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Relations between tangle and I concurrence for even n-qubit states

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Gilad Gour and Nolan R Wallach [*J. Math. Phys.* **51** 112201 (2010)] have proposed the 4-tangle and the square of the I concurrence. They also gave the relationship between the 4-tangle and the square of the I concurrence. In this paper, we give the expression of the square of the I concurrence and the n-tangle for six-qubit and eight-qubit by some local unitary transformation invariant. We prove that in six-qubit and eight-qubit states there exist strict monogamy laws for quantum correlations. We elucidate the relations between the square of the I concurrence and the n-tangle for six-qubit and eight-qubits. Especially, using this conclusion, we can show that 4-uniform states do not exist for eight-qubit states.

Keywords: quantum correlations, entangled relations, n-tangle, six-qubit and eight-qubit

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1. Introduction

Entanglement is considered as the central resource for quantum information and computation,[1–4] and numerous theoretical and experimental researches have been conducted in this field.[5–8] In particular, the search for maximally entangled states has received a great deal of attention.[9–24] Then there comes a fundamental question which states are maximally entangled. In the case of 2 qubits, it is known that Bell states are maximally entangled with respect to any measures of entanglement.[1] The concurrence has been shown to be a useful entanglement measure for pure two qubits. In 2000, Coffman, Kundu, and Wootters[9] used concurrence to examine three-qubit quantum systems and introduced the concept of "residual entanglement", or the 3-tangle. In 2001, Wong and Christensen[20] gave the definition of n-tangle for even n qubits. The n-tangle of even n qubits is invariant under permutations of the qubits, which is an entanglement monotone. An expression for 4-tangle was obtained by examining the negativity fonts present in a four-way partial transpose under local unitary operations in 2019.[21]

For higher number of qubits, the problem is no longer simple and depends in general on the entanglement measure. In 2010, Gilad Gour and Nolan R Wallach[10] proposed the tangle or the square of the I concurrence. In four qubits there are four bipartite cuts consisting of one qubit *versus* the remaining three quibts and three bipartite cuts consisting of two qubits *versus* the remaining two qubits. Denoting the four qubits by A, B, C, and D, they are defined as

$$\tau_1 = \frac{1}{4}\left(\tau_{A(BCD)} + \tau_{B(ACD)} + \tau_{C(ABD)} + \tau_{D(ABC)}\right), \quad (1)$$

$$\tau_2 = \frac{1}{3}\left(\tau_{AB(CD)} + \tau_{AC(BD)} + \tau_{AD(BC)}\right), \quad (2)$$

where $\tau_{A(BCD)} = 2\left(1 - \mathrm{tr}\rho_A^2\right)$ with $\rho_A = \mathrm{tr}\rho_{BCD}\left(|\psi\rangle\langle\psi|\right)$ being the reduced density matrix and $\tau_{AB(CD)} = 2\left(1 - \mathrm{tr}\rho_{AB}^2\right)$ with $\rho_{AB} = \mathrm{tr}\rho_{CD}\left(|\psi\rangle\langle\psi|\right)$ being the reduced density matrix. Also they are related by

$$\tau_{ABCD} = 4\tau_1 - 3\tau_2, \quad (3)$$

where τ_{ABCD} is the 4-tangle, which is defined by Wong and Christensen[20] as follows:

$$\tau_{ABCD} = \left|\langle\psi|\,\sigma_y \otimes \sigma_y \otimes \sigma_y \otimes \sigma_y\,|\psi^*\rangle\right|^2. \quad (4)$$

In this paper, we investigate the relation between the n-tangle with $\tau_1, \tau_2, \ldots$ for a six-qubit system and those for an eight-qubit system.

2. Relation between n-tangle and square of I concurrence

2.1. Relation between n-tangle and square of I concurrence of six-qubit

In the following, we show how specific monogamy relations can be deduced from local unitary transformation invariant.

For a six-qubit pure state,

$$|\varphi\rangle_{ABCDEF} = a_0|000000\rangle + a_1|000001\rangle + \cdots + a_{63}|111111\rangle, \quad (5)$$

Like Gilad Gour and Nolan R Wallach's definition, for six-qubit pure states, the square of the I concurrence can be given as

$$\begin{aligned}\tau_1 = {} & \frac{1}{6}\left(\tau_{A(BCDEF)} + \tau_{B(ACDEF)} + \tau_{C(ABDEF)}\right. \\ & \left. + \tau_{D(ABCEF)} + \tau_{E(ABCDF)} + \tau_{F(ABCDE)}\right), \quad (6)\\ \tau_2 = {} & \frac{1}{15}\left(\tau_{AB(CDEF)} + \tau_{AC(BDEF)} + \tau_{AD(BCEF)} + \tau_{AE(BCDF)}\right. \\ & \left. + \tau_{AF(BCDE)} + \tau_{BC(ADEF)} + \tau_{BD(ACEF)} + \tau_{BE(ACDF)}\right)\end{aligned}$$

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$$+\tau_{BF(ACDE)}+\tau_{CD(ABEF)}+\tau_{CE(ABDF)}+\tau_{CF(ABDE)}$$
$$+\tau_{DE(ABCF)}+\tau_{DF(ABCE)}+\tau_{EF(ABCD)}), \quad (7)$$

$$\tau_3 = \frac{1}{10}(\tau_{ABC(DEF)}+\tau_{ABD(CEF)}+\tau_{ABE(CDF)}+\tau_{ABF(CDE)}$$
$$+\tau_{ACD(BEF)}+\tau_{ACE(DBF)}+\tau_{ACF(DBE)}$$
$$+\tau_{ADE(BCF)}+\tau_{ADF(BCE)}+\tau_{AEF(BCD)}), \quad (8)$$

where $\tau_{A(BCDEF)} = 2\left(1-\mathrm{tr}\,\rho_A^2\right)$, $\tau_{AB(CDEF)} = 2\left(1-\mathrm{tr}\,\rho_{AB}^2\right)$, $\tau_{ABC(DEF)} = 2\left(1-\mathrm{tr}\,\rho_{ABC}^2\right)$, *etc.*

And the 6-tangle is

$$\tau_{ABCDEF} = \left|\langle\psi^*|\,\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\,|\psi\rangle\right|^2. \quad (9)$$

In Ref. [25] it has been shown that

$$\mathrm{Tr}_i\,\rho_i^2 = \frac{1}{2}+\frac{1}{2}F_i,$$
$$\mathrm{Tr}_{ij}\,\rho_{ij}^2 = \frac{1}{4}+\frac{1}{4}\left(F_i+F_j+F_{ij}\right),$$
$$\mathrm{Tr}_{ijk}\,\rho_{ijk}^2 = \frac{1}{8}+\frac{1}{8}\left(F_i+F_j+F_k+F_{ij}+F_{ik}+F_{jk}+F_{ijk}\right), \quad (10)$$

where

$$F_i = \langle\psi|\,\hat{\sigma}_{ix}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{iy}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\,|\psi\rangle^2,$$
$$F_{ij} = \langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jx}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jy}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jz}}\,|\psi\rangle^2$$
$$+\langle\psi|\,\hat{\sigma}_{iy}\hat{\sigma}_{\mathrm{jx}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{iy}\hat{\sigma}_{\mathrm{jy}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{iy}\hat{\sigma}_{\mathrm{jz}}\,|\psi\rangle^2$$
$$+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\hat{\sigma}_{\mathrm{jx}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\hat{\sigma}_{\mathrm{jy}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\hat{\sigma}_{\mathrm{jz}}\,|\psi\rangle^2,$$
$$F_{ijk} = \langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jx}}\hat{\sigma}_{\mathrm{kx}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jx}}\hat{\sigma}_{\mathrm{ky}}\,|\psi\rangle^2$$
$$+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jx}}\hat{\sigma}_{\mathrm{kz}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jy}}\hat{\sigma}_{\mathrm{kx}}\,|\psi\rangle^2$$
$$+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jy}}\hat{\sigma}_{\mathrm{ky}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{ix}\hat{\sigma}_{\mathrm{jy}}\hat{\sigma}_{\mathrm{kz}}\,|\psi\rangle^2$$
$$+\cdots$$
$$+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\hat{\sigma}_{\mathrm{jz}}\hat{\sigma}_{\mathrm{kx}}\,|\psi\rangle^2+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\hat{\sigma}_{\mathrm{jz}}\hat{\sigma}_{\mathrm{ky}}\,|\psi\rangle^2$$
$$+\langle\psi|\,\hat{\sigma}_{\mathrm{iz}}\hat{\sigma}_{\mathrm{jz}}\hat{\sigma}_{\mathrm{kz}}\,|\psi\rangle^2. \quad (11)$$

It is obvious that such invariants satisfy $F_i \geq 0$, $F_{ij} \geq 0$, and $F_{ijk} \geq 0$.

From Eqs. (6)–(10), we can obtain

$$\tau_{A(BCDEF)} = 2\left(1-\mathrm{tr}\,\rho_A^2\right) = 1-F_A,\ldots,$$
$$\tau_{AB(CDEF)} = 2\left(1-\mathrm{tr}\,\rho_{AB}^2\right)$$
$$= \frac{1}{2}[(1-F_A)+(1-F_B)+(1-F_{AB})],\ldots,$$
$$\tau_{ABC(DEF)} = 2\left(1-\mathrm{tr}\,\rho_{ABC}^2\right)$$
$$= \frac{1}{4}[(1-F_A)+(1-F_B)+(1-F_C)+(1-F_{AB})$$
$$+(1-F_{AC})+(1-F_{BC})+(1-F_{ABC})],\ldots \quad (12)$$

From Eqs. (6)–(12), we can obtain

$$\tau_1 = 1-\frac{1}{6}\left(F_A+F_B+F_C+F_D+F_E+F_F\right). \quad (13)$$

It is easy to show that $0 \leq \tau_1 \leq 1$.

Similarly, from Eqs. (6)–(12), we can obtain

$$\tau_2 = \frac{3}{2}-\frac{1}{6}\left(F_A+F_B+F_C+F_D+F_E+F_F\right)$$
$$-\frac{1}{30}\left(F_{AB}+F_{AC}+\cdots+F_{DE}+F_{DF}+F_{EF}\right). \quad (14)$$

Therefore, one can show that $0 \leq \tau_2 \leq 3/2$.

Also, from Eqs. (8)–(12), we can obtain

$$\tau_3 = \frac{7}{4}-\frac{1}{8}\left(F_A+F_B+F_C+F_D+F_E+F_F\right)$$
$$-\frac{1}{20}\left(F_{AB}+F_{AC}+\cdots+F_{DF}+F_{EF}\right)$$
$$-\frac{1}{80}\left(F_{ABC}+F_{ABD}+\cdots+F_{CEF}+F_{DEF}\right). \quad (15)$$

Thus, we have $0 \leq \tau_3 \leq 7/4$.

On the other hand, we can show

$$8\tau_{ABCDEF}+\left(F_{ABC}+F_{ABD}+\cdots+F_{CDF}+F_{CEF}+F_{DEF}\right)$$
$$= 2\left(F_A+F_B+F_C+F_D+F_E+F_F\right)+8. \quad (16)$$

From Eqs. (11)–(14), we have

$$\tau_{ABCDEF} = 6\tau_1+10\tau_3-15\tau_2. \quad (17)$$

We can measure the entanglements of the product, GHZ, and maximally entangled state by this relation.

For the product state, we have $F_A = F_B = F_C = F_D = F_E = 1$, $F_{AB} = F_{AC} = \cdots = F_{DE} = 1$, $F_{ABC} = F_{ABD} = \cdots = F_{DEF} = 1$.

From Eqs. (9)–(15), we can obtain $\tau_{ABCDEF} = 0$, $\tau_1 = \tau_2 = \tau_3 = 0$, obviously, and it satisfies Eq. (17). For the GHZ state,

$$|\varphi\rangle_{ABCDEF} = \frac{1}{\sqrt{2}}(|000000\rangle+|111111\rangle),$$

one can have

$$F_A = F_B = F_C = F_D = F_E = F_F = 0,$$
$$F_{AB} = F_{AC} = F_{AD} = \cdots = F_{EF} = 1,$$
$$F_{ABC} = F_{ABD} = \cdots = F_{DEF} = 0.$$

From Eqs. (9)–(14), we can obtain $\tau_{ABCDEF} = 1$, $\tau_1 = \tau_2 = \tau_3 = 1$, and it also satisfies Eq. (17).

For the maximally six-qubit entangled state[26]

$$|\psi_{\mathrm{M}}\rangle_{1234567}$$
$$= \frac{1}{4}(|000000\rangle+|000111\rangle-|001001\rangle+|001110\rangle$$
$$-|010011\rangle+|010100\rangle+|011010\rangle+|011101\rangle$$
$$-|100010\rangle+|100101\rangle-|101011\rangle-|101100\rangle$$
$$+|110001\rangle+|110110\rangle+|111000\rangle-|111111\rangle),$$

we can show that

$$F_A = F_B = F_C = F_D = F_E = F_F = 0,$$
$$F_{AB} = F_{AC} = F_{AD} = \cdots = F_{EF} = 0,$$

$$F_{ABC} = F_{ABD} = F_{ABE} = F_{ABF} = \cdots = F_{CDF} = F_{CEF} = F_{DEF} = 0.$$

From Eqs. (9)–(15), we can obtain $\tau_1 = 1$, $\tau_2 = 3/2$, $\tau_3 = 7/4$, $\tau_{ABCDEF} = 1$, and it also satisfies Eq. (17).

2.2. Relation between *n*-tangle and square of I concurrence of eight-qubit

For an eight-qubit state

$$|\varphi\rangle_{ABCDEFGH} = a_0|00000000\rangle + a_1|00000001\rangle + \cdots + a_{255}|11111111\rangle. \quad (18)$$

Similarly, we can have

$$\tau_1 = \frac{1}{8}\big(\tau_{A(BCDEFGH)} + \tau_{B(ACDEFGH)} + \tau_{C(ABDEFGH)} + \tau_{D(ABCEFGH)} + \tau_{E(ABCDFGH)} + \tau_{F(ABCDEGH)} + \tau_{G(ABCDEFH)} + \tau_{h(ABCDEFG)}\big), \quad (19)$$

$$\tau_2 = \frac{1}{28}(\tau_{AB(CDEFGH)} + \tau_{AC(BDEFGH)} + \cdots + \tau_{GH(ABCDEF)}), \quad (20)$$

$$\tau_3 = \frac{1}{56}(\tau_{ABC(DEFGH)} + \tau_{ABD(CEFGH)} + \cdots + \tau_{FGH(ABCDE)}), \quad (21)$$

$$\tau_4 = \frac{1}{70}(\tau_{ABCD(EFGH)} + \tau_{ABCE(DFGH)} + \cdots + \tau_{EFGH(ABCD)}), \quad (22)$$

$$\tau_{ABCDEFGH} = \big|\langle\psi^*|\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y\otimes\sigma_y|\psi\rangle\big|^2. \quad (23)$$

Similarly, we can obtain

$$\tau_1 = 1 - \frac{1}{8}(F_A + F_B + F_C + F_D + F_E + F_F + F_G + F_H), \quad (24)$$

$$\tau_2 = \frac{3}{2} - \frac{1}{8}(F_A + F_B + \cdots + F_G + F_H) - \frac{1}{56}(F_{AB} + F_{AC} + F_{AD} + \cdots + F_{GH}), \quad (25)$$

$$\tau_3 = \frac{7}{4} - \frac{3}{32}(F_A + F_B + \cdots + F_G + F_H) - \frac{3}{112}(F_{AB} + F_{AC} + F_{AD} + \cdots + F_{GH}) - \frac{1}{224}(F_{ABC} + F_{ABD} + F_{ABE} + \cdots + F_{FGH}), \quad (26)$$

$$\tau_4 = \frac{15}{8} - \frac{1}{16}(F_A + F_B + \cdots + F_G + F_H) - \frac{3}{112}(F_{AB} + F_{AC} + F_{AD} + \cdots + F_{GH}) - \frac{1}{112}(F_{ABC} + F_{ABD} + F_{ABE} + \cdots + F_{FGH}) - \frac{1}{560}(F_{ABCD} + F_{ABCE} + \cdots + F_{EFGH}), \quad (27)$$

$$(F_{1234} + \cdots + F_{5678}) - 9(\mathrm{F}_1 + \cdots + \mathrm{F}_6) - (F_{12} + \cdots + F_{78}) + (F_{123} + \cdots + F_{678}) = 26 + 16\tau_{ABCDEFGH}. \quad (28)$$

From Eqs. (21)–(25), we have

$$\tau_{ABCDEFGH} = 15\tau_1 - 35\tau_2 + 56\tau_3 - 35\tau_4. \quad (29)$$

For the product state, we have

$$F_A = F_B = F_C = F_D = F_E = 1,$$
$$F_{AB} = F_{AC} = F_{AD} = F_{AE} = F_{BC} = F_{BD} = F_{BE} = F_{CD} = F_{CE} = F_{DE} = 1,$$
$$F_{ABC} = F_{ABD} = F_{ABE} = F_{ABF} = \cdots = F_{CDF} = F_{CEF} = F_{DEF} = 1,$$
$$F_{ABCD} = F_{ABCE} = F_{EFGH} = 1.$$

From Eqs. (23)–(27), we can obtain $\tau_{ABCDEFGH} = 0$, $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$, obviously, it satisfies Eq. (29).

Similarly, for the GHZ state, $|\varphi\rangle_{ABCDEFGH} = (1/\sqrt{2})(|00000000\rangle + |11111111\rangle)$, one can have

$$F_A = F_B = F_C \cdots F_H = 0,$$
$$F_{AB} = F_{AC} = F_{AD} = \cdots = F_{GH} = 1,$$
$$F_{ABC} = F_{ABD} = F_{ABE} \cdots = F_{CEF} = F_{FGH} = 0,$$
$$F_{ABCD} = F_{ABCE} = \cdots = F_{EFGH} = 1.$$

From Eqs. (23)–(27), we can obtain $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 1$, $\tau_{ABCDEFGH} = 1$. Obviously, it satisfies Eq. (29).

3. Discussion

For a six-qubit pure state, from Eq. (17), we know that

$$\tau_{ABCDEF} = 6\tau_1 + 10\tau_3 - 15\tau_2.$$

For the 3-uniform state, it must have

$$F_A = F_B = F_C = F_D = F_E = F_F = 0,$$
$$F_{AB} = F_{AC} = F_{AD} = \cdots = F_{EF} = 0,$$
$$F_{ABC} = F_{ABD} = F_{ABE} = F_{ABF} = \cdots = F_{CDF} = F_{CEF} = F_{DEF} = 0,$$

then, we have

$$\tau_1 = 1, \quad \tau_2 = \frac{3}{2}, \quad \tau_3 = \frac{7}{4}.$$

Therefore, we obtain $\tau_{ABCDEF} = 1$. This result is just consistent with maximally six-qubit entangled state. Therefore, we can say that maximally six-qubit entangled state is a 3-uniform state, and its 6-tangle is 1.

For the eight-qubit state, from Eq. (29), we know that

$$\tau_{ABCDEFGH} = 15\tau_1 - 35\tau_2 + 56\tau_3 - 35\tau_4.$$

Assuming that the eight-qubit state is a 4-uniform states, we have

$$F_A = F_B = F_C \cdots = F_H = 0,$$

$$F_{AB} = F_{AC} = F_{AD} = \cdots = F_{GH} = 0,$$
$$F_{ABC} = F_{ABD} = F_{ABE} \cdots = F_{CEF} = F_{FGH} = 0,$$
$$F_{ABCD} = F_{ABCE} = \cdots = F_{EFGH} = 0,$$
$$\tau_1 = 1, \quad \tau_2 = \frac{3}{2}, \quad \tau_3 = \frac{7}{4}, \quad \tau_4 = \frac{15}{8}.$$

From Eq. (29), we obtain $\tau_{ABCDEFGH} = -41/8$. Because $\tau_{ABCDEFGH} \geq 0$, therefore, it is impossible have a 4-uniform state for the eight-qubit state. This conclusion is consistent with the result in Ref. [27].

4. Conclusions

We have derived two relations of pure-state six-qubit and eight-qubit entanglement, the central results are Eqs. (17) and (29). We first introduce a relationship between the n-tangle and the multipartite entanglement of even n qubits. For a maximally six-qubit entangled state, we can find that it is a 3-uniform state with the 6-tangle equal to 1. Therefore, we can say that n-tangle equal to 1 is the necessary condition of maximally six-qubit entangled state. But for an eight-qubit state, using this relation, we find that it is impossible to have a 4-uniform state. Therefore, maximally entangled eight-qubit is also a 3-uniform state with the 8-tangle equal to 0.

Quantum metrology is the most promising technology originating from quantum information,[28,29] and already shows a shining future. Later, we will calculate the quantum capture information in the case, and try to find the relationship between these relevant information and quantum capture information.

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