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Accelerate Bose–Einstein condensate by interaction*

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In recent years, accelerating waves have attracted great research interests both due to their unique properties and tempting applications. Here we investigate the effect of the inter-particle interaction on accelerating of Bose–Einstein condensate (BEC). We show that spatially homogeneous interactions will have no accelerating effect on BEC regardless of the interaction form (contact, dipole–dipole, or any others). But spatially inhomogeneous interactions may lead to an accelerating motion of the condensate. As an example, the accelerating dynamic of BEC under a spatially linear modulated contact interaction is studied in detail. It is found that such an interaction will accelerate the condensate with a time varying acceleration. Furthermore, an interaction engineering scheme to achieve constantly accelerating BEC is proposed and studied numerically. Numerical results suggest that this engineering scheme can also suppress profile changing of the condensate during its evolution, thus realize an accelerating profile-keeping matter wave packet. Our analysis also applies to optical waves with Kerr nonlinearity.

Keywords: Bose–Einstein condensate, interaction, acceleration

PACS: 67.85.–d, 03.67.Bg, 05.30.Jp **DOI:** 10.1088/1674-1056/ab4e8a

1. Introduction

Manipulation of atomic Bose–Einstein condensate (BEC) is very important in many applications, such as atom laser,[1,2] matter-wave interferometry,[3,4] atomic holography,[5–7] and lithography.[8,9] In particular, to obtain a matter wave with specific de Broglie wavelength, accelerating or decelerating of BEC is regularly needed. Very basic mechanical knowledge states that a free wave packet can only keep still or move at a constant velocity, to accelerate it an external potential should be applied. Along this line of thought, many accelerating schemes have been proposed, for example, using of gradient magnetic field,[10] time-dependent harmonic traps,[11] frequency-chirped optical lattice,[12] spatial array of quadrupole traps,[12] and modulated optical fields.[13]

Moreover, it is found that certain kind of accelerating waves can also be realized without any external potential applied. In the year 1979, Berry and Balazs theoretically investigated the free space Schrödinger equation and showed that an Airy wave packet can undergo an accelerating motion in free space.[14] This miraculous property of the Airy wave packet originates from its non-integrability (in other words, an Airy wave packet carries infinite amount of matter and energy, therefore is not physically realizable). After Berry and Balazs's pioneering work, extensive theoretical investigations were done on such kind of waves (for a review see Ref. [15]). And it was pointed out that an Airy wave packet can be made integrable (thus physical realizable) and at the same time holding its main feature by truncating to a finite range.[16] In 2007, such self-accelerating truncated Airy wave packets were experimentally demonstrated in an optical system.[17] The idea was soon introduced back to the field of matter wave. Accelerating electron beams have already experimentally achieved.[18] Recently, the generation of accelerating BEC Airy wave packets by using amplitude or phase imprinting techniques was also proposed.[19]

Now, the question is besides these two ways, are there any other methods to accelerate a BEC wave packet? For atomic BEC, the inter-atom interaction can be tuned by the Feshbach resonance technique,[20,21] and is also an important and widely used tool for the manipulation of BEC.[22–31] But it is generally believed to have no effect on accelerating of a matter wave packet,[11] since the interaction between two particles usually conserves the total momentum. In this article, at first, we theoretically show that this belief is valid only for spatially homogeneous interactions (regardless of the particular interaction form and wave packet shape), while in the case of spatially inhomogeneous interactions it is broken. As an example, dynamic of BEC under spatially linear modulated contact interaction is studied. We show that an initial steady BEC wave packet will undergo an accelerating motion under such an interaction. But the acceleration is a time varying one. To overcome this blemish, we also propose an engineering scheme of the interaction to achieve a constantly accelerating matter wave packet. Numerical results demonstrate that this scheme works well, and at the same time this scheme also suppresses profile varying of a wave packet during its evolution, thus produces a profile-keeping accelerating matter wave packet.

The rest of this article is organized as follows. In Section 2, the effect of the inter-atom interaction on accelerating

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dynamic of a BEC wave packet is studied theoretically. In Section 3, accelerating motion of a BEC wave packet under spatially linear modulated contact interaction is studied numerically. In this part, a scheme to realize a constantly accelerating profile-keeping wave packet is also proposed. In Section 4, the main results of this work are summarized.

2. Theory

To simplify the discussion, here we consider a system of BEC which is tightly trapped along the y and z directions with an external potential ($V_{\rm ext}(y,z)=\omega_\perp^2\left(y^2+z^2\right)/2$), yet can freely move along the x direction. Such a system can be reduced to one dimension,[32,33] and under the mean field theory the dynamic can be well described by the following Gross–Petaevskii equation:

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial}{\partial t}\psi &= -\frac{1}{2m}p_x^2\psi(x,t) \\ &\quad+\left[N\int V\left(x,x'\right)\left|\psi\left(x',t\right)\right|^2\mathrm{d}x'\right]\psi(x,t), \end{aligned} \tag{1}$$

where $\hbar$ is the reduced Planck constant, $p_x=-\mathrm{i}\hbar\partial/\partial x$ is the momentum operator along x direction, m is the mass of the condensated atom, N is the total number of atoms contained in the condensate, $V\left(x,x'\right)$ is the interaction of two atoms at positions x and x' respectively, and ψ is a wave function describing the macroscopic quantum state of the condensate which is normalized to 1, i.e., $\int_{-\infty}^{\infty}|\psi(x)|^2\,\mathrm{d}x=1$.

By using Gross–Petaevskii equation (1), the equations governing the evolution of the mean values of the BEC wave packet position and momentum, $\langle x\rangle$ and $\langle p\rangle$, can be derived as (Ehrenfest's theorem[34,35])

$$v = \frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t}=\frac{\langle p_x\rangle}{m}, \tag{2}$$

$$a = \frac{1}{m}\frac{\mathrm{d}\langle p_x\rangle}{\mathrm{d}t}=\frac{1}{\mathrm{i}\hbar m}\left\langle\left[p_x,V_{\rm eff}(x,t)\right]\right\rangle, \tag{3}$$

with $V_{\rm eff}(x,t)=N\int V\left(x,x'\right)\left|\psi\left(x',t\right)\right|^2\mathrm{d}x'$ being an effective potential produced by the inter-atom interaction. Comparing with the classical correspondence, it is clear that v and a are the moving speed and acceleration of the BEC wave packet, respectively. If $a=0$, the wave packet will stay still or moving at a constant speed. However, if $a\neq 0$, the wave packet will move in an accelerating manner. Using commutation relation $\left[p_x,f(x)\right]=-\mathrm{i}\hbar\mathrm{d}f(x)/\mathrm{d}x$, equation (3) can be further calculated as follows:

$$\begin{aligned} a &= -\frac{1}{m}\int|\psi(x)|^2\frac{\partial V_{\rm eff}(x)}{\partial x}\mathrm{d}x \\ &= -\frac{N}{m}\iint|\psi(x)|^2\frac{\partial V\left(x,x'\right)}{\partial x}\left|\psi\left(x'\right)\right|^2\mathrm{d}x\mathrm{d}x'. \end{aligned} \tag{4}$$

Since there are two space variables x and x' (positions of two interacting atoms) in the expression of acceleration, it would be convenient to introduce new variables

$$X = \frac{x-x'}{2}, \tag{5}$$

$$x_{\rm c} = \frac{x+x'}{2}. \tag{6}$$

In terms of X and $x_{\rm c}$, acceleration (4) reads

$$\begin{aligned} a &= -\frac{N}{4m}\iint|\psi(x_{\rm c}+X)|^2\left[\frac{\partial V\left(X,x_{\rm c}\right)}{\partial X}\right. \\ &\quad\left.+\frac{\partial V\left(X,x_{\rm c}\right)}{\partial x_{\rm c}}\right]|\psi(x_{\rm c}-X)|^2\,\mathrm{d}X\,\mathrm{d}x_{\rm c}, \end{aligned} \tag{7}$$

which can be split into two parts

$$a=a_1+a_2, \tag{8}$$

with

$$\begin{aligned} a_1 &= -\frac{N}{4m}\iint|\psi(x_{\rm c}+X)|^2\frac{\partial V\left(X,x_{\rm c}\right)}{\partial X} \\ &\quad\times|\psi(x_{\rm c}-X)|^2\,\mathrm{d}X\,\mathrm{d}x_{\rm c}, \end{aligned} \tag{9}$$

$$\begin{aligned} a_2 &= -\frac{N}{4m}\iint|\psi(x_{\rm c}+X)|^2\frac{\partial V\left(X,x_{\rm c}\right)}{\partial x_{\rm c}} \\ &\quad\times|\psi(x_{\rm c}-X)|^2\,\mathrm{d}X\,\mathrm{d}x_{\rm c}. \end{aligned} \tag{10}$$

Because we are considering the interaction between two identical particles, switching their positions will make no difference, i.e., regardless of the special form of interaction we always have $V\left(X,x_{\rm c}\right)=V\left(-X,x_{\rm c}\right)$. This is to say, the integrand in Eq. (9) is an odd function with respect to variable X, thus the value of a_1 will always be zero. It will have no contribution to the acceleration of the BEC wave packet. When the interaction is spatially homogeneous, that is, the interaction potential V has no relation with variable $x_{\rm c}$, or in other words, $\partial V\left(X,x_{\rm c}\right)/\partial x_{\rm c}$ equals zero. Then acceleration a_2 will also be zero. So, a spatially homogeneous interaction will have no accelerating effect on the BEC wave packet regardless of the specific form of the interaction (contact, dipole–dipole, or some others). This is the usually well known case.

Here it should be pointed out that for a self-accelerating Airy wave packet, because its wave function is not square integrable, consequently there does not exist proper definitions of $\langle x\rangle$ and $\langle p\rangle$, and the above derivation fails fundamentally. And for the square integrable truncated Airy wave packet, although the wave packet seems to undergo an accelerating motion, its mean position $\langle x\rangle$ will have no change, and its acceleration a will keep zero during its propagation (see Fig. 1). That is, the truncated Airy wave packet in fact does not accelerate, therefore we call the motion a seemingly accelerating motion.

However, when the interaction is spatially inhomogeneous, a_2 can have a non-zero value, and the condensate will be accelerated by the interaction. One such example can be easily fabricated

$$V\left(x,x'\right)=g(x)\,\delta\left(x-x'\right), \tag{11}$$

where $\delta(\cdot)$ is the usual Dirac delta function (the interaction is a commonly considered contact interaction), and $g(x)$ is a space-dependent interaction strength. Bu using the Feshbach resonance technique,[20,21] such an interaction can be realized by applying a spatially modulated magnetic field.[23–31] Here we study a simple linear form of the interaction modulation

$$g(x) = g_0(x - x_{g=0}), \tag{12}$$

where the parameter $g_0 = 2\hbar\omega_\perp a_s/l_0$ (a_s is the s-wave scattering length of the condensate atom, and l_0 is the length scale of the interaction gradient) is an interaction strength scale, and $x_{g=0}$ defines the point at which the interaction strength is tuned to 0.

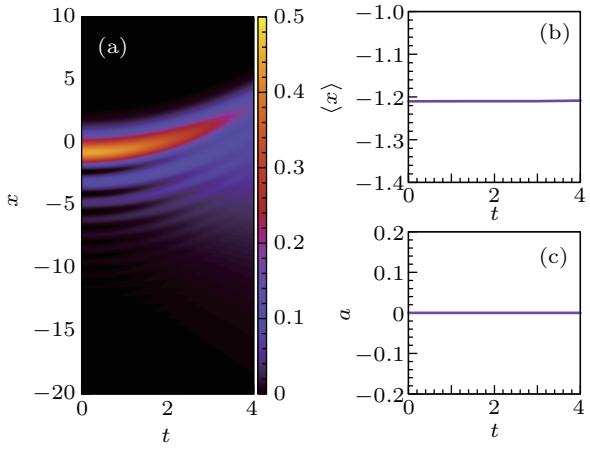


Fig. 1. Seemingly accelerating motion of an exponentially truncated Airy wave packet in free space. The initial wave packet is $\psi = C\exp(0.2x)\mathrm{Airy}(x)$, where C is a normalization constant. The evolution of this wave packet according to Eq. (1) with $V(x,x') = 0$ is plotted, and natural units $m = \hbar = 1$ are used in the figures. (a) The evolution of $|\psi(x,t)|^2$. The density profile undergoes a seemingly accelerating motion. (b) and (c) The mean position $\langle x\rangle$ and acceleration a. The mean position of the wave packet keeps still, and the acceleration is 0 all the time, thus in fact the wave packet does not accelerate.

Firstly, to roughly get some physical insight, we consider a Gaussian wave packet

$$\psi = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4}\exp\left(-\frac{(x-x_0)^2}{4\sigma_0^2}\right), \tag{13}$$

with x_0 and σ_0 being the center position and width of the wave packet, respectively. For such a wave packet, the acceleration can be calculated analytically, and it is

$$a_0 = -\frac{N}{m}\int |\psi(x)|^2\frac{\partial V_{\mathrm{eff}}(x)}{\partial x}\mathrm{d}x = -\frac{Ng_0}{4\sqrt{\pi}m\sigma_0}, \tag{14}$$

which is obviously not zero, indicating that the BEC wave packet can be accelerated by the interaction induced effective potential. Considering a ^{39}K BEC with 10^6 atoms[36] transversely trapped by a harmonic trap with $\omega_\perp = 1000$ Hz and having length $\sigma_0 = 5$ μm along x-direction, if the s-wave scattering length a_s is tuned from 0 to $-25a_0$ within length $l_0 = 1$ cm (for ^{39}K, $a_s = -33a_0(1 - 52/(B - 402.4))$ with B in units of G,[37] so this can be achieved by nonlinearly tuning the magnetic field from 454.4 G to 604.4 G within length of 1 cm, the corresponding magnetic gradient is in the order of magnitude of 150 G/cm, which is ordinarily used in BEC experiments[38–40]), the acceleration is estimated to be about ~ 0.08 m/s. This value can be greatly increased if a large atom number (for example, $N = 10^8$[41,42]) BEC or a magnetic field with large gradient (for example, 200 T/m[43]) is used.

It should be noted that unlike accelerated by a linear external potential, here the acceleration not only depends on slope g_0 of the interaction potential, but is also related to width σ_0 of the wave packet. Under the same interaction, a narrower (smaller value of σ_0) wave packet will have a larger acceleration. Moreover, if the evolution of such an initial wave packet is considered, as the width of the wave packet will usually change during its evolution due to the dispersion and interaction, the interaction induced acceleration will also be a time varying one (this will be shown in the next section in detail numerically).

Although equation (14) comes from a Gaussian wave packet assumption, it is valid qualitatively for wave packets with other mathematical forms, such as sech which is the mathematical form of a bright soliton.[22,44,45] This can be explained as follows. Because of the normalizing restriction (i.e., conservation of particles number), a smaller value of wave packet width σ_0 will produce a sharper effective potential, and it will induce a larger force and hence acceleration on the wave packet.

3. Numerical results

To study the dynamic of such a system, one substitutes the specific interaction potential (11) into Eq. (1), and immediately obtains the following nonlinear Schrödinger equation:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + Ng(x)|\psi(x)|^2\psi(x,t). \tag{15}$$

It is usually hard to solve this equation analytically, so here we solve it numerically using the extensively used operator splitting and fast Fourier transform technique.[46,47] And in the calculations, natural units $\hbar = m = 1$ will be used.

In Fig. 2, we take function (13) with parameters $x_0 = 0$ and $\sigma_0 = 1$ as the initial wave packets and show their evolutions under different forms of interaction. In Figs. 2(a1)–2(a4) and 2(b1)–2(b4), the interactions are set to $Ng(x) = 0.3(x+10)$ and $Ng(x) = -0.3(x+10)$, respectively. It is clear that in these two cases, the BEC wave packet undergoes an accelerating motion, the $\langle x\rangle$–t relation shows a curved shape and the acceleration a has a non-zero value. At the very beginning time, the shape of the wave packet has not changed

much, thus the value of acceleration a agrees well with the value given by Eq. (14), $a = -0.042$ and 0.042 for the two cases, respectively. In Figs. 2(a1)–2(a4), the interaction has a net repulsive effect in the condensate occupied space. Both the interaction and dispersion effects tend to broad the wave packet during its evolution, thus according to Eq. (14), the magnitude of acceleration decreases. While, in Figs. 2(b1)–2(b4), the interaction has a net attractive effect in the BEC occupied space. In such a case, the attractive interaction and the dispersion effects compete with each other, as a result the width of the wave packet experiences an oscillation during the evolution, and consequently the acceleration oscillates as well. To show this relation between the wave packet width and acceleration, in Fig. 2(b4) we also plot the full width at half maximum (FWHM) of the wave packet. The correlation between a and FWHM can be seen clearly in this figure. For comparison, in Figs. 2(c1)–2(c4) we plot the evolution of the same initial wave packet under a spatially homogeneous interaction $Ng(x) = -3$. We see that although the wave packet breathes during its evolution, it does not accelerate. Its mean position $\langle x \rangle$ and acceleration a keep 0 all the time.

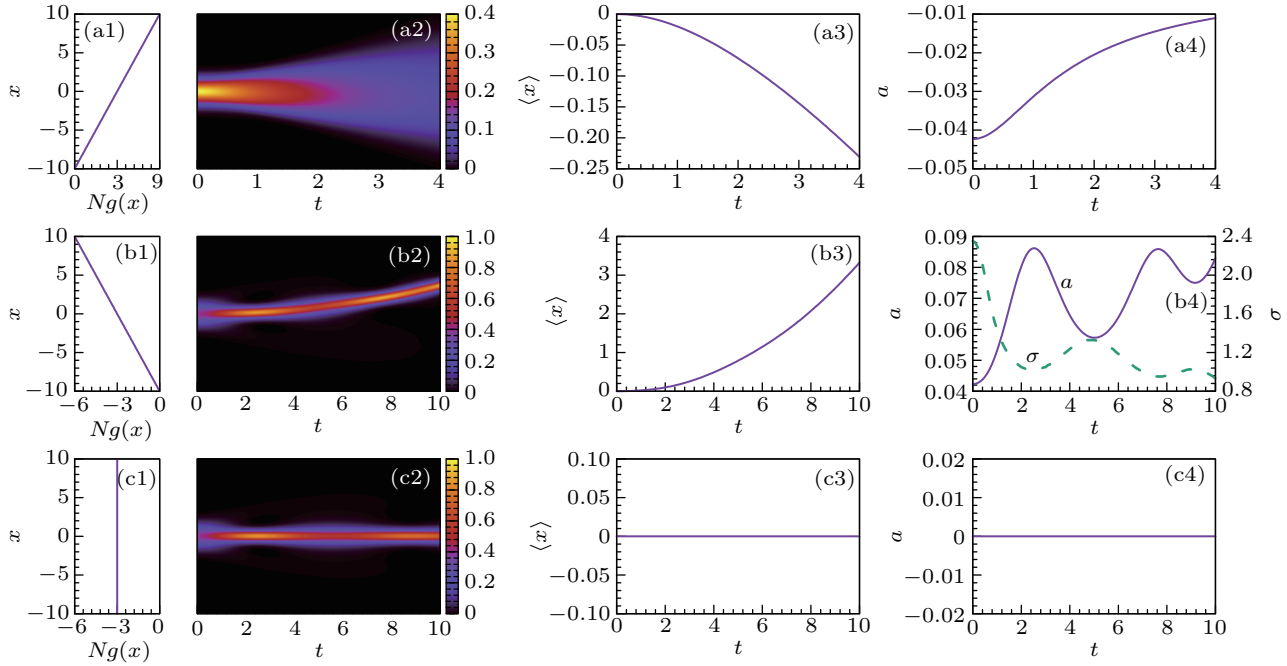


Fig. 2. Interaction induced accelerating motion of initial steady Gaussian BEC wave packets (13). In panels (a1)–(a4), (b1)–(b4), and (c1)–(c4), the interactions are set to be (a1) $Ng(x) = 0.3(x+10)$, (b1) $-0.3(x+10)$, and (c1) -3, respectively. Panels (a2)–(c2), (a3)–(c2), and (a4)–(c4) show the evolution of BEC density $|\psi(x,t)|^2$, mean values of wave packet position $\langle x \rangle$ and acceleration a defined by Eq. (3), respectively. In (b4), the dashed line represents the FWHM of the wave packet. All the three initial wave packets have the same parameters $\sigma_0 = 1$ and $x_0 = 0$.

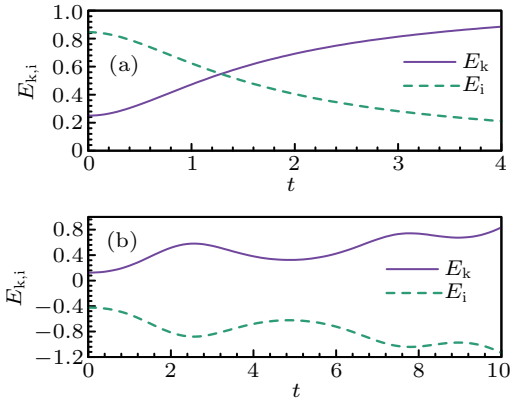


Fig. 3. (a) and (b) Kinetic and interaction energies of the accelerating wave packets shown in Figs. 2(a) and 2(b), respectively. The solid line represents the kinetic energy E_k, while the dashed line represents the interaction energy E_i.

To demonstrate that the acceleration in Figs. 2(a) and 2(b) is really caused by the interaction, we plot the corresponding kinetic and interaction energies separately in Fig. 3. From the figure, we see that the increasing of the kinetic energy is always accompanied by a decreasing of the interaction energy. This fact clearly indicates that the acceleration of the wave packet is due to the inter-atom interaction. The oscillation of $E_{k,i}$ in Fig. 3(b) is also due to breathing of the wave packet.

From Figs. 2(a4) and 2(b4), it also can been seen that the accelerations are time varying during the evolution. That is, the motions of the wave packets are not usually interested simple constantly accelerating ones. This blemish can be overcome by engineering the interaction strength factor g_0 over time using the Feshbach resonance technique.[20,21] To achieve a constantly accelerating motion with acceleration $a = a_c$, according to Eq. (14), initially one should set the interaction strength to be

$$g_0(t=0) = g_c = -4\sqrt{\pi}\sigma_0 a_c, \tag{16}$$

and at time t one needs to tune the interaction strength to

$$g_0(t) = a_c/\tilde{a}_t, \tag{17}$$

with

$$\tilde{a}_t = -\int |\psi(x,t)|^2 \frac{\partial}{\partial x}\left[g_c(x-x_{g=0})|\psi(x,t)|^2\right] \mathrm{d}x. \tag{18}$$

In Fig. 4, we plot the engineered $g_0(t)$ and the corresponding dynamical evolution of a wave packet (13) under the attractive interaction case. From the figure, we see that under the engineered interaction the acceleration keeps its initial value, and the mean position increases in a perfect parabolic manner. Thus, a constantly accelerating wave packet is achieved as expected.

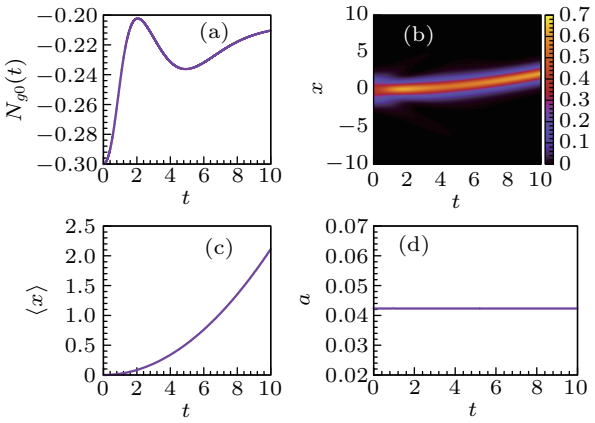


Fig. 4. Constantly accelerating attractive BEC wave packet realized by interaction engineering. (a) The engineered interaction parameter $g_0(t)$. Panels (b)–(d) show the evolution of BEC density $|\psi(x,t)|^2$, mean values of wave packet position $\langle x\rangle$ and acceleration a, respectively. The initial interaction is $Ng(x,t=0) = -0.3(x+10)$, and the initial wave packet is Eq. (13) with $x_0 = 0$ and $\sigma_0 = 1$, the same as that in Fig. 2(b2).

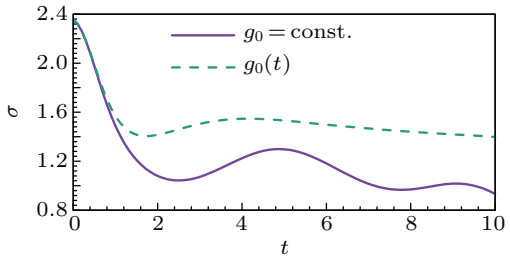


Fig. 5. Suppression of BEC wave packet width varying by interaction engineering. The solid line represents the FWHM of the wave packet under a timely constant attractive interaction $Ng(x) = -0.3(x+10)$, while the dashed line represents the FWHM of the wave packet under a timely engineered interaction. The corresponding densities are shown in Figs. 2(b2) and 4(b).

Moreover, comparing Fig. 4(b) with Fig. 2(b2), we can roughly notice that the interaction engineering can also suppress shape varying of the wave packet during its evolution. This can be seen more clearly from Fig. 5 where we explicitly compare the FWHM of the same wave packet under constant and engineered interactions during the evolution. From this figure, we see that after $t = 2$ the width of the wave packet changes very slightly under the engineered interaction. Thus, the achieved constantly accelerating wave packet also has a nearly shape-keeping feature. These manners make the system potentially useful in the field of creating atom laser[1,2] and atomic analogy of light bullet.[44,48]

Although for the repulsive case, one can also get a constantly accelerating BEC by engineering g_0 in a similar way (see Fig. 6), the engineering will at the same time speed up the broadening of the wave packet (to keep the acceleration constant, the broadening of the wave packet needs a stronger interaction to balance it, and a stronger repulsive interaction will make the wave packet broad more quickly), see Fig. 7 where we compare the FWHM of wave packets during evolution under constant and engineered interactions, i.e., the wave packets in Figs. 2(a2) and 6(b). The broadening of the wave packet will greatly reduce the intensity of the matter wave, therefore may make it less significant in practice.

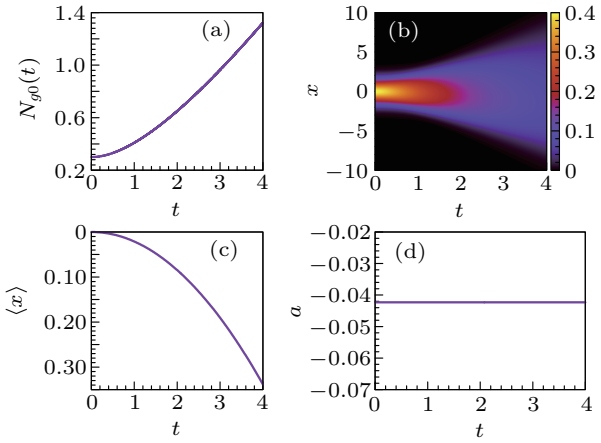


Fig. 6. Constantly accelerating repulsive BEC wave packet realized by interaction engineering. (a) The engineered interaction parameter $g_0(t)$. Panels (b)–(d) show the evolution of BEC density $|\psi(x,t)|^2$, mean values of wave packet position $\langle x\rangle$ and acceleration a, respectively. The initial interaction is $Ng(x,t=0) = 0.3(x+10)$, and the initial wave packet is Eq. (13) with $x_0 = 0$ and $\sigma_0 = 1$, the same as that in Fig. 2(a2).

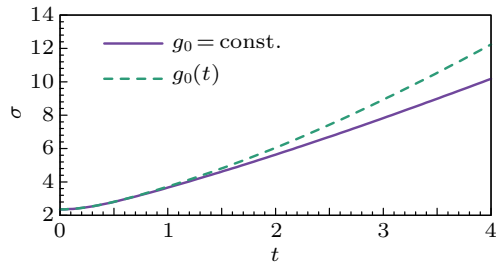


Fig. 7. Speeding up of BEC wave packet broadening caused by interaction engineering. The solid line represents the FWHM of the wave packet under a timely constant repulsive interaction $Ng(x) = 0.3(x+10)$, while the dashed line represents the FWHM of the wave packet under a timely engineered interaction. The corresponding densities are shown in Figs. 2(a2) and 6(b).

4. Conclusion

We discussed the role of interaction in accelerating a BEC wave packet. We showed that if the interaction between two atoms is spatially homogeneous (or in other words, only depends on their separating distance), it will have no acceleration

effects on the atomic BEC. However, if this is not the case, the interaction may cause an accelerating motion of the condensate. One example of such interaction induced accelerating BEC was fabricated. We showed that if we linearly modulate the usual contact interaction in space, an initial steady BEC wave packet will undergo an accelerating motion with time varying acceleration. We also proposed that a constantly accelerating BEC wave packet can be obtained by timely engineering the interaction strength. At last, the analysis here can also be straightforwardly extended to the field of nonlinear fiber optics where a light pulse obeys the similar nonlinear Schrödinger equation.[49]

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