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Landau-like quantized levels of neutral atom induced by a dark-soliton shaped electric field*

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Motivated by the fascinating progresses in the cold atom experiments and theories, especially the artificial gauge field induced spin–orbit coupling of neutral atoms, we present a novel dispersion of neutral atoms carrying a non-vanishing magnetic moment in a special gauge field, an external electric field of dark-soliton shaped profile. By means of WKB approximation, we obtain discrete quantized landau-like energy levels, which is instructive for the quantum Hall effect of neutral particles. The observability of the results is also discussed.

Keywords: Landau level, cold atom, Rashba spin–orbit coupling

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1. Introduction

It is well known that in quantum mechanics Landau quantization is referred to the quantization of the cyclotron orbits of charged particles in magnetic fields. That is to say, the charged particles can only occupy the orbits of discrete energy values, i.e., Landau levels, which is due to the interference of the orbital motion. As usual, when non-interacting electrons in two-dimension xy plane are subjected to a perpendicular z-directional magnetic field, the Landau levels are degenerate due to the minimal coupling of the applied electromagnetic gauge field with the momentum of the particle, and massive degeneracies provide rich opportunities for more elaborate electron correlations to develop. The Landau levels play important roles in some physical problems especially the quantum Hall effect.[1] Landau quantization is fully responsible for the cyclotron motion of electrons as a function of the external magnetic field. A question may be asked whether the similar Landau quantization might be realized for neutral particles? The answer is yes. Previously, it has been shown that neutral particles with a magnetic moment exhibit the Aharonov–Bohm effect in certain circumstances,[2,3] which has been observed in a gravitational neutron interferometer in 1989[4] and in a neutral atomic Ramsey interferometer.[5] Later the Aharonov–Casher interaction was used to generate a similar Landau quantization for a neutral particle with a magnetic dipole moving in an electric field.[6] Landau-like energy levels were also studied for a neutral particle with a permanent electric dipole moment in the presence of an external magnetic field.[7] In recent years, the synthetic magnetic field and a restricted class of spin–orbit coupling (SOC) have been successfully realized in ultra-cold atoms.[8–12] Suitable combinations of laser beams can make neutral atoms behave like electrons in a magnetic field.[13] In addition, many schemes have been proposed to create general gauge fields.[14] The high controllability of ultra-cold atoms opens up many avenues to explore some fundamental phenomena at the forefront of condensed matter physics.[15–19] Recently, discrete Landau like states were obtained in neutral particles in a linear electromagnetic gauge, i.e., particular electric field configurations by means of interaction of neutral atoms and electric (synthetic) fields.[20] Along with the similar scheme, we introduce an external electric field to couple with the center-of-mass motion of neutral particles and investigate the the effect of the synthetic gauge fields on the neutral atoms. In this work, we mainly focus on the formation of discrete landau-like energy levels for neutral atoms.

The outline of this paper is as follows. In Section 2, we demonstrate our theoretic model: a neutral particle in an applied electric field of which the envelope can be expressed as a hyperbolic secant square function. In Section 3, using WKB approximation we obtain the eigen energy levels, discrete quantized Landau-like energy levels. This is the main results of this paper. Finally, we give a summary for the full text in Section 4.

2. Model

A neutral particle moving with velocity $\boldsymbol{v}$ in an electric field $\mathcal{E}$ will feel an effective magnetic field $\boldsymbol{B} = -(\boldsymbol{v} \times \mathcal{E})/c^2$, which is due to the relativistic effect, such that a Rashba type

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interaction can be engineered. We consider that the neutral atom confined in xy two-dimensional plane is submitted to the electric field and the Hamiltonian reads as

$$\begin{aligned}\mathcal{H} &= \frac{p^2}{2m} + \alpha\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\mathcal{E}) \\ &= \frac{p^2}{2m} + \alpha\boldsymbol{p}\cdot\boldsymbol{A}_{\text{eff}},\end{aligned} \tag{1}$$

where $\alpha \approx g\mu_{\text{B}}/(2mc^2)$, $\boldsymbol{p}$ and m are the momentum and the mass of the atom, c is the speed of light, g is the Land'e g-factor, μ_{B} denotes the Bohr magneton, $\boldsymbol{\sigma}$ represents the magnetic moment, and the effective "gauge" field $\boldsymbol{A}_{\text{eff}} = \mathcal{E}\times\boldsymbol{\sigma}$. It can be seen that the momentum of the neutral atom is coupled with the gauge field $\boldsymbol{A}_{\text{eff}}$, which is analogous to the spin–orbit coupling of a charged particle in an electromagnetic Abelian gauge field. Here the analogous gauge field is relevant to the physical electric field, which forms an SU(2) representation as the Pauli matrices do. We know that for the general Landau levels of two-dimension electron gas, the simple and familiar Landau gauge is applied, whereas for the fractional quantum Hall effect, the symmetry gauge is necessary. In this work, we adopt a new "gauge", a y-direction hyperbolic secant squared electric field $\mathcal{E} = \mathcal{E}_0\boldsymbol{e}_y/\cosh^2(y/y_0)$, possessing a profile like a dark-soliton. Thus Hamiltonian (1) of the system can be rewritten as

$$\mathcal{H} = p_x^2/2m + p_y^2/2m + \frac{\alpha\mathcal{E}_0 p_x\sigma_z}{\cosh^2(y/y_0)}, \tag{2}$$

which can be expressed in short as

$$\mathcal{H} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + V(y), \tag{3}$$

where $V(y) = V_0\sigma_z/\cosh^2(y/y_0)$ with $V_0 = \alpha\mathcal{E}_0 p_x$ dependent on the pseudo-spin σ_z and x-component momentum p_x. Due to the special synthetic gauge, Hamiltonian (3) is independent of x, and commute with x-component momentum operator p_x, i.e., p_x is a constant of center-of-mass motion in x-direction and can be viewed as a good number, thus one can separate the neutral particle's motional wave-function in the form $\psi(x,y) = \exp(\mathrm{i}p_x x/\hbar)\phi(y)$. By plugging $\psi(x,y)$ into Eq. (3), an effective one-dimension Hamiltonian acting on $\phi(y)$ can be written as

$$\mathcal{H}^{\pm} = -\frac{\hbar^2}{2m}\nabla_y^2 \pm \frac{\alpha\mathcal{E}_0 p_x}{\cosh^2(y/y_0)}. \tag{4}$$

It can be seen that Hamiltonian (3) naturally divides into two branches $\mathcal{H}^{\pm}$ ($\pm$ corresponding to σ_z eigenvalues ±1). It is not difficult to understood that only in the cases of $p_x > 0$, $\sigma_z = -1$ or $p_x < 0$, $\sigma_z = 1$, these two channels supply a bound potential energy such that bound states are allowed, otherwise the particle will be scattered and behave like free particles. In the following, we consider the case in which $p_x > 0$, $\sigma_z = -1$ by means of WKB approximation.

3. WKB solution

Following the preceding parts, we obtian a one-dimensional Schrödinger equation after factoring out $\exp(\mathrm{i}p_x/\hbar)$,

$$\mathcal{H}^{-} = -\frac{\hbar^2}{2m}\nabla_y^2 - \frac{V_0}{\cosh^2(y/y_0)}. \tag{5}$$

The above stationary Schrödinger equation can be solved exactly by reducing to a hypergeometric equation.[20] To avoid the mathematical complexities, we seek the solutions of eigen energy spectrum using WKB approximation. We use the following ansatz to seek the solution of the Schrödinger equation. First we assign the y-direction momentum of the neutral atom to be zero for reaching the classic turning point with $\xi = V(y)$ (see the potential energy curve in Fig. 1), which leads to

$$p(y) = \sqrt{2m[\xi - V(y)]} = 0, \tag{6}$$

this means

$$\left[2m\left(\xi + \frac{V_0}{\cosh^2(y/y_0)}\right)\right]^{1/2} = 0. \tag{7}$$

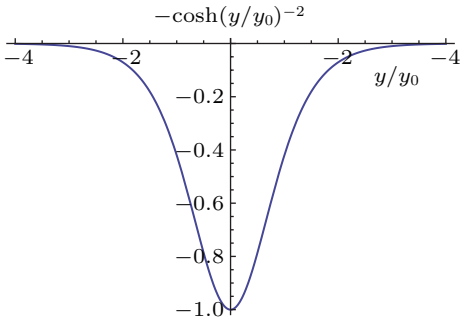


Fig. 1. The bound potential energy curve of a hyperbolic secant square potential.

Hence we can obtain two turning points of the hyperbolic secant square potential for the particle to crossover from classic regime to quantum regime, or verse. We denote them as y_a (y_b) with the relation $y_a = -y_b = y_0\operatorname{arccosh}\sqrt{-V_0/\xi}$, that is to say, the regime $y_b < y < y_a$ is the classically allowed region in which the energy of the particle is greater than the potential energy, whereas the other regions $y < y_b$ and $y > y_a$ are classically forbidden regions where the potential energy is greater than the energy, however in these regimes the quantum tunneling occurs.

Next we define the integration

$$\begin{aligned}J(\xi) &= \int_{y_b}^{y_a} p(y)\,\mathrm{d}y \\ &= \int_{y_b}^{y_a}\left[2m\left(\xi + \frac{V_0}{\cosh^2(y/y_0)}\right)\right]^{1/2}\mathrm{d}y.\end{aligned} \tag{8}$$

In order to evaluate the above integration, we take the derivative for both sides of Eq. (8) with respect to ξ first

$$\mathrm{d}J(\xi)/\mathrm{d}\xi = m\int_{y_b}^{y_a}\left[2m\left(\xi + \frac{V_0}{\cosh^2(y/y_0)}\right)\right]^{-1/2}\mathrm{d}y. \tag{9}$$

Here we perform a variable substitution with $z=\sinh(y/y_0)$, then we have $\mathrm{d}z=\cosh(y/y_0)\mathrm{d}y/y_0$, hence the above expression (9) becomes

$$\begin{aligned}\mathrm{d}J(\xi)/\mathrm{d}\xi &= my_0\int_{z_b}^{z_a}\left[2m\xi\left(1+z^2+V_0/\xi\right)\right]^{-1/2}\mathrm{d}z\\ &= \pi my_0/\sqrt{-2m\xi},\end{aligned}\tag{10}$$

where $z_a=-z_b=\sqrt{-1-V_0/\xi}$. So after the integration of Eq. (10), we can arrive at

$$J(\xi)=-\pi y_0\sqrt{-2m\xi}+K.\tag{11}$$

Here K is an integration constant. Considering the special case when $\xi=-V_0$ the integration domain of z in Eq. (8) contracted to a point, so we have $J(-V_0)=-\pi y_0\sqrt{2mV_0}+K=0$, hence the constant K is determined as $K=\pi y_0\sqrt{2mV_0}$. For simplicity, we define an energy scaling $V_b=\hbar^2/(2my_0^2)$. According to the WKB approximation, we have

$$J(\xi)=\left(n+\frac{1}{2}\right)\pi\hbar.\tag{12}$$

Thus the energies ξ can be solved as

$$\xi=-V_b\left[\sqrt{V_0/V_b}-\left(n+\frac{1}{2}\right)\right]^2,\quad (n=0,1,2,\ldots).\tag{13}$$

With the x-component kinetic energy included, the total energy of the particle becomes

$$\xi_n=-V_b\left[\sqrt{V_0/V_b}-\left(n+\frac{1}{2}\right)\right]^2+\frac{p_x^2}{2m},\quad (n=0,1,2,\ldots).\tag{14}$$

Up to now, we have obtained the quantized Landau-like energy levels of a neutral atom in a analogous gauge field induced by a hyperbolic secant shaped electric field, and this is also the main results of this paper. The energy-dispersion relation of the neutral particle for the $\sigma_z=-1$ branch reads as

$$\begin{aligned}\xi_n &= -\left[\sqrt{\frac{2m\alpha E_0y_0}{\hbar}\left(\frac{k_x}{k_0}\right)}-\left(n+\frac{1}{2}\right)\right]^2+\left(\frac{k_x}{k_0}\right)^2,\\ &\quad (n=0,1,2,\ldots),\end{aligned}\tag{15}$$

where ξ_n and k_x are scaled dimensionless by V_b and $k_0=1/y_0$, respectively. We plot the quantized landau-like energy levels as a function of k_x. From Fig. 2, one can see that the energy separation becomes more and more larger with the increasing energy level n. The energy separation at the same velocity can be estimated for the atom ^{87}Rb, and the related parameters are $\alpha=3.6\times10^{-16}$, electric field amplitude $E_0=10^{10}$ V/m, and atomic velocity $v_x=1$ cm/s. As an example, the energy separation between the levels $n=4$ and $n=3$ amounts to 19 nK or so, which is observable within nowadays advanced cold atom experiment. It is also evident that the neutral atoms feature degenerate energy levels, which are the characteristics of realistic Landau levels. Besides, the higher energy levels possess more degeneracies. The hyperbolic secant shaped static electric field can be generated by means of stable continuous optics waves. Analogously, the other branch with $p_x<0$ and $\sigma_z=1$ behaves in the same manner. In semi-classical approximation, the wave function adopts the following form in the classically accessible region $(y_b<y<y_a)$:

$$\phi_n(y)=\frac{A_\mathrm{n}}{\sqrt{p_y}}\cos\left(\frac{1}{\hbar}\int_{y_b}^{y}p_y\mathrm{d}y-\frac{\pi}{4}\right),\quad (y_b<y<y_a),\tag{16}$$

where A_n is the normalized constant and $p_y=\sqrt{2m[\xi-V(y)]}$. Thus the total wave function reads as

$$\begin{aligned}\psi_n(x,y) &= \frac{A_\mathrm{n}}{\sqrt{p_y}}\cos\left(\frac{1}{\hbar}\int_{y_b}^{y}p_y\mathrm{d}y-\frac{\pi}{4}\right)\exp(\mathrm{i}p_xx/\hbar),\\ &\quad (y_b<y<y_a).\end{aligned}$$

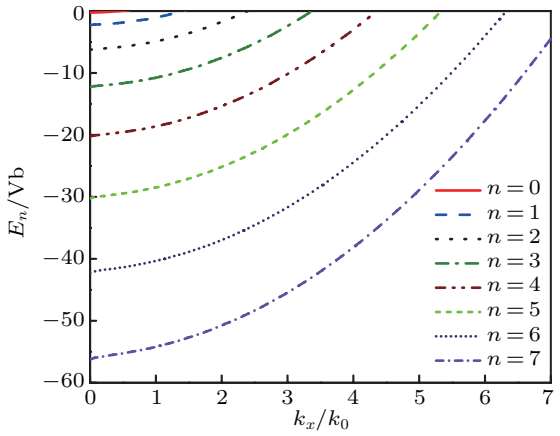


Fig. 2. The quantized Landau-like levels of the neutral atom in hyperbolic secant squared gauge field.

4. Summary

Based on the fascinating developments of cold atom experiments and theories, we investigate neutral atoms with a magnetic moment submitted to a hyperbolic secant squared electric field. By means of WKB approximation, we obtain the eigen energy of the system, quantized landau-like levels. The energy spacing becomes larger with increasing energy level n and should be observable within up-to-date experimental researches, which is useful for the further studies of the Hall effect of neutral particles. The hyperbolic secant squared potential can be obtained without much difficulties by a laser beam shaping method using acousto-optic deflection of light,[21] in which the examples are particularly given to the dipole trapping of ultracold atoms.

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