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Double-passage mechanical cooling in a coupled optomechanical system*

Qing-Xia Mu(穆青霞)[1], Chao Lang(郎潮)[1], and Wen-Zhao Zhang(张闻钊)[2,†]

[1] *Mathematics and Physics Department, North China Electric Power University, Beijing* 102206, *China*
[2] *Beijing Computational Science Research Center (CSRC), Beijing* 100193, *China*

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We consider a three-mode optomechanical system where two cavity modes are coupled to a common mechanical oscillator. We focus on the resolved sideband limit and illustrate the relation between the significant parameters of the system and the instantaneous-state mean phonon number of the oscillator cooled to the ground state, particularly at the early stage of the evolution. It is worth noting that the optical coupling sets up a correlation between the two cavity modes, which has significant effect on the cooling process. Using numerical solutions, we find that the inter-cavity coupling will decrease the cooling effect when both cavities have the same effective optomechanical coupling. However, when the effective optomechanical couplings are different, the cooling effect will be strongly improved by selecting appropriate range of inter-cavity coupling.

Keywords: optomechanical system, ground-state cooling, quantum optics

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1. Introduction

Cavity optomechanics[1–4] is a rapidly developing field that explores the interaction between light and mechanical motion based on radiation pressure, which plays an important role in realizing quantum control of mechanical oscillators such as high-precision measurement[5,6] and quantum information processing.[7–9] The key to achieve this process is to cool the mechanical system to the quantum ground state where the final mean phonon number of the mechanical resonator is less than one.[10,11] In recent years, ground state cooling of a mechanical resonator has received increasing attention in theoretical and experimental research fields, since the preparation for a resonator in its ground state not only enables one to exploit its quantum behaviors in the field of quantum technologies, but also has many potential applications in biological sensing,[12] mechanical displacement[13] and high-precision detection of gravitational waves.[14] So far, many schemes have been proposed to achieve the ground-state cooling of mechanical oscillator such as the resolved sideband cooling,[15–17] feedback cooling[18,19] and back-action cooling.[20] More significantly, several cooling protocols have already been experimentally demonstrated.[21,22]

Recently, various approaches to optimize ground-state cooling in optomechanical systems have been studied through cavity-assisted laser cooling,[23,24] dynamical dissipative cooling,[25] atom-assisted cooling,[26] and cooing in non-Markovian regime.[27] Of particular interest are multimode optomechanical systems incorporating more than one mechanical oscillator[28–30] or cavity mode,[24,31–34] which are emerging platforms for studying the fundamental properties of matter near the quantum ground state. The degree of quantum control in this hybrid optomechanical system can be further enhanced by replacing the mirror with a double-face reflective membrane within the cavity.[35,36] On the other hand, coupled cavities can be used to study the quantum coherence effects involving multiple optical modes which can generate various quantum phenomena. This hybrid systems would also open up practical avenues to achieve the ground-state cooling in the resolved sideband regime.

In this paper, we try to study the optomechanical cooling scheme from a novel perspective, which has been rarely discussed in the literature. Motivated by the fast teleportation protocol proposed in Refs. [37] and [38], we show the possibility of cooling the mechanical oscillator to the ground state via resonant oscillations of the optomechanical system. For a coupled optomechanical system shown in Fig. 1(a), the cooling mechanism has been widely discussed as illustrated in Fig. 1(b). By choosing appropriate driven frequencies, the initial state $|0,0,n\rangle$ tends to follow the red path and evolves to $|0,0,n-1\rangle$. This process will adiabatically lower the energy of the oscillator and evolve towards the ground state ($|0,0,n\rangle \rightarrow |0,0,n-1\rangle \rightarrow \cdots \rightarrow |0,0,0\rangle$). However, we will show that the non-adiabatic resonant oscillation can also result in an extremely low phonon number at the early stage of the evolution. We use numerical simulations to show the probability of obtaining a low-phonon-number state under threshold

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†Corresponding author. E-mail: zhangwz@csrc.ac.cn

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can be notably large, which indicates that one can probabilistically achieve the ground state much faster than adiabatically lower down the phonon number. Then, we investigate in detail that how to achieve a higher probability by manipulating the system parameters including the inter-cavity coupling and the optomechanical coupling. It is suggested that asymmetric optomechanical couplings may be helpful to increase the probability at the early stage of the evolution. It is our hope that our results may encourage the researchers to re-examine the optomechanical cooling scheme from another angle.

2. Model and solution

We focus on a three-mode optomechanical system as shown in Fig. 1(a), in which a mechanical resonator interacts with two driven cavities. The Hamiltonian for this two-cavity optomechanical system can be written as ($\hbar = 1$)

$$\begin{aligned} H &= \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \omega_m b^\dagger b + J\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) \\ &+ \left(g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2\right)\left(b + b^\dagger\right) + \Omega_1\left(a_1 e^{i\omega_{L1}t} + a_1^\dagger e^{-i\omega_{L1}t}\right) \\ &+ \Omega_2\left(a_2 e^{i\omega_{L2}t} + a_2^\dagger e^{-i\omega_{L2}t}\right). \end{aligned} \tag{1}$$

Here the first three terms are the free Hamiltonian of the two cavity modes and the mechanical oscillator with frequency ω_i ($i = 1,2$) and ω_m, respectively. The operators a_i ($i = 1,2$) and b are the corresponding annihilation operators. The fourth term represents the interaction between the two optical modes by the tunnel coupling parameter J.[31,32,40–42] The fifth term describes the single-photon optomechanical interaction with coupling coefficient g_i. The last two terms account for the two external driving lasers with frequency ω_{Li} and the related driven strength Ω_i, respectively.

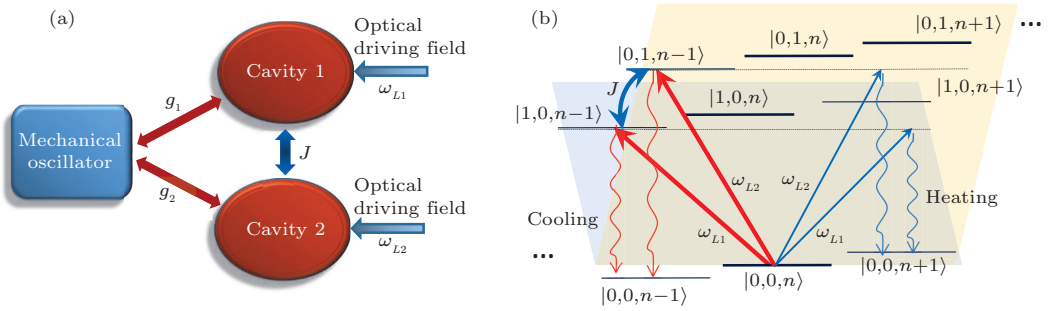


Fig. 1. (a) Schematic of a three-mode optomechanical system. Two cavities (cavity 1 and cavity 2) interact with a common mechanical oscillator, which are also driven by two external laser fields with frequency ω_{L1} and ω_{L2}, respectively. Further, a tunnel coupling J between the two cavity modes is allowed. (b) Level scheme of the linearized Hamiltonian [Eq. (3)] in the displaced frame. Here $|0\rangle$, $|1\rangle$, $|n\rangle$ denote the states of the two cavity modes and the mechanical oscillator, respectively. The blue double arrow represents the transition from the state $|0,1,n-1\rangle$ to the state $|1,0,n-1\rangle$ with coupling strength J.

We consider that the two driving fields have the same frequency $\omega_{L1} = \omega_{L2} = \omega_L$, then the system Hamiltonian (1) in the rotating frame with frequency ω_L can be rewritten as

$$\begin{aligned} H &= \Delta_1' a_1^\dagger a_1 + \Delta_2' a_2^\dagger a_2 + \omega_m b^\dagger b + J\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) \\ &+ \left(g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2\right)\left(b + b^\dagger\right) + \Omega_1\left(a_1 + a_1^\dagger\right) \\ &+ \Omega_2\left(a_2 + a_2^\dagger\right), \end{aligned} \tag{2}$$

where $\Delta_1' = \omega_1 - \omega_L$ and $\Delta_2' = \omega_2 - \omega_L$ describe the detunings of the two cavity modes from the driving fields, respectively.

Following the usual linearized method in the coupled optomechanical systems,[42,43] we can assume that the driving fields are strong enough that the system can be linearized around their mean pump amplitudes α_i and β, then yields Hamiltonian (see Appendix A for details)

$$\begin{aligned} H_L &= \Delta_1 a_1^\dagger a_1 + \Delta_2 a_2^\dagger a_2 + \omega_m b^\dagger b + J\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) \\ &+ \left(G_1^* a_1 + G_1 a_1^\dagger\right)\left(b + b^\dagger\right) \\ &+ \left(G_2^* a_2 + G_2 a_2^\dagger\right)\left(b + b^\dagger\right), \end{aligned} \tag{3}$$

where $G_i = g_i\alpha_i$ is the cavity-enhanced coupling rate and $\Delta_i = \omega_i - \omega_{Li} + g_i(\beta + \beta^*)$ describes the optomechanical-coupling modified detuning.

The cooling mechanism is shown in Fig. 1(b), where the energy levels of the system is plotted. Here $|0\rangle, |1\rangle, |n\rangle$ are the states of the optomechanical cavity 1, optomechanical cavity 2, and mechanical oscillator, respectively. The red arrows in Fig. 1(b) represent the cooling processes due to the energy swapping and counter-rotating-wave interaction, while the blue arrows denote the heating mechanism corresponding to the energy swapping and quantum backaction. It should be noted that the cooling processes occur between two different excitation pathways, from $|0,0,n\rangle$ to $|1,0,n-1\rangle$ and from $|0,0,n\rangle$ to $|0,1,n-1\rangle$, respectively, which will lead to constructive or destructive quantum interference and the cooling effect will be strengthened or weakened correspondingly. Meanwhile, the transition from the state $|1,0,n\rangle$ to the state $|0,1,n\rangle$ is caused by the cavity-cavity coupling strength J, which will have significant impact on the cooling process.

To explore the cooling potential of the linearized Hamiltonian H_L, we consider the case that the cavity modes are linearly coupled to the vacuum bath and the mechanical oscillator experiences thermal bath. Then we apply the standard input-output theory[44,45] to derive the following quantum Langevin equations for optical and mechanical modes as

$$\begin{aligned}
\dot{a}_1 &= -(\mathrm{i}\Delta_1+\tfrac{\kappa_1}{2})a_1-\mathrm{i}G_1(b+b^\dagger)-\mathrm{i}Ja_2+\sqrt{\kappa_1}a_1^{\mathrm{in}},\\
\dot{a}_2 &= -(\mathrm{i}\Delta_2+\tfrac{\kappa_2}{2})a_2-\mathrm{i}G_2(b+b^\dagger)-\mathrm{i}Ja_1+\sqrt{\kappa_2}a_2^{\mathrm{in}},\\
\dot{b} &= -(\mathrm{i}\omega_m+\tfrac{\gamma}{2})b-\mathrm{i}[(G_1^*a_1+G_1a_1^\dagger)+(G_2^*a_2+G_2a_2^\dagger)]+\sqrt{\gamma}b^{\mathrm{in}},
\end{aligned} \tag{4}$$

where κ_i and γ are the decay rates of the cavity modes and mechanical oscillator, respectively. The operators a_1^{in} and a_2^{in} are the input vacuum noise of the two cavities satisfying the correlations $\left\langle a_i^{\mathrm{in}}(t), a_i^{\mathrm{in}^\dagger}(t')\right\rangle = \delta_{ii'}\delta(t-t')$, for $i,i'=1,2$. The term b^{in} corresponds to the noise operator of the mechanical mode which follows the relation $\left\langle b^{\mathrm{in}}(t)b^{\mathrm{in}^\dagger}(t')\right\rangle = (n_{\mathrm{th}}+1)\delta(t-t')$, and $n_{\mathrm{th}} = (e^{\hbar\omega_m/k_{\mathrm{B}}T}-1)^{-1}$ is the thermal phonon number related to the frequency of the mechanical mode at the environment temperature T.

In the following we focus on the resolved sideband limit $\kappa \ll \omega_m$. In this regime the optimal cooling is achieved near $\Delta_i = \omega_m$, where the rotating wave approximation can be used and thus Eq. (4) is simplified to

$$\begin{aligned}
\dot{a}_1 &= -\left(\mathrm{i}\Delta_1+\frac{\kappa_1}{2}\right)a_1-\mathrm{i}G_1b-\mathrm{i}Ja_2+\sqrt{\kappa_1}a_1^{\mathrm{in}},\\
\dot{a}_2 &= -\left(\mathrm{i}\Delta_2+\frac{\kappa_2}{2}\right)a_2-\mathrm{i}G_2b-\mathrm{i}Ja_1+\sqrt{\kappa_2}a_2^{\mathrm{in}},\\
\dot{b} &= -\left(\mathrm{i}\omega_m+\frac{\gamma}{2}\right)b-\mathrm{i}\left(G_1^*a_1+G_2^*a_2\right)+\sqrt{\gamma}b^{\mathrm{in}}.
\end{aligned} \tag{5}$$

In order to calculate the mean phonon number, we need to determine the mean value of all the second-order moments, $N_1 = \left\langle a_1^\dagger a_1\right\rangle$, $N_2 = \left\langle a_2^\dagger a_2\right\rangle$, $N_{\mathrm{b}} = \langle b^\dagger b\rangle$, $\left\langle a_1a_2^\dagger\right\rangle$, $\langle a_1b^\dagger\rangle$ and $\langle a_2b^\dagger\rangle$, which are governed by the following differential equations

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}N_1 &= -\kappa_1N_1+\mathrm{i}\left(G_1^*\langle a_1b^\dagger\rangle-G_1\langle a_1b^\dagger\rangle^*\right)\\
&\quad+\mathrm{i}J\left(\left\langle a_1a_2^\dagger\right\rangle-\left\langle a_1a_2^\dagger\right\rangle^*\right),
\end{aligned} \tag{6}$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}N_2 &= -\kappa_2N_2+\mathrm{i}\left(G_2^*\langle a_2b^\dagger\rangle-G_2\langle a_2b^\dagger\rangle^*\right)\\
&\quad+\mathrm{i}J\left(\left\langle a_1a_2^\dagger\right\rangle^*-\left\langle a_1a_2^\dagger\right\rangle\right),
\end{aligned} \tag{7}$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}N_{\mathrm{b}} &= -\gamma N_{\mathrm{b}}+\mathrm{i}\left(G_1\langle a_1b^\dagger\rangle^*-G_1^*\langle a_1b^\dagger\rangle\right)\\
&\quad+\mathrm{i}\left(G_2\langle a_2b^\dagger\rangle^*-G_2^*\langle a_2b^\dagger\rangle\right)+\gamma n_{\mathrm{th}},
\end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left\langle a_1a_2^\dagger\right\rangle &= -\left[\mathrm{i}(\Delta_1-\Delta_2)+\frac{\kappa_1+\kappa_2}{2}\right]\left\langle a_1a_2^\dagger\right\rangle\\
&\quad-\mathrm{i}\left(G_1\langle a_2b^\dagger\rangle^*-G_2^*\langle a_1b^\dagger\rangle\right)\\
&\quad+\mathrm{i}J(N_1-N_2),
\end{aligned} \tag{9}$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\langle a_1b^\dagger\rangle &= -\left[\mathrm{i}(\Delta_1-w_m)+\frac{\kappa_1+\gamma}{2}\right]\langle a_1b^\dagger\rangle\\
&\quad-\mathrm{i}G_1(N_{\mathrm{b}}+1)+\mathrm{i}G_1(N_1+1)\\
&\quad+\mathrm{i}G_2\left\langle a_1a_2^\dagger\right\rangle-\mathrm{i}J\langle a_2b^\dagger\rangle,
\end{aligned} \tag{10}$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\langle a_2b^\dagger\rangle &= -\left[\mathrm{i}(\Delta_2-w_m)+\frac{\kappa_2+\gamma}{2}\right]\langle a_2b^\dagger\rangle\\
&\quad-\mathrm{i}G_2(N_{\mathrm{b}}+1)+\mathrm{i}G_2(N_2+1)\\
&\quad+\mathrm{i}G_1\left\langle a_1a_2^\dagger\right\rangle^*-\mathrm{i}J\langle a_1b^\dagger\rangle.
\end{aligned} \tag{11}$$

3. Cooling at early stage

We proceed to discuss the dynamical evolution of the phonon number under the linearized Hamiltonian in Eq. (3). To obtain the optical cooling effect, we then analyze and discuss the influence of the parameters, such as the coupling strength of the two cavities and the effective optomechanical-couplings, on the occupation number of the mechanical resonator. In Fig. 2, we plot the time evolution of the exact numerical results of the average numbers of N_1, N_2, and N_{b}. It shows the well-known Rabi oscillation for cavity modes a_1, a_2 and mechanical oscillator b, which reveals the maximum energy exchange from the mechanical oscillator to these two cavity modes since the rotating-wave interaction denoted by the term $G_i(a_i^\dagger b+a_ib^\dagger)$ is on resonance near the optimal effective detuning $\Delta_i=\omega_m$ and thus the corresponding cooling process is prominent. Due to the mechanism shown in Fig. 1(b), the phonon number N_{b} for the mechanical oscillator is gradually lower down to the steady state value (below 1) at $\omega_m t = 300$. The mean phonon occupancy of the mechanical oscillator can be cooled down to below 1 from the initial number 100, which means that ground-state cooling of mechanical mode may be achieved. Meanwhile, one may also notice that at the early stage of the evolution, there is a strong energy exchange between the mechanical oscillator and the cavities, resulting in a large oscillation of N_{b}. If we define the phonon number $N_{\mathrm{b}} < 1$ as a threshold of achieving the ground state cooling, it is shown that there are already many low-phonon-number points that reach the threshold when $\omega_m t < 100$.

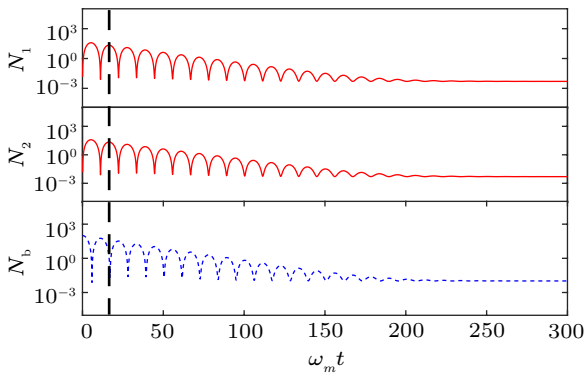


Fig. 2. Time evolution of the mean occupation numbers of N_1 (red solid curve), N_2 (red solid curve) and N_{b} (blue dotted curve) for the following parameters: $\kappa_1/\omega_m = \kappa_2/\omega_m = 0.1$, $\gamma/\omega_m = 10^{-5}$, $\Delta_1/\omega_m = \Delta_2/\omega_m = 1$, $G_1/\omega_m = 0.2$, $G_2/\omega_m = 0.1$, $J/\omega_m = 0.05$, and $n_{\mathrm{th}} = 100$.

Now, we focus on these low-phonon-number points in the short-time evolution $\omega_m t < 100$. In Fig. 3, we plot the time evolution of the mean phonon number N_b with the parameters $G_1/\omega_m = G_2/\omega_m = 0.2$, $J/\omega_m = 0.02$, and the dimensionless time is cut off to $\omega_m t = 100$. The phonon number oscillates quickly in a relatively short period, which leads to a below threshold ($N_b = 1$) phonon number periodically. Meanwhile, due to the anti-Stokes process, the revival peak phonon numbers in each period become smaller and smaller. The fast oscillating originates from exchange of energy between the photon and phonon, and the slow decreasing of the peak phonon numbers in each period is the consequence of the anti-Stokes process. The gray square area in Fig. 3 indicates the time domain that the phonon number temporarily falls down below the threshold value ($N_b \leq 1$).

We define a parameter P to describe the probability of obtaining $N_b < 1$ at early stage. The phonon number as a physical observable can be measured at time t. If we can precisely control the evolution time and measure $N_b(t)$ exactly at a particular time (for example, at $\omega t = 81$ to $\omega t = 88$), then the mechanical oscillator will be in the ground state. However, in a practical case, it is extremely difficult to control evolution time, so that we must use a parameter P to describe the probability of obtaining a ground state if we perform the measurement at a random time point $0 < \omega t < 100$. The meaning of P is that in the early stage of evolution, there is a notable probability that the system is on the ground state, even if we can not control the measurement time point.

The cooling mechanism shown in Fig. 1(b) will indeed guide the final state of the mechanical oscillator towards the ground state. However, this is a relatively slow process. Inspired by the investigation in Refs. [37] and [38], where the authors showed the probability of implementing fast qubit gate by utilizing the non-adiabatic evolution in an adiabatic process, the oscillation of the phonon number at the early stage of the evolution may also imply a faster cooling of the mechanical oscillator. One may worry about the precise control of the evolution time that the state is just evolve into the low-phonon-number states. However, the numerical results show that even if the evolution time is completely out of control, a random evolution time will also ensure a notable probability that the phonon number is under threshold. As shown in Fig. 3, the gray region of $N_b < 1$ is notable. For instance, if we consider the region $50 < \omega_m t < 100$, the possibility will be nearly 50%, which means that it is more likely to obtain a ground state of the mechanical oscillator than the excited states. We would like to point out that the cooling caused by dissipation of cavity is a slow process. However, the resonant evolution which is a much faster process could also drive the oscillator to the ground state in the early stage evolution. More importantly, the probability of getting to the ground state is not small.

Certainly, the strong oscillation at the early stage is not a universal feature of the evolution for such an optomechanical cooling system. For other sets of parameters, the phenomenon that the strong oscillation leads to below threshold phonon numbers will become weaker or even disappear. Therefore, it is valuable for us to investigate for what a configuration (parameters), we will obtain a ground state at the early stage with a higher probability.

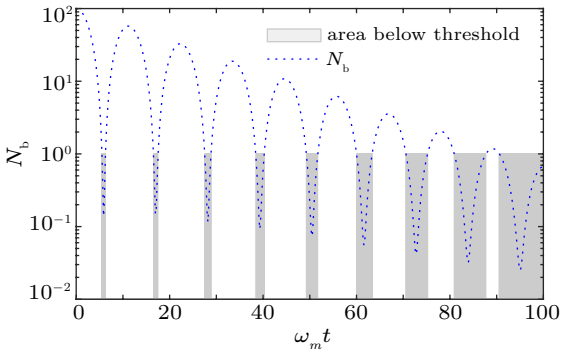


Fig. 3. Time evolution of the mean phonon number N_b for $\kappa_1/\omega_m = \kappa_2/\omega_m = 0.1$, $\gamma/\omega_m = 10^{-5}$, $\Delta_1/\omega_m = \Delta_2/\omega_m = 1$, $J/\omega_m = 0.02$, $G_1/\omega_m = G_2/\omega_m = 0.2$, and $n_{th} = 100$. Gray area indicates the time domain that the mean phonon number N_b temporarily falls down below the threshold value 1.

In Fig. 4, we analyze the effect of the coupling strength J on the probability of cooling the mechanical mode under the threshold $N_b < 1$ in the case of the short-time limit. We draw a function of P (P stands for the probability of cooling the mechanical mode to the ground state with the mean phonon number less than 1) in terms of the coupling strength J (in units of ω_m) as shown in Fig. 4. One can find that the envelope of P decreases gradually with the increase of the coupling strength J, at last reaches and remains 0 when $G_1 = G_2$. That is to say, the coupling strength will greatly reduce the mechanical mode cooled to the ground state at the early stage. This is due to the fact that when J is 0, half each phonon's energy is transferred to two photons and the cooling processes occur from $|0,0,n\rangle$ to $|1,0,n-1\rangle$ and from $|0,0,n\rangle$ to $|0,1,n-1\rangle$ when $G_1 = G_2$. With the continuous increase of the coupling between the two cavities, the energy transferred from each phonon to two photons is less than half so that the number of phonons cooled to the ground state will decrease correspondingly. Therefore, in order to achieve a better cooling effect, we should control the coupling strength as small as possible when $G_1 = G_2$. In contrast, when $G_1 \neq G_2$, it can be clearly observed that with the increase of the coupling strength J, the cooling possibility P can be enhanced significantly compared with the case of equal optomechanical-coupling and reaches the maximum value at $J = 0.15$. That is to say, it is possible to manipulate the inter-cavity coupling to optimize the cooling effect in this case. Another obvious feature is that the cooling possibility P tends to zero when J increases big enough in both the cases whether G_1 and G_2 are equal or not. This is due to the fact that,

when J is so large that the energy of the two cavity modes only exchange with each other but not with the oscillator, the cooling effect will be suppressed. A significantly large coupling rate will reduce the amount of photons that interact with the mechanical resonator.

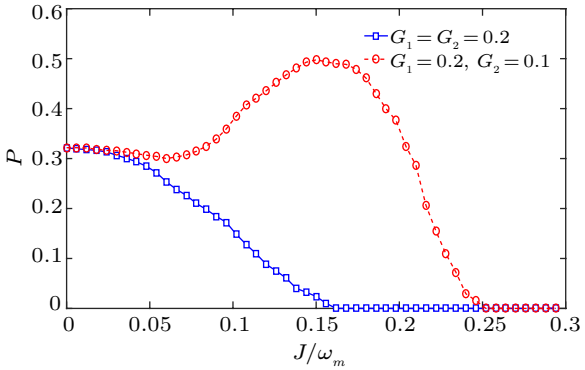


Fig. 4. The cooling possibility P as a function of coupling strength J (in units of ω_m) when the optomechanical coupling rates are equal to $G_1/\omega_m = G_2/\omega_m = 0.2$ (blue scatter plot) or different $G_1/\omega_m = 0.2$, $G_2/\omega_m = 0.1$ (red scatter plot). The dimensionless time is cut off to $\omega_m t = 100$, and the other parameters are the same as those in Fig. 3.

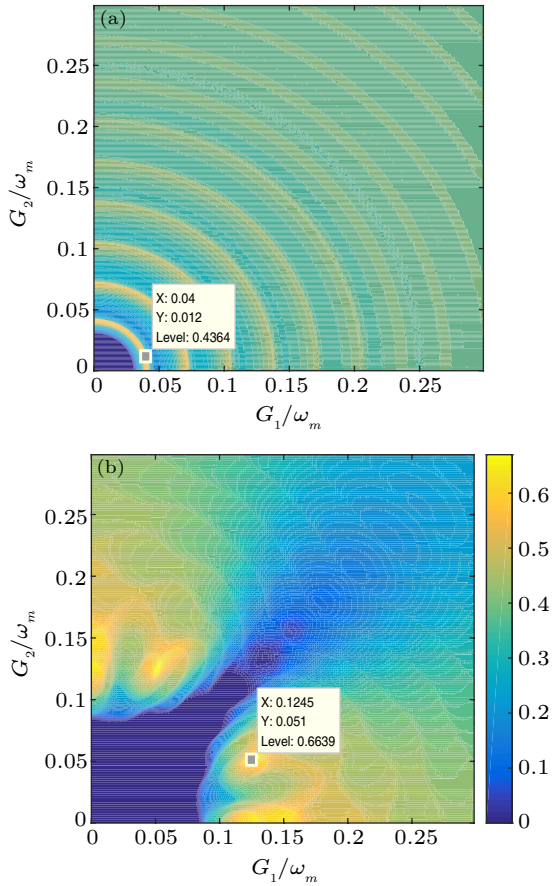


Fig. 5. The probability P as a function of G_1 (in units of ω_m) and G_2 (in units of ω_m) for $J = 0$ (a) and $J/\omega_m = 0.1$ (b) with $\kappa_1/\omega_m = \kappa_2/\omega_m = 0.1$, $\gamma/\omega_m = 10^{-5}$, $\Delta_1/\omega_m = \Delta_2/\omega_m = 1$, $\omega_m t = 100$, and $n_{\rm th} = 100$.

To further analyze the influence of optomechanical coupling rates G_1 and G_2 on the cooling effect, we plot the probability P as a function of G_1 and G_2 for $J = 0$ (Fig. 5(a)) and $J/\omega_m = 0.1$ (Fig. 5(b)). It shows that when there is no coupling between the two cavity modes, the cooling probability P shows interference fringe with the relation of G_1 and G_2. However, if the coupling is considered, the maximum value of probability P is enhanced significantly in the appropriate range of G_1 and G_2, which implies that there is a notable probability to obtain a low-phonon-number state below the threshold at the early stage of evolution.

4. Discussion and conclusion

In summary, we have presented a scheme of double-passage mechanical cooling in an optomechanical system where two interacted cavities are coupled to the same mechanical resonator. Instead of focusing on the long time cooling limit, we have paid more attention to the short time evolution at the early stage, which empower us to cool the phonon number below the threshold probabilistically. We show that there is a notable probability to obtain a low-phonon-number state below the threshold at the early stage of the evolution. Then, we investigate how the system parameters affect the probability of obtaining a below-threshold state. It reveals that when the two optomechanical couplings are different, it is possible to manipulate the inter-cavity coupling to obtain a higher probability. We also show how to choose the optomechanical coupling with or without the inter-cavity coupling.

Appendix A: Linearization of the optomechanical Hamiltonian

Considering all the damping and noise effects for the cavity modes and mechanical oscillator, the quantum Langevin equations can be derived from Hamiltonian (2) as follows:

$$\begin{aligned}
\dot{a}_1 &= -\left(\mathrm{i}\Delta_1' + \frac{\kappa_1}{2}\right) a_1 - \mathrm{i}g_1 a_1 (b + b^\dagger) - \mathrm{i}J a_2 - \mathrm{i}\Omega_1 + \sqrt{\kappa_1} a_1^{\rm in}, \\
\dot{a}_2 &= -\left(\mathrm{i}\Delta_2' + \frac{\kappa_2}{2}\right) a_2 - \mathrm{i}g_2 a_2 (b + b^\dagger) - \mathrm{i}J a_1 - \mathrm{i}\Omega_2 + \sqrt{\kappa_2} a_2^{\rm in}, \\
\dot{b} &= -\left(\mathrm{i}\omega_m + \frac{\gamma}{2}\right) b - \mathrm{i}\left(g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2\right) + \sqrt{\gamma} b^{\rm in},
\end{aligned} \tag{A1}$$

where $\Delta_1' = \omega_1 - \omega_L$ and $\Delta_2' = \omega_2 - \omega_L$.

Under the strong driving condition, we make use of the linear approximation that each operator can be expressed as the sum of steady-state mean values and their fluctuations, e.g., $a_1 = \alpha_1 + \delta a_1$, $a_2 = \alpha_2 + \delta a_2$, $b = \beta + \delta b$, then the above Langevin equations can be separated by the following classical and quantum components,

$$\begin{aligned}
\dot{\alpha}_1 &= -\left(\mathrm{i}\Delta_1' + \frac{\kappa_1}{2}\right) \alpha_1 - \mathrm{i}g_1 \alpha_1 (\beta + \beta^*) - \mathrm{i}J\alpha_2 - \mathrm{i}\Omega_1, \\
\dot{\alpha}_2 &= -\left(\mathrm{i}\Delta_2' + \frac{\kappa_2}{2}\right) \alpha_2 - \mathrm{i}g_2 \alpha_2 (\beta + \beta^*) - \mathrm{i}J\alpha_1 - \mathrm{i}\Omega_2, \\
\dot{\beta} &= -\left(\mathrm{i}\omega_m + \frac{\gamma}{2}\right) \beta - \mathrm{i}\left(g_1 |\alpha_1|^2 + g_2 |\alpha_2|^2\right), \\
\delta\dot{a}_1 &= -\left(\mathrm{i}\Delta_1 + \frac{\kappa_1}{2}\right) \delta a_1 - \mathrm{i}g_1 \alpha_1 (\delta b + \delta b^\dagger) \\
&\quad - \mathrm{i}g_1 \delta a_1 (\delta b + \delta b^\dagger) - \mathrm{i}J\delta a_2 + \sqrt{\kappa_1} a_1^{\rm in},
\end{aligned} \tag{A2}$$

$$\begin{aligned}\dot{\delta a_2} =& -\left(\mathrm{i}\Delta_2+\frac{\kappa_2}{2}\right)\delta a_2+\mathrm{i}g_2\alpha_2(\delta b+\delta b^\dagger)\\ &-\mathrm{i}g_2\delta a_2(\delta b+\delta b^\dagger)-\mathrm{i}J\delta a_1+\sqrt{\kappa_2}a_2^{\mathrm{in}},\\ \dot{\delta b} =& -\left(\mathrm{i}\omega_m+\frac{\gamma}{2}\right)\delta b-\mathrm{i}g_1\left(\alpha_1^*\delta a_1+\alpha_1\delta a_1^\dagger\right)\\ &-\mathrm{i}g_2\left(\alpha_2^*\delta a_2+\alpha_2\delta a_2^\dagger\right)-\mathrm{i}g_1\delta a_1\delta a_1^\dagger\\ &-\mathrm{i}g_2\delta a_2\delta a_2^\dagger+\sqrt{\gamma}b^{\mathrm{in}},\end{aligned}\tag{A3}$$

where we have defined the modified detunings $\Delta_i=\Delta_i'+g_i(\beta+\beta^*)$.

According to Eq. (A2), the steady-state solution of the system can be derived as

$$\begin{aligned}\alpha_1 &= \frac{-J\Omega_2-\mathrm{i}\Omega_1(\mathrm{i}\Delta_2+\kappa_2/2)}{(\mathrm{i}\Delta_1+\kappa_1/2)(\mathrm{i}\Delta_2+\kappa_2/2)+J^2},\\ \alpha_2 &= \frac{-J\Omega_1-\mathrm{i}\Omega_2(\mathrm{i}\Delta_1+\kappa_1/2)}{(\mathrm{i}\Delta_1+\kappa_1/2)(\mathrm{i}\Delta_2+\kappa_2/2)+J^2},\\ \beta &= -\frac{\mathrm{i}g_1|\alpha_1|^2+\mathrm{i}g_2|\alpha_2|^2}{(\mathrm{i}\omega_m+\frac{\gamma}{2})},\end{aligned}\tag{A4}$$

which are dominated by a few controllable parameters such as Δ_i, κ_i, and Ω_i.

Considering the strong driving condition, one can neglect all the nonlinear terms $\mathrm{i}g_i\delta a_i(\delta b+\delta b^\dagger)$ and $\mathrm{i}g_2\delta a_2\delta a_2^\dagger$ in Eq. (A3), then the corresponding effective Hamiltonian for the above Langevin equations can be written as

$$\begin{aligned}H_L =&\Delta_1a_1^\dagger a_1+\Delta_2a_2^\dagger a_2+\omega_mb^\dagger b+J\left(a_1^\dagger a_2+a_2^\dagger a_1\right)\\ &+\left(G_1^*a_1+G_1a_1^\dagger\right)\left(b+b^\dagger\right)\\ &+\left(G_2^*a_2+G_2a_2^\dagger\right)\left(b+b^\dagger\right),\end{aligned}\tag{A5}$$

where $G_i=g_i\alpha_i$ is the cavity-enhanced coupling rate, and the notation δ has been dropped for all the fluctuation operators for simplicity, e.g., $\delta a_1\to a_1$.

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