

Quantum interference of a time-dependent wave packet of atom irradiated by an ultra-short laser pulse

Wen-Min Yan(闫文敏)[1,2], Ji-Gen Chen(陈基根)[3], Jun Wang(王俊)[1,2], Fu-Ming Guo(郭福明)[1,2], Yu-Jun Yang(杨玉军)[1,2]

Citation: Chin. Phys. B . 2020, 29(1): 013202 . **doi:** 10.1088/1674-1056/ab5a3c
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Effect of laser intensity on quantum trajectories in the macroscopic high-order harmonic generation

Yun Pan(潘云)[1,2], Fuming Guo(郭福明)[1,2], Yujun Yang(杨玉军)[1,2], Dajun Ding(丁大军)[1,2]

Chin. Phys. B . 2019, 28(11): 113201 . **doi:** 10.1088/1674-1056/ab47f8

Helicity of harmonic generation and attosecond polarization with bichromatic circularly polarized laser fields

Jun Zhang(张军), Tong Qi(齐桐), Xue-Fei Pan(潘雪飞), Jing Guo(郭静), Kai-Guang Zhu(朱凯光), Xue-Shen Liu(刘学深)

Chin. Phys. B . 2019, 28(10): 103204 . **doi:** 10.1088/1674-1056/ab4176

Selection of right-circular-polarized harmonics from *p* orbital of neon atom by two-color bicircular laser fields

Chang-Long Xia(夏昌龙), Yue-Yue Lan(兰悦跃), Qian-Qian Li(李倩倩), Xiang-Yang Miao(苗向阳)

Chin. Phys. B . 2019, 28(10): 103203 . **doi:** 10.1088/1674-1056/ab4278

Interpulse interference of electron emission from an atom irradiated by sinusoidally phase-modulated pulse

Hai-Ying Yuan(袁海颖), Fu-Ming Guo(郭福明), Di-Yu Zhang(张頔玉), Jun Wang(王俊), Ji-Gen Chen(陈基根), Yu-Jun Yang(杨玉军)

Chin. Phys. B . 2018, 27(10): 103201 . **doi:** 10.1088/1674-1056/27/10/103201

Simultaneous study of the lower order harmonic and photoelectron emission from an atom in intense laser pulse

Yan Yan(闫妍), Fu-Ming Guo(郭福明), Jun Wang(王俊), Ji-Gen Chen(陈基根), Yu-Jun Yang(杨玉军)

Chin. Phys. B . 2018, 27(8): 083202 . **doi:** 10.1088/1674-1056/27/8/083202

Quantum interference of a time-dependent wave packet of atom irradiated by an ultra-short laser pulse*

Wen-Min Yan(闫文敏)[1,2], Ji-Gen Chen(陈基根)[3], Jun Wang(王俊)[1,2], Fu-Ming Guo(郭福明)[1,2], and Yu-Jun Yang(杨玉军)[1,2,†]

[1] *Institute of Atomic and Molecular Physics, Jilin University, Changchun* 130012, *China*
[2] *Jilin Provincial Key Laboratory of Applied Atomic and Molecular Spectroscopy (Jilin University), Changchun* 130012, *China*
[3] *Zhejiang Provincial Key Laboratory for Cutting Tools, Taizhou University, Taizhou* 318000, *China*

(Received 16 September 2019; revised manuscript received 5 November 2019; accepted manuscript online 22 November 2019)

The wave packet evolution of an atom irradiated by an intense laser pulse is systematically investigated by using the numerical solution of the time-dependent Schrödinger equation. There are two types of spatial interference structures in the time-dependent evolution of the atomic wave packet. With the increasing of the evolution time, the interference fringe spacing for type I (type II) becomes larger (smaller). As the wavelength of the incident laser increases, the interference of the wave packet is changed from type II to type I, and the shift of interference type can be attributed to the contribution of excited states by using the energy analysis of the time-dependent wave function.

Keywords: high-order harmonic generation, above-threshold ionization

PACS: 32.80.Rm, 42.50.Hz **DOI:** 10.1088/1674-1056/ab5a3c

1. Introduction

With advances in laser technologies, the intensity for an ultra-short laser pulse has approached or exceeded the strength of the electric field experienced by an electron of the atom.[1,2] When the atom is irradiated by the strong laser pulse, many novel nonlinear phenomena were observed, such as non-sequential double ionization, above-threshold ionization, and high-order harmonic generation (HHG).[3–14] These strong field processes have important applications in the ultrafast measurement, for example, HHG is the main route for producing attosecond time-scale optical pulses.[15–21] Therefore, the strong field atomic physics has become the focus of the recent studies.

The processes of strong-field atomic physics can be understood by the semi-classical three-step model.[22,23] The bound electron tunnels through the potential barrier formed by the action of the atomic potential and the laser electric field, and then the electron driven by the electric field is away from the nucleus. Part of the ionized wave packet has a chance to return to the parent ion under the influence of the driving light field. If the elastic scattering between the ionized electron and the parent ion occurs, the above-threshold ionization with high energy can be observed; if the ionized electron can recombine with its parent ion, harmonic photons are emitted. However, it is necessary to solve the time-dependent Schrödinger equation (TDSE) in order to understand mechanisms behind these physical processes.[24–28] There is no analytical solution for TDSE of an atom irradiated by the strong laser pulse, which can only rely on numerical schemes.

Many numerical methods were adopted to clarify nonlinear phenomena in the strong field atomic physics. For example, one can calculate trajectories of particles by using the generalized quantum-trajectory Monte Carlo method[29–35] or the Bohmian trajectories[36–39] from the time-dependent wave function. In addition, by directly using the time-dependent wave function, one can not only obtain measurable parameters of the system but also intuitively understand the physics mechanism behind the interaction process between the atom and the strong laser field. By analyzing the radial distribution of the time-dependent wave packet, Chen *et al.* investigated the relation of the high harmonic generation and the spatial structure of the atomic Rydberg state.[40] Based on directly observing the interaction between the rescattering wave packet and the parent ion, Tong *et al.* analyzed the motion relation between the classical electron and the electronic wave packet.[41] Through comparing distributions of wave packet densities of a long range potential and a short range potential, Zhang *et al.* investigated the effect of soft collision on the generation of THz radiation.[42] However, there is a lack of systematic research on the time evolution of the wave packet from the interaction between the atom and the intense laser pulse.

In this work, we systematically investigate the influence of the laser intensity and the wavelength on the evolution of the wave packet by numerically solving TDSE. It is found

*Project partially supported by the National Key Research and Development Program of China (Grant Nos. 2019YFA0307700 and 2017YFA0403300), the National Natural Science Foundation of China (Grant Nos. 11627807, 11534004, 11975012, and 11774129), the Jilin Provincial Research Foundation for Basic Research, China (Grant No. 20170101153JC), and the Science and Technology Project of the Jilin Provincial Education Department, China (Grant No. JJKH20190183KJ).

†Corresponding author. E-mail: yangyj@jlu.edu.cn

© 2020 Chinese Physical Society and IOP Publishing Ltd

that there are clear interference structures of the density distribution in the electronic wave packet evolution. The feature of the interference structure changes with the variation of the wavelength and the intensity of the incident laser. By analyzing the time-dependent evolution of the wave packet with different energies, it is demonstrated that the excited state plays very important role on the evolution of the interference structure (atomic unit (a.u.) is used throughout this paper unless otherwise indicated).

2. Model and method

In the length gauge and the dipole approximation, the time-dependent Schrödinger equation for an atom irradiated by the laser pulse is

$$\mathrm{i}\frac{\partial}{\partial t}\psi(x,t) = \hat{H}(x,t)\psi(x,t), \tag{1}$$

where

$$\hat{H}(x,t) = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_{\mathrm{atom}}(x) + xE(t). \tag{2}$$

Here, the soft Coulomb potential is $V_{\mathrm{atom}}(x) = -1/\sqrt{x^2+a}$, where $a = 1.1418$. The lowest eigenenergy of this potential is -0.579, which is corresponding to the ground energy of argon. The interaction between the atom and the laser electric field is $xE(t)$, where $E(t) = E_0 \sin(\omega t)\,\mathrm{e}^{-(t-1.5T)^2/\sigma^2}$. Here, E_0 is the peak amplitude of the laser electric field, ω is the frequency, T is the duration of one optical cycle, and the width $\sigma = 0.75T$ of the laser pulse. Equation (1) is numerically solved with the split operator method.[28–31] To analyze the interference of the wave packet, amplitudes of bound and continuum states are calculated from the projection of the time-dependent wave function to the basis function without the electric field

$$c_i(t) = \langle \psi_i(x)|\psi(x,t)\rangle, \tag{3}$$

$$c_\varepsilon(t) = \langle \psi_\varepsilon(x)|\psi(x,t)\rangle. \tag{4}$$

By using amplitude information, one can obtain populations of excited and continuum states of the atom irradiated by laser pulse:

$$P_{\mathrm{excited}}(t) = \sum_{i=2}^{n} |c_i(t)|^2, \tag{5}$$

$$P_{\mathrm{ion}}(t) = \int_0^{E_{\max}} |c_\varepsilon(t)|^2\,\mathrm{d}\varepsilon. \tag{6}$$

3. Results and discussion

Figure 1 shows the time-dependent evolution of the laser electric field and the wave packet, the ground state of which is subtracted in the calculation. Here, the incident laser wavelength is 1000 nm and its peak amplitude $E_0 = 0.08$. It can be noticed from Fig. 1 that, the atomic ionization mainly occurs at the two peaks ($t = 1.25T$ for peak 1 and $t = 1.75T$ for peak 2) of the laser electric field. When the atom is ionized at peak 1, its ionization wave packet is driven away from the atomic nucleus by the laser electric field. The amplitude of the ionization wave packet is continuously reduced due to the dispersion effect. After the second ionization at peak 2, one can observe a clear interference in the wave packet evolution, and there are two patterns of the wave packet interference. With the increasing of the time, if the spatial width between interference fringes becomes larger as shown by the white dotted line in Fig. 1, this interference is called as type I; if the spatial width is decreased as shown by the white dashed line in Fig. 1, the interference is named as type II. Especially, one can see from Fig. 1, the interference pattern is gradually changed from type II to type I.

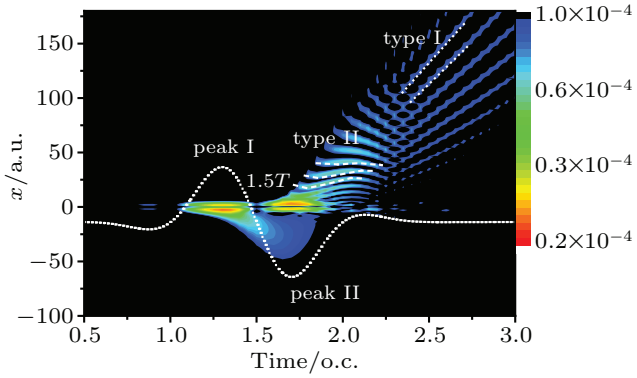


Fig. 1. The variation of atomic potential energy with the time-dependent evolution of the laser electric field (wavelength is 1000 nm and peak amplitude $E_0 = 0.08$) and the wave packet (the ground state is subtracted). The unit o.c. is short for optical cycle.

In order to understand the physical mechanism of the interference pattern, we study the effect of laser parameters on the interference pattern. We first investigate the laser wavelength effect on the evolution of the wave packet. Figure 2 exhibits the time-dependent wave packet evolution of atom irradiated by different wavelength (from 600 nm to 1600 nm) laser pulses with peak amplitude $E_0 = 0.08$. It can be seen from Fig. 2 that, there are two main ionization instants in the evolution of wave packets. Furthermore, with the increase of the wavelength, the pattern of the wave packet interference is changed from type I to type II. When the driving wavelength is 600 nm, the interference pattern only contains the type I. For the longer driving wavelength (1600 nm), the interference type II is only retained. In addition to the influence of the laser wavelength on the wave packet evolution, the intensity of the laser pulse will also play an important role in the evolution process.

Figure 3 presents the wave packet evolution of the atom in different peak amplitude (0.07–0.1) laser pulses with 800-nm wavelength. As the laser intensity enhances, the amplitude of the ionized wave packet gradually increases. For the 800-nm driven laser pulse, when the laser intensity is weaker than 0.08, the interference pattern of the time dependent wave

packet is mainly dominated by type I stripes. If the laser intensity is stronger than 0.08, the interference type II fringes gradually play a role in the evolution dynamics. Especially, for $E_0 = 0.1$, the interference type II has a major role in the interval from 1.6 o.c. to 2.2 o.c. When the time is beyond 2.5 o.c., the interference fringes convert to type I for the wave packet evolution of the atom driven by the same laser pulse.

For the longer wavelength (1000 nm), the wave packet evolution shows different features with the increase of the laser intensity, as shown in Fig. 4. For this wavelength, one can clearly observe that, both types of interference patterns exist in the evolution of the wave packet at the same time. For the laser intensity calculated in the figure, the interference characteristics of the wave packet are dominated by type II. When the time is beyond 2.5 o.c., the observed interference structure is dominated by the interference between type I and type II. According to the above analyses, as the increase of the wavelength or the intensity of the laser pulse, the interference structure in the evolution of the wave packet exhibits a transition from type I to type II stripe.

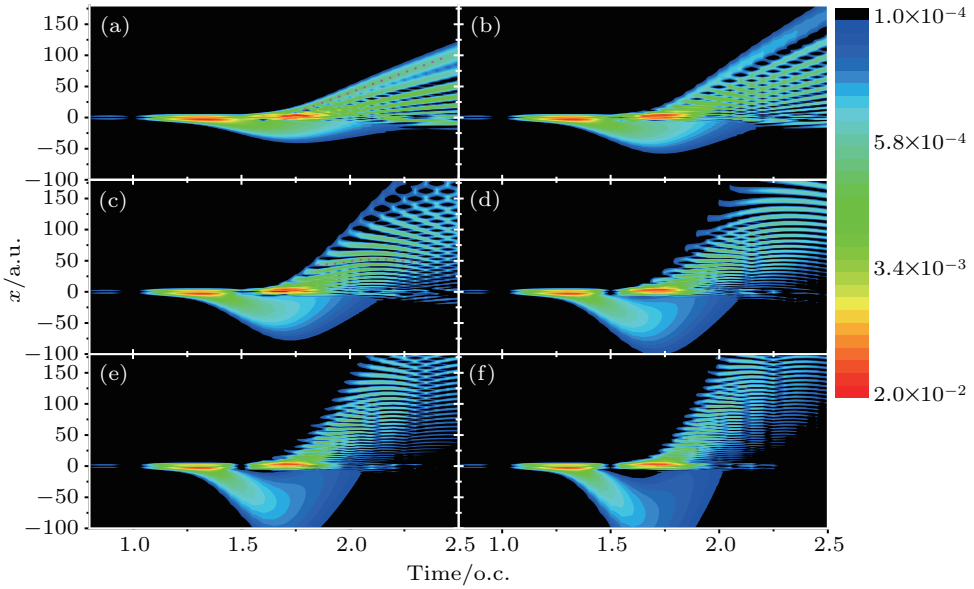


Fig. 2. Evolution of wave packet for an atom irradiated by different wavelength [(a) 600 nm, (b) 800 nm, (c) 1000 nm, (d) 1200 nm, (e) 1400 nm, (f) 1600 nm] laser pulses with peak amplitude $E_0 = 0.08$.

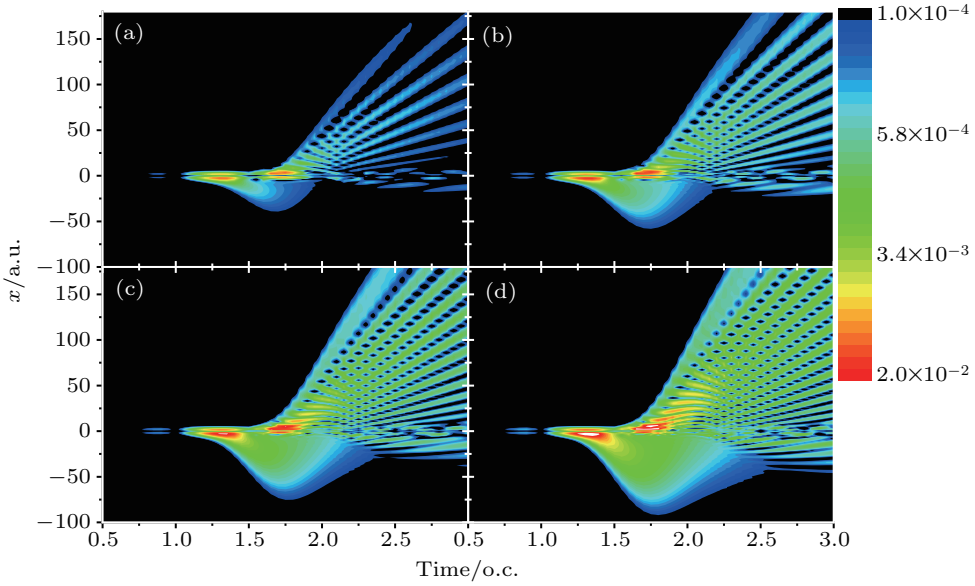


Fig. 3. Wave packet evolution of an atom irradiated by different peak amplitude [(a) $E_0 = 0.07$, (b) $E_0 = 0.08$, (c) $E_0 = 0.09$, (d) $E_0 = 0.1$] laser pulses with 800-nm wavelength.

Through above studies on the changing law of interference pattern, it is found that the variation of interference pattern from type I to type II is attributed to the increase of the pondermotive energy of the laser pulse. The transition of the interference pattern corresponds to the changing of ionization mechanism from multi-photon ionization to tunnel ionization. In order to clarify the transition mechanism of the interference fringes changing with laser parameters, we altered populations

of different atomic eigenstates of the wave packet in the middle of the incident pulse. Since there are mainly two ionization moments, these interference fringes are essentially generated by the interference between the two ionized wave packets. By changing compositions of the wave packet after the first ionized moment, one can directly observe the interference from different energy wave packets. At $1.5T$ in the evolution process, the population composition in the wave function is artificially changed and continued to evolve the corresponding time-dependent wave function.

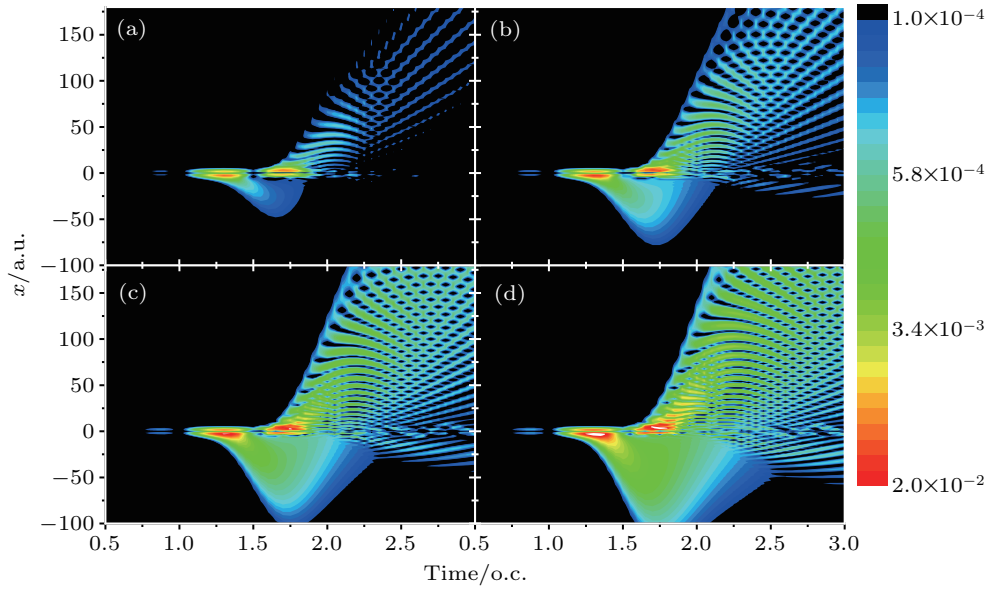


Fig. 4. Wave packet evolution of an atom irradiated by different intensity [(a) $E_0 = 0.07$, (b) $E_0 = 0.08$, (c) $E_0 = 0.09$, (d) $E_0 = 0.1$] laser pulses with 1000-nm wavelength.

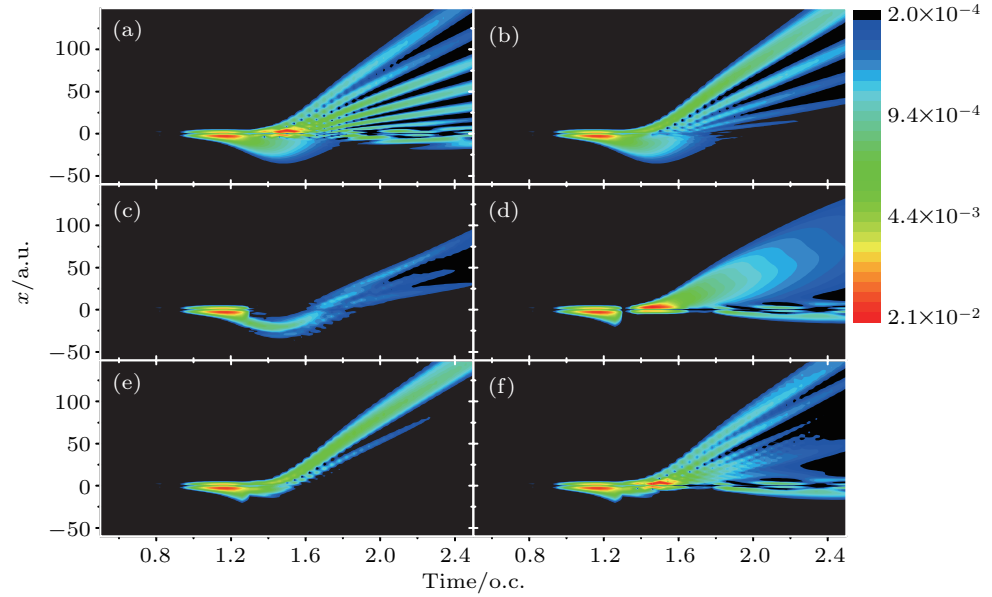


Fig. 5. Wave packet evolution of an atom irradiated by the 800-nm laser pulse with $E_0 = 0.08$ at an instant $1.5T$ for the wave function by: (a) doing nothing, (b) subtracting the ground state, (c) subtracting bound states, (d) only containing ground state, (e) only containing excited states, and (f) containing bound states.

Figure 5 shows wave packet evolution results from different energy populations of the atom irradiated by a 800-nm laser pulse, and figure 5(b) shows results of the time-dependent wave packet retaining continuous and excited states at $1.5T$. It can be seen from the figure that, the number of interference fringes in the evolution process is reduced, especially for the wave packet near the origin position. If the continuum part of the wave function is only retained at $1.5T$, number of the interference fringes is further reduced and only a clear one can be seen (Fig. 5(c)). From the comparison between Fig. 5(b) and Fig. 5(c), one can explicitly notice the contribution to the interference fringes from excited states and continuum states. However, one cannot observe the interference fringes if only the ground state population is kept in the time-dependent wave function at $1.5T$ (Fig. 5(d)). While only the excited state population is retained, a relatively strong sin-

gle interference fringe can be seen from Fig. 5(e). When the total bound state populations are preserved in the wave packet, one can find that the wave packet distribution (Fig. 5(f)) is similar as the original wave packet evolution as shown in Fig. 5(a). In terms of the above analyses, we demonstrated the interference patterns generated by the interference between the ionized wave packet from the ground state at peak 1 and the ionization part from the bound states at peak 2. The interference farthest from the nucleus is generated by the ionization from the excitation at peak 1.

Figure 6 shows the time-dependent wave packet evolution from the different energy populations of the atom irradiated by a 1200-nm laser pulse. Compared with the result from the 800 nm laser pulse, the spatial position of the wave packet is far away from the nucleus. The interference pattern in the wave packet evolution is dominated by the type II. When the ground state is removed at $1.5T$, there is no interference, as presented in Fig. 6(b). When the bound states are further subtracted, a weaker interference type I can be seen from Fig. 6(c). Due to the longer wavelength of the incident laser pulse, the ionized wave packet spreads in a larger spatial region. If the ground state is only retained, the ionization from the ground state is concentrated at peak 2, as shown in Fig. 6(d). When the driver pulse's wavelength is longer, the ionization from the excited states plays the minor role as shown in Fig. 6(e).

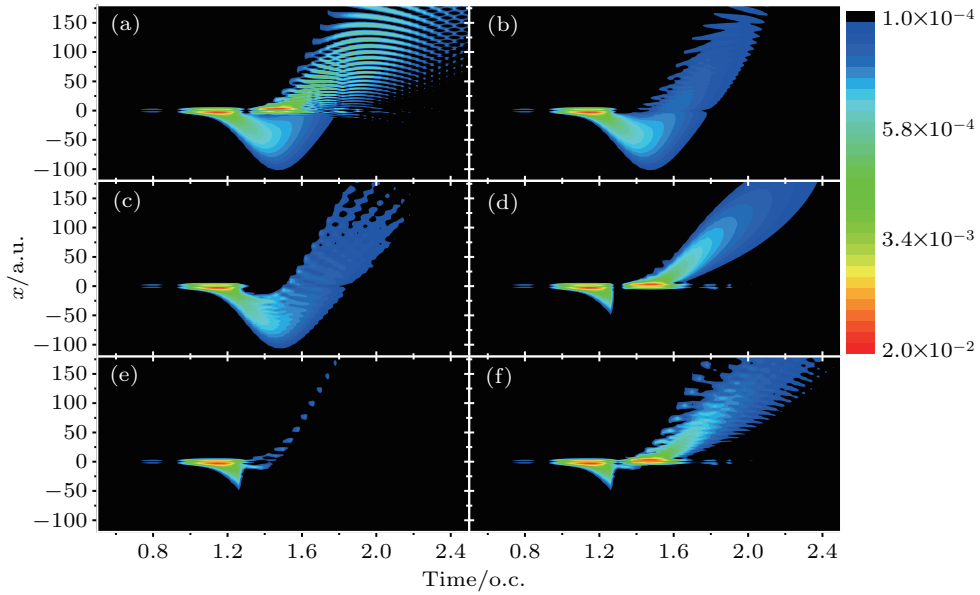


Fig. 6. Wave packet evolution of the atom irradiated by the 1200-nm laser pulse at the instant $1.5T$ for the wave function by: (a) doing nothing, (b) subtracting the ground state, (c) subtracting bound states, (d) only containing ground state, (e) only containing excited states, and (f) containing bound states.

From the above discussions, it can be found that the excitation states play an important role in the interference pattern for the case of the small pondermotive energy. In this laser parameter regime, the main ionization mechanism of the atom is the multiphoton ionization. After driven by the subsequent laser electric field, the excited state is ionized in a short duration. Therefore, the ionized wave packet carries spatial distribution information of the excited state. Driven by the laser electric field, this part of the ionized wave packet is far away from the nuclear region, resulting in the appearance of the interference type I. For the laser pulse with larger pondermotive energy, ionization wave packets are produced mainly through the tunneling process and the excitation plays little role in the ionization. Part of the ionized wave packets can return to the parent ion driven by the laser electric field. The returned wave packets with the lower energy firstly pass the parent ion and intersect with the wave packet generated from the ionization in the ensuing half optical cycle. Due to the interference of the wave packet with the lower energy, the spatial width of interference fringes becomes larger. At a later instant, the returned wave packets with higher energy intersect with the wave packet from the second half optical cycle, which leads to the smaller spatial width of interference fringes. Spatial width variation of interference fringes results in the appearance of the type II structure. In order to confirm this conclusion, we systematically studied the population of the ionization and excitation at the middle ($1.5T$) and the end of the pulse ($3T$). Figures 7(a) and 7(b) show the variation of excited states and ionization populations for different intensity laser pulses with wavelength 600 nm and 1200 nm at instants $t = 1.5T$ and $t = 3T$, respectively. For the longer wavelength laser pulse, the ionization mainly comes from the ground state. Thus, the ionization population at the middle of the pulse is half of the population at the end of the laser pulse. The population of the excited states is smaller than that of the ionization in the whole range of laser intensity. For the short wavelength driving pulse, the excitation has a large influence on the overall ionization. The wave packet is excited during the first half of

the optical cycle and is ionized in the second half of the optical cycle.

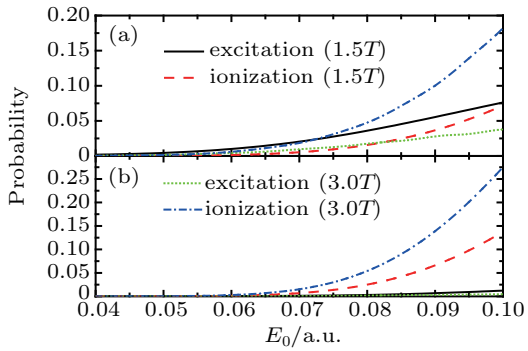


Fig. 7. Populations of the excited states and ionizations at instants $t = 1.5T$ and $t = 3T$ for different intensity laser pulses with wavelength 600 nm (a) and 1200 nm (b), respectively.

4. Conclusion

In summary, we theoretically investigated the wave packet evolution of the atom in an ultra-short laser pulse by solving the time-dependent Schrödinger equation. Two interference patterns are observed in the wave packet evolution process. With the increase of the laser intensity and the wavelength, the interference structure is changed from type I to type II. By analyzing the energy component in the time-dependent wave packet, it is found that the excited states play important role for the conversion of the interference pattern. Through the direct analysis of the wave function evolution, one can deeply understand the physical mechanism behind the dynamic process of the interaction between the atom and the intense laser pulse.

Acknowledgement

We acknowledge the High Performance Computing Center of Jilin University for supercomputer time.

References

[1] Di Piazza A, Müller C, Hatsagortsyan K Z and Keitel C H 2012 *Rev. Mod. Phys.* **84** 1177
[2] Mitrofanov A V, Voronin A A, Sidorov-Biryukov D A, Mitryukovsky S I, Fedotov A B and Serebryannikov E E 2016 *Optica* **3** 299
[3] Protopapas M, Keitel C H and Knight P L 1997 *Rep. Prog. Phys.* **60** 389
[4] Brabec T and Krausz F 2000 *Rev. Mod. Phys.* **72** 545
[5] Winterfeldt C, Spielmann C and Gerber G 2008 *Rev. Mod. Phys.* **80** 117
[6] Fu L B, Xin G G, Ye D F and Liu J 2012 *Phys. Rev. Lett.* **108** 103601
[7] Chen Z J, Liang Y Q and Lin C D 2010 *Phys. Rev. Lett.* **104** 253201
[8] Yu B H and Li Y B 2012 *J. Mod. Opt.* **59** 1797
[9] Zhou Y M, Huang C, Liao Q and Lu P X 2012 *Phys. Rev. Lett.* **109** 053004
[10] Von Conta A, Tehlar A, Schletter A, Arasaki Y, Takatsuka K and Worner H J 2018 *Nat. Commun.* **9** 3162
[11] Trabert D, Hartung A, Eckart S, Trinter F, Kalinin A, Schoffler M, Schmidt L Ph H, Jahnke T, Kunitski M and Dorner R 2018 *Phys. Rev. Lett.* **120** 043202
[12] Porat G, Alon G, Rozen S, Pedatzur O, Kruger M, Azoury D, Natan A, Orenstein G, Bruner B D, Vrakking M J J and Dudovich N 2018 *Nat. Commun.* **9** 2805
[13] Yun H, Mun J H and Hwang S I Park S B, Ivanov I A, Nam C H and Kim K T 2018 *Nat. Photon.* **12** 620
[14] Zhao Y T, Ma S Y, Jiang S C, Yang Y J, Zhao X and Chen J G 2019 *Opt. Express* **27** 34393
[15] Li X F, L'Huillier A and Ferray M, Lompre L and Mainfray G 1989 *Phys. Rev. A* **39** 5751
[16] Altucci C, Velotta R, Heesel E, Springste E, Marangos J P, Vozzi C, Benedetti E, Calegari F, Sansone G, Stagira S, Nisoli M and Tosa V 2006 *Phys. Rev. A* **73** 043411
[17] Ishii N, Kaneshima K, Kitano K, Kanai T, Watanabe S and Itatani J 2014 *Nat.Commun.* **5** 3331
[18] Silva F, Teichmann S M, Cousin S L, Hemmer M and Biegert J 2015 *Nat.Commun.* **6** 6611
[19] Marangos J P 2016 *J. Phys. B* **49** 132001
[20] Gardner D F, Tanksalvala M, Shanblatt E R, Zhang X S, Galloway B R, Porter C, Jr R K, Bevis C, Adams D E, Kapteyn H C, Murnane M M and Mancini G F 2017 *Nat. Photon.* **11** 259
[21] Tadesse G K, Eschen W, Klas R, Hilbert V, Schelle D, Nathanael A, Zilk M, Steinert M, Schrempel F, Pertsch T, Tvnnermann A, Limpert J and Rothhardt J 2018 *Sci. Rep.* **8** 8677
[22] Krause J L, Schafer K J and Kulander K C 1992 *Phys. Rev. Lett.* **68** 3535
[23] Corkum P B 1993 *Phys. Rev. Lett.* **71** 1994
[24] Wang J, Wang B B, Guo F M, Li S Y, Ding D J, Chen J G, Zeng S L and Yang Y J 2014 *Chin. Phys. B* **23** 053201
[25] Zhai Z, Yu R F, Liu X S and Yang Y J 2008 *Phys. Rev. A* **78** 041402
[26] Guo F M, Yang Y J, Jin M X, Ding D J and Zhu Q R 2009 *Chin. Phys. Lett.* **26** 053201
[27] Chen J G, Zeng S L and Yang Y J 2010 *Phys. Rev. A* **82** 043401
[28] Chen J G, Yang Y J, Zeng S L and Liang H Q 2011 *Phys. Rev. A* **83** 023401
[29] Lin C, Zhang H T, Sheng Z H, Yu X H, Liu P, Xu J W, Song X H, Hu S L, Chen J and Yang W F 2016 *Acta Phys. Sin.* **65** 223207 (in Chinese)
[30] Song X H, Lin C, Sheng Z H, Liu P, Chen Z J, Yang W F, Hu S L, Lin C D and Chen J 2016 *Sci. Rep.* **6** 28392
[31] Yang W F, Zhang H T, Lin C, Xu J W, Sheng Z H, Song X H, Hu S L and Chen J 2016 *Phys. Rev. A* **94** 043419
[32] Gong X C, Lin C, He F, Song Q Y, Lin K, Ji Q Y, Zhang W B, Ma J Y, Lu P F, Liu Y Q, Zeng H P, Yang W F and Wu J 2017 *Phys. Rev. Lett.* **118** 143203
[33] Song X H, Xu J W, Lin C, Sheng Z H, Liu P, Yu X H, Zhang H T, Yang W F, Hu S L, Chen J, Xu S P, Quan W and Liu X J 2017 *Phys. Rev. A* **95** 033426
[34] Song X H, Shi G L, Zhang G J, Xu J W, Lin C, Chen J and Yang W F 2018 *Phys. Rev. Lett.* **121** 103201
[35] Zuo R X, Song X H, Liu X W, Yang S D, and Yang W F 2019 *Chin. Phys. B* **28** 094208
[36] Feit M D and Fleck J A Jr 1983 *J. Chem. Phys.* **78** 301
[37] Wei S S, Li S Y, Guo F M, Yang Y J and Wang B B 2013 *Phys. Rev. A* **87** 063418
[38] Song Y, Li S Y, Liu X S, Guo F M and Yang Y J 2013 *Phys. Rev. A* **88** 053419
[39] Song Y, Guo F M, Li S Y, Chen J G, Zeng S L and Yang Y J 2012 *Phys. Rev. A* **86** 033424
[40] Chen J G, Yang Y J, Chen J and Wang B 2015 *Phys. Rev. A* **91** 043403
[41] Tong X M, Watahiki S, Hino K and Toshima N 2007 *Phys. Rev. Lett.* **99** 093001
[42] Zhang D W, Lv H, Meng C, Du X Y, Zhou Z Y, Zhao Z X and Yuan J M 2012 *Phys. Rev. Lett.* **109** 243002