Theoretical framework for geoacoustic inversion by adjoint method*

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Traditional geoacoustic inversions are generally solved by matched-field processing in combination with meta-heuristic global searching algorithms which usually need massive computations. This paper proposes a new physical framework for geoacoustic retrievals. A parabolic approximation of wave equation with non-local boundary condition is used as the forward propagation model. The expressions of the corresponding tangent linear model and the adjoint operator are derived, respectively, by variational method. The analytical expressions for the gradient of the cost function with respect to the control variables can be formulated by the adjoint operator, which in turn can be used for optimization by the gradient-based method.

Keywords: geoacoustic inversion, adjoint method, parabolic equation, non-local boundary condition

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1. Introduction

Sound wave is the only information carrier that can travel long distances in the ocean. The exploration and development of marine resources, underwater target detection and identification, and environmental monitoring are all dependent on underwater acoustic detection technology.[1–3] Accurate acquisition of geoacoustic parameters is a prerequisite for the effective implementation of this technology. Due to the complexity of the marine environment, it is difficult for traditional *in situ* methods to obtain large-scale geoacoustic parameters in real time. With the advancement of sonar technology and inverse problem theory, the use of acoustic signals to invert the parameters of underwater acoustic environments has received more and more attention.[4–6]

Most previous geoacoustic studies have focused on using matched-field processing methods in combination with heuristic searching methods, such as genetic algorithm,[7] simulated annealing,[8] and sequential Monte Carlo sampling,[9] which always need huge computing resources because of too many forward propagation model runs. The adjoint method coming from optimal control theory is known to give accurate and efficient data assimilation processes in oceanography and meteorology.[10,11] However, it has rarely been applied in underwater acoustics for inversion purposes.[12]

For underwater acoustics modelling, a downgoing radiation condition must be imposed on the transmitted component of the field since the seabed is penetrable to sound waves, especially at low frequencies. Although many acoustic propagation tools are now available, the parabolic equation (PE) model has the advantages of dealing with the range-dependent environments efficiently and obtaining the full wave solution of the sound field. In the PE models, a downgoing radiation condition can be done by appending an absorbing layer[13] or alternatively by applying a non-local boundary condition (NLBC).[14] The latter is more attractive for limiting the computations in the interested region and time saving.

The remainder of this paper is organized as follows: The forward propagation model, *i.e.*, a non-local boundary condition for the finite-difference PE solver, is briefly introduced in Section 2. Section 3 presents the detailed theoretical derivations of the geoacoustic inversion by variational approach, including the tangent linear model, adjoint operator, and the gradients of the cost function with respect to the control variables. The iteration process is given in Section 4.

2. Forward propagation model

With the far-field assumption and paraxial approximation, the standard PE can be written as[13]

$$\frac{\partial^2 u(x,z)}{\partial z^2} + 2\mathrm{i}k_0 \frac{\partial u(x,z)}{\partial x} + k_0^2 \left[n^2(z) - 1\right] u(x,z) = 0, \quad (1)$$

where x is the range and z is the depth, $u(x,z)$ is a slowly varying function related to the complex acoustic pressure field $p(x,z)$ as $p(x,z) = \mathrm{H}_0^1(k_0 x) \cdot u(x,z)$, $\mathrm{H}_0^1(k_0 x)$ is the zeroth order Hankel function, $k_0 = 2\pi f / c_0$ is a reference wave number, f is the source frequency, and c_0 is a reference sound speed. Term $n(z) = c_0/c(z)$ is the refractive index, and $c(z)$ is the sound speed of the water column.

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To obtain a well-posed initial boundary value problem of Eq. (1), an initial field $u(0,z)=S(z_s,z)$ (S is an initial field function and z_s is the source depth) and a perfectly reflecting interface condition $u(x,0)=0$ at the sea surface are used. The NLBC formulation at the water–sediment interface $z=z_b$, proposed by Yevick and Thomson,[14] is used as the lower boundary. Dividing the propagation range $0\to x+\Delta x$ into $L+1$ intervals of width Δx, and denoting $u_j=u(x-j\Delta x,z_b)$ and $x=(L+1)\Delta x$, the NLBC can be expressed as

$$\left(\frac{\partial}{\partial z}-\mathrm{i}\beta\right)u(x,z_b)=\mathrm{i}\beta\sum_{j=1}^{L+1}g_j u_j, \tag{2}$$

$$\beta=\frac{\rho_w}{\rho_b}k_0\sqrt{N_b^2-1+\nu^2}, \tag{3}$$

$$\Gamma_0^2=k_0^2\left(N_b^2-1+\nu^2\frac{1-\Re}{1+\Re}\right), \tag{4}$$

where ρ_w and ρ_b indicate the density of water column and sediment, respectively. $N_b=(c_0/c_b)\cdot[1+\mathrm{i}\alpha_b]$, where c_b and α_b are the sound speed and the absorption loss of the sediment, respectively. $\nu^2=4\mathrm{i}/(k_0\Delta x)$ and g_j are the coefficients derived from the Taylor expansion of the vertical wave number operator, Γ_0 in powers of the translation operator $\Re=\exp(-\Delta x\partial x)$. Thus, the standard PE model with initial and boundary conditions can be summarized as

$$\begin{cases}\dfrac{\partial^2u(x,z)}{\partial z^2}+2\mathrm{i}k_0\dfrac{\partial u(x,z)}{\partial x}+k_0^2\left(n^2-1\right)u(x,z)=0,\\ u(0,z)=S(z_s,z),\\ u(x,0)=0,\\ \left(\dfrac{\partial}{\partial z}-\mathrm{i}\beta\right)u(x,z_b)=\mathrm{i}\beta\sum\limits_{j=1}^{L+1}g_j u_j.\end{cases} \tag{5}$$

Equation (5) can be solved by the implicit Crank–Nicolson finite difference method. Once the field $u(x,z)$ is obtained, the transmission loss $T(x,z)$ in unit dB is then given by

$$T(x,z)=-20\log_{10}|u(x,z)|+10\log_{10}(x). \tag{6}$$

3. Framework of the inverse problem

3.1. Cost function

Note the computation domain be $[0,R]\times[0,z_b]$, and a vertical line array locating at a given range R receive the field measurements $T^{\mathrm{obs}}(R,z)$. Then, the control procedure consists of finding the optimal control variables which minimize a cost function $J[\boldsymbol{X}]$ as

$$J[\boldsymbol{X}]=\int_0^{z_b}\left[T(R,z,\boldsymbol{X})-T^{\mathrm{obs}}(R,z)\right]^2\mathrm{d}z, \tag{7}$$

where $\boldsymbol{X}=[\boldsymbol{c},c_b,\rho_b,\alpha_b]$ is the vector of control parameters, $\boldsymbol{c}$ is a vector describing the sound speed of the water column, c_b, ρ_b, and α_b are the parameters describing the sediment layer property. Based on these parameters, $T(R,z,\boldsymbol{X})$ can be computed by the forward propagation model given in Section 2.

3.2. Tangent linear model

Set perturbations to $c(z)$, c_b, ρ_b, and α_b as $\tilde{c}(z)=c(z)+t\hat{c}(z)$, $\tilde{c}_b=c_b+t\hat{c}_b$, $\tilde{\rho}_b=\rho_b+t\hat{\rho}_b$, and $\tilde{\alpha}_b=\alpha_b+t\hat{\alpha}_b$, then the corresponding perturbations of β and g_j can be described as $\tilde{\beta}=\beta+t\hat{\beta}$ and $\tilde{g}_j=g_j+t\hat{g}_j$, respectively. Note $\hat{u}(x,z)$ is the Gâteaux differential of $u(x,z)$

$$\hat{u}(x,z)=\lim_{t\to0}\frac{\tilde{u}(x,z)-u(x,z)}{t}. \tag{8}$$

Then $\hat{u}(x,z)$ satisfies the following tangent linear model (TLM)

$$\begin{cases}\dfrac{\partial^2\hat{u}(x,z)}{\partial z^2}+2\mathrm{i}k_0\dfrac{\partial\hat{u}(x,z)}{\partial x}+k_0^2\left(n^2-1\right)\hat{u}(x,z)\\ \quad+2k_0^2u(x,z)n\dfrac{\partial n}{\partial c}\hat{c}=0,\\ \hat{u}(0,z)=0,\\ \hat{u}(x,0)=0,\\ \left(\dfrac{\partial}{\partial z}-\mathrm{i}\beta\right)\hat{u}(x,z_b)-\mathrm{i}\hat{\beta}u(x,z_b)=\mathrm{i}\hat{\beta}\sum\limits_{j=1}^{L+1}g_ju_j+\mathrm{i}\beta\sum\limits_{j=1}^{L+1}\hat{g}_ju_j,\end{cases} \tag{9}$$

where

$$\hat{\beta}=\frac{\partial\beta}{\partial\rho_b}\hat{\rho}_b+\frac{\partial\beta}{\partial N_b}\hat{N}_b,\quad \hat{N}_b=\frac{\partial N_b}{\partial c_b}\hat{c}_b+\frac{\partial N_b}{\partial\alpha_b}\hat{\alpha}_b,\quad \hat{g}_j=\frac{\partial g_j}{\partial N_b}\hat{N}_b.$$

3.3. Adjoint model

Define inner product as $\langle f,h\rangle=\int_0^{z_b}\int_0^R f(x,z)\cdot\bar{h}(x,z)\mathrm{d}x\mathrm{d}z$. Multiply the conjugate of tangent linear equation by $w(x,z)$ (*i.e.*, the adjoint operator) and integrate it on the domain $[0,R]\times[0,z_b]$,

$$\begin{aligned}&\left\langle w,\frac{\partial^2\hat{u}}{\partial z^2}\right\rangle+\left\langle w,2\mathrm{i}k_0\frac{\partial\hat{u}}{\partial x}\right\rangle+\left\langle w,k_0^2\left(n^2-1\right)\hat{u}\right\rangle\\&+\left\langle w,2k_0^2n\frac{\partial n}{\partial c}u\hat{c}\right\rangle=0,\end{aligned} \tag{10}$$

in which

$$\begin{aligned}\left\langle w,\frac{\partial^2\hat{u}}{\partial z^2}\right\rangle=&\left\langle\frac{\partial^2w}{\partial z^2},\hat{u}\right\rangle\\&+\int_0^R\left[w(x,z_b)\frac{\partial\bar{\hat{u}}(x,z_b)}{\partial z}-\frac{\partial w(x,z_b)}{\partial z}\bar{\hat{u}}(x,z_b)\right]\mathrm{d}x\\&-\int_0^R w(x,0)\frac{\partial\bar{\hat{u}}(x,0)}{\partial z}\mathrm{d}x,\end{aligned} \tag{11}$$

$$\left\langle w,2\mathrm{i}k_0\frac{\partial\hat{u}}{\partial x}\right\rangle=\left\langle 2\mathrm{i}k_0\frac{\partial w}{\partial x},\hat{u}\right\rangle-2\mathrm{i}k_0\int_0^{z_b}w(R,z)\bar{\hat{u}}(R,z)\mathrm{d}z, \tag{12}$$

$$\left\langle w,k_0^2\left[n^2-1\right]\hat{u}\right\rangle=\left\langle k_0^2\left(n^2-1\right)w,\hat{u}\right\rangle, \tag{13}$$

$$\left\langle w,2k_0^2n\frac{\partial n}{\partial c}u\hat{c}\right\rangle=\left\langle 2k_0^2n\frac{\partial n}{\partial c}w\bar{u},\hat{c}\right\rangle. \tag{14}$$

Combining the above four equations and considering the lower boundary layer of the tangent linear model,

$$\left\langle\frac{\partial^2w}{\partial z^2}+2\mathrm{i}k_0\frac{\partial w}{\partial x}+k_0^2\left(n^2-1\right)w,\hat{u}\right\rangle$$

$$
\begin{aligned}
&-2\mathrm{i}k_0\int_0^{z_\mathrm{b}} w(R,z)\bar{\tilde{u}}(R,z)\mathrm{d}z+\left\langle 2k_0^2 n\frac{\partial n}{\partial c}w\bar{u},\hat{c}\right\rangle\\
&-\int_0^R\left[\mathrm{i}\bar{\beta}w(x,z_\mathrm{b})+\frac{\partial w(x,z_\mathrm{b})}{\partial z}\right]\bar{\tilde{u}}(x,z_\mathrm{b})\mathrm{d}x\\
&-\int_0^R w(x,0)\frac{\partial\bar{\tilde{u}}(x,0)}{\partial z}\mathrm{d}x\\
&-\int_0^R f_1 w(x,z_\mathrm{b})\mathrm{d}x\cdot\hat{c}_\mathrm{b}-\int_0^R f_2 w(x,z_\mathrm{b})\mathrm{d}x\cdot\hat{\rho}_\mathrm{b}\\
&-\int_0^R f_3 w(x,z_\mathrm{b})\mathrm{d}x\cdot\hat{\alpha}_\mathrm{b}=0,
\end{aligned}\tag{15}
$$

where

$$
\begin{aligned}
f_1&=\mathrm{i}\left[\overline{\frac{\partial\beta}{\partial N_\mathrm{b}}}\left(\bar{u}(x,z_\mathrm{b})+\sum_{j=1}^{L+1}\bar{g}_j\bar{u}_j\right)+\bar{\beta}\sum_{j=1}^{L+1}\overline{\frac{\partial g_j}{\partial N_\mathrm{b}}}\bar{u}_j\right]\overline{\frac{\partial N_\mathrm{b}}{\partial c_\mathrm{b}}},\\
f_2&=\mathrm{i}\overline{\frac{\partial\beta}{\partial\rho_\mathrm{b}}}\left[\bar{u}(x,z_\mathrm{b})+\sum_{j=1}^{L+1}\bar{g}_j\bar{u}_j\right],\\
f_3&=\mathrm{i}\left[\overline{\frac{\partial\beta}{\partial N_\mathrm{b}}}\left(\bar{u}(x,z_\mathrm{b})+\sum_{j=1}^{L+1}\bar{g}_j\bar{u}_j\right)+\bar{\beta}\sum_{j=1}^{L+1}\overline{\frac{\partial g_j}{\partial N_\mathrm{b}}}\bar{u}_j\right]\overline{\frac{\partial N_\mathrm{b}}{\partial\alpha_\mathrm{b}}}.
\end{aligned}
$$

Now return to the cost function, the Gâteaux differential of $J[\boldsymbol{X}]$ with respect to $\hat{X}$ at point $\boldsymbol{X}$ is

$$
\begin{aligned}
&J'[\boldsymbol{X};\hat{X}]\\
&=\lim_{t\to0}\frac{J[\tilde{X}]-J[\boldsymbol{X}]}{t}\\
&=2\int_0^{z_\mathrm{b}}\left[T(R,z,\boldsymbol{X})-T^{\mathrm{obs}}(R,z)\right]\hat{T}(R,z,\boldsymbol{X})\mathrm{d}z\\
&=\Re\left\{-\frac{40}{\ln10}\int_0^{z_\mathrm{b}}\left[\frac{T(R,z,\boldsymbol{X})-T^{\mathrm{obs}}(R,z)}{\bar{u}(R,z)}\right]\bar{\tilde{u}}(R,z)\mathrm{d}z\right\}.
\end{aligned}\tag{16}
$$

On the other hand, according to the definition we can get

$$
\begin{aligned}
J'[\boldsymbol{X};\hat{X}]&=\nabla_{\boldsymbol{X}}J\cdot\hat{X}\\
&=\int_0^{z_\mathrm{b}}\nabla_c J\cdot\hat{c}\mathrm{d}z+\nabla_{c_\mathrm{b}}J\cdot\hat{c}_\mathrm{b}\\
&\quad+\nabla_{\rho_\mathrm{b}}J\cdot\hat{\rho}_\mathrm{b}+\nabla_{\alpha_\mathrm{b}}J\cdot\hat{\alpha}_\mathrm{b}.
\end{aligned}\tag{17}
$$

According to Eq. (16) and Eq. (17),

$$
\begin{aligned}
&\int_0^{z_\mathrm{b}}\nabla_c J\cdot\hat{c}\mathrm{d}z+\nabla_{c_\mathrm{b}}J\cdot\hat{c}_\mathrm{b}+\nabla_{\rho_\mathrm{b}}J\cdot\hat{\rho}_\mathrm{b}+\nabla_{\alpha_\mathrm{b}}J\cdot\hat{\alpha}_\mathrm{b}\\
&+\Re\left\{\int_0^{z_\mathrm{b}}B(R,z)\bar{\tilde{u}}(R,z)\mathrm{d}z\right\}=0,
\end{aligned}\tag{18}
$$

where

$$
B(R,z)=\frac{40}{\ln10}\frac{T(R,z)-T^{\mathrm{err}}(R,z)}{\bar{u}(R,z)}.
$$

Combining Eq. (15) with Eq. (18),

$$
\begin{aligned}
&\Re\left\{\left\langle\frac{\partial^2 w}{\partial z^2}+2\mathrm{i}k_0\frac{\partial w}{\partial x}+k_0^2\left(n^2-1\right)w,\hat{u}\right\rangle\right\}\\
&+\Re\left\{\int_0^{z_\mathrm{b}}\left[B(R,z)-2\mathrm{i}k_0 w(R,z)\right]\bar{\tilde{u}}(R,z)\mathrm{d}z\right\}\\
&-\Re\left\{\int_0^R\left[\mathrm{i}\bar{\beta}w(x,z_\mathrm{b})+\frac{\partial w(x,z_\mathrm{b})}{\partial z}\right]\bar{\tilde{u}}(x,z_\mathrm{b})\mathrm{d}x\right\}\\
&-\Re\left\{\int_0^R w(x,0)\frac{\partial\bar{\tilde{u}}(x,0)}{\partial z}\mathrm{d}x\right\}\\
&+\int_0^{z_\mathrm{b}}\nabla_c J\cdot\hat{c}\mathrm{d}z+\Re\left\{\left\langle 2k_0^2 n\frac{\partial n}{\partial c}w\bar{u},\hat{c}\right\rangle\right\}+\nabla_{c_\mathrm{b}}J\cdot\hat{c}_\mathrm{b}\\
&-\Re\left\{\int_0^R f_1 w(x,z_\mathrm{b})\mathrm{d}x\right\}\cdot\hat{c}_\mathrm{b}\\
&+\nabla_{\rho_\mathrm{b}}J\cdot\hat{\rho}_\mathrm{b}-\Re\left\{\int_0^R f_2 w(x,z_\mathrm{b})\mathrm{d}x\right\}\cdot\hat{\rho}_\mathrm{b}+\nabla_{\alpha_\mathrm{b}}J\cdot\hat{\alpha}_\mathrm{b}\\
&-\Re\left\{\int_0^R f_3 w(x,z_\mathrm{b})\mathrm{d}x\right\}\cdot\hat{\alpha}_\mathrm{b}=0.
\end{aligned}\tag{19}
$$

Owing to $\hat{u}$, $\hat{c}$, $\hat{c}_\mathrm{b}$, $\hat{\rho}_\mathrm{b}$, and $\hat{\alpha}_\mathrm{b}$ are arbitrary and equation (19) equals zero, the adjoint operator satisfies the following initial boundary condition model as

$$
\begin{cases}
\dfrac{\partial^2 w(x,z)}{\partial z^2}+2\mathrm{i}k_0\dfrac{\partial w(x,z)}{\partial x}+k_0^2\left(n^2-1\right)w(x,z)=0,\\
w(R,z)=\dfrac{B(R,z)}{2\mathrm{i}k_0},\\
w(x,0)=0,\\
\dfrac{\partial w(x,z_\mathrm{b})}{\partial z}+\mathrm{i}\bar{\beta}w(x,z_\mathrm{b})=0,
\end{cases}\tag{20}
$$

and the corresponding gradients of the cost function respective to the control variables are

$$
\nabla_c J=-2k_0^2 n\frac{\partial n}{\partial c}\int_0^R\Re\left\{w(x,z)\bar{u}(x,z)\right\}\mathrm{d}x,\tag{21}
$$

$$
\nabla_{c_\mathrm{b}}J=\int_0^R\Re\left\{f_1\cdot w(x,z_\mathrm{b})\right\}\mathrm{d}x,\tag{22}
$$

$$
\nabla_{\rho_\mathrm{b}}J=\int_0^R\Re\left\{f_2\cdot w(x,z_\mathrm{b})\right\}\mathrm{d}x,\tag{23}
$$

$$
\nabla_{\alpha_\mathrm{b}}J=\int_0^R\Re\left\{f_3\cdot w(x,z_\mathrm{b})\right\}\mathrm{d}x.\tag{24}
$$

4. Iteration process

With these gradients, optimization could be generally accomplished through using the iterative gradient-based methods,

$$
s^{j+1}=s^j-\nabla_s J|_{s^j}\cdot a_\mathrm{s}^j,\tag{25}
$$

where a_s^j is the iteration step for s^j, and s can be any of the variables c, c_b, ρ_b, and α_b.

The flowchart of the whole iteration process is shown in Fig. 1. Starting with a first guess of the control vector $\boldsymbol{X}$, the forward model runs to give the corresponding complex field $u(x,z)$ and the replica transmission loss $T(R,z)$. Then, the resulting mismatches between model results and observations are used to initiate the adjoint model which can also be solved by the implicit Crank–Nicolson finite difference method. With the adjoint information thus obtained, the gradients of the cost function are determined and the current control vector $\boldsymbol{X}$ is updated accordingly to start the next iteration until the criteria for the solutions are met.

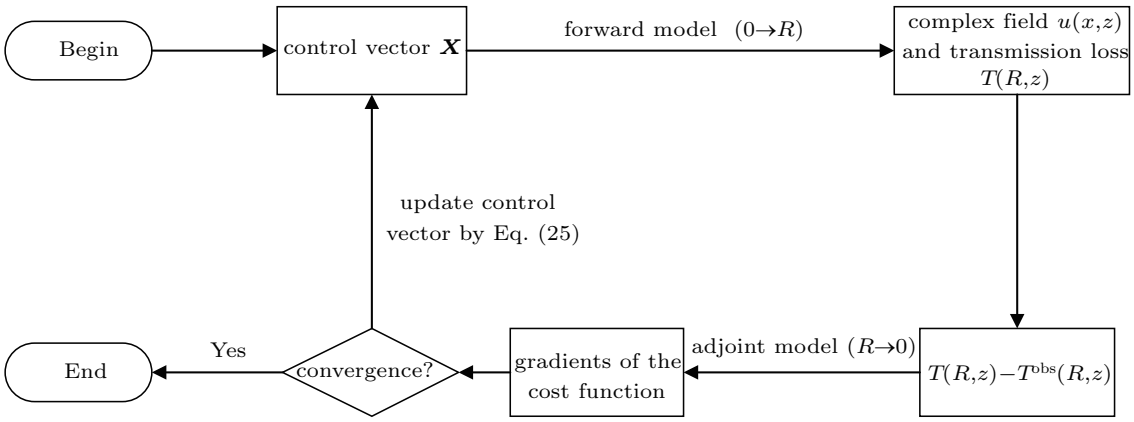


Fig. 1. The flowchart of the iteration process for geoacoustic inversion by adjoint NLBCPE method, where "$0 \rightarrow R$" means integrating the forward model along the range 0 to R, and "$R \rightarrow 0$" means integrating the adjoint model in the reverse direction.

5. Discussion and conclusion

The analytical theory framework for geoacoustic inversion by the adjoint PE method is introduced above. It should be noted that parameter estimation problems have demonstrated to be ill-posed, especially when the problems are severely nonlinear and the observed data are contaminated by noise.[15] These problems usually have multiple local optimal solutions. To deal with the ill-posedness of the inversion, an efficient method is to carry out regularization that can make the cost function be convex. Detailed regularization techniques for inverse problem can refer to Ref. [16].

On the other hand, the theoretical derivations are based on small perturbation approach. Therefore, the adjoint process is a local optimization method and the inversion accuracy is dependent on the initial guess,[17] especially for severely nonlinear inverse problems. If the initial guess is deviated far from the true solution, the iterations will be always trapped to a local optimal solution. In practical operations, a good initial guess can be obtained from historical observations and/or empirical values.

Geoacoustic parameters are very important for underwater acoustic detection. How to acquire these parameters accurately from the perspective of adjoint PE method needs systematical research. This paper only gives the basic theoretical framework, and future work will be focused on discussing the uniqueness and stability of the solution, as well as collecting the experimental data to validate this theoretical method.

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