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Quantum optical interferometry via general photon-subtracted two-mode squeezed states*

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We investigate the sensitivity of phase estimation in a Mach–Zehnder interferometer with photon-subtracted two-mode squeezed vacuum states. Our results show that, for given initial squeezing parameter, both symmetric and asymmetric photon subtractions can further improve the quantum Cramér–Rao bound (*i.e.*, the ultimate phase sensitivity), especially for single-mode photon subtraction. On the other hand, the quantum Cramér–Rao bound can be reached by parity detection for symmetric photon-subtracted two-mode squeezed vacuum states at particular values of the phase shift, but it is not valid for asymmetric photon-subtracted two-mode squeezed vacuum states. In addition, compared with the two-mode squeezed vacuum state, the phase sensitivity via parity detection with asymmetric photon-subtracted two-mode squeezed vacuum states will be getting worse. Thus, parity detection may not always be the optimal detection scheme for nonclassical states of light when they are considered as the interferometer states.

Keywords: quantum optics, phase sensitivity, quantum metrology

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1. Introduction

Optical interferometry and phase estimation are the basis for many precision measurement applications. In general, the sensitivity of the phase estimation within these settings crucially depends on the input states as well as the detection schemes. It is well known that the phase sensitivity with coherent-based interferometry is limited by the shot-noise limit (SNL) of $\Delta\phi = 1/\sqrt{\bar{n}}$,[1,2] where ϕ is the unknown phase to be estimated and $\bar{n}$ is the total number of the photons used to perform the estimation. In order to go beyond the SNL, or even reach the so-called Heisenberg limit (HL) $\Delta\phi = 1/\bar{n}$,[3,4] one has to resort to interferometry with nonclassical states of light. Up to the present, it has been demonstrated that, with parity detection,[5] the HL sensitivity can be approached in a lossless Mach–Zehnder interferometer (MZI) by using nonclassical states, such as twin-Fock states,[6] NOON states,[7] entangled coherent states.[8] Squeezed vacuum state, which is the brightest experimentally availble nonclassical light, has also received much attention.[9–16] In particular, it is found that an MZI with a two-mode squeezed vacuum state (TMSVS) and parity detection can offer sub-Heisenberg sensitivity.[10]

In recent years, it is of particular interest to investigate the enhancement given by non-Gaussian states or operations for quantum communication, metrology, imaging, and information processing.[17–28] For example, it has been found that those non-Gaussian squeezed states generated by photon subtraction or addition can improve the phase sensitivity for a given initial squeezing in quantum metrology.[22–26] In 2012, Carranza and Gerry[22] found that compared with the TMSVS, the symmetric photon-subtracted TMSVS (PS-TMSVS) generated by subtracting an equal number photons from each mode of TMSVS can further improve the phase sensitivity and the degree of superresolution in an MZI. These applications of photon-subtracted or added single-mode squeezed vacuum states in quantum precision implemented by an MZI or SU(1,1) interferometer are also explored.[23–26] In view of efforts towards complete understanding of non-Gaussian quantum squeezed states for quantum information, we also investigate whether the enhancement is still valid under energy-constraint for quantum metrology.[27,28] Very recently, symmetric PS-TMSVS with tunable mean photon numbers by utilizing a bright spontaneous-parametric-down-conversion in combination with photon-number-resolving detectors has been generated in the experiment,[29,30] which can enrich quantum information protocols relying on multi-photon interference. While compared with symmetric PS-TMSVS, it is easier to product the asymmetric PS-TMSVS (especially the single-mode photon subtracted TMSVS). Therefore, it may be interesting to investigate whether the asymmetric PS-TMSVS can further enhance the phase sensitivity on some particular occasions?

Different from that work in Ref. [22], we consider a general PS-TMSVS as the input state of an MZI, and will investigate the phase sensitivity in detail by the quantum Fisher information and parity detection, respectively. It is known that those sources of TMSVS can be generated by the process of spontaneous-parametric-down-conversion.[31] Mathe-

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matically, a TMSVS is generated by performing the two-mode squeezed operator $\hat{S}_2(r)=\exp\left[r\left(\hat{a}^\dagger\hat{b}^\dagger-\hat{a}\hat{b}\right)\right]$ on a two-mode vacuum state, *i.e.*, $|z\rangle=\left(1-z^2\right)^{1/2}\exp\left[z\hat{a}^\dagger\hat{b}^\dagger\right]|0,0\rangle$ with squeezing parameter $z=\tanh r$. Then, the normalized general PS-TMSVS can be obtained by subtracting different number of photons from each mode of TMSVS,[32] *i.e.*,

$$|z,-k,-l\rangle=\frac{\left(1-z^2\right)^{1/2}}{N_{k,l}^{1/2}}\hat{a}^k\hat{b}^l\exp\left[z\hat{a}^\dagger\hat{b}^\dagger\right]|0,0\rangle, \tag{1}$$

where $N_{k,l}$ is the normalization factor (without loss of generality, assuming $k\geq l$)

$$N_{k,l}=k!l!\left(\frac{z^2}{1-z^2}\right)^k P_l^{k-l,0}\left(\frac{1+z^2}{1-z^2}\right), \tag{2}$$

and the total mean photon number of the PS-TMSVS is $\bar{N}=\left(N_{k,l+1}+N_{k+1,,l}\right)/N_{k,l}$. Photon subtraction and addition are the typical non-Gaussian operations used to generate non-Gaussian states. Agarwal and Tara[33] theoretically studied the non-classical properties of photon-added coherent states, which was implemented[34] via a nondegenerate parametric amplifier with small coupling strength. The photon subtraction was implemented[35] with a beam splitter of high transmissivity. For a review of quantum-state engineering with photon addition and subtraction, we refer to Refs. [17] and [18]. The paper is organized as follows. In Section 2, for an MZI with the PS-TMSVS as input states, we obtain the quantum Fisher information. Via quantum Fisher information, we investigate how the photon subtraction affects the quantum Cramér–Rao bound (QCRB), the ultimate limit of the phase estimation. In Section 3, we study the parity detection and phase sensitivity. Our results shows that the QCRB can be reached by implementing the parity detection for symmetric PS-TMSVS at particular value of the phase shift. However, the phase sensitivity with asymmetric PS-TMSVS can not reach the QCRB. Thus, parity detection is not the optimal detection scheme when the asymmetric PS-TMSVS is considered as an interferometer state. Our main results are summarized in Section 4.

2. Quantum Fisher information in an MZI

The balanced MZI considered here is mainly composed of two 50:50 beam splitters and two phase shifters. As was shown by Yurke *et al.*,[36] the input beam splitter BS1 is described by the transformation $\hat{U}_{\rm BS1}=\exp\left[-\mathrm{i}\pi\left(\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger\right)/4\right]$. And the operator representation of the output beam splitter BS2 is taken as $\hat{U}_{\rm BS2}=\exp\left[\mathrm{i}\pi\left(\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger\right)/4\right]$. The operator $\hat{U}(\varphi)=\exp\left[\mathrm{i}\varphi\left(\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}\right)/2\right]$ denotes the two phase shifters, the angle φ being the phase shift between the two arms to be estimated. According to the work in Ref. [36], the unitary transformation associated with such balanced MZI can be written as

$$\begin{aligned}\hat{U}&=\exp\left(\mathrm{i}\frac{\pi}{2}\hat{J}_1\right)\exp\left(-\mathrm{i}\varphi\hat{J}_3\right)\exp\left(-\mathrm{i}\frac{\pi}{2}\hat{J}_1\right)\\&=\exp\left(-\mathrm{i}\varphi\hat{J}_2\right),\end{aligned} \tag{3}$$

where these operators consisted of two sets of Boson operators

$$\begin{aligned}\hat{J}_1&=\frac{1}{2}\left(\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger\right),\ \hat{J}_2=\frac{1}{2\mathrm{i}}\left(\hat{a}^\dagger\hat{b}-\hat{a}\hat{b}^\dagger\right),\\\hat{J}_3&=\frac{1}{2}\left(\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b}\right),\end{aligned} \tag{4}$$

are the angular momentum operators in the well-known Schwinger representation,[36] They satisfy the commutation relation $\left[\hat{J}_i,\hat{J}_j\right]=\mathrm{i}\varepsilon_{ijk}\hat{J}_k$ $(i,j,k=1,2,3)$, and commute with the Casimir operator, *i.e.*, $\left[\hat{J}_0,\hat{J}_i\right]=0$. For an input state propagating through the MZI, the resulted output state can be written as

$$|\mathrm{out}\rangle_{\rm MZI}=\mathrm{e}^{-\mathrm{i}\varphi\hat{J}_2}|\mathrm{in}\rangle. \tag{5}$$

Applying the following transformation relations

$$\begin{aligned}\mathrm{e}^{-\mathrm{i}\varphi\hat{J}_2}\hat{a}^\dagger\mathrm{e}^{\mathrm{i}\varphi\hat{J}_2}&=\hat{a}^\dagger\cos\frac{\varphi}{2}+\hat{b}^\dagger\sin\frac{\varphi}{2},\\\mathrm{e}^{-\mathrm{i}\varphi\hat{J}_2}\hat{b}^\dagger\mathrm{e}^{\mathrm{i}\varphi\hat{J}_2}&=\hat{b}^\dagger\cos\frac{\varphi}{2}-\hat{a}^\dagger\sin\frac{\varphi}{2},\end{aligned} \tag{6}$$

and the relation $\mathrm{e}^{-\mathrm{i}\varphi\hat{J}_2}|0\rangle_a|0\rangle_b=|0\rangle_a|0\rangle_b$, in principle, one can obtain the explicit form of the output state the MZI.

Different from that work in Ref. [22], here we first investigate the quantum Fisher information in the MZI with PS-TMSVS as interferometer states. It is well known that the QCRB via quantum Fisher information sets the ultimate limit for the phase sensitivity.[37] Here, we first give the quantum Fisher information for the interferometer with the PS-TMSVS. The ultimate limit of phase sensitivity is given by the QCRB[3,37]

$$\Delta\varphi_{\rm QCRB}=\frac{1}{\sqrt{F_{\rm Q}}}. \tag{7}$$

For pure states injected into an MZI, the quantum Fisher information $F_{\rm Q}$ can be obtained by[38]

$$F_{\rm Q}=4\left[\left\langle\psi'(\varphi)\right|\left.\psi'(\varphi)\right\rangle-\left|\left\langle\psi'(\varphi)\right|\left.\psi(\varphi)\right\rangle\right|^2\right], \tag{8}$$

where $|\psi(\varphi)\rangle=\mathrm{e}^{-\mathrm{i}\varphi\hat{J}_3}\mathrm{e}^{-\mathrm{i}\pi\hat{J}_1/2}|\mathrm{in}\rangle$ is the state just before the second beam splitter of the MZI, and $|\psi'(\varphi)\rangle=\partial|\psi(\varphi)\rangle/\partial\varphi$. In terms of the input state, the quantum Fisher information becomes

$$F_{\rm Q}=4\left[\langle\mathrm{in}|\hat{J}_2^2|\mathrm{in}\rangle-\left|\langle\mathrm{in}|\hat{J}_2|\mathrm{in}\rangle\right|^2\right], \tag{9}$$

and thus the quantum Fisher information is, up to factor of 4, the variance of the operator $\hat{J}_2$. Noting that the expectation value of a general product of operators $\hat{a}^{\dagger q}\hat{a}^p\hat{b}^{\dagger j}\hat{b}^h$ in the TMSVS[19]

$$N_{p,q,h,j}=\sum_m^{\min[p,h]}\frac{p!q!h!j!(1-z^2)^{-(p+j)}z^{2p+h+j-2m}}{m!(p-m)!(h-m)!}$$

$$\times \frac{\delta_{p+j,q+h}}{(j-h+m)!} \tag{10}$$

and Eq. (2), we can obtain the corresponding quantum Fisher information when the PS-TMSVS is considered as the interferometer state, *i.e.*,

$$F_{\rm QS} = \frac{2N_{k+1,l+1} + N_{k,l+1} + N_{k+1,l}}{N_{k,l}}. \tag{11}$$

Particularly, for single-mode photon subtracted TMSVS, equation (11) reduces to a simple form, *i.e.*,

$$F_{\rm QS}|_{l=0} = 2\frac{(k+1)(k+2)z^2}{(1-z^2)^2} + k. \tag{12}$$

While for the case of $k = l = 0$, we can further obtain the quantum Fisher information for input state with TMSVS

$$F_{\rm QS}|_{k=l=0} = \frac{4z^2}{(1-z^2)^2}, \tag{13}$$

which is just that result in Ref. [10] with $z^2 = \bar{n}/(\bar{n}+2)$ ($\bar{n}$ is the mean photon number of the TMSVS). Compared with Eq. (13), equation (12) clearly indicates that, for a given initial squeezing parameter z, single-mode photon subtraction can further enhance the quantum Fisher information.

In order to see clearly the influence of the photon subtraction on the ultimate limit of phase sensitivity of an MZI, we plot $\Delta\varphi_{\rm QCRB}$ varies with squeezing parameter z and the total mean photon number of the PS-TMSVS for some different values of (k,l) in Fig. 1, respectively. We can see from Fig. 1(a) that, for a given initial squeezing parameter z, it is better to perform asymmetric photon subtraction (especially single-mode photon subtraction) than symmetric photon subtraction in order to improve the QCRB, the ultimate limit of phase sensitivity. This may be because, for a given total number of the photon subtraction $k+l$, it is always better to perform asymmetric photon subtraction rather than symmetric one in order to increase the total mean photon number of the PS-TMSVS as shown in Fig. 2. Therefore, compared with TMSVS and symmetric PS-TMSVS, the asymmetric PS-TMSVS can give the higher ultimate limit of the phase sensitivity for given initial squeezing parameter. Next, one just need to find the optimal detection which enables the phase sensitivity to reach the QCRB. It should be pointed out that the quantum Fisher information (or the QCRB) is independent of the phase shift, while the phase sensitivity derived by the concrete detection methods generally depends on the values of the phase shift.

Within a constrain on the total mean photon number, Fig. 1(b) shows that the TMSVS gives a better phase sensitivity than the PS-TMSVS. However, the TMSVS generated by process of spontaneous-parametric-down-conversion is at very low brightness and its mean photon number is also small.[29,30] For the quantum metrology, the mean photon number of input states is an important factor. One advantage of photon subtraction is that it can increase the mean photon number of states as shown in Fig. 2. Thus, asymmetric photon subtraction may be also useful in quantum precision measurement. In the following, we will investigate the phase sensitivity based on the parity detection.

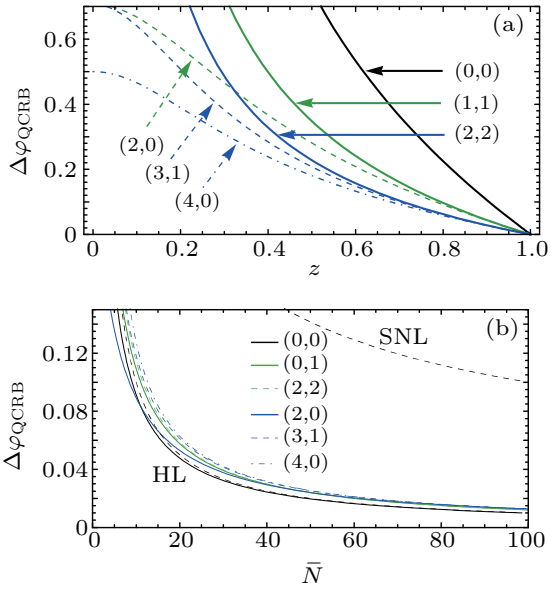


Fig. 1. The ultimate limit of phase sensitivity $\Delta\varphi_{\rm QCRB}$ of an MZI with the PS-TMSVS. (a) The $\Delta\varphi_{\rm QCRB}$ as a function of the initial squeezing parameter z for some values of (k,l); (b) The $\Delta\varphi_{\rm QCRB}$ varies with the total mean photon number for some values of (k,l). The upper black line denotes the SNL limit, while the below black dashed line represents the HL limit.

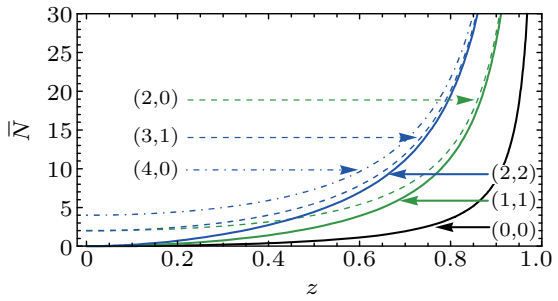


Fig. 2. The total mean photon number of the PS-TMSVS varies with the squeezing parameter z for some values of (k,l).

3. Parity detection and phase sensitivity

There are several detection methods for extracting phase information from the output states of the MZI, *e.g.*, intensity detection, homodyne detection, and parity detection.[5] Up to represent, it has been known that the QCRB can be reached at particular values of the phase shift by detecting the photon number parity on one of the output modes.[5–12,15,39] In the following, we will shows a counterexample that the QCRB can not be reached via parity detection with some nonclassical states of light. In experiments, the parity detection using a photon-number resolving detector with coherent states has also been demonstrated.[40] Here, we use parity detection

to investigate the phase sensitivity of the general PS-TMSVS. And then, we analyze whether the phase sensitivity obtained by parity detection can approach the QCRB.

Actually, the parity detection is to obtain the expectation value of the parity operator in the output state of the MZI. Different from the calculation method used in Refs. [10] and [22], we directly calculate the signal of the parity detection. For the sake of convenience, we rewrite the parity operator on the mode a as follows:[41]

$$\hat{\Pi}_a = (-1)^{\hat{a}^\dagger \hat{a}} = \int \frac{\mathrm{d}^2\gamma}{\pi} |\gamma\rangle\langle -\gamma|, \tag{14}$$

where $|\gamma\rangle$ is a coherent state. Then the expectation value of the parity operator can be obtained by

$$\langle \hat{\Pi}_a(\varphi)\rangle = {}_{\mathrm{MZI}}\langle \mathrm{out}| \int \frac{\mathrm{d}^2\gamma}{\pi} |\gamma\rangle\langle -\gamma|\mathrm{out}\rangle_{\mathrm{MZI}}. \tag{15}$$

Thus, if one knows the output state of the MZI, one can present the parity-based phase estimation scheme with calculation of the average signal $\langle \hat{\Pi}_a\rangle$ and phase sensitivity. When the general PS-TMSVS is injected into an MZI, we obtain the expectation value of the parity operator in the output state (See Appendix A)

$$\langle \Pi_a(\varphi)\rangle = \frac{\langle \hat{\Pi}_a\rangle_0}{N_{k,l}} \frac{\partial^{2k+2l}}{\partial x^k \partial y^l \partial t^k \partial \tau^l} \exp\left[\frac{(y^2+\tau^2-x^2-t^2)z^3\sin 2\varphi}{2(1-2z^2\cos 2\varphi+z^4)} + \frac{(xy+t\tau)(z-z^3\cos 2\varphi)}{1-2z^2\cos 2\varphi+z^4} - \frac{(xt-y\tau)(z^2+z^4)\sin\varphi}{1-2z^2\cos 2\varphi+z^4} + \frac{(x\tau+yt)(z^2-z^4)\cos\varphi}{1-2z^2\cos 2\varphi+z^4}\right]\Bigg|_{x,y,t,\tau=0}, \tag{16}$$

where an additional $\pi/2$ phase shift was introduced. In Eq. (16), $\langle \hat{\Pi}_b\rangle_0$ is the corresponding expectation value of the parity operator for the input state with TMSVS

$$\langle \hat{\Pi}_a(\varphi)\rangle_0 = \frac{1-z^2}{\sqrt{1-2z^2\cos 2\varphi+z^4}}, \tag{17}$$

which is just that result in Ref. [10] with $z^2 = \bar{n}/(\bar{n}+2)$. Particularly, in the case of single-mode photon subtraction, for example $l=0$, equation (16) reduces to the expectation value of parity operator for input state with single-mode photon subtracted TMSVS

$$\langle \Pi_a(\varphi)\rangle|_{l=0} = \langle \hat{\Pi}_a\rangle_0^{k+1} \sin^k\varphi P_k\left(\frac{-(1+z^2)}{\sqrt{1-2z^2\cos 2\varphi+z^4}}\right), \tag{18}$$

where we have used the normalized constant $N_{k,0} = k!z^{2k}/(1-z^2)^k$ and the generating function of the Legendre polynomial[42]

$$\frac{\partial^{2k}}{\partial t^k \partial \tau^k} \mathrm{e}^{-t^2-\tau^2+2xt\tau/(x^2-1)^{1/2}}\Big|_{t=\tau=0} = \frac{2^k k!}{(x^2-1)^{k/2}} \mathrm{P}_k(x). \tag{19}$$

Consequently, in the case of $k=l$, when the symmetric PS-TMSVS is considered as the interferometer state of a Mach–Zehnder interferometer, we can prove that our equation (16) is completely identical to Eq. (14) in Ref. [22] by numerical method. Therefore, our investigation extends the work of the parity-based phase estimation scheme with the symmetric PS-TMSVS in Ref. [22].

We plot Fig. 3 to show how the signal of the parity detection varies with the phase shift for different values of (k,l). From Fig. 3, we can see that, for symmetric PS-TMSVS, the values of $\hat{\Pi}_a(\varphi)$ approach to the maximum at $\varphi=0$. In addition, the central peak of $\hat{\Pi}_a(\varphi)$ at $\varphi=0$ narrows as k (or l) increases, which may indicate that symmetric photon subtraction can enhance suppersolution for given initial squeezing parameter z. However, for asymmetric PS-TMSVS with $l=0$, from Eq. (18) one clearly see that $\langle \Pi_a(\varphi)\rangle|_{l=0} = 0$ at $\varphi=0$. For general asymmetric PS-TMSVS ($k \neq l$), we can numerically prove that the signal of parity detection is also equal to zero at $\varphi=0$ as shown in Fig. 3. In addition, figure 3 also shows that, for asymmetric PS-TMSVS with different values of (k,l), the phase shift corresponding to the maximum values of $\hat{\Pi}_a(\varphi)$ is also different.

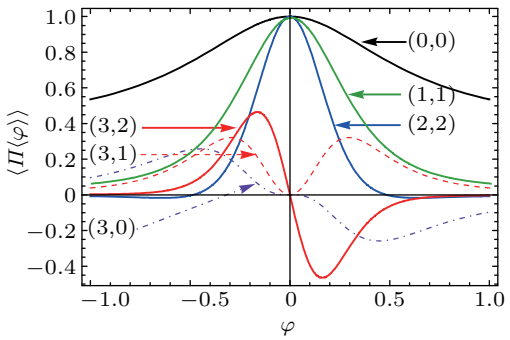


Fig. 3. Plot of the signal value of the parity detection against the phase shift for $z=0.6$ and some values of photon subtraction (k,l).

On the other hand, for the measurement of photon number parity, the phase sensitivity determined from the error propagation calculus is given by

$$\Delta\varphi = \frac{\sqrt{1-\langle \hat{\Pi}_a(\varphi)\rangle^2}}{\left|\partial\langle \hat{\Pi}_a(\varphi)\rangle/\partial\varphi\right|}. \tag{20}$$

In the case of $k=l=0$, we have

$$\Delta\varphi|_{k=l=0} = \frac{1+z^4-2z^2\cos 2\varphi}{2z(1-z^2)|\cos\varphi|}, \tag{21}$$

which also reduces that result in Ref. [10] with $z^2 = \bar{n}/(\bar{n}+2)$.

For the symmetric PS-TMSVS as the interferometer state, combining Eqs. (11) and (20), we can prove based on the numerical method that the QCRB can be reached by parity detection at $\varphi = 0$. Symmetric photon subtraction can enhance the phase sensitivity of an MZI for a given initial squeezing parameter as shown in Fig. 4. When the photon subtraction deviates from the symmetric operation, particularly single-mode photon subtraction, the phase sensitivity with such asymmetric PS-TMSVS as the interferometer state is getting worse. Only when the photon subtraction slightly deviates from the symmetric operation, for example $k = l+1$, can the photon subtraction improve the phase sensitivity.

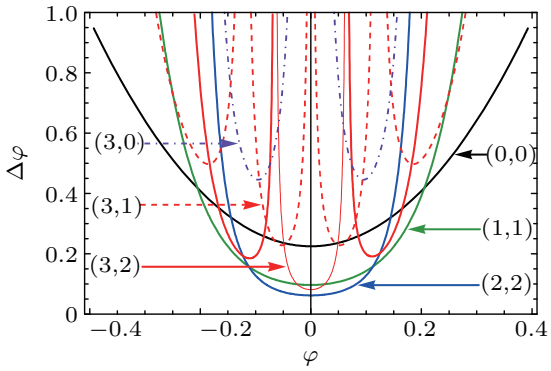


Fig. 4. Phase sensitivity $\Delta\varphi$ as a function of the phase shift φ for $z = 0.8$ and some values of (k,l) when the PS-TMSVS is considered as interferometer states.

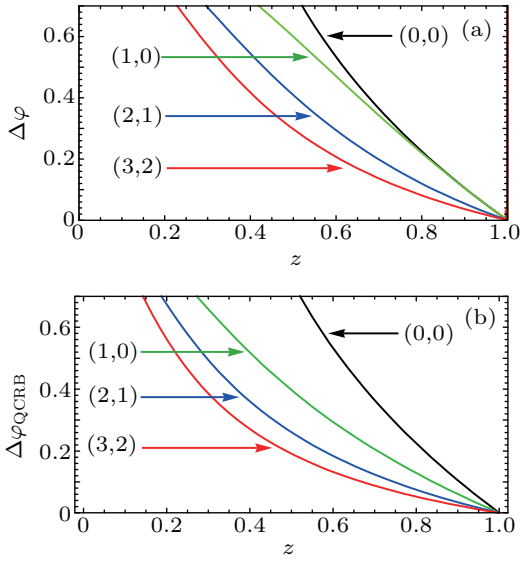


Fig. 5. Phase sensitivity $\Delta\varphi$ as a function of the initial squeezing parameter z for different values of (k,l) at $\varphi = 10^{-3}$. (a) The phase sensitivity obtained by the parity detection; (b) The ultimate limit of phase sensitivity $\Delta\varphi_{\mathrm{QCRB}}$ obtained by the quantum Fisher information.

In Fig. 5, we plot phase sensitivity obtained by the parity detection and the quantum Fisher information, respectively, with the asymmetric photon subtraction with $k = l+1$. We can see that, although the phase sensitivity via the parity detection do not reach the QCRB, the asymmetric photon subtraction with $k = l+1$ can further improve the phase sensitivity for a given initial squeezing parameter. It is known that the quantum Fisher information is independent of the phase shift. We have shown that the quantum Fisher information or $\Delta\varphi_{\mathrm{QCRB}}$ can be effectively improved by single-mode photon subtraction as shown in Fig. 1(a). Therefore, our results imply that, for the general asymmetric PS-TMSVS considered as the interferometer state, the parity detection is not the optimal measure scheme.

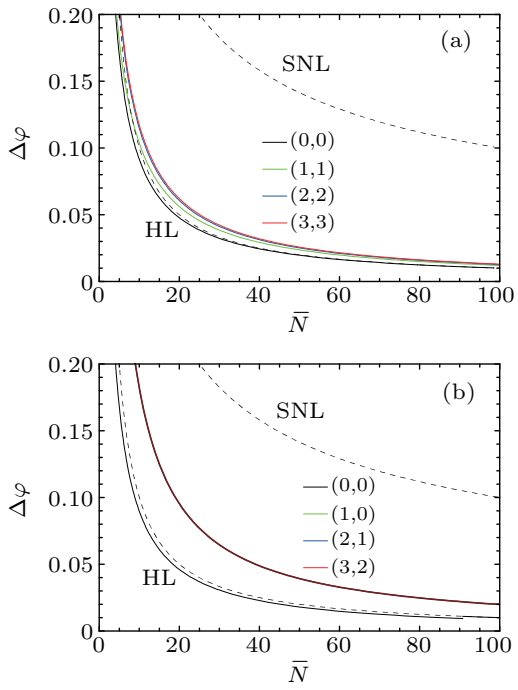


Fig. 6. Plots of phase sensitivity $\Delta\varphi$ as a function of total mean photon number of the PS-TMSVS for different values of (k,l) at $\varphi = 10^{-3}$. (a) Symmetric PS-TMSVS, $k = l$; (b) Asymmetric PS-TMSVS, $k = l+1$. The upper black line denotes the SNL limit, while the below black dashed lines represents the HL limit.

Finally, for a given constraint on the total mean photon number of the PS-TMSVS, we investigate how the phase sensitivities obtained by parity detection change with the total mean photon number. In Fig. 6, we plot the phase uncertainty $\Delta\varphi$ against the mean photon number of input states at $\varphi = 10^{-3}$, along with the corresponding curves for both SNL and HL limits. One can see from Fig. 6 that, within a constraint on the total mean photon number, compared with TMSVS, photon-subtracted operations do not enhance the phase sensitivity. On the other hand, within a constraint on the total mean photon number, compared with the asymmetric PS-TMSVS, the symmetric PS-TMSVS can give better phase sensitivity. In addition, figure 6(b) shows that these curves are overlapped with each other. This indicates that the phase sensitivity for asymmetric PS-TMSVS with $k = l+1$ are almost the same. In order to improve the phase sensitivity of an MZI, one may like to enhance the squeezing parameter to enhance the mean photon number of the TMSVS. But the largest achievable two-mode squeezing parameter in a stable optical configuration

is about $z \approx 0.82$[43] *i.e.*, the mean photon number is about 4. While the photon subtraction can increase the mean photon number of the states, which was explained in detail very recently by Stephen *et al.*[44] and references herein. Hence, techniques that improve the performance of the phase estimation without demanding higher initial squeezing may be still useful in quantum metrology. In this regard, for a given two-mode squeezing parameter z, the TMSVS that is engineered by multiple-photon subtraction (especially single-mode photon subtraction), contains more photons. In addition, the quantum Fisher information of these resulted non-Gaussian entangled states can be enlarged. Thus, non-Gaussion operations may still be advantageous for quantum metrology.

4. Conclusions

In summary, we have studied the performance of the general PS-TMSVS in the phase estimation based on an MZI interferometer. Via the quantum Fisher information, our numerical analysis shows that, for a given initial squeezing parameter, it is better to perform asymmetric photon subtraction (especially single-mode photon subtraction) than symmetric photon subtraction in order to improve the QCRB, the ultimate limit of the phase sensitivity. This may be because that it is always better to perform asymmetric photon subtraction rather than symmetric one in order to increase the total mean photon number of the PS-TMSVS. Compared the generation of symmetric PS-TMSVS, it may be easier to generate single-mode photon subtracted TMSVS in experiments. Therefore, asymmetric PS-TMSVS, especially single-mode photon subtracted TMSVS, may be useful in quantum precision measurement.

Although our results show that the QCRB phase sensitivity can be reached by parity detection at $\varphi = 0$ for symmetric PS-TMSVS, it is not valid for asymmetric PS-TMSVS (particularly single-mode photon subtracted TMSVS). Therefore, for general asymmetric PS-TMSVS considered as the interferometer state, the parity detection is still not the optimal detection scheme. One may look for other detection schemes to extracting phase information from the output states of an MZI, such as difference intensity detection and homodyne detection.[5] However, for symmetric PS-TMSVS as the input states, we can prove that signals of both difference intensity detection and homodyne detection are zero. Although the signal of the single-mode intensity detection is not zero, its derivative with respect to the phase shift is zero. Thus, for our considered input states, it is not appropriate to extracting the information of the phase shift by intensity detection or homodyne detection. Up to present, many studies show that the QCRB can be reached via parity detection in an MZI with many nonclassical states.[5–11,15,39] Here, we give a counterexample that the QCRB cannot be reached via parity detection with the asymmetric PS-TMSVS.

Appendix A: Derivation of Eq. (16)

In the basis of the coherent state, the TMSVS can be expressed by

$$|z\rangle = \left(1-z^2\right)^{1/2} \int \frac{\mathrm{d}^2\alpha \mathrm{d}^2\beta}{\pi^2} \exp\left[-\frac{|\alpha|^2+|\beta|^2}{2} + \alpha^*\beta^* z\right] |\alpha,\beta\rangle\,. \tag{A1}$$

Then, the PS-TMSVS can be expressed by

$$\begin{aligned} |z,-k,-l\rangle = \frac{\left(1-z^2\right)^{1/2}}{N_{k,l}^{1/2}} \hat{a}^k \hat{b}^l \exp\left[z\hat{a}^\dagger \hat{b}^\dagger\right] |0,0\rangle = \frac{\left(1-z^2\right)^{1/2}}{N_{k,l}^{1/2}} \int \frac{\mathrm{d}^2\alpha \mathrm{d}^2\beta}{\pi^2} \frac{\partial^{k+l}}{\partial x^k \partial y^l} \\ \times \exp\left[-\frac{|\alpha|^2+|\beta|^2}{2} + \alpha^*\beta^* z + x\alpha + y\beta\right] |\alpha,\beta\rangle \Big|_{x,y=0}. \end{aligned} \tag{A2}$$

When the PS-TMSVS is injected into the MZI, by Eqs. (5) and (6), the resulted output state can be written as

$$\begin{aligned} |\mathrm{out}\rangle_{\mathrm{MZI}} = \frac{\left(1-z^2\right)^{1/2}}{N_{k,l}^{1/2}} \int \frac{\mathrm{d}^2\alpha \mathrm{d}^2\beta}{\pi^2} \frac{\partial^{k+l}}{\partial x^k \partial y^l} \exp\left[-\left(|\alpha|^2+|\beta|^2\right) + \alpha^*\beta^* z + x\alpha + y\beta\right] \\ \times \exp\left[\hat{a}^\dagger\left(\alpha\cos\frac{\varphi}{2} - \beta\sin\frac{\varphi}{2}\right) + \hat{b}^\dagger\left(\alpha\sin\frac{\varphi}{2} + \beta\cos\frac{\varphi}{2}\right)\right] |0,0\rangle \Big|_{x,y=0}, \end{aligned} \tag{A3}$$

which is the state of light at the output of the MZI. Then substituting Eq. (A3) into Eq. (15), and applying the integral formula[45]

$$\int \frac{\mathrm{d}^2 z}{\pi} \exp\left[\zeta |z|^2 + \xi z + \eta z^*\right] = -\frac{1}{\zeta} \exp\left[\frac{-\xi\eta}{\zeta}\right], \tag{A4}$$

whose convergent condition is $\mathrm{Re}(\zeta) < 0$, as well as the integral formula[45]

$$\int \frac{\mathrm{d}^2 z}{\pi} \exp\left(\zeta |z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}\right) = \frac{1}{\sqrt{\zeta^2 - 4fg}} \exp\left(\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}\right), \tag{A5}$$

whose convergent condition is $\mathrm{Re}(\xi \pm f \pm g) < 0$ and $\mathrm{Re}\left(\frac{\zeta^2 - 4fg}{\xi \pm f \pm g}\right) < 0$, after long calculations, we obtain the expectation value of the parity operator in the output state of the MZI as shown in Eq. (16).

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