ISSN 1674-1056
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Citation: Chin. Phys. B . 2019, 28(10): 100303 . **doi:** 10.1088/1674-1056/ab3e64
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Manipulating transition of a two-component Bose–Einstein condensate with a weak δ-shaped laser*

Bo Li(李博)[1,2], Xiao-Jun Jiang(蒋小军)[1], Xiao-Lin Li(李晓林)[1,†], Wen-Hua Hai(海文华)[3], and Yu-Zhu Wang(王育竹)[1]

[1] *Key Laboratory for Quantum Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai* 201800, *China*
[2] *University of Chinese Academy of Sciences, Beijing* 100049, *China*
[3] *Department of Physics and Key Laboratory of Low-dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha* 410081, *China*

(Received 19 July 2019; revised manuscript received 15 August 2019; published online 30 September 2019)

We theoretically study the transition dynamics of a two-component Bose–Einstein condensate driven by a train of weak δ-shaped laser pulses. We find that the atomic system can experience peculiar resonant transition even under weak optical excitations and derive the resonance condition by the perturbation method. Employing this mechanism, we propose a scheme to obtain an atomic ensemble with desired odd/even atom number and also a scheme to prepare a nonclassical state of the many-body system with fixed atom number.

Keywords: two-component Bose–Einstein condensate, quantum kicked top model, nonclassical state

PACS: 03.75.Lm, 03.75.Mn, 03.65.Aa **DOI:** 10.1088/1674-1056/ab3e64

1. Introduction

The two-mode quantum system is a basic model to study the transition dynamics in quantum physics. With the experimental realization of the double-well Bose–Einstein condensate (BEC)[1] and the two-states BEC,[2,3] these ultracold atoms are an ideal platform for studying and manipulating transition dynamics and entanglement of atoms on a macroscopic scale.[4] Different from the single-atom system, the transition dynamics of the many-body BEC system are remarkably affected by collision interactions.[5] This nonlinear interaction has been found to play an important role in nonlinear Josephson oscillation,[6–8] nonlinear Landau–Zener tunneling,[9–12] nonlinear Rosen–Zener tunneling,[13,14] and coherent destruction of tunneling (CDT),[15,16] and affects the generation of many-body entangled states.[17–19]

In recent years, periodic modulation technique has been frequently used for controlling the transition dynamics of the BEC systems. For example, researchers proposed two ways to precisely control the number of bosons allowed to tunnel in a double-well BEC system by periodically modulating the collision interactions[15] and the external force,[16] respectively. The δ-shaped periodic kick, which is a specific periodic modulation, exhibits a series of dynamical phenomena in chaos research, such as quantum chaos owing to the existence of the two-body interaction,[20,21] the emerging field of the excited-state quantum phase transition owing to dynamic instability,[22] and the quantum resonance.[23–25] Recently, the δ-shaped kick was realized with a pair of frequency combs and was used to manipulate the transition dynamics of a single atomic qubit[26] and the entanglement[27,28] of two-atomic qubits. But it remains a difficult task to precisely control the transition (tunneling) in a many-body system.

In this work, we investigate the transition dynamics of the BEC with two internal states driven by weak and periodic δ-shaped laser pulses. The same as the treating in the chaos research, we deal with this system in a quantum kicked top model. Adopting the perturbation method, we find that the transition probability is small in the general case due to the weak pulse intensity, but in some special cases the weak laser pulses could cause resonance and lead to transition. Using this special resonance phenomenon, we propose a scheme to control the atom numbers to be odd or even which is independent of the atom number. We also propose a scheme to obtain a desired number of atoms by tuning the parameters to be atom number dependent.

2. Theoretical model and solution

We consider a two-mode BEC system that consists of N ^{87}Rb atoms with two hyperfine states $|1\rangle \equiv |F=1, m_f=-1\rangle$ and $|2\rangle \equiv |F=2, m_f=1\rangle$.[3] The atoms are trapped in a single well and the two levels are coupled through the laser-induced Raman transition. In the form of second quantization, the Hamiltonian of the system can be given as follows:[29,30]

$$\begin{aligned} H &= H_1 + H_2 + H_{\rm int} + H_{\rm las}, \\ H_i &= \int {\rm d}\boldsymbol{r} \hat{\Psi}_i^{\dagger}(\boldsymbol{r}) \Big[-\frac{1}{2}\nabla^2 + \frac{1}{2}\lambda_i^2 r^2 \end{aligned}$$

*Project supported by the National Key Research and Development Program of China (Grant No. 2016YFA0301504) and the National Natural Science Foundation of China (Grant Nos. 61835013 and 11774362).

†Corresponding author. E-mail: xiaolin_li@siom.ac.cn

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$$+\frac{U_{ii}}{2}\hat{\Psi}_i^\dagger(\boldsymbol{r})\hat{\Psi}_i(\boldsymbol{r})\Big]\hat{\Psi}_i(\boldsymbol{r}),\quad (i=1,2),$$
$$H_{\rm int} = U_{1,2}\int \mathrm{d}\boldsymbol{r}\hat{\Psi}_1^\dagger(\boldsymbol{r})\hat{\Psi}_2^\dagger(\boldsymbol{r})\hat{\Psi}_2(\boldsymbol{r})\hat{\Psi}_1(\boldsymbol{r}),$$
$$H_{\rm las} = -\Omega_0\int \mathrm{d}\boldsymbol{x}[\hat{\Psi}_2^\dagger(\boldsymbol{r})\hat{\Psi}_1(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\Delta t} + \hat{\Psi}_1^\dagger(\boldsymbol{r})\hat{\Psi}_2(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\Delta t}]. \quad (1)$$

The above Hamiltonian has been rescaled by an appropriate $\hbar\omega_0$ and all the variables are dimensionless.[29] $\hat{H}_i$ describes the Hamiltonian of the atoms in states $|1\rangle$ and $|2\rangle$ in the absence of the interactions ($\hat{H}_{\rm int}$) between atoms in different internal states. $\hat{H}_{\rm las}$ represents the laser-induced coupling with a Rabi frequency Ω_0 and a laser detuning Δ. The Rb atoms are confined in a harmonic trap at frequencies λ_i. $U_{11,22,12} = 4\pi a^{\rm sc}_{11,22,12}/x_0$ describe the interaction strengths of the intra-component and inter-component two-body collisions, with the corresponding s-wave scattering lengths $a^{\rm sc}_{11,22,12}$ and the length of the harmonic oscillator $x_0 = \sqrt{\hbar/M\omega_0}$. The field operators $\hat{\Psi}_i(\boldsymbol{r})$ and $\hat{\Psi}_i^\dagger(\boldsymbol{r})$ annihilate and create atoms at $\boldsymbol{r}$ in states $|1\rangle$, $|2\rangle$ and satisfy the commutation relation $[\hat{\Psi}_i(\boldsymbol{r}),\hat{\Psi}_i^\dagger(\boldsymbol{r}')] = \delta_{i,j}\delta(\boldsymbol{r}-\boldsymbol{r}')$. For simplicity, we assume the scattering lengths satisfy $a^{\rm sc}_{11} = a^{\rm sc}_{22} = a^{\rm sc}_{12} = a_0$.

To focus on the population change in the two internal levels and neglect the external motion of the atoms, we take a two-mode approximation $\hat{\Psi}_1(\boldsymbol{r}) = a_1\phi_1(\boldsymbol{r})$, $\hat{\Psi}_2(\boldsymbol{r}) = a_2\phi_2(\boldsymbol{r})$, where a_1 and a_2 are the annihilation operators of states $|1\rangle$ and $|2\rangle$, respectively, and obey the commutation relations $[a_i,a_j] = 0$, $[a_i,\ a_j^\dagger] = \delta_{i,j}$; and $\phi_i(\boldsymbol{r})$ expresses the spatial and normalized mode functions. The two-mode approximation performances well when the scale of the condensate is small, otherwise the atoms will not stay in the ground state of the harmonic oscillator because of the collisional interactions and the two-mode approximation breaks down. As shown in Ref. [31], the number of the atoms should satisfy the condition $N \ll r_0/|a_0|$, where r_0 is the position uncertainty in a harmonic oscillator ground state. After dropping the c-number terms as the total number of the atoms, $N = a_1a_1^\dagger + a_2a_2^\dagger$ is conserved, the two-mode Hamiltonian becomes[30]

$$\hat{H} = \frac{g}{4}(a_1a_1^\dagger - a_2a_2^\dagger)^2 + \gamma(a_1a_1^\dagger - a_2a_2^\dagger) + \frac{\Omega(t)}{2}(a_1a_2^\dagger + a_2a_1^\dagger). \quad (2)$$

Here the effective nonlinearity $g = u_{11} + u_{22} - 2u_{12}$, the effective detuning $\gamma = \bar{\omega}_1 + \Delta - \bar{\omega}_2 + (N-1)(u_{11}-u_{22})$, and the coupling strength $\Omega(t) = \Omega_0\int \mathrm{d}\boldsymbol{r}\phi_1^*(\boldsymbol{r})\phi_2(\boldsymbol{r})$, with $u_{ij} = U_{ij}\int \mathrm{d}\boldsymbol{r}|\phi_i(\boldsymbol{r})|^2|\phi_j(\boldsymbol{r})|^2$ and $\bar{\omega}_i = \int \mathrm{d}\boldsymbol{r}\phi^*(\boldsymbol{r})(-\frac{1}{2}\nabla^2 + \frac{1}{2}\lambda_i^2r^2)\phi_i(\boldsymbol{r})$. If we switch the coupling laser on and off rapidly and periodically, $\Omega(t)$ will take the form of δ-function pulse[32]

$$\Omega(t) = \nu\sum_{n=0}^{\infty}\delta(t-nT),\quad (\nu = \sqrt{\pi}\sigma\Omega_0). \quad (3)$$

Here the evolution time t and the pulse separation T are the dimensionless variables in units of $1/\omega_0$ and the pulse width $\sigma \to 0$, while $\sigma\Omega_0$ remains finite. Moreover, we reduce the Hamiltonian to a simpler form by employing an angular momentum representation

$$\hat{J}_x = \frac{1}{2}(a_1^\dagger a_2 + a_2^\dagger a_1),$$
$$\hat{J}_y = \frac{i}{2}(a_2^\dagger a_1 - a_1^\dagger a_2),$$
$$\hat{J}_z = \frac{1}{2}(a_1^\dagger a_1 - a_2^\dagger a_2), \quad (4)$$

which obeys the usual angular momentum commutation relations and fulfills $\hat{J}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2 = N/2(N/2+1)$. Finally, we obtain a Hamiltonian in the form of a nonlinear quantum top model

$$\hat{H} = g\hat{J}_z^2 + \gamma\hat{J}_z + \nu\sum_{n=1}^{\infty}\delta(t-nT)\hat{J}_x. \quad (5)$$

The previous works toward this quantum kicked model were mostly focused on quantum entanglement, chaos,[33,34] and dynamical instability.[20,22] Generally, the iterated mapping method is preferred for solving this model.[35–38] However, this iterated mapping method, which has frequently been used for the chaos problem, loses quantum characteristics, such as the eigenfunction and the eigenenergy,[35] which are very important for controlling the atoms precisely. Differently, we attempt to study the transition dynamics of this model using a perturbation method as shown in Ref. [39].

Assuming a weak laser coupling, we study the transition probability of the atoms among different eigenstates $|l,m\rangle$ ($l = N/2,\ m = -l,-l+1,\ldots,l$) of the angular momentum component J_z through a time-dependent perturbation method. According to the perturbation theory, we divide the Hamiltonian into two parts $\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$, where the time-independent part is $\hat{H}_0 = g\hat{J}_z^2 + \gamma\hat{J}_z$ and the time-dependent perturbation is $\hat{H}_1(t) = \nu\sum_{n=1}^{\infty}\delta(t-nT)\hat{J}_x$. Next, we carry out perturbation expansion of the state vector as

$$|\psi_k(t)\rangle = |\psi_k^{(0)}(t)\rangle + |\psi_k^{(1)}(t)\rangle = \sum_{m'=-N/2}^{N/2}\Big[C^{(0)}_{m',k}(t) + C^{(1)}_{m',k}(t)\Big]|l,m'\rangle. \quad (6)$$

Here the letter k labels the initially occupied state $|l,k\rangle$, and superscripts (0) and (1) indicate the perturbation orders. Clearly, $|\psi_k^{(0)}(t)\rangle = \sum_{m'}C^{(0)}_{m',k}(t)|l,m'\rangle$ satisfies the time-dependent Schrödinger equation $\mathrm{i}\partial|\psi_k^{(0)}(t)\rangle/\partial t = H_0|\psi_k^{(0)}(t)\rangle$. It is easy to obtain the wave function of the zeroth order as

$$|\psi_k^{(0)}(t)\rangle = \sum_{m'}C^{(0)}_{m',k}(0)\,\mathrm{e}^{-\mathrm{i}(\gamma m'+gm'^2)t}|l,m'\rangle = \mathrm{e}^{-\mathrm{i}(\gamma k+gk^2)t}|l,k\rangle. \quad (7)$$

Such a state is an eigenstate of H_0 with eigenenergy $E_k = \gamma k + gk^2$. Next, we solve the first-order equation $\mathrm{i}\partial|\psi_k^{(1)}(t)\rangle/\partial t = H_0|\psi_k^{(1)}(t)\rangle + H_1|\psi_k^{(1)}(t)\rangle$, which is equivalent to

$$\begin{aligned}&\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\sum_{m'}C^{(1)}_{m',k}(t)|l,m'\rangle\\ &= H_0\sum_{m'}C^{(1)}_{m',k}(t)|l,m'\rangle + H_1\,\mathrm{e}^{-\mathrm{i}(\gamma k+gk^2)t}|l,k\rangle. \end{aligned}\tag{8}$$

Multiplying $\langle l,m|$ on the left side of Eq. (8), and noticing $\langle m,l|l,m'\rangle = \delta_{mm'}$ and $J_x|l,k\rangle = 1/2[\sqrt{(l+k+1)(l-k)}\times|l,k+1\rangle + \sqrt{(l-k+1)(l+k)}|l,k-1\rangle]$, we arrive at

$$\begin{aligned}\mathrm{i}\dot{C}^{(1)}_{m,k}(t) &= (\gamma m+gm^2)C^{(1)}_{m,k} + \frac{\nu}{2}\Big[\sqrt{(l+k+1)(l-k)}\delta_{m,k+1}\\ &\quad+\sqrt{(l-k+1)(l+k)}\delta_{m,k-1}\Big]\\ &\quad\times\sum_{n=1}^{\infty}\delta(t-nT)\,\mathrm{e}^{-\mathrm{i}(\gamma k+gk^2)t}. \end{aligned}\tag{9}$$

Under the initial condition $C^{(1)}_{m,k}(0) = 0$, its solution is

$$\begin{aligned}C^{(1)}_{m,k}(t) &= -\mathrm{i}\frac{\nu}{2}\mathrm{e}^{-\mathrm{i}(\gamma m+gm^2)t}\Big[\sqrt{(l+k+1)(l-k)}\delta_{m,k+1}\\ &\quad+\sqrt{(l-k+1)(l+k)}\delta_{m,k-1}\Big]\\ &\quad\times\int\mathrm{e}^{\mathrm{i}[\gamma(m-k)+g(m^2-k^2)]t}\sum_{n=1}^{\infty}\delta(t-nT)\,\mathrm{d}t\\ &= -\mathrm{i}\frac{\nu}{2}\mathrm{e}^{-\mathrm{i}(\gamma m+gm^2)t}\Big[\sqrt{(l+k+1)(l-k)}\delta_{m,k+1}\\ &\quad+\sqrt{(l-k+1)(l+k)}\delta_{m,k-1}\Big]\\ &\quad\times\sum_{n=1}^{[t]}\mathrm{e}^{\mathrm{i}[\gamma(m-k)+g(m^2-k^2)]nT}. \end{aligned}\tag{10}$$

Here $[t]$ denotes an integer obeying $[t]\geq 1$ and $t-1<[t]<t$, and the summation vanishes for $t<1$. Given Eq. (10), we obtain the analytical solutions of the transition probability amplitudes

$$\begin{aligned}C^{(1)}_{k\pm1,k}(t) &= -\mathrm{i}\frac{\nu}{2}\sqrt{(l\pm k+1)(l\mp k)}\;\mathrm{e}^{-\mathrm{i}[\gamma(k\pm1)+g(k\pm1)^2]t}\\ &\quad\times\sum_{n=1}^{[t]}\mathrm{e}^{\mathrm{i}\{\gamma[(k\pm1)-k]+g[(k\pm1)^2-k^2]\}nT}. \end{aligned}\tag{11}$$

The corresponding transition probabilities read

$$P_{k\pm1,k}(t) = \frac{\nu^2}{4}(l\pm k+1)(l\mp k)\Big|\sum_{n=1}^{[t]}\mathrm{e}^{\pm i[\gamma+g(2k\pm1)]nT}\Big|^2. \tag{12}$$

The probabilities $P_{k\pm1,k}(t)$ describe the quantum transitions from the initial state $|l,k\rangle$ to the final states $|l,k\pm1\rangle$.

From Eq. (12), one can conclude that the following condition has to be satisfied to make the transition probability grow with the kick numbers adds:

$$[\gamma+g(2k\pm1)]T = 2\pi n_{\pm}, \tag{13}$$

where $n_{\pm}$ is integer. As shown in Fig. 1(a), the transition probability will lead to resonance when the parameters meet the resonance condition (different peaks relate to different values of $n_{\pm}$). Additionally, the resonance conditions of the transition between different angular momentum eigenstates are diverse because they are k-dependent. The specific evolution process of the transition probability is shown in Fig. 1(b). When the resonance condition is satisfied, the transition probability increases monotonically with the kick time. Otherwise, it will oscillate periodically and remain to be a negligible value. It should be noted that the resonance condition of $C^{(1)}_{k\pm1,k}(t)$ is equal to $C^{(1)}_{k,k\pm1}(t)$ according to the first-order time-dependent perturbation theory. Therefore, the system could evolve back from state $|l,k\pm1\rangle$ to $|l,k\rangle$ if the transition $|l,k\rangle\rightarrow|l,k\pm1\rangle$ is allowed.

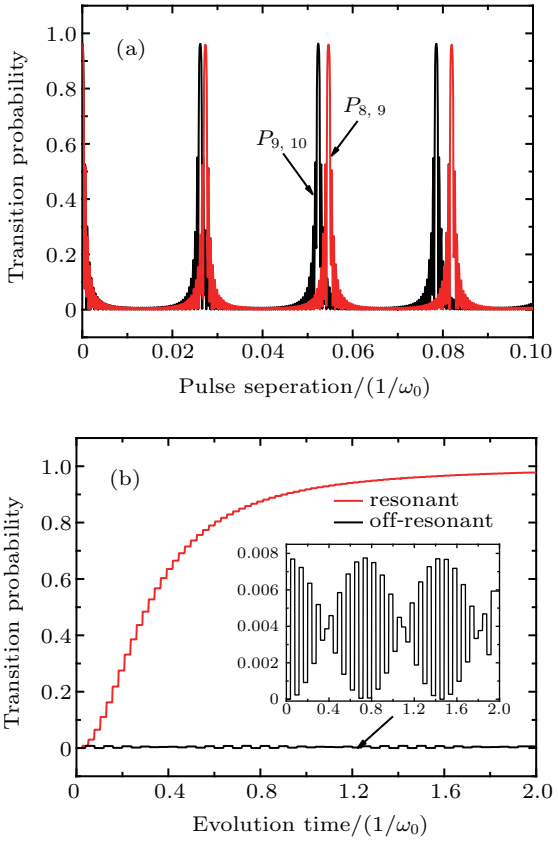


Fig. 1. (a) The transition probabilities of $P_{9,10}$ (black line) and $P_{8,9}$ (red line) after 50 kicks with $N = 20$, $\nu = 0.01$, $\gamma = 50$, and $g = 10$. (b) The specific evolution processes of the transition probabilities with $T = 2\pi/240$ (resonant situation) and $T = 0.04$ (off-resonant situation) respectively (assuming a trap frequency ω_0 of $2\pi\times10$ kHz). $P_{j,i}$ expresses the probability of transiting from initial state $|l,i\rangle$ to $|l,j\rangle$, and all the variables are dimensionless.

3. Manipulating transition via different resonances

Quantum transition between stationary states is a basic problem in quantum mechanics. The idea of resonance transition is associated with the driving frequency fitting a level difference between two internal electronic states[40] or two exter-

nal motional states.[41–44] But, additional resonance condition is required to allow the transition to happen when the coupling strength is weak. Quantum chaos studies determined the resonance condition of a linear kick-rotor system as $\omega_{\pm}T = 2\pi n_{\pm}$, with $\omega_{\pm}$ denoting the energy difference of the vibrational levels and $n_{\pm}$ being the positive rational numbers.[45] Recall the eigenenergy $E_k = \gamma k + gk^2$, we have the level differences

$$\omega_{\pm} = |E_{k\pm1} - E_k| = |\gamma + g(2k \pm 1)|. \tag{14}$$

Here, the resonance condition allowing the transition between different angular momentum eigenstates is similar to that in the quantum chaos case, but with $n_{\pm}$ being an integer.

It is well known that the state vector of a Hamiltonian system (5) also can be expressed by the Fock bases $|N_1, N_2\rangle$ with N_i $(i = 1, 2)$ being the number of atoms occupying the i-th internal hyperfine state. Employing the equations $\langle l, k|J_z|l, k\rangle = k = \langle N_1, N_2|J_z|N_1, N_2\rangle = \frac{1}{2}(N_1 - N_2)$ and $N_1 + N_2 = N$, we find $N_1 = \frac{N}{2} + k$ and $N_2 = \frac{N}{2} - k$. This means that the initial and final states $|l, k \pm j\rangle$, $j = 0, 1$ correspond to the Fock states $|N/2 + (k \pm j), N/2 - (k \pm j)\rangle$. Consequently, the transitions from $|l, k\rangle$ to $|l, k \pm 1\rangle$ are equivalent to those from $|N/2 + k, N/2 - k\rangle$ to $|N/2 + (k \pm 1), N/2 - (k \pm 1)\rangle$, which means an atom jumps between the two internal levels. For the resonance case, we list two types of resonance parameters as follows.

3.1. The N-independent odd–even sensitive transition

Although the resonant condition is k-dependent as shown in Eq. (13), this dependence could be removed in some special cases. Considering the parameters fulfill a special condition

$$\gamma T = n_1 \pi, \quad gT = n_2 \pi, \tag{15}$$

where n_1 and n_2 are integers, we find three types of k-independent resonances: 1) If n_1 and n_2 are both even, all transitions between different angular momentum eigenstates are resonant, which means every atom is allowed to transition. 2) If n_1 and n_2 are both odd, the resonance condition is fulfilled when $2k = N_1 - N_2$ is even, i.e., the transition is allowed when the population difference is even. 3) Similarly, if n_1 is even and n_2 is odd, the resonance condition is fulfilled when the population difference is odd. Based on this unique phenomenon, we can control whether the number of atoms is even or odd.

As an example, we consider a two-level BEC with all the atoms prepared in state $|1\rangle$, i.e., the Fock state $|N, 0\rangle$, at first (as shown in Fig. 2(a)). Secondly, we shine a continuous laser onto the condensate to pump the atoms in state $|2\rangle$ to a high-field-seeking Zeeman sublevel and eject them from the trap.[46] Next, we set the parameters as $\nu = 0.01$, $\gamma = 200\pi$, $g = 100\pi$, and $T = 0.01$. Consequently, if the number of atoms is odd, the transition between the states $|1\rangle$ and $|2\rangle$ is allowed. Once the system evolves from the Fock state $|N, 0\rangle$ to $|N-1, 1\rangle$, the pump laser will evolve the system from $|N-1, 1\rangle$ to $|N-1, 0\rangle$ (as shown in Fig. 2(b)). After this, further transition is forbidden because the number of atoms in the condensate is now even (as shown in Fig. 2(c)). Therefore, we can prepare a condensate with an even number of atoms in state $|1\rangle$. The procedure to obtain an odd number of atoms is similar.

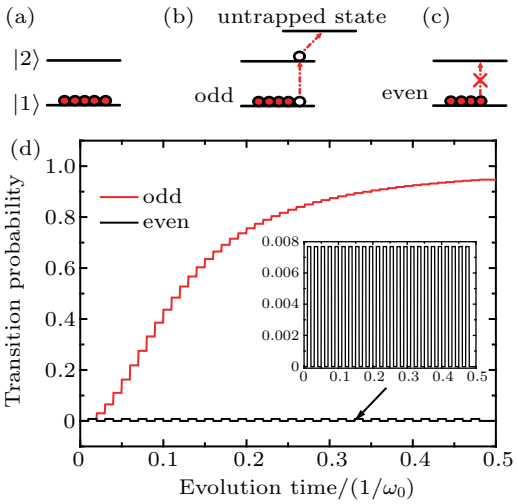


Fig. 2. The procedure to obtain an even number of atoms. (a) The atoms are prepared in the internal state $|1\rangle$ at first. (b) With the resonance condition of the odd-allowed transition fulfilled, one atom is allowed to transition from state $|1\rangle$ to $|2\rangle$ and then be pumped to an untrapped state. (c) The resonance condition is no longer fulfilled as the number of atoms has changed from odd to even. (d) The evolution of transition probability of the atoms in the Fock states $|21, 0\rangle$ (odd) and $|20, 0\rangle$ (even) with $\nu = 0.01$, $\gamma = 200\pi$, $g = 100\pi$, and $T = 0.01$.

3.2. The N-dependent population transform

Precisely controlling the atom number of the BEC remains difficult even with the feedback control technique.[47] Here, we propose a scheme to precisely control the atom number of the condensate using the N-dependent resonance condition. For a condensate with atom number $N \pm a$, where a is the atom number uncertainty (here we set $N = 22$ and $a = 2$), we can obtain a condensate of atom number $N = 20$ as follows: Firstly, we prepare all of the atoms in state $|1\rangle$ and then turn on the pump laser, in the same way as mentioned in the above subsection. As shown in Fig. 3(d), the resonance condition is separated when the atom number in state $|1\rangle$ is different. Furthermore, the separation will increase as $n_{\pm}$ in the resonance condition (16) increases. Therefore, for example, we can adjust the parameters to $\nu = 0.01$, $\gamma = 50$, $g = 10$, and $T = 8\pi/280$ to meet the resonance condition of $|24, 0\rangle \rightarrow |23, 1\rangle$ only without arousing the transition $|23, 0\rangle \rightarrow |22, 1\rangle$ and other transitions as well. If there are 24 atoms in state $|1\rangle$, with the pump laser continuously pumping the atoms in state $|2\rangle$ out of the condensate, one atom will transition from $|1\rangle$ to $|2\rangle$ after several kicks and then leave the condensate. Next, we adjust the pulse separation $T = 8\pi/270$, which satisfies the resonance

condition of $|23,0\rangle \rightarrow |22,1\rangle$, to pump another atom out of the condensate. Repeating the process, we can also eliminate the possibility of $|22,0\rangle \rightarrow |21,0\rangle$ and get a desired population distribution $|20,0\rangle$. Thus, we could prepare a nonclassical state in the many-body system with fixed atom number, which is important in quantum computing and metrology.[48] It is difficult to measure the atom numbers in state $|1\rangle$ owing to the indistinguishability of the bosons, but we can confirm it by detecting the atoms in state $|2\rangle$ because the atoms would not show up in state $|2\rangle$ unless the atom numbers in state $|1\rangle$ satisfied the resonance condition.

When the resonance condition is satisfied, the transition speed w can be expressed as $w = \partial P/\partial t = 1/T \times \partial P/\partial[t] = \chi[t]/T$, where $\chi = \nu^2(l \pm k + 1)/4$. Obviously, a shorter pulse separation T will lead to a faster transition speed and thus a shorter preparation time. Recalling the unit of time $1/\omega_0$ and assuming a typical trap frequency $\omega_0 = 2\pi \times 10$ kHz, we only need tens of μs or even less to realize a transition as shown in Figs. 1(b) and 2(d), which is much shorter than the BEC lifetime.

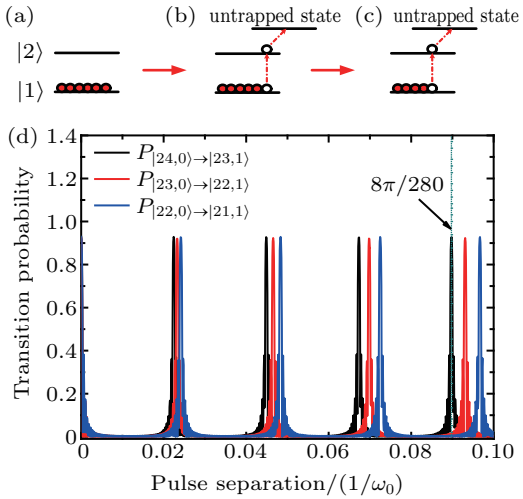


Fig. 3. The procedure to obtain the desired number of atoms. (a) The atoms are prepared in state $|1\rangle$ at first. (b) With the resonance condition of $|24,0\rangle \rightarrow |23,1\rangle$ fulfilled, one atom is allowed to transition from state $|1\rangle$ to $|2\rangle$ and be pumped to an untrapped state. (c) The further transition is stopped because the population has changed, but we can restart it by altering the parameters to fulfill the resonance condition of $|23,0\rangle \rightarrow |22,1\rangle$, etc. (d) The transition probability after 60 kicks with $\nu = 0.01$, $\gamma = 50$, and $g = 10$.

4. Conclusion

We have calculated the transition dynamics of a two-level BEC system through a perturbation method. From the analytical solution, we found that the transition probability induced by a single weak kick could be ignored. However, when the pulse separation T and the energy differences of the angular momentum eigenstates $\gamma + g(2k \pm 1)$ fulfilled the resonance condition $[\gamma + g(2k \pm 1)]T = 2\pi n_{\pm}$, the transition probability would quickly add to a considerable value as the kick number increased. It has been found that the resonance can be N-independent or N-dependent in some special cases. For the N-independent resonance, the transition has an odd–even sensitivity to the population difference. Based on this, we proposed a scheme to control the atoms in one of the internal states to be odd or even. while for the N-dependent resonance, we proposed a scheme to precisely control the atom numbers. The manipulations of the transitions can be experimentally tested in the existing setups. Adjustment of the pulse separation T could be achieved easily through the acousto-optic modulator. As for the two-body interaction, we can adjust it through the Feshbach resonance.[49]

Our findings provide a new route to the manipulation of transition dynamics. The precise control of the atom number could improve the precision of collision shifts and therefore of atomic clocks.[50] Additionally, the preparation of the nonclassical state with certain atom numbers is essential for the sensitivity of two-mode interferometers [51] and the quantum metrology.[48] Furthermore, our study can be extended to the double-well system easily.

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