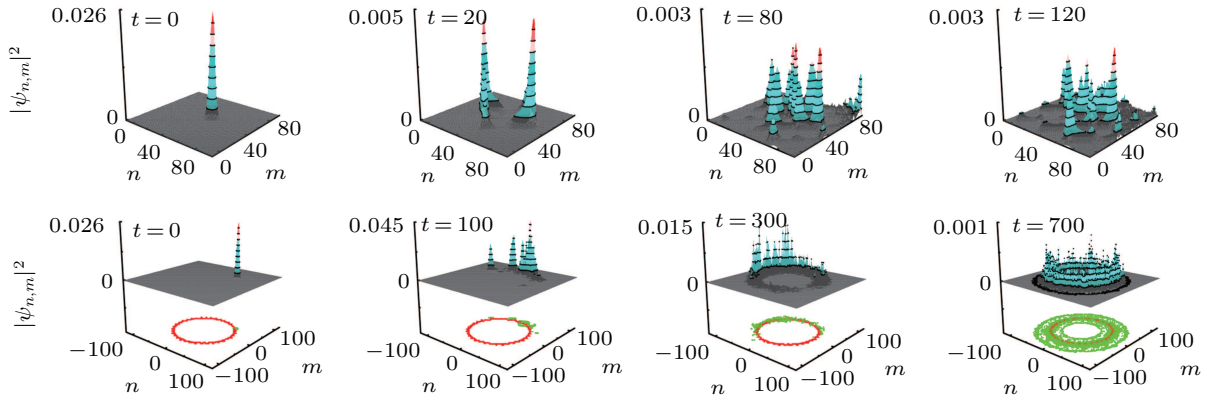


Fig. 2. The dynamics of the condensate with $\phi = 0.189$. From the first row to the second row: $\gamma = 0, 0.001$, respectively. The cyclotron orbit (red lines) of the second row in the m–n plane is given by Eqs. (14) and (15). The initial conditions are $(\xi_{x_0}, \xi_{y_0}) = (50, 50), (p_{x_0}, p_{y_0}) = (\pi, \pi)$.

In order to have a deeper understanding of the cyclotron dynamics of the condensate, the two eigenmodes of ω_1 and ω_2 given by Eq. (16) varying with ϕ are shown by the black lines in Fig. 3, where different lines represent different γ. When the harmonic trap potential is absent, *i.e.*, $\gamma = 0$, $\omega_1 \equiv 0$, the condensate does single periodic oscillations with frequency $\omega_2/(2\pi)$. In this case, only the single periodic cyclotron orbit exists. However, when the harmonic trap potential is present, the cyclotron dynamics of the condensate is the composition of the two different eigenmodes, one is a low-frequency mode with frequency $\omega_1/(2\pi)$, and another one is a high-frequency mode with frequency $\omega_2/(2\pi)$, which cause a series of complex dynamics phenomena. The frequency of the two modes increases with γ. As shown by the black lines in Fig. 3, the low-frequency mode ω_1 is almost independent of the lattice and decreases monotonously with ϕ, and $\omega_1 \to 0$ when $\phi \to \infty$. It means that for sufficiently larger artificial magnetic field ($\phi \to \pi/2$), the cyclotron dynamics of the system approaches to a single periodic orbit with eigenmode ω_2. However, when the lattice is present, the high-frequency mode ω_2 changes nonlinearly with ϕ, *i.e.*, the high-frequency mode ω_2 drops down to the value of $\phi = 0$ after it reaches the maximum, while when the lattice potential is absent, *i.e.*, $D_1 = D_2 = 1$, ω_2 is monotonically increasing with ϕ (see the red lines in the Fig. 3). The lattice potential significantly modifies the mode ω_2, which makes rich cyclotron dynamics of the condensate. We can get rich cyclotron orbits from Eqs. (14) and (15) which depend on the coupling effects of the lattice potential, artificial magnetic field, and harmonic trap potential. The cyclotron orbit of the condensate will be multi-periodic or quasi-periodic depending on ω_1/ω_2 being a rational number or not.

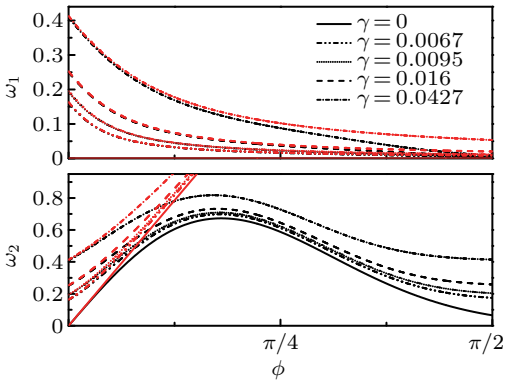


Fig. 3. The frequency of the two eigenmode against ϕ for different γ. The different curves correspond to different γ. The black lines represent the case when the lattice potential is present. The red lines denote the situation when the lattice potential does not exist.

4. Complex cyclotron dynamics

In order to more clearly understand the dependence of cyclotron dynamics on the coupling effects of the lattice potential, artificial magnetic field, and harmonic trap potential, we obtain many kinds of cyclotron orbits from Eqs. (14) and (15). The orbit of the condensate is multi-periodic when ω_1/ω_2 is a rational number, or quasi-periodic when ω_1/ω_2 is an irrational number. In addition, there are single periodic elliptical orbits in some special cases. Next, we will discuss these three kinds of orbits respectively.

4.1. Single periodic orbit

We can obtain single periodic orbit by setting $c_1 = c_2 = 0$ or $c_3 = c_4 = 0$ according to the asymptotical solution of Eqs. (14) and (15). Taking $c_3 = c_4 = 0$ as an example, γ and ϕ need to satisfy the relationship as follows:

$$D_2\phi^2+\gamma-\frac{D_2\gamma}{D_1}+\sqrt{\left(\gamma+\frac{D_2\gamma}{D_1}+D_2\phi^2\right)^2-\frac{4D_2\gamma^2}{D_1}}=\frac{2D_2\gamma\phi\xi_{y_0}}{D_1\sin p_{x_0}}, \tag{17}$$

$$D_2\phi^2-\gamma+\frac{D_2\gamma}{D_1}-\sqrt{\left(\gamma+\frac{D_2\gamma}{D_1}+D_2\phi^2\right)^2-\frac{4D_2\gamma^2}{D_1}}=\frac{2D_2\phi\sin\left(p_{y_0}+\phi\xi_{x_0}\right)}{\xi_{x_0}}. \tag{18}$$

So, the orbit of the condensate can be written as

$$\xi_x^2+\frac{\xi_y^2}{[-\omega_1^3/8\gamma\phi D_1D_2+\omega_1(\phi/2\gamma+1/2D_2\phi)]^2}=c_1^2+c_2^2. \tag{19}$$

It shows that when γ and ϕ satisfy Eqs. (17) and (18), the trajectory of the mass center of the condensate is an ellipse. The cyclotron orbit is a single periodic orbit with frequency $\omega=\omega_1/(2\pi)$ and period $T=2\pi/\omega_1$.

To confirm the above theoretical predictions, as an example, we set the initial position $(\xi_{x_0},\xi_{y_0})=(-5,5)$ and the initial momenta $(p_{x_0},p_{y_0})=(0.5,1.0)$. Then we can obtain the corresponding strength of the harmonic trapping $\gamma=0.018$ and artificial magnetic field $\phi=0.094$ from Eqs. (17)–(18). The asymptotical orbit of the condensate given by Eq. (19) and the corresponding direct numerical simulation of Eq. (3) are presented in Fig. 4. Under such conditions, the condensate makes clockwise elliptical motion, because we choose $p_{x_0}>0$ and $p_{y_0}>0$. From Fig. 4 we can find that the asymptotical result is in good agreement with the numerical result.

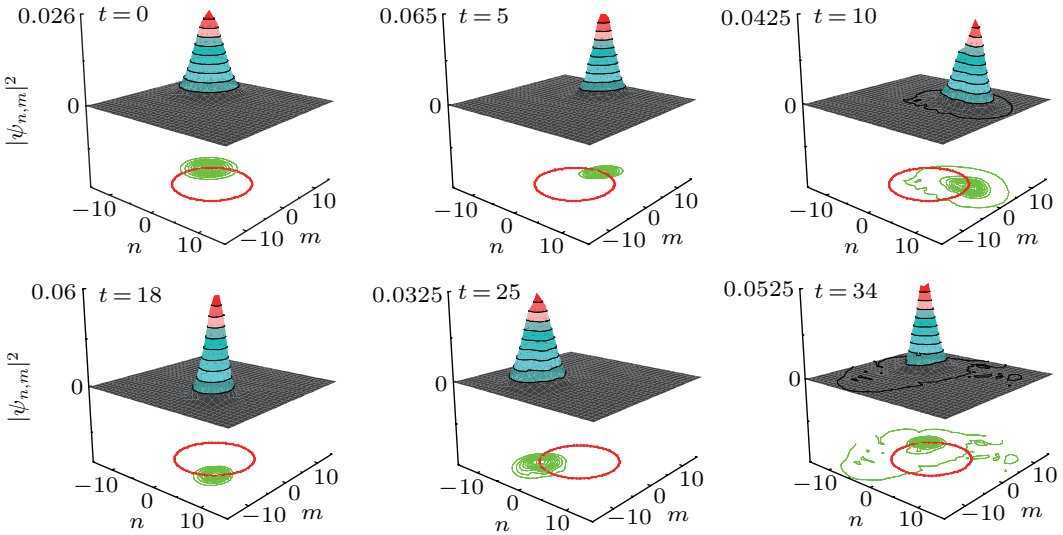


Fig. 4. The dynamics of the condensate and its contour plot with $\gamma=0.018$, $\phi=0.094$. The elliptical orbit (red lines) is our theoretical prediction of Eq. (19). The initial position is $(\xi_{x_0},\xi_{y_0})=(-5,5)$ and the initial momenta are $(p_{x_0},p_{y_0})=(0.5,1.0)$.

4.2. Multi-periodic orbit

If ω_1/ω_2 is a rational number, the cyclotron trajectory given by Eqs. (14) and (15) will be multi-periodic. Setting $\omega_1/\omega_2=1/Q$, then from Eq. (16) we can obtain

$$\gamma=\frac{D_2\phi^2}{[(Q^2+1)/Q]\sqrt{D_2/D_1}-1-D_2/D_1}, \tag{20}$$

where $1/Q$ means a rational number. So given the initial conditions and the magnetic field intensity, we can get the corresponding strength of the harmonic trap potential which makes $\omega_1/\omega_2=1/Q$ a rational number. We plot the orbits given by Eqs. (14) and (15) with $\omega_1/\omega_2=1/2$, $1/3$, $1/4$, $1/5$ respectively in Fig. 5, where the initial position is $(\xi_{x_0},\xi_{y_0})=(3,3)$. Using the strength of the artificial magnetic field $\phi=0.15$, the corresponding harmonic confinement $\gamma=0.0427,0.016,0.0095,0.0067$ can be calculated from Eq. (20). The initial momenta are shown in Fig. 5. We predict

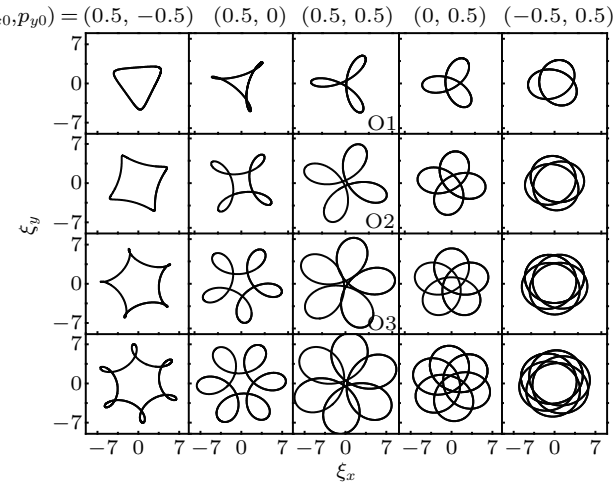


Fig. 5. The cyclotron orbits in the ξ_x–ξ_y plane with $\phi=0.15$ and $(\xi_{x_0},\xi_{y_0})=(3,3)$. From the first row to the fourth row: $\gamma=0.0427,0.016,0.0095,0.0067$, respectively. From the first column to the fifth column: $(p_{x_0},p_{y_0})=(0.5,-0.5),(0.5,0),(0.5,0.5),(0,0.5),(-0.5,0.5)$, respectively.

that neutral atoms exposed to an artificial magnetic field and harmonic trap potential in optical lattices have various multi-periodic orbits as harmonic trap potential and initial momenta vary. Obviously, the orbit of the condensate is not only closed, but also has many intersections. That is to say, the orbit is multi-periodic. Under this condition, the larger the γ is, the less number of the intersection points is.

We also plot the temporal evolution and the contour plot of the wave packet in Fig. 6 by direct numerical simulation of Eq. (3) for the cases of O1–O3 marked in Fig. 5. The red lines in m–n plane are our theoretical results. As time goes by, the mass center of the condensate is moving along the asymptotical orbit given by Eqs. (14) and (15) although the Gaussian shape of the condensate is no longer maintained. After one period, the mass center of the condensate returns to its initial position. The theoretical prediction is in good agreement with the numerical simulation.

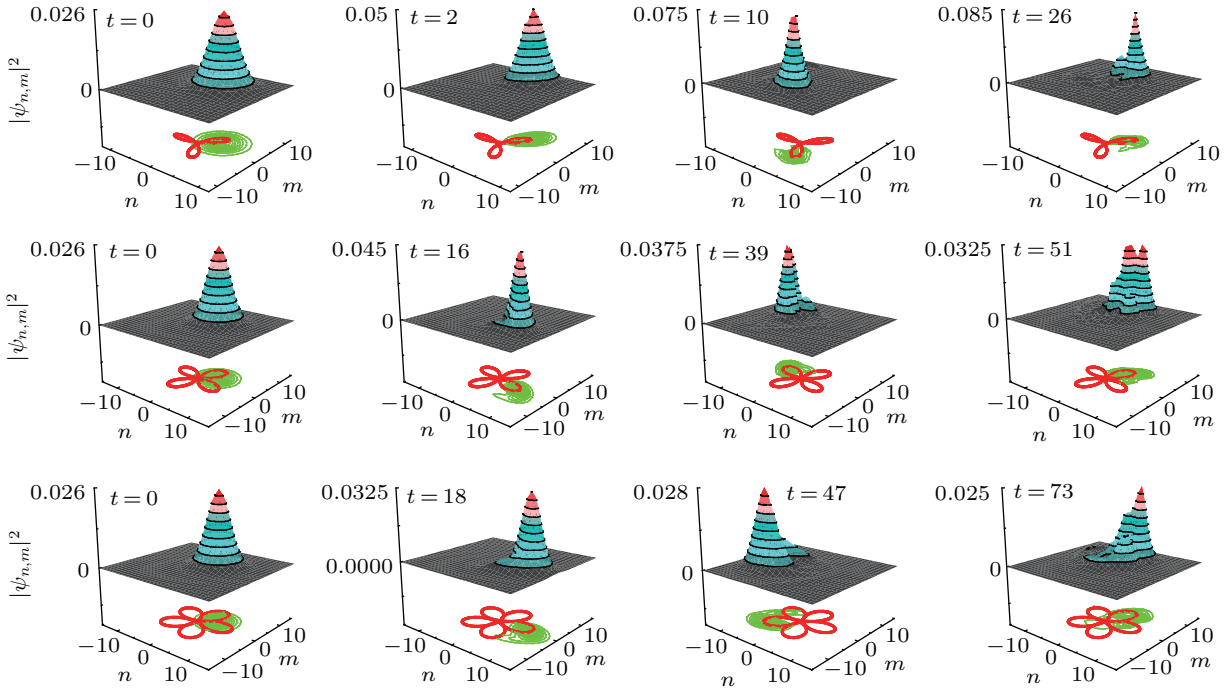


Fig. 6. The dynamics of the condensate and its contour plot given by numerical simulation of Eq. (3) for the cases marked by O1, O2, and O3 in Fig. 5. The red orbits are given by Eqs. (14) and (15).

4.3. Quasi-periodic orbit

If ω_1/ω_2 is an irrational number, the asymptotical cyclotron orbit of the condensate given by Eqs. (14) and (15) will be quasi-periodic. The presence of the lattice potential, artificial magnetic field, and harmonic confinement leads to abundant and complex dynamics phenomena. Figure 7 illustrates the asymptotical cyclotron orbits with $(\xi_{x_0}, \xi_{y_0}) = (5,5)$ and $\gamma = 0.0067$ for different ϕ. In this case, ω_1/ω_2 is an irrational number. At the first column, the initial momenta $(p_{x_0}, p_{y_0}) = (0,0)$, and at the second column, the initial momenta $(p_{x_0}, p_{y_0}) = (\pi,\pi)$. In these cases, the cyclotron orbit of the mass center of the condensate is quite complex. The condensate is doing a non-repetitive cyclotron motion in an annular region. The trajectory of the condensate is not closed and intersected at some points, *i.e.*, the orbit is quasi-periodic. We can find from Fig. 3 that $\omega_1 \to 0$ when $\phi = 0.98$, so the cyclotron dynamics of the system approaches to a single periodic orbit with eigemode ω_2 (see the second row in Fig. 7). The temporal evolution of the condensate given by direct numerical simulation of Eq. (3) for the case of O4 marked in Fig. 7 is presented in Fig. 8, where red lines in m–n plane are our theoretical results. The numerical results are also agree with our theoretical results.

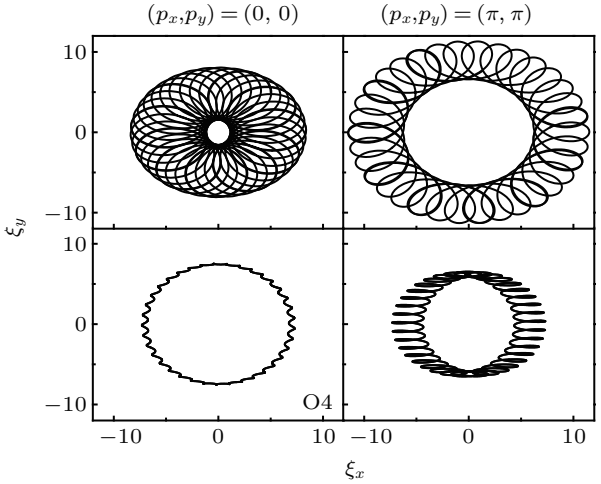


Fig. 7. The cyclotron orbits in the ξ_x–ξ_y plane with $(\xi_{x_0}, \xi_{y_0}) = (5,5), \gamma = 0.0067$. In the first row $\phi = 0.252$ and in the second row $\phi = 0.98$. From the first column to the second column: $(p_{x_0}, p_{y_0}) = (0,0), (\pi,\pi)$, respectively.

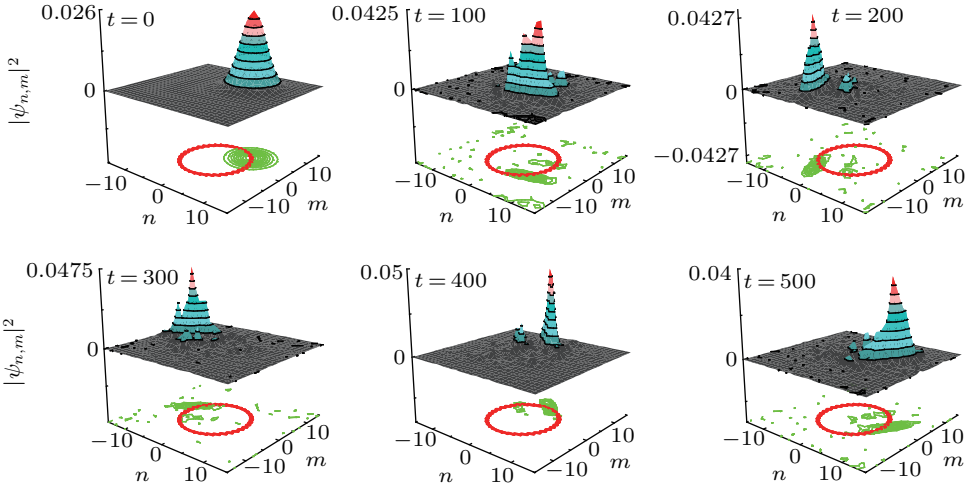


Fig. 8. The dynamics of the condensate and its contour plot given by Eq. (3) for the case marked by O4 in Fig. 7.

5. Conclusion

In summary, we considered the effects of the artificial magnetic field and harmonic trap potential on the cyclotron dynamics of the neutral atoms in optical lattices. Rich and complex dynamics phenomena of neutral atoms in optical lattices are predicted under the coupling effects of lattice potential, artificial magnetic field, and harmonic confinement.

In the absence of the harmonic trap potential, there are multiple equilibrium points with different stability characters in the system and the stability of the system depends on the stability of the equilibrium point. In this case, the stable orbit of the condensate is only the single periodic orbit, which circling around the stable equilibrium point. However, the existence of the harmonic trap potential significantly modifies the equilibrium state properties and the stability of the condensate in optical lattice exposed to the artificial magnetic field. There is only one equilibrium point in the system, so the cyclotron center is only one as well. The asymptotical cyclotron dynamics of the condensate is the composition of the two different eigenmodes. The lattice potential significantly modifies the high-frequency mode, which causes rich dynamics phenomena of the condensate. Therefore, the coupling effects of lattice potential, artificial magnetic field, and harmonic trap potential lead to single periodic, multi-periodic or quasi-periodic cyclotron orbit of neutral atoms. In these cases, although the condensate may not keep its initial Gaussian shape as time goes on, the mass center of the condensate follows the theoretically predicted trajectory. So we can manipulate the cyclotron dynamics of neutral atoms in optical lattice by adjusting the strength of the artificial magnetic field, harmonic trap potential, and initial conditions.

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