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# Dynamical evolution of photon-added thermal state in thermal reservoir*

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The dynamical behavior of a photon-added thermal state (PATS) in a thermal reservoir is investigated by virtue of Wigner function (WF) and Wigner logarithmic negativity (WLN), where this propagation model is abstracted as an input–output problem in a thermal-loss channel. The density operator of the output optical field at arbitrary time can be expressed in the integration form of the characteristics function of the input optical field. The exact analytical expression of WF is given, which is closely related to the Laguerre polynomial and is dependent on the evolution time and other interaction parameters (related with the initial field and the reservoir). Based on the WLN, we observe the dynamical evolution of the PATS in the thermal reservoir. It is shown that the thermal noise will make the PATS lose the non-Gaussianity.

**Keywords:** quantum statistics, master equation, photon-added thermal state, thermal reservoir, Wigner logarithmic negativity

**PACS:** 03.65.–w, 05.30.–d **DOI:** 10.1088/1674-1056/ab457d

## 1. Introduction

Evolution of pure states into mixed states is one of the major topics in quantum statistical mechanics.[1] This evolution usually happens when a system is immersed in a thermal reservoir. Similarly, the propagation of the optical field is an important topics in the field of optics. Especially, in quantum optics, people often come across the problem such that a quantum state passes through a quantum channel.[2,3] In fact, the propagation of the optical field inside a reservoir is a dynamic problem of an open quantum system.[4] Furthermore, optical field damping plays an important role in many problems and arose interests of researchers.

Mathematically, one can describe the dynamical evolution of the optical field by using the master equation or the Liouville equation.[5–7] The Liouville equation can be applied in quantum optics and statistical mechanics. Generally speaking, the decay (or decoherence) due to the interaction between a system and its environment can be described by a Liouville (super) operator. However, the Liouville (super) operator describes a non-unitary time evolution, which cannot trivially be integrated through the standard Lie-algebraic techniques. By resorting to quasiprobability representation of the density operator,[8] or upon eigen density matrices,[9] or using group-theoretical approach,[10] or the thermo-entangled state representation,[11–17] one can obtain the analytical solutions of the master equation. Unlike the above mentioned methods, we use the beam-splitter (BS) formalism to solve the master equation.[18] Following this formalism, we abstract the propagation of the optical field as an input–output process in a time evolution channel.[19]

Two decades ago, Agarwal and Tara introduced the photon-added thermal state (PATS), which exhibits no squeezing or sub-Poissonian statistics.[20] After that, the single PATS has been experimentally prepared by Zavatta[21] and theoretically investigated by Li.[22] Likewise, we have explored the decoherence of the PATS in a thermal environment.[23] For the same problem, we shall adopt another method to study its evolution in a thermal reservoir in this paper. Rather than solving the master equation by an algebraic method, we abstract it as an input–output problem in a thermal-loss channel. The output density operator has been linked to the input characteristic function. Our adopted method is a clever method to a certain extent. Furthermore, we discuss the time evolution of the optical field by virtue of the Wigner function (WF) and the Wigner logarithmic negativity (WLN).

This paper is arranged as follows. In Section 2, we introduce the theoretical model of field damping in a thermal reservoir. Here, we abstract the equivalent relationships of different ways including the master equation, the BS formalism, the loss channel, and the evolution channel. Here we give the input–out or time-evolution formulas on density operator in the characteristic function (CF) formalism. In Section 3, taking the PATS as the initial optical field, we derive the time-evolution density operator for the optical fields at arbitrary time. In Section 4, we give the Wigner function for the output state and study a figure of merit (i.e., Wigner logarithmic negativity) to

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judge the evolution of the optical field. Our conclusions are summarized in the last section.

## 2. Theoretical model of field damping in thermal reservoir

The dynamical evolution of the density operator $\rho(t)$ of the optical field is described by the master equation (or the Liouville equation)[24,25]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\kappa}{2}(n_\mathrm{e}+1)\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right) + \frac{\kappa}{2}n_\mathrm{e}\left(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger\right), \tag{1}$$

where $\kappa$ is the reservoir damping constant and $n_\mathrm{e}$ denotes the thermal average photon number of a thermal reservoir state

$$\rho_\mathrm{e}(n_\mathrm{e}) = \sum_{n=0}^{\infty}\frac{n_\mathrm{e}^n}{(n_\mathrm{e}+1)^{n+1}}|n\rangle\langle n|. \tag{2}$$

Of course, when $n_\mathrm{e}=0$, $\rho_\mathrm{e}(n_\mathrm{e})$ will reduce to the vacuum reservoir. That is to say, this equation is an effective equation describing optical field damping in a thermal reservoir (or environment).

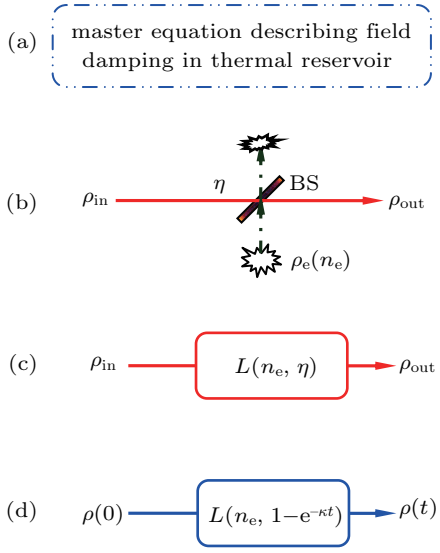



**Fig. 1.** (a) Master equation describing field damping in a thermal reservoir like Eq. (1), (b) beam-splitter formalism. (c) thermal-loss channel $L(n_\mathrm{e},\eta)$, and (d) time-evolution channel $L(n_\mathrm{e},1-\mathrm{e}^{-\kappa t})$ of the optical field.

In fact, the above process described by Eq. (1) can be modeled as the BS formalism,[18] where the input field $\rho_\mathrm{in}$ and the thermal state $\rho_\mathrm{e}(n_\mathrm{e})$ (denoting the reservoir or environment) are injected into a variable BS with reflectivity $\eta$, then the output field $\rho_\mathrm{out}$ is obtained after tracing over the reservoir variables, see Fig. 1(b). The BS formalism can be abstracted as the thermal-loss channel $L(n_\mathrm{e},\eta)$, where $\rho_\mathrm{out}$ is output after $\rho_\mathrm{in}$ passing through the channel (see Fig. 1(c)) and $\eta$ is called as the loss factor. The relationship between $\rho_\mathrm{in}$ and $\rho_\mathrm{out}$ can be linked by

$$\rho_\mathrm{out} = \int\frac{\mathrm{d}^2\alpha}{\pi}\chi_{\rho_\mathrm{in}}(\sqrt{1-\eta}\alpha) \times \mathrm{e}^{-(n_\mathrm{e}+1/2)\eta|\alpha|^2}D(-\alpha), \tag{3}$$

where $\chi_{\rho_\mathrm{in}}(\alpha)=\mathrm{Tr}[\rho_\mathrm{in}D(\alpha)]$ is the characteristic function (CF) of the input state $\rho_\mathrm{in}$ and $D(\alpha)$ is the displacement operator. The expression in Eq. (3) has been derived in our previous work.[26] If $n_\mathrm{e}=0$, then thermal-loss channel $L(n_\mathrm{e},\eta)$ is reduced to the loss channel $L(\eta)$.

During the propagation of the optical field, the energy loss within time $t$ is described by $1-\mathrm{e}^{-\kappa t}$, and this loss should be equivalent to the BS loss, so that the thermal-loss channel can be further abstracted as the time-evolution channel $L(n_\mathrm{e},1-\mathrm{e}^{-\kappa t})$ of the optical field. Through this channel, the initial state $\rho(0)$ evolutes to the state $\rho(t)$ at moment $t$. Linking by $\eta \Longleftrightarrow 1-\mathrm{e}^{-\kappa t}$, $\rho_\mathrm{in} \Longleftrightarrow \rho(0)$, and $\rho_\mathrm{out} \Longleftrightarrow \rho(t)$, we obtain the density operator of the optical field at moment $t$

$$\rho(t) = \int\frac{\mathrm{d}^2\alpha}{\pi}\chi_{\rho(0)}(\alpha\mathrm{e}^{-\frac{\kappa t}{2}}) \times \mathrm{e}^{-(n_\mathrm{e}+1/2)\left(1-\mathrm{e}^{-\kappa t}\right)|\alpha|^2}D(-\alpha), \tag{4}$$

where $\chi_{\rho(0)}(\alpha)=\mathrm{Tr}[\rho(0)D(\alpha)]$ is the CF of the initial state $\rho(0)$. Obviously, the two formulas in Eqs. (3) and (4) are equivalent to each other. As long as we know the initial CF, we can obtain the density operator of the optical field at any time. For the sake of convenience, we apply Eq. (3) to complete our following work.

## 3. Photon-added thermal state and its evolution in thermal reservoir

In this section, we consider how the PATS evolves in a thermal reservoir. The PATS can be described by the density operator[27]

$$\rho = N^{-1}a^{\dagger m}\exp\left(-\varepsilon a^\dagger a\right)a^m, \tag{5}$$

where $N$ is a normalization constant and $m$ is an integer. In fact, the operator $\exp\left(-\varepsilon a^\dagger a\right)$ describes a thermal state, whose normalized density operator is

$$\rho_\mathrm{th} = (1-\mathrm{e}^{-\varepsilon})\exp\left(-\varepsilon a^\dagger a\right) = \sum_{k=0}^{\infty}\frac{n_\mathrm{s}^k}{(n_\mathrm{s}+1)^{k+1}}|k\rangle\langle k|, \tag{6}$$

where the thermal average photon number $n_\mathrm{s}=\mathrm{Tr}\left(\rho_\mathrm{th}a^\dagger a\right)$ is related to $\varepsilon=\frac{\hbar\omega}{k_\mathrm{B}T}$ by

$$n_\mathrm{s} = \frac{1}{1-\mathrm{e}^{\varepsilon}} \quad \text{or} \quad \varepsilon = \ln\frac{n_\mathrm{s}+1}{n_\mathrm{s}}. \tag{7}$$

In the course of statistical and thermal physics, the state in Eq. (6) is an equilibrium state with finite temperature $T$.[28] Here $k_\mathrm{B}$ is the Boltzmann constant, $\hbar$ is the Planck constant,

and $\omega$ is the frequency. It needs to mention that we must distinguish the thermal state with $n_{\rm e}$ in Eq. (2) and that with $n_{\rm s}$ in Eq. (6).

Using the operator identity $\mathrm{e}^{-\lambda a^{\dagger}a} =: \mathrm{e}^{-(1-\mathrm{e}^{-\lambda})a^{\dagger}a}:$ ($:\cdots:$ denotes normal form) and the relations $a^{\dagger m} = \partial_{\mu}^{m}\mathrm{e}^{\mu a^{\dagger}}|_{\mu=0}$, $a^{m} = \partial_{\nu}^{m}\mathrm{e}^{\nu a}|_{\nu=0}$ (notice that $\partial_{\mu}^{m} = \frac{\mathrm{d}^{m}}{\mathrm{d}\mu^{m}}, \partial_{\nu}^{m} = \frac{\mathrm{d}^{m}}{\mathrm{d}\nu^{m}}$), we can write $\rho$ in Eq. (5) as

$$\rho = N^{-1}\partial_{\mu}^{m}\partial_{\nu}^{m}:\mathrm{e}^{\mu a^{\dagger}+\nu a-(1-\mathrm{e}^{-\varepsilon})a^{\dagger}a}:|_{\mu=\nu=0}. \tag{8}$$

Equation (8) is just the normally ordered form of the PATS. To ensure $\mathrm{Tr}(\rho)=1$, we can obtain the normalization constant $N = m!\,(n_{\rm s}+1)^{m+1}$. According to the definition $\chi = \mathrm{Tr}[\rho D(\alpha)]$, we obtain the CF of the PATS

$$\begin{aligned}\chi(\alpha) &= \frac{1}{m!\,(n_{\rm s}+1)^{m}}\partial_{\mu}^{m}\partial_{\nu}^{m}\\ &\quad\times\mathrm{e}^{(n_{\rm s}+1)(\alpha\nu-\mu\alpha^{*}+\mu\nu)-(n_{\rm s}+\frac{1}{2})|\alpha|^{2}}|_{\mu=\nu=0}.\end{aligned} \tag{9}$$

Obviously, the PATS is related with parameters $n_{\rm s}$ and $m$. In particular, when $n_{\rm s}=0$, $\rho$ is reduced to the Fock state $|m\rangle$.

Considering the PATS as the input (initial) optical field and substituting Eq. (9) into Eq. (3), we obtain the output optical field (i.e., the final density operator at time $t$)

$$\begin{aligned}\rho_{\rm out} &= \frac{1}{m!\,(n_{\rm s}+1)^{m}}\partial_{\mu}^{m}\partial_{\nu}^{m}\mathrm{e}^{(n_{\rm s}+1)\mu\nu}\\ &\quad\times\int\frac{\mathrm{d}^{2}\alpha}{\pi}\mathrm{e}^{-(n_{\rm s}-\eta n_{\rm s}+\eta n_{\rm e})|\alpha|^{2}+A(\nu\alpha-\mu\alpha^{*})}\\ &\quad\times\exp(\alpha^{*}a)\exp\left(-\alpha a^{\dagger}\right)|_{\mu=\nu=0}\end{aligned} \tag{10}$$

with $A = (n_{\rm s}+1)\sqrt{1-\eta}$. Knowing the density operator in Eq. (10), we can study any properties of the optical field at any time $t$. Obviously, the optical field is dependent on parameters ($m$, $n_{\rm s}$, $n_{\rm e}$, $\eta$) or ($m$, $n_{\rm s}$, $n_{\rm e}$, $\kappa t$).

## 4. Evolution of Wigner logarithmic negativity

In this section, the analytical expression of the Wigner function (WF) is derived and the dynamical evolution is discussed according to Wigner logarithmic negativity.

### 4.1. Wigner function

The WF $W(\beta)$ is a weight function for the expansion of the density operator $\rho$ in terms of the operator $O(\beta)$,[29] that is

$$\rho = \int W(\beta)\,O(\beta)\,\mathrm{d}^{2}\beta \tag{11}$$

with operator

$$O(\beta) = \frac{2}{\pi}D(\beta)(-1)^{a^{\dagger}a}D^{\dagger}(\beta), \tag{12}$$

where $D(\beta) = \exp\left(\beta a^{\dagger}-\beta^{*}a\right)$ is the usual displacement operator with $\beta = (x+\mathrm{i}y)/\sqrt{2}$. Therefore, the Wigner function is the expectation value of the operator

$$W(\beta) = \mathrm{Tr}[\rho O(\beta)]. \tag{13}$$

Based on the above definition, the description of evolution of the density operator can be replaced by its WF's evolution in phase space. Substituting Eq. (10) into Eq. (13), we obtain

$$\begin{aligned}W_{\rm out}(\beta) &= \frac{1}{\pi m!\,(n_{\rm s}+1)^{m}B}\mathrm{e}^{-\frac{1}{B}|\beta|^{2}}\\ &\quad\times\partial_{\mu}^{m}\partial_{\nu}^{m}\mathrm{e}^{-(n_{\rm s}+1)\frac{J}{B}\mu\nu+\frac{A}{B}(\beta\nu+\beta^{*}\mu)}|_{\mu=\nu=0},\end{aligned} \tag{14}$$

where we have set $B = (1-\eta)n_{\rm s}+\eta n_{\rm e}+1/2$ and $J = 1/2-(1+n_{\rm e})\eta$. The expression in Eq. (14) is just the analytical expression of the time evolution of WF. In our numerical calculations, we will apply such derivative form by the scientific computing software MATHEMATICA. Further, using the relations of Laguerre function

$$L_{m}(xy) = \frac{(-1)^{m}}{m!}\partial_{\mu}^{m}\partial_{\nu}^{m}\mathrm{e}^{-\mu\nu+\mu x+\nu y}|_{\mu=\nu=0}, \tag{15}$$

we have

$$W_{\rm out}(\beta) = \frac{(-J)^{m}}{\pi B^{m+1}}\mathrm{e}^{-\frac{1}{B}|\beta|^{2}}L_{m}\left(\frac{KJ}{B}|\beta|^{2}\right) \tag{16}$$

with $K = (n_{\rm s}+1)(1-\eta)$. Equation (16) shows that the WF of PATS in the thermal-loss channel is closely related to the Laguerre polynomials.

Two extreme cases and two special cases should be mentioned.

**Extreme case I** When $\kappa t\to 0$, leading to $\eta\to 0, A\to n_{\rm s}+1$, $B\to n_{\rm s}+1/2$, $J\to 1/2$, $K\to n_{\rm s}+1$, equation (16) reduces to

$$\begin{aligned}W_{\kappa t\to 0}(\beta) &= \frac{2(-1)^{m}}{\pi(2n_{\rm s}+1)^{m+1}}\mathrm{e}^{-\frac{2|\beta|^{2}}{2n_{\rm s}+1}}\\ &\quad\times L_{m}\left(\frac{n_{\rm s}+1}{2n_{\rm s}+1}|\beta|^{2}\right),\end{aligned} \tag{17}$$

which is just the Wigner function of the input (i.e., initial) PATS as expected.

**Extreme case II** At long time $\kappa t\to\infty$, leading to $\eta\to 1$, $A\to 0$, $B\to n_{\rm e}+1/2$, $J\to -(n_{\rm e}+1/2)$, $K\to 0$, equation (16) reduces to

$$W_{\kappa t\to\infty}(\beta) = \frac{2}{\pi(2n_{\rm e}+1)}\mathrm{e}^{-\frac{2}{2n_{\rm e}+1}|\beta|^{2}}, \tag{18}$$

which is just the WF of the thermal state in the thermal reservoir. This indicates that the WF losses non-Gaussianity in nature and becomes Gaussian at long time. It is easy to check that $\rho_{\kappa t\to\infty} = \rho_{\rm e}(n_{\rm e})$.

**Special case I** When $m=0$, due to $L_{0}(x)=1$, equation (16) reduces to

$$W_{\rm out}(\beta) = \frac{1}{\pi B}\mathrm{e}^{-\frac{1}{B}|\beta|^{2}}, \tag{19}$$

which corresponds to the evolution of the thermal state in the thermal-loss channel. This always has the Gaussian form at any time.

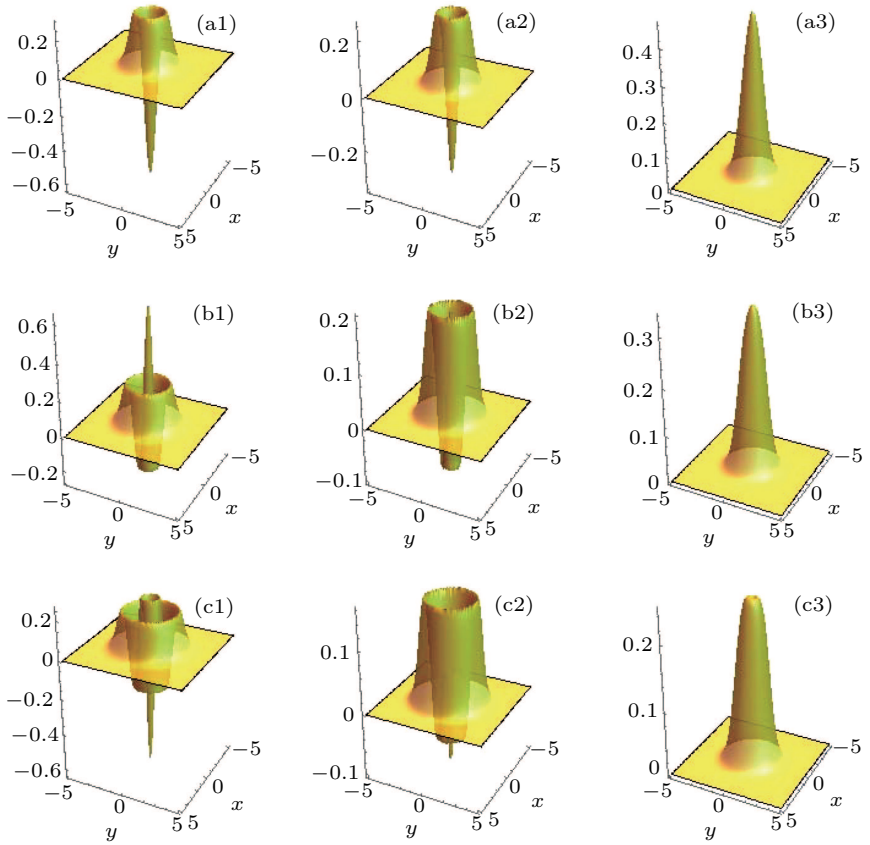



**Fig. 2.** Evolution of Wigner functions as a function of $\beta = x + \mathrm{i}y$ ($x, y \in [-5, 5]$) for PATS in thermal-loss channel with fixed $n_{\mathrm{e}} = 0$, $n_{\mathrm{s}} = 0$ and (a) $m = 1$, (b) $m = 2$, and (c) $m = 3$ at different time $\kappa t = 0$ (column 1), 0.25 (column 2), and 2 (column 3). Indeed, this corresponds to the evolution of Fock state in loss channel.

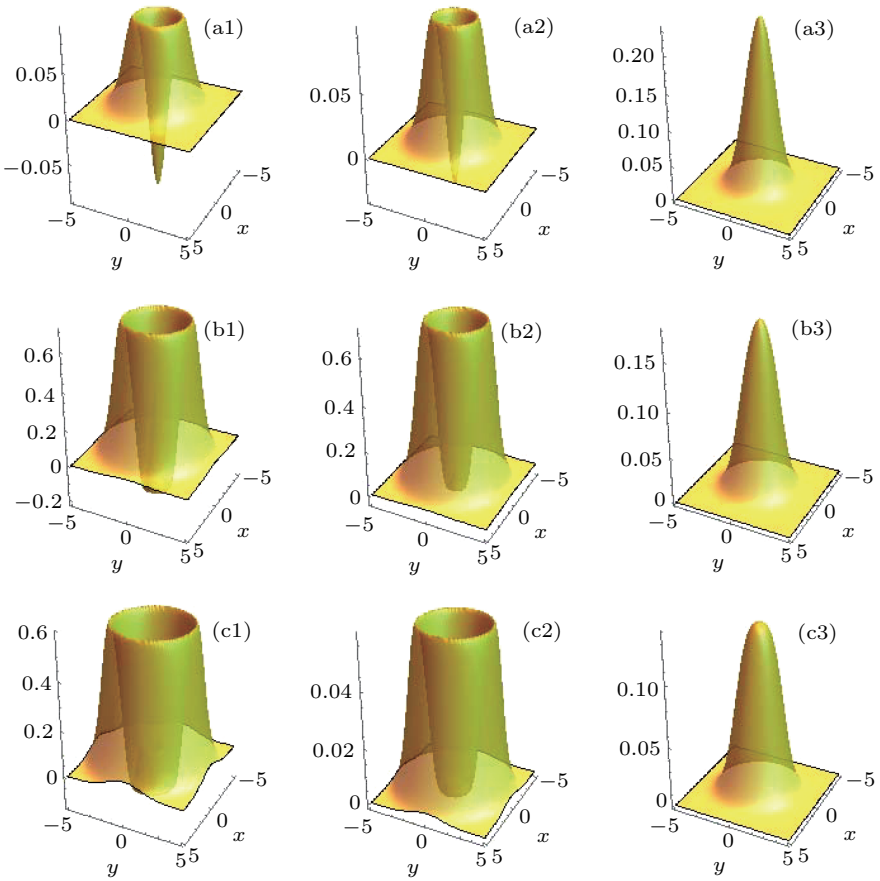



**Fig. 3.** Evolution of Wigner functions as a function of $\beta = x + \mathrm{i}y$ ($x, y \in [-5, 5]$) for PATS in thermal-loss channel with fixed $n_{\mathrm{e}} = 0.5$, $n_{\mathrm{s}} = 0.8$ and (a) $m = 1$, (b) $m = 2$, and (c) $m = 3$ at different time $\kappa t = 0$ (column 1), 0.25 (column 2), and 2 (column 3).

**Special case II** For $m=1$, due to $L_1(x)=1-x$, equation (16) reduces to

$$W_{\text{out}}(\beta)=\frac{A^2|\beta|^2-BJ(1+n_{\text{s}})}{\pi(n_{\text{s}}+1)B^3}\,\mathrm{e}^{-\frac{1}{B}|\beta|^2}, \tag{20}$$

which corresponds to the evolution of the single PATS in the thermal-loss channel. Obviously, if the condition $|\beta|^2 < BJ(1+n_{\text{s}})/A^2$ is satisfied, the Wigner negativity will appear.

According to Eq. (14), we plot some Wigner functions of the optical fields at some given time with given parameters. In Fig. 2, we fix $n_{\text{e}}=0$ and $n_{\text{s}}=0$ and plot the Wigner functions at three decay time $\kappa t$. In fact, this case corresponds to the dynamical evolution of the Fock state $|m\rangle$ in the loss channel. Similarly, by giving $n_{\text{e}}=0.5$ and $n_{\text{s}}=0.8$, the time evolution of the Wigner function with the PATS in the thermal reservoir is plotted for three different decay time $\kappa t$ in Fig. 3. At the moment $\kappa t=0$, the optical field is just the PATS, whose Wigner function will exhibit partial negativity around the origin. With time evolution, the negative region of the WF gradually diminishes and the WF tends to a Gaussian form at last. That is to say, when $\kappa t\to\infty$, the interaction will lead to that the PATS is decoherent as the thermal state of reservoir.

### 4.2. Wigner logarithmic negativity

The Wigner function (WF) is a quasi-probability distribution, which may have negative values. So negativity of the WF is an important quantum feature of quantum states. In particular, the volume of the negative part has been introduced as a nonclassicality quantifier.[30] Recently, Albarelli[31] and Takagi[32] have defined a computable resource monotone, i.e., Wigner logarithmic negativity (WLN)

$$\mathcal{N}_{\text{L}}(\rho)=\ln\int \mathrm{d}^2\beta\,|W(\beta)| \tag{21}$$

to characterize the non-Gaussianity of the continuous-variable optical fields.

As we know, the unity of the Wigner function can be verify by $\int \mathrm{d}^2\beta W(\beta)=1$. However, the integration value $\int \mathrm{d}^2\beta W(\beta)$ in phase space is just that the volume of the positive part $V_+$ minus the volume of the negative part $V_-$ (i.e., $V_+-V_-=1$). While the integration value $\int \mathrm{d}^2\beta\,|W(\beta)|$ in phase space is that the volume of the positive part $V_+$ plus the volume of the negative part $V_-$ . That is $V_++V_-=1+2V_-\geq 1$. Therefore, the integration value $\int \mathrm{d}^2\beta\,|W(\beta)|$ is always larger than or equal to the unity 1 (i.e., $\int \mathrm{d}^2\beta\,|W(\beta)|\geq 1$), which is dependent on the volume of the negative part $V_-$. So, if the WF $W(\beta)$ is always non-negative, the value $\int \mathrm{d}^2\beta\,|W(\beta)|$ is equal to $\int \mathrm{d}^2\beta W(\beta)$, i.e., 1, which also leads that the quantity $\mathcal{N}_{\text{L}}(\rho)$ is equal to zero.

Figure 4 shows the WLN $\mathcal{N}_{\text{L}}(\rho)$ versus the dimensionless time $\kappa t$ with fixing (a) $n_{\text{e}}=0$, $n_{\text{s}}=0$ and (b) $n_{\text{e}}=0.5$, $n_{\text{s}}=0.8$ but changing different $m$. The numerical results show that the thermal noise can deteriorate the partial negativity $V_-$ and the WLN $\mathcal{N}_{\text{L}}(\rho)$ monotonically decreases with the decay time.

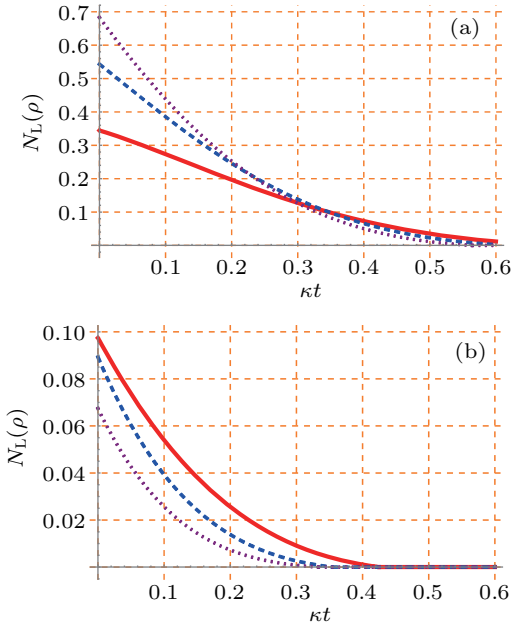



**Fig. 4.** Wigner logarithmic negativity versus evolution time $\kappa t$ with (a) $n_{\text{e}}=0$, $n_{\text{s}}=0$; (b) $n_{\text{e}}=0.5$, $n_{\text{s}}=0.8$, but different $m$. Here the red solid line, blue dashed line, and brown dot-dashed line are corresponding to $m=1$, $m=2$, and $m=3$, respectively.

## 5. Conclusion

To summarize, we have explored the propagation of the PATS in a thermal reservoir. Rather than solving the master equation, we abstract this model as an input–output problem in a thermal-loss channel. According to our previous formula, we obtain the input–output relation of the density operator in the CF formalism, which enables us to easily obtain the time evolution of the optical field at any time. We have derived the analytical expressions of the WF and have studied the dynamic behavior of the WLN. The results show that (i) the expression for time evolution of the WF is closely related to the Laguerre polynomials. (ii) As the PATS is influenced by the thermal reservoir, it loses the non-Gaussianity and shows the Gaussian character of the reservoir.

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