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Quantum discord of two-qutrit system under quantum-jump-based feedback control*

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This paper studies quantum discord of two qutrits coupled to their own environments independently and coupled to the same environment simultaneously under quantum-jump-based feedback control. Our results show that spontaneous emission, quantum feedback parameters, classical driving, initial state, and detection efficiency all affect the evolution of quantum discord in a two-qutrit system. We find that under the condition of designing proper quantum-jump-based feedback parameters, quantum discord can be protected and prepared. In the case where two qutrits are independently coupled to their own environments, classical driving, spontaneous emission, and low detection efficiency have negative effect on the protection of quantum discord. For different initial states, it is found that the evolution of quantum discord under the control of appropriate parameters is similar. In the case where two qutrits are simultaneously coupled to the same environment, the classical driving plays a positive role in the generation of quantum discord, but spontaneous emission and low detection efficiency have negative impact on the generation of quantum discord. Most importantly, we find that the steady discord depends on feedback parameters, classical driving, and detection efficiency, but not on the initial state.

Keywords: qutrit, quantum-jump-based feedback, quantum discord

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1. Introduction

In science, the quantification and classification of correlations are of fundamental importance. In the emerging field of quantum information, various correlation metrics have recently been introduced and studied, such as entanglement,[1–3] quantum correlation, and classical correlation.[4,5] In particular, Ollivier and Zurek[4] and Henderson and Vedral[5,6] have introduced the quantum discord as a measure of quantum correlation beyond entanglement. Recently, quantum discord has been widely explored, including using different alternative methods to guide quantum discord,[7,8] studying quantum discord in different space-time,[9,10] and exploring quantum discord under different interactions.[11–14] At the same time, other studies on higher-dimensional and higher spin systems[15–18] and experimental advances[19] have also increased interest in quantum discord.

The system considered here consists of two parts, then all correlations can be defined as entropic quantities.[4] In the following, we will define the total correlation by means of measures on one of the subsystems. The quantum system $\hat{\rho}_{AB}$ has quantum and classical correlations. All the correlations can be characterized by the quantum mutual information[4,5]

$$I(\hat{\rho}_{AB}) = S(\hat{\rho}_A) + S(\hat{\rho}_B) - S(\hat{\rho}_{AB}), \tag{1}$$

where $S(\hat{\rho}) = -\mathrm{Tr}[\hat{\rho}\log_2(\hat{\rho})]$ is the von Neumann entropy. Based on this expression, correlations are generally considered to be separated according to their classical and quantum natures, respectively.[4] In this way, the quantum discord has been introduced as

$$D(\hat{\rho}_{AB}) = I(\hat{\rho}_{AB}) - C(\hat{\rho}_{AB}), \tag{2}$$

where $C(\hat{\rho}_{AB})$ is the classical correlation[4,5] defined by the following maximization procedure. A complete set of projector operators $\{\hat{\Pi}_k\}$ must be built for the subsystem B. Then the quantity

$$C(\hat{\rho}_{AB}) = \max_{\{\hat{\Pi}_k\}}[S(\hat{\rho}_A) - S(\hat{\rho}_{AB} \mid \{\hat{\Pi}_k\})] \tag{3}$$

must be maximized with respect to the variation of the set of $\{\hat{\Pi}_k\}$, where $p_k = \mathrm{Tr}(\hat{\rho}_{AB}\hat{\Pi}_k)$, $\hat{\rho}_k = \mathrm{Tr}_B(\hat{\Pi}_k\hat{\rho}_{AB}\hat{\Pi}_k)/p_k$, and $S(\hat{\rho}_{AB} \mid \{\hat{\Pi}_k\}) = \sum_k p_k S(\hat{\rho}_k)$.

Since the measurements can be performed on either qutrits, the classical correlation will be calculated by performing measurements on qutrit B. To this end, we consider using the following base vectors for measurement[20]

$$\begin{aligned}
|V_1\rangle &= \exp(\mathrm{i}\chi_1)\sin\theta\cos\phi|0\rangle \\
&\quad + \exp(\mathrm{i}\chi_2)\sin\theta\sin\phi|1\rangle + \cos\theta|2\rangle, \\
|V_2\rangle &= \exp(\mathrm{i}\chi_1)\cos\theta\cos\phi|0\rangle \\
&\quad + \exp(\mathrm{i}\chi_2)\cos\theta\sin\phi|1\rangle - \sin\theta|2\rangle, \\
|V_3\rangle &= -\exp(\mathrm{i}\chi_1)\sin\phi|0\rangle + \exp(\mathrm{i}\chi_2)\sin\phi|1\rangle,
\end{aligned}$$

where $(0 \le \theta, \phi \le \pi/2)$ and $(0 \le \chi_1, \chi_2 \le 2\pi)$.

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The purpose of this paper is to investigate the effect of a strategic quantum feedback control on the evolution of quantum discord. Quantum feedback is a typical method of controlling quantum system dynamics through external control. Markovian quantum feedback control introduced by Wiseman and Milburn[21,22] has been widely used to feed back measurements to the system to modify the future dynamics of the system, such as suppressing decoherence,[23–26] protecting quantum correlations,[27] improving steady state entanglement,[28–33] manipulating the geometric phase[34] and stationary state[35] of the dissipative two-level system, preparing a three-atom singlet,[36] W state, and GHZ state,[37,38] and enhancing the accuracy of parameter estimations.[39] Recently, quantum-jump-based feedback control has been used to accelerate quantum evolution,[40] enhance exciton transmission,[41] study on the dissipative stabilization of multipartite entanglement with Rydberg atoms,[42] maneuver Einstein–Podolsky–Rosen Steering,[43] and so on. According to different measurement strategies, homodyne-based and quantum-jump-based feedback controls are proposed. Reference [31] shows that the latter is superior in protecting entangled states. In this paper, we will study the quantum discord of two qutrits under quantum-jump-based feedback control.

In the following, we will study the quantum discord in both feedback and no feedback regimes. The structure of this paper is arranged as follows: In Section 2, we will investigate the evolution of the quantum discord between two qutrits independently coupled to their own environment under quantum-jump-based feedback control. In Section 3, we will investigate the evolution of the quantum discord between two qutrits simultaneously coupled to one environment under quantum-jump-based feedback control. Finally, we give a concise conclusion and discussion.

2. Protection of quantum discord

2.1. Effective master equation

We consider a system consisting of a qutrit which couple resonantly to a damped cavity and is driven by two classical fields. The atomic levels and transitions are depicted in Fig. 1(a). The transition between the levels $|3\rangle \leftrightarrow |2\rangle$ ($|2\rangle \leftrightarrow |1\rangle$) is coupled to the cavity mode a resonantly with the coupling strength g, driven by the classical field with the Rabi frequency Ω. The atom can spontaneously decay from $|3\rangle$ to $|2\rangle$ ($|2\rangle$ to $|1\rangle$) with rate γ_1 (γ_2), and the cavity mode is damped with rate κ_a. The complete master equation describing dynamics of our system reads

$$\dot{\rho} = \mathcal{L}\rho - \mathrm{i}g[(J^{\dagger}a + J^{-}a^{\dagger}),\rho] + \kappa_a \mathcal{D}[a]\rho, \tag{4}$$

here, $\mathcal{L}$ is a superoperator given as

$$\mathcal{L}\rho = -\mathrm{i}\Omega[J^{\dagger} + J^{-},\rho] + \gamma_1 \mathcal{D}[\sigma_{23}]\rho + \gamma_2 \mathcal{D}[\sigma_{12}]\rho \tag{5}$$

and the superoperator $\mathcal{D}$ acting on an operator c is defined as $\mathcal{D}[c]c \equiv c\rho c^{\dagger} - (c^{\dagger}c\rho + \rho c^{\dagger}c)/2$. The amplitude damping operator is defined as $J^{-} = \sigma_{12} + \sigma_{23}$, where $\sigma_{kl} = |k\rangle\langle l|$. In what follows, the effective master equation will be derived under the condition of a strongly dissipative cavity, which can be obtained by considering the dynamic evolution of each matrix element of the density operator in the system. In the limit of large decay rate, highly excited modes can be safely ignored, so we only need to expand the density matrix ρ in terms of the a mode number states with small photon numbers.[18,28,36] Therefore, we can get the density operator ρ as

$$\begin{aligned}\rho \approx\ & \rho_0|0\rangle_a\langle 0| + \rho_1|1\rangle_a\langle 0| + \rho_1^{\dagger}|0\rangle_a\langle 1| \\ & + \rho_2|1\rangle_a\langle 1| + \rho_3|2\rangle_a\langle 0| + \rho_3^{\dagger}|0\rangle_a\langle 2|.\end{aligned} \tag{6}$$

Substituting the above equation into Eq. (4), we can obtain a set of coupled equations of motion for the field-matrix elements

$$\begin{aligned}\dot{\rho}_0 &= \mathcal{L}\rho_0 - \mathrm{i}g[J^{\dagger}\rho_1 - \rho_1^{\dagger}J^{-}] + \kappa_a\rho_2, \\ \dot{\rho}_1 &= \mathcal{L}\rho_1 - \mathrm{i}g[\sqrt{2}J^{\dagger}\rho_3 + J^{-}\rho_0 - \rho_2 J^{-}] - \frac{\kappa_a}{2}\rho_1, \\ \dot{\rho}_2 &= \mathcal{L}\rho_2 - \mathrm{i}g[J^{-}\rho_1^{\dagger} - \rho_1 J^{\dagger}] - \kappa_a\rho_2, \\ \dot{\rho}_3 &= \mathcal{L}\rho_3 - \mathrm{i}g[\sqrt{2}J^{-}\rho_1] - \kappa_a\rho_3.\end{aligned} \tag{7}$$

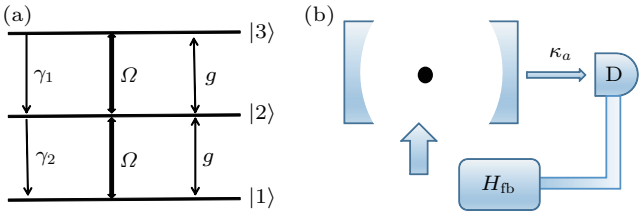


Fig. 1. Schematic view of the model. A qutrit is trapped in a cavity. When a photon from the leaky cavity is detected by a detection apparatus, a feedback pulse is triggered.

Under the condition $\kappa_a \gg g$, we make the assumption that $\dot{\rho}_1 = \dot{\rho}_3 = 0$ and substitute the corresponding results into the expressions of $\dot{\rho}_0$ and $\dot{\rho}_2$. After neglecting the terms proportional to second order $O(g^2/\kappa_a^2)$, we can obtain

$$\begin{aligned}\dot{\rho}_0 &= \mathcal{L}\rho_0 - \frac{2g^2}{\kappa_a}[J^{\dagger}J^{-}\rho_0 + \rho_0 J^{\dagger}J^{-} - 2J^{\dagger}\rho_2 J^{-}] + \kappa_a\rho_2, \\ \dot{\rho}_2 &= \mathcal{L}\rho_2 - \frac{2g^2}{\kappa_a}[J^{-}J^{\dagger}\rho_2 + \rho_2 J^{-}J^{\dagger} - 2J^{-}\rho_0 J^{\dagger}] - \kappa_a\rho_2.\end{aligned} \tag{8}$$

Solving the above two equations and adiabatically eliminating[18,28,36,44] the elements ρ_2, we can obtain the effective master equation prompting the evolution of the atom

$$\begin{aligned}\dot{\rho} =\ & -\mathrm{i}\Omega[J^{\dagger} + J^{-},\rho] + \gamma_1\mathcal{D}[\sigma_{23}]\rho \\ & + \gamma_2\mathcal{D}[\sigma_{12}]\rho + \Gamma\mathcal{D}[J^{-}]\rho,\end{aligned} \tag{9}$$

where $\Gamma = 4g^2/\kappa_a$ describes the effective decay rate of the atom and the cavity mode a. Moreover, if the collective decay rate Γ is much larger than the atomic spontaneous emission

rate $\gamma_{1(2)}$, we can ignore the effect of $\gamma_{1(2)}$. The final effective master equation of the system can be expressed as

$$\dot{\rho} = -\mathrm{i}\Omega[J^{\dagger}+J^{-},\rho]+\Gamma\mathcal{D}[J^{-}]\rho. \tag{10}$$

2.2. Implementation of the feedback

Now let us consider the system with feedback, as shown in Fig. 1(b). The output from the leaky cavity is measured by a detector D, then the measurement is used to trigger a feedback Hamiltonian $H_{\mathrm{fb}} = I(t)B$. For a feedback scheme based on photodetection measurement, the stochastic master equation for the qutrit system can be described by

$$\dot{\rho} = -\mathrm{i}\Omega[J^{\dagger}+J^{-},\rho]+\Gamma\mathcal{D}[UJ^{-}]\rho, \tag{11}$$

where $U = \exp[-\mathrm{i}\lambda\sigma_x]$ is feedback control with the control strength λ, where $\sigma_x = \left(\mathrm{i}/\sqrt{2}\right)\left(J^{-}+J^{\dagger}\right)$. For simplicity, we have assumed $\gamma_1 = \gamma_2 = \gamma$.

We consider a system formed by two non-interacting parts A and B. The consistency of each part is shown in Fig. 1. Under this condition, the master equation for the two-qutrit system becomes

$$\dot{\rho} = \sum_{k=A,B} -\mathrm{i}\Omega[J_k^{\dagger}+J_k^{-},\rho]+\Gamma[UJ_k^{-}\rho J_k^{\dagger}U^{\dagger} - \frac{1}{2}(J_k^{\dagger}J_k^{-}\rho+\rho J_k^{\dagger}J_k^{-})]. \tag{12}$$

In this section, we first assume that two qutrits are in a particular quantum state, then substituting this quantum state into Eq. (12). Finally, we use Eq. (2) to numerically simulate the evolution process of the quantum discord under quantum feedback control.

Now we discuss the case where the initial state is $|\Psi\rangle = \sqrt{1/3}|11\rangle+\sqrt{1/3}|22\rangle+\sqrt{1/3}|33\rangle$. In Fig. 2(a), we plot the quantum discord of the two qutrits as a function of the dimensionless quantity Γt and the feedback parameter λ. It can be seen that the evolution of the quantum discord is different under different feedback parameters, and the quantum discord is a periodic function of λ with period $T = 2\pi$. Meanwhile, we also find that the decay of the quantum discord can be retarded when some appropriate feedback parameters are chosen. As can be seen from Fig. 2(b), compared with the case of spontaneous emission effects, the quantum discord in the case without spontaneous emission effects has a smaller decay rate. Consequently, the spontaneous emission is a detrimental factor for our current model.

In order to demonstrate the dependency of $D(\hat{\rho}_{AB})$ on classical driving strength Ω, we present Fig. 3, where the initial state is $|\Psi\rangle$ and the plot is drawn at $\lambda = 0.4\pi$. We here choose $\Omega = 0$, $\Omega/\Gamma = 2.5$, and $\Omega/\Gamma = 5$. It is easy to find that the classical driving plays a negative role in protecting quantum discord, and as the classical driving strength increases, the impact of the quantum feedback on the dynamics of the quantum discord is complicated. That is to say, our numerical results show that classical driving cannot protect the quantum discord when the quantum-jump-based feedback control exists. In the following, we will focus on the evolution of the quantum discord without classical driving.

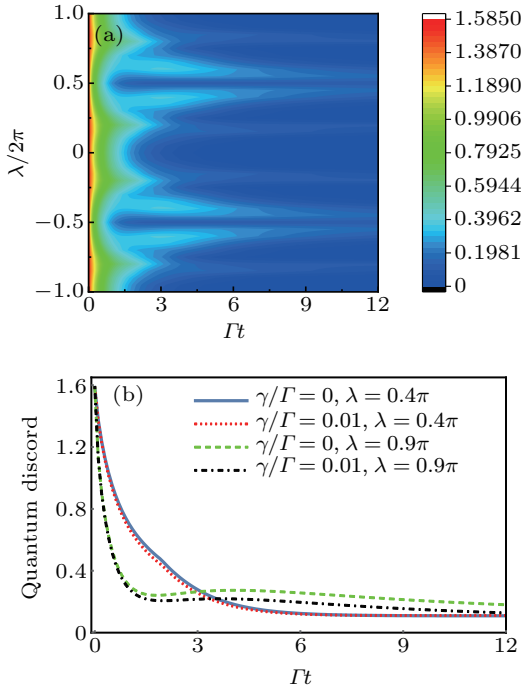


Fig. 2. Quantum discord for the initial state $|\Psi\rangle$ as a function of the dimensionless quantity Γt and λ with $\Omega = 0$, (a) without ($\gamma/\Gamma = 0$) and (b) with ($\gamma/\Gamma = 0.01$) spontaneous emission effects.

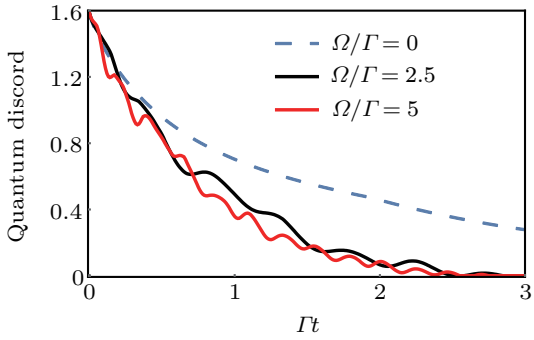


Fig. 3. Time evolution of the quantum discord, with the initial state being $|\Psi\rangle$, with $\lambda = 0.4\pi$, for the cases of $\Omega/\Gamma = 0$, $\Omega/\Gamma = 2.5$, and $\Omega/\Gamma = 5$.

Figure 4 illustrates the quantum discord for different initial states ($|\Psi\rangle$, $|\Phi\rangle$, and $|\phi\rangle$) as a function of the dimensionless quantity Γt, where $|\Psi\rangle = \sqrt{1/3}|11\rangle+\sqrt{1/3}|22\rangle+\sqrt{1/3}|33\rangle$, $|\Phi\rangle = \sqrt{1/3}|13\rangle+\sqrt{1/3}|31\rangle+\sqrt{1/3}|22\rangle$, $|\phi\rangle = \sqrt{1/3}|11\rangle+\sqrt{1/3}|32\rangle+\sqrt{1/3}|23\rangle$, $\Omega = 0$, and $\lambda = 0.4\pi$. What is shown in Fig. 4 is the quantum discord protection not only depends on the feedback parameter λ and classical driving strength Ω but also on the initial state chosen. At the same time, we use the research methods in Figs. 2 and 3 to study the initial quantum states $|\Phi\rangle$ and $|\phi\rangle$. It is found that the feedback parameters, spontaneous emission, and classical driving have similar effects on the evolution process.

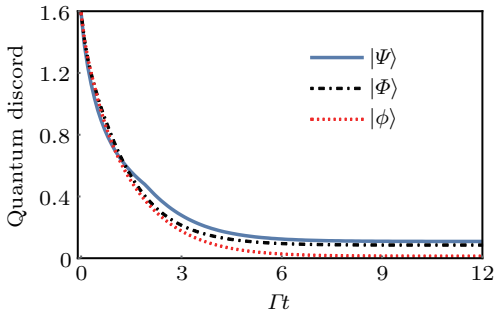


Fig. 4. Evolution of the quantum discord of an initial state $|\Psi\rangle$, $|\Phi\rangle$ or $|\phi\rangle$ with $\Omega = 0$ and $\lambda = 0.4\pi$.

The detection efficiency is another important factor affecting the feedback control process. In order to consider this factor, we can modify the corresponding master equation as

$$\begin{aligned}\dot{\rho} &= \sum_{k=A,B} -\mathrm{i}\Omega[J_k^{\dagger}+J_k^{-},\rho]+\Gamma\eta\left[UJ_k^{-}\rho J_k^{\dagger}U^{\dagger}\right.\\ &\quad\left.-\frac{1}{2}(J_k^{\dagger}J_k^{-}\rho+\rho J_k^{\dagger}J_k^{-})\right]\\ &\quad+\Gamma(1-\eta)\left[J_k^{-}\rho J_k^{\dagger}-\frac{1}{2}(J_k^{\dagger}J_k^{-}\rho+\rho J_k^{\dagger}J_k^{-})\right], \qquad (13)\end{aligned}$$

where η represents the detection efficiency, and $(1-\eta)$ corresponds to the case where the detector is not clicked and the control action is not applied. In Fig. 5, we plot the quantum discord *versus* the effect of different detection efficiencies with $\Omega = 0$, $\lambda = 0.4\pi$ and two qutrits initially in the state $|\Psi\rangle$. It is shown that the spontaneous emission effect plays a negative role in protecting quantum discord, and the detection efficiency largely determines the protection effect of the quantum-jump-based feedback control on the quantum discord. The protection of the quantum discord is the best when the detection is perfect and there is no spontaneous emission effect. Similarly, we find that other two-qutrit states ($|\Phi\rangle$ and $|\phi\rangle$) show similar behavior to $|\Psi\rangle$ under the appropriate parameters.

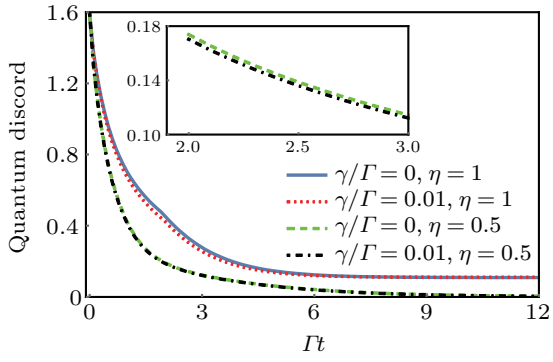


Fig. 5. The quantum discord *versus* the effect of different detection efficiencies with $\Omega = 0$, $\lambda = 0.4\pi$ and two qutrits initially in the state $|\Psi\rangle$.

3. Preparation of quantum discord

As can be seen from the above discussion, for the model in Section 2, quantum-jump-based feedback control can protect the quantum discord, that is, reduce the attenuation rate of the quantum discord, but the quantum discord can only be maintained for a limited time. So how do we prepare the quantum discord of two-qutrit system and make it stable? We will discuss it in this section.

3.1. Effective master equation

We consider another system consisting of two qutrits coupled resonantly to a damped cavity. The transition between the levels $|2\rangle \leftrightarrow |1\rangle$ and $|3\rangle \leftrightarrow |2\rangle$ are coupled to cavity mode a resonantly with the coupling strength g, driven by classical fields with the Rabi frequency Ω. The atom can spontaneously decay from $|3\rangle$ to $|2\rangle$ ($|2\rangle$ to $|1\rangle$) with rate γ_1 (γ_2), and the cavity mode is damped with rate κ_a.

The complete master equation describing dynamics of our system can be written as

$$\dot{\rho} = \mathcal{L}\rho - \mathrm{i}g[(J^{\dagger}a+J^{-}a^{\dagger}),\rho]+\kappa_a\mathcal{D}[a]\rho, \qquad (14)$$

here, $\mathcal{L}$ is a superoperator given as

$$\begin{aligned}\mathcal{L}\rho &= -\mathrm{i}\Omega[J^{\dagger}+J^{-},\rho]+\gamma_1^1\mathcal{D}[\sigma_{23}^1]\rho+\gamma_2^1\mathcal{D}[\sigma_{12}^1]\rho\\ &\quad+\gamma_1^2\mathcal{D}[\sigma_{23}^2]\rho+\gamma_2^2\mathcal{D}[\sigma_{12}^2]\rho, \qquad (15)\end{aligned}$$

and the superoperator $\mathcal{D}$ acting on an operator c is defined by $\mathcal{D}[c]\rho \equiv c\rho c^{\dagger} - (c^{\dagger}c\rho + \rho c^{\dagger}c)/2$. The amplitude damping operator is defined as $J^{-} = \sigma_{12}^1+\sigma_{23}^1+\sigma_{12}^2+\sigma_{23}^2$, where $\sigma_{kl} = |k\rangle\langle l|$. In the limit of large cavity dissipative rates, we can neglect highly excited modes safely.[18,28,36,44] Similar to the analysis procedure of Subsection 2.1, we can obtain the stochastic master equation

$$\dot{\rho} = -\mathrm{i}\Omega[J^{\dagger}+J^{-},\rho]+\Gamma\mathcal{D}[J^{-}]\rho. \qquad (16)$$

3.2. Implementations of the feedback

Now let us consider the system with feedback, as shown in the following Fig. 6.

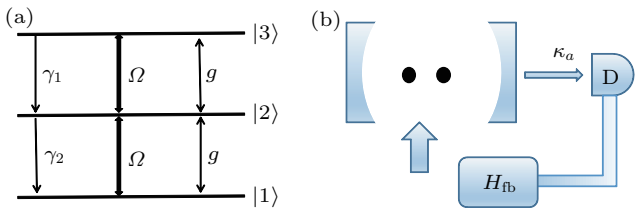


Fig. 6. Schematic view of the model. Two qutrits are trapped in a cavity. When a photon from the leaky cavity is detected by a detection apparatus, a feedback pulse is triggered.

The output from the leaky cavity is measured by a detector D, then the measurement is used to trigger a feedback Hamiltonian $H_{\mathrm{fb}} = I(t)B$. For a feedback scheme based on photodetection measurement, the stochastic master equation for the two-qutrit system can be written as

$$\dot{\rho} = -\mathrm{i}\Omega[J^{\dagger}+J^{-},\rho]+\Gamma\mathcal{D}[U_{\mathrm{fb}}J^{-}]\rho, \qquad (17)$$

where $U_{\rm fb}=\exp(-{\rm i}\lambda\sigma_x^1)$, with $\sigma_x^1=(\sqrt{2}/2)(\sigma_{32}^1+\sigma_{23}^1+\sigma_{12}^1+\sigma_{21}^1)$. For simplicity, we have assumed $\gamma_1^i=\gamma_2^i=\gamma$. We choose the local feedback that, like the spontaneous emission term, cannot maintain the symmetry of the atomic exchange. This can allow feedback to move the population between subspaces, limiting the likelihood of destructive interference.

The quantum discord as a function of feedback and driving strength is shown in Fig. 7. Compared with the case of spontaneous emission effects, the quantum discord of the composed system in the case without spontaneous emission effects has a large maximum value in steady state, therefore we hold the point that the atomic spontaneous emission is a detrimental factor for our current model. And it is obvious that a nearly perfect quantum discord can be achieved by quantum-jump-based feedback.

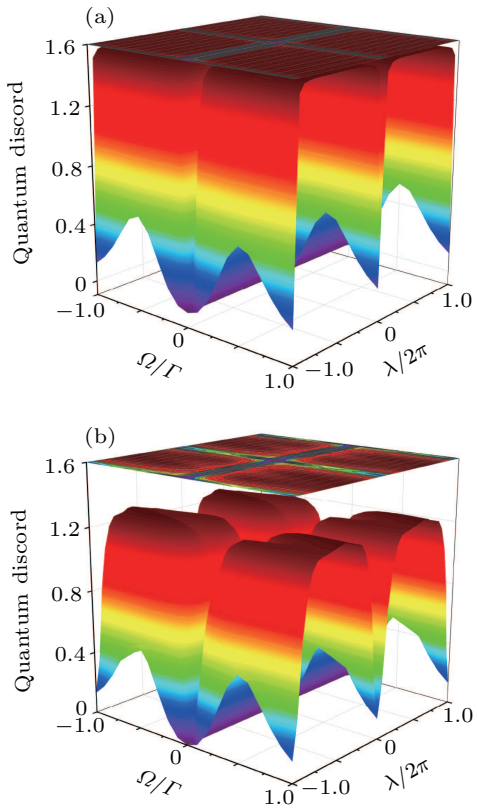


Fig. 7. The quantum discord as a function of feedback and driving strength (a) without ($\gamma/\Gamma=0$) and (b) with ($\gamma/\Gamma=0.01$) spontaneous emission effects.

In order to demonstrate the dependency of the quantum discord on classical driving strengths Ω, we present Fig. 8, where the initial state is $|\Psi\rangle$ and the plot is drawn at $\lambda=\pi$. We here choose $\Omega=0$, $\Omega/\Gamma=2.5$, and $\Omega/\Gamma=5$. We can see from Fig. 8 that when the feedback parameter $\lambda=\pi$ is chosen, the effect of classical driving on the dynamics of the quantum discord is complicated. But it is easy to find that the classical driving plays a positive role in the preparation of the quantum discord, and with the advent of classic driving, a nearly perfect maximum value of discord appears. That is to say, our numerical results show that when the appropriate quantum-jump-based feedback control is present, the classical driving is very beneficial for the preparation of the quantum discord.

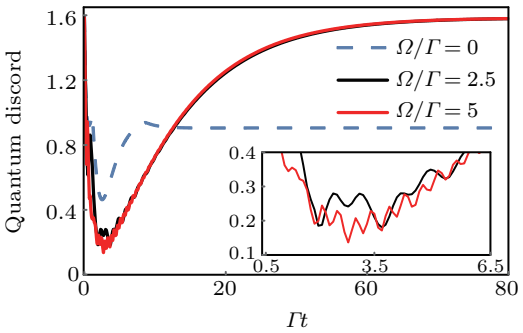


Fig. 8. Time evolution of the quantum discord, with the initial state being $|\Psi\rangle$, with $\lambda=\pi$, for the cases of $\Omega/\Gamma=0$, $\Omega/\Gamma=2.5$, and $\Omega/\Gamma=5$.

Figure 9 illustrates the quantum discord for different initial states ($|\Psi\rangle$ and $|\Phi\rangle$) as a function of the dimensionless quantity Γt, where $|\Psi\rangle=\sqrt{1/3}|11\rangle+\sqrt{1/3}|22\rangle+\sqrt{1/3}|33\rangle$, $|\Phi\rangle=|11\rangle$, $\Omega/\Gamma=1$, and $\lambda=\pi$. Figure 9 shows that the quantum discord evolution process not only depends on the feedback parameter λ and the classical driving strength Ω but also on the initial state chosen. For any initial state, the long-term evolution of the quantum discord can reach the maximum.

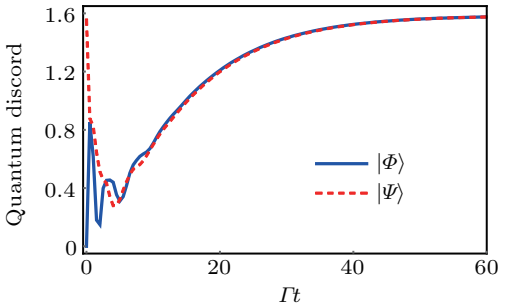


Fig. 9. Evolution of the quantum discord of an initial state $|\Psi\rangle$ or $|\Phi\rangle$ with $\Omega=\Gamma$ and $\lambda=\pi$.

The detection efficiency is another important factor affecting the feedback control process. In order to consider this factor, we can modify the corresponding master equation as

$$\begin{aligned}\dot{\rho} &= -{\rm i}\Omega[J^{\dagger}+J^{-},\rho]+\Gamma\eta\mathcal{D}[U_{\rm fb}J^{-}]\rho \\ &\quad +\Gamma(1-\eta)\mathcal{D}[J^{-}]\rho, \end{aligned} \tag{18}$$

where η represents the detection efficiency, and $(1-\eta)$ corresponds to the case where the detector is not clicked and the control action is not applied. In Fig. 10, we plot the quantum discord *versus* the effect of different detection efficiencies with $\Omega=\Gamma$, $\lambda=\pi$, and two qutrits initially in the state $|\Psi\rangle$. Obviously, the system always achieves the maximum value of discord without atomic decay, but the time required depend on the efficiency of the detection process. Furthermore, the presence of spontaneous emission leads to a reduction in the final

quantum discord, which is also affected via the inefficiency of the detector, this conclusion is consistent with the conclusion in the literature.[36]

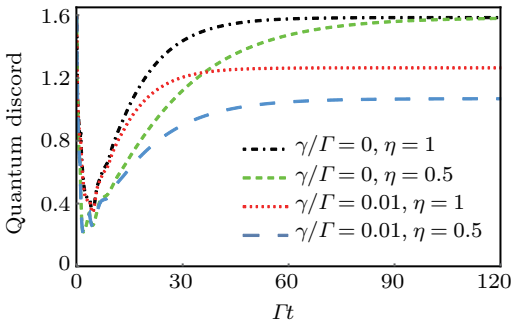


Fig. 10. The quantum discord *versus* the effect of different detection efficiencies with $\Omega = \Gamma$, $\lambda = \pi$, and two qutrits initially in the state $|\Psi\rangle$. Without decay the system always reaches the maximum value of discord, with the time depending on the efficiency, while the presence of spontaneous emission results in a decrease in the final quantum discord.

To consider the correctness of the adiabatic approximation, we can rewrite the main equation (14) as

$$\begin{aligned}\dot{\rho} \;=\; & -\mathrm{i}\Omega[J^{\dagger}+J^{-},\rho]+\gamma_1^1\mathcal{D}[\sigma_{23}^1]\rho+\gamma_2^1\mathcal{D}[\sigma_{12}^1]\rho \\ & +\gamma_1^2\mathcal{D}[\sigma_{23}^2]\rho+\gamma_2^2\mathcal{D}[\sigma_{12}^2]\rho \\ & -\mathrm{i}g[(J^{\dagger}a+J^{-}a^{\dagger}),\rho]+\kappa_a\mathcal{D}[U_{\mathrm{fb}}a]\rho. \end{aligned} \tag{19}$$

In the adiabatic regime with $\kappa_a = 1600\gamma$, $g = 200\gamma$, $\Omega = 100\gamma$, and $\lambda = \pi$, the discord of the steady state is about 1.39 that is close to the result of effective model denoted by the red line in Fig. 10.

4. Discussion and conclusions

In general, the existence of decoherence makes it an obstacle to the quantum information science to experimentally produce faithful and reliable entangled states, because quantum system will inevitably interact with their surrounding and cause information loss. In the context of cavity quantum electrodynamics, the typical decoherence factors include spontaneous emission of atoms and cavity decay. In the absence of atomic spontaneous emission, the reason why quantum feedback can protect quantum discord is that the fast damping rate κ_a of the cavity makes it easy for photon to leak from the cavity rather than interact with the atom. Once the detector detects a leaked photon, the feedback control will act on the system to modify the future dynamics of the system. In this way, proper feedback control suppresses the decoherence of the system and the surrounding environment, and protects the quantum discord of the quantum system. The presence of spontaneous emission will aggravate the loss of information in the quantum system and have a negative impact on the protection of quantum discord. In the preparation of quantum discord, we adopt the basic idea of direct quantum feedback, that is, in a system consisting of largely decay cavity and negligible atomic spontaneous emission, the feedback with not preserving the symmetry with respect to exchange of atoms can be used to design an approximate maximum discord steady state. When the spontaneous emission of an atom occurs, the steady state quantum discord value decreases. It is due to a competition between the spontaneous emission and the feedback rate.[18,29] So the atomic decay still plays a negative role in the current scheme.

In experiments, for an optical cavity approaching the bad cavity limit, where the atom–cavity cooperativity parameter is $C = g^2/\kappa_a\gamma \approx 19$[45] with $(\kappa_a, g, \gamma) \sim 2\pi \times (255.05, 70, 1)$ MHz,[46] the quantum discord is about 1.24 based on Eq. (19) in the case of $\Omega = g$ and $\lambda = \pi$. The value is relatively high for the discord. In the future cavity quantum electrodynamics experiments, it is expected to obtain a better bad cavity structure. The atomic configuration involved in our scheme may be implemented with metastable Krypton, whose relevant atomic levels are shown in Ref. [47]. However, its structure does not fully satisfy our theoretical assumptions. Therefore, we also hope that in the near future, the cascaded three-level atom that fully satisfies our hypothesis can be found.

In summary, we find that a quantum-jump-based feedback scheme can protect and prepare the quantum discord of qutrit-qutrit system. In Section 2, we show that quantum feedback can protect the quantum discord, and the emergence of spontaneous emission and classical driving are negative factors. In addition, we find that the feedback has different protection effects on the quantum discord of different initial states, and the protection effect is the best for the quantum discord in the initial state $|\Psi\rangle$. Finally, we discuss the effects of low efficiency and spontaneous emission on quantum discord during the detection process. We find that high detection efficiency is more conducive to the protection of the quantum discord, and the existence of spontaneous emission will undermine the protection of the quantum discord. In Section 3, we show that another quantum-jump-based feedback scheme can produce stable maximum quantum discord. First, the quantum jump-based feedback scheme can improve the steady-state discord of the two-qutrit system, but the spontaneous emission in the current scheme plays a negative role. In addition, we explore the effects of classical driving on quantum discord and find that the existence of classical driving is a positive factor and a necessary condition for preparing the maximum steady-state discord. Later, we find that the feedback can produce a stable maximum quantum discord, regardless of the initial state. Finally, we discuss in detail the effects of low efficiency and spontaneous emission on quantum discord during the detection process. On the one hand, the system always achieves the maximum value of discord without atomic decay, but the time required depends on the efficiency of the detection pro-

cess. On the other hand, the presence of spontaneous emission leads to a reduction in the final quantum discord, which is also affected by the inefficiency of the detector. Therefore, in the two-qutrit system, quantum-jump-based feedback is an effective scheme for protecting and preparing quantum discord.

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