ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理
Chinese Physics B

Geometrical quantum discord and negativity of two separable and mixed qubits

Tang-Kun Liu(刘堂昆), Fei Liu(刘飞), Chuan-Jia Shan(单传家), Ji-Bing Liu(刘继兵)

Citation: Chin. Phys. B . 2019, 28(9): 090304 . **doi:** 10.1088/1674-1056/28/9/090304
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Monogamy quantum correlation near the quantum phase transitions in the two-dimensional *XY* spin systems

Meng Qin(秦猛), Zhongzhou Ren(任中洲), Xin Zhang(张欣)
Chin. Phys. B . 2018, 27(6): 060301 . **doi:** 10.1088/1674-1056/27/6/060301

Dynamical decoupling pulses on the quantum correlations for the system of superconducting quantum circuit

Wang Dong-Mei, Qian Yi, Xu Jing-Bo, Yu You-Hong
Chin. Phys. B . 2015, 24(11): 110304 . **doi:** 10.1088/1674-1056/24/11/110304

Rise of quantum correlations in non-Markovian environments in continuous-variable systems

Liu Xin, Wu Wei
Chin. Phys. B . 2014, 23(7): 070303 . **doi:** 10.1088/1674-1056/23/7/070303

Measurement-induced disturbance in Heisenberg *XY* spin model with Dzialoshinskii-Moriya interaction under intrinsic decoherence

Shen Cheng-Gao, Zhang Guo-Feng, Fan Kai-Ming, Zhu Han-Jie
Chin. Phys. B . 2014, 23(5): 050310 . **doi:** 10.1088/1674-1056/23/5/050310

Correlation dynamics of a qubit–qutrit system in a spin-chain environment with Dzyaloshinsky–Moriya interaction

Yang Yang, Wang An-Min
Chin. Phys. B . 2014, 23(2): 020307 . **doi:** 10.1088/1674-1056/23/2/020307

Geometrical quantum discord and negativity of two separable and mixed qubits*

Tang-Kun Liu(刘堂昆)†, Fei Liu(刘飞), Chuan-Jia Shan(单传家), and Ji-Bing Liu(刘继兵)

College of Physics and Electronic Science, Hubei Normal University, Huangshi 435002, *China*

(Received 8 May 2019; revised manuscript received 9 July 2019; published online 23 August 2019)

We studied quantum correlation and quantum entanglement of a quantum system in which a coherent state light field interacts with two qubits that are initially prepared in a separable and mixed state. The influence of mean photon number of the coherent field and distribution probability of the atom on the geometrical quantum discord and the negativity are discussed. Our results show that the mean photon number of light field and distribution function of the atom can regulate and control the quantum correlation and quantum entanglement.

Keywords: geometrical quantum discord, quantum correlation, negativity, quantum entanglement

PACS: 03.65.Ud, 42.50.Pq **DOI:** 10.1088/1674-1056/ab37f6

1. Introduction

The correlation is a universal phenomenon in the nature. Quantum correlation is a special nonclassical property of quantum systems, and quantum entanglement is an important quantum correlation in quantum information processing. As a resource, quantum entanglement plays a key role in many quantum information processing tasks.[1] The quantum entangled states with different degrees of entanglement will play different roles in quantum information processing. In the past few decades, many advances have been made in the study of quantum entanglement.[2–7] With the development of researches, it has been found that quantum entanglement cannot cover all non-classical correlations, that is to say, there is no entanglement but there is the correlation between quantum systems, so a more perfect measure of quantum correlation needs to be found. Whereupon, in 2001 year, Ollivier and Zurek[8] proposed the concept of quantum discord. It introduces a new measure of all the quantum correlations of the system. The quantum discord characterizes non-classical correlations in quantum mechanics, similar to the entanglement, quantum discord can also capture the fundamental features of quantum states. Since the quantum discord was put forward, and soon it has been found the non-classical correlations are more widespread than the entanglement. The quantum discord was investigated quite intensively in recent years.[9–16] In practice, due to the quantum discord maximizing involved in the calculation process, it is difficult to get the analytic expression. At present, only two-body Bell diagonal state, X-type 2×2 mixed state and highly symmetric manuscript dimensional state can be analyzed and calculated, this has greatly hindered the research progress of the quantum discord. In order to overcome this difficulty, Dakic *et al.* put forward a new method of measuring quantum correlations in 2010 year,[17] namely the geometrical quantum discord (GQD). The GQD is a definition of the smallest Hilbert–Schmidt distance between a given state and quantum discord for zero state. So far, some new research progress has been made for the GQD in different quantum systems.[18–22] However, the GQD and the negativity under the association measure of separable and mixed quantum correlation problems are rarely studied. Therefore, in this paper, we investigate the quantum correlation and the quantum entanglement in a system of separable and mixed qubits interacting with the coherent state light field. We use the GQD to measure the quantum correlation between separable and mixed qubits, and the negativity to measure the quantum entanglement between the separable and mixed qubits, respectively. This paper is organized as follows. In the next section, we give our theoretical model and density matrix of the quantum system. In Section 3, we investigate the GQD and the negativity between the separable and mixed qubits. We also discuss numerical results. Finally, our results are summarized in Section 4.

2. Model and density matrix

We consider a quantum system composed of two separable and mixed qubits interacting with a cavity field simultaneously, and suppose that the two qubits are coupled to the a cavity field with the coupling constant g. Under the rotating wave approximation, the Hamiltonian of the system can be written as ($\hbar=1$)[23]

$$\hat{H}=\Omega\hat{a}^{\dagger}\hat{a}+\frac{1}{2}\omega_0\sum_{i=A}^{B}\hat{\sigma}_i^z+g\sum_{i=A}^{B}(\hat{a}^{\dagger}\hat{\sigma}_i^{-}+\hat{a}\hat{\sigma}_i^{+}). \tag{1}$$

*Project supported by the National Natural Science Foundation of China (Grant No. 11604090).

†Corresponding author. E-mail: tkliu@hbnu.edu.cn

© 2019 Chinese Physical Society and IOP Publishing Ltd

Here Ω is the field frequency, ω_0 is the atomic transition frequency; $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operators of a cavity mode with frequency Ω, $\hat{\sigma}_i^z$ is the inversion operator of the i-th qubit, $\hat{\sigma}_i^+ = |e_i\rangle\langle g_i|$, and $\hat{\sigma}_i^- = |g_i\rangle\langle e_i|$ are the raising and the lowering operators between the excited $|e_i\rangle$ and the ground $|g_i\rangle$ states of the i-th qubit ($i = A, B$), respectively.

In the interaction picture and resonance case ($\Omega = \omega_0$), the Hamiltonian of the system is

$$\hat{H}_{\rm I} = \hat{H}_{\rm IA} + \hat{H}_{\rm IB} = g\left(\hat{a}^\dagger\hat{\sigma}_A^- + \hat{a}\hat{\sigma}_A^+\right) + g\left(\hat{a}^\dagger\hat{\sigma}_B^- + \hat{a}\hat{\sigma}_B^+\right). \quad (2)$$

When the two qubits are prepared in $\hat{\rho}_{\rm atoms}(0)$, and the field $\hat{\rho}_{\rm field}(0)$, the density operator of the system at time t is

$$\hat{\rho}(t) = \hat{U}(t)(\hat{\rho}_{\rm atoms}(0) \otimes \hat{\rho}_{\rm field}(0))\hat{U}^\dagger(t). \quad (3)$$

For the interaction of two qubits with a cavity field, using Eq. (2) and Taylor expansions, we obtain the analytical form of the evolution operator given in the four-dimensional atomic space

$$\hat{U}(t) = \begin{pmatrix} \hat{C}_1^2 & -\mathrm{i}\hat{C}_1\hat{S}_1 & -\mathrm{i}\hat{S}_1\hat{C}_1 & -\hat{S}_1^2 \\ -\mathrm{i}\hat{C}_1\hat{S}_2 & \hat{C}_1\hat{C}_2 & -\hat{S}_1\hat{S}_2 & -\mathrm{i}\hat{S}_1\hat{C}_2 \\ -\mathrm{i}\hat{S}_2\hat{C}_1 & -\hat{S}_2\hat{S}_1 & \hat{C}_2\hat{C}_1 & -\mathrm{i}\hat{C}_2\hat{S}_1 \\ -\hat{S}_2^2 & -\mathrm{i}\hat{S}_2\hat{C}_2 & -\mathrm{i}\hat{C}_2\hat{S}_2 & \hat{C}_2^2 \end{pmatrix}, \quad (4)$$

where

$$\hat{C}_1 = \cos(\sqrt{\hat{a}^\dagger\hat{a}+1}gt),$$
$$\hat{S}_1 = \hat{a}\frac{\sin(\sqrt{\hat{a}^\dagger\hat{a}}gt)}{\sqrt{\hat{a}^\dagger\hat{a}}}, \quad \hat{S}_2 = \frac{\sin(\sqrt{\hat{a}^\dagger\hat{a}}gt)}{\sqrt{\hat{a}^\dagger\hat{a}}}\hat{a}^\dagger,$$
$$\hat{C}_2 = \cos(\sqrt{\hat{a}^\dagger\hat{a}}gt).$$

We imagine that the two atomic qubits to be initially prepared in the state

$$\begin{aligned}\hat{\rho}_{\rm atoms}(0) &= [P|e_A\rangle\langle e_A| + (1-P)|g_A\rangle\langle g_A|] \\ &\quad \otimes [P|e_B\rangle\langle e_B| + (1-P)|g_B\rangle\langle g_B|] \\ &= \begin{pmatrix} P^2 & 0 & 0 & 0 \\ 0 & P(1-P) & 0 & 0 \\ 0 & 0 & P(1-P) & 0 \\ 0 & 0 & 0 & (1-P)^2 \end{pmatrix}. \end{aligned} \quad (5)$$

Here P is the probability distribution of the atomic qubits ($0 \le P \le 1$). When $P = 0$, both qubits are initially in the ground states; when $P = 1/2$, both qubits are initially in the maximally mixed states; and when $P = 1$, both qubits are initially in the excited states. The density matrix of the radiation field is initially prepared in a single-mode coherent state

$$\hat{\rho}_{\rm field}(0) = \sum_{n=0,m=0}^{\infty} F_n F_m^* |n\rangle\langle m|, \quad (6)$$

where $F_i = \exp(-\frac{1}{2}|\alpha|^2)\frac{\alpha^i}{\sqrt{i!}}$, $(i = n, m)$. $\alpha = |\alpha|\,\mathrm{e}^{\mathrm{i}\varphi}$, $|\alpha|$ is the mean photon number of the light field, and φ is the phase angle of the light field (for convenience, we take $\varphi = 0$).

According to Eq. (3), we can obtain the reduced density operator $\hat{\rho}_{\rm atoms}(t)$ for the qubit subsystem

$$\begin{aligned}\hat{\rho}_{\rm atoms}(t) &= {\rm Tr}_{\rm field}\,\hat{\rho}_{\rm system}(t) \\ &= \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}. \end{aligned} \quad (7)$$

The matrix elements ρ_{ij} $(i = 1, \ldots, 4,\ j = 1, \ldots, 4)$ are given in the Appendix A.

3. Numerical results and discussion

3.1. The quantum correlation between two qubits

For a two-body quantum systems, the quantum correlation can be defined in terms of the GQD[17]

$$GD(t) = \frac{1}{4}(\| X \|^2 + \| Y \|^2 - K_{\max}), \quad (8)$$

where $\| X \|^2 = \sum_{i=1}^{3} X_i^2$, $Y = Y_{ij}$ is an incidence matrix, $\| Y \|^2 = {\rm Tr}(Y^{\rm T}Y)$, $K_{\max}$ is the largest eigenvalue of the matrix $K = XX^{\rm T} + YY^{\rm T}$, superscript T denotes transpose of vector X or matrix Y. $\| X \|^2$ and $\| Y \|^2$ are Hilbert–Schmidt norm.

For an atomic system initially in a separable and mixed qubits, we select different atomic distribution probabilities in this paper, so are different initial states. In the numerical processing, the evolution of the $GD(t)$ is prominent. The temporal evolution of the $GD(t)$ is shown in Figs. 1–3.

When $P = 0$, and two qubits are initially in the ground states, the evolution characteristics of the $GD(t)$ *versus* different $|\alpha|$ are shown in Fig. 1. The results show that the period of the $GD(t)$ evolution becomes longer with $|\alpha|$, and meanwhile the GQD value began to evolve randomly between 0 and 0.55. On the other hand, from Fig. 1(d), we can see that the mean photon number of light field is large, the quantum correlation evolution of two qubits is always within a small negative range. In addition, when $P = 1$, and both qubits are initially in the excited states, the temporal evolution curves of the $GD(t)$ are not much different from Fig. 1.

When $P = 0.25$, the two qubits are initially in the mixed states. Its time evolution curved lines of the $GD(t)$ are also similar, which are shown in Fig. 2. The numerical results show that the maximum value of the GQD is 0.188, when the two qubits are initially in the mixed states. Through numerical calculation, we found that there was no significant difference between the GQD evolution curve at the initial state of $P = 0.75$ and $P = 0.25$.

Finally, we choose the two qubits are initially in the maximally mixed states, that is to say when $P = 0.5$. The time evolution of the $GD(t)$ is shown in Fig. 3. With the change of $|\alpha|$, the time evolution curved line of the $GD(t)$ is clearly different from Figs. 1 and 2. The numerical results show that the maximum value of the GQD is 0.106.

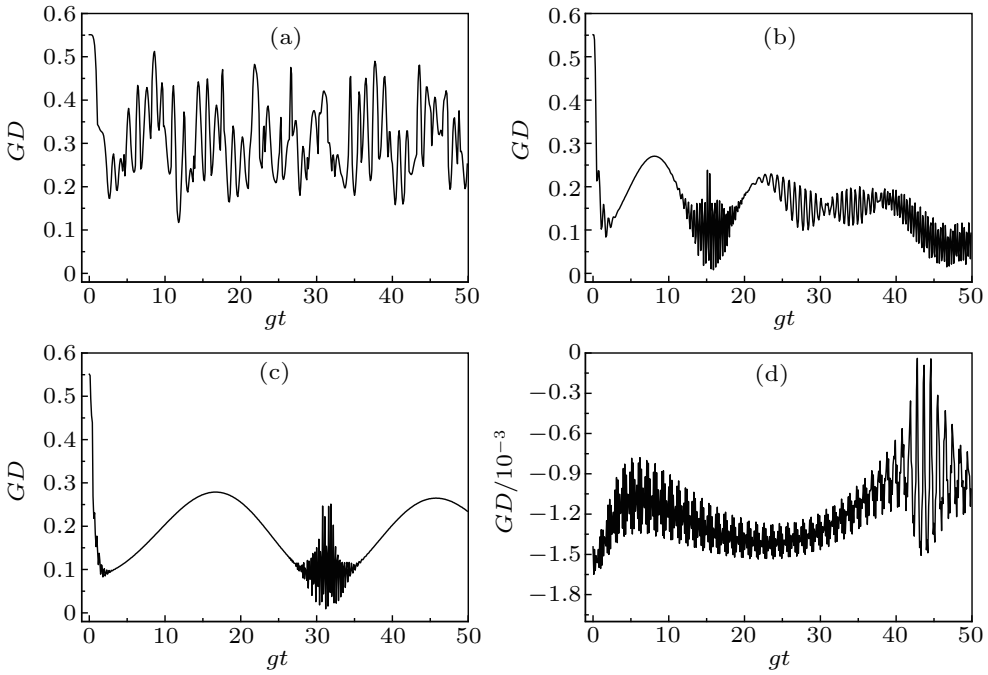


Fig. 1. The $GD(t)$ for $P = 0$ against the sealed time gt. In panels (a)–(d), $|\alpha| = 1, 5, 10, 15$, respectively.

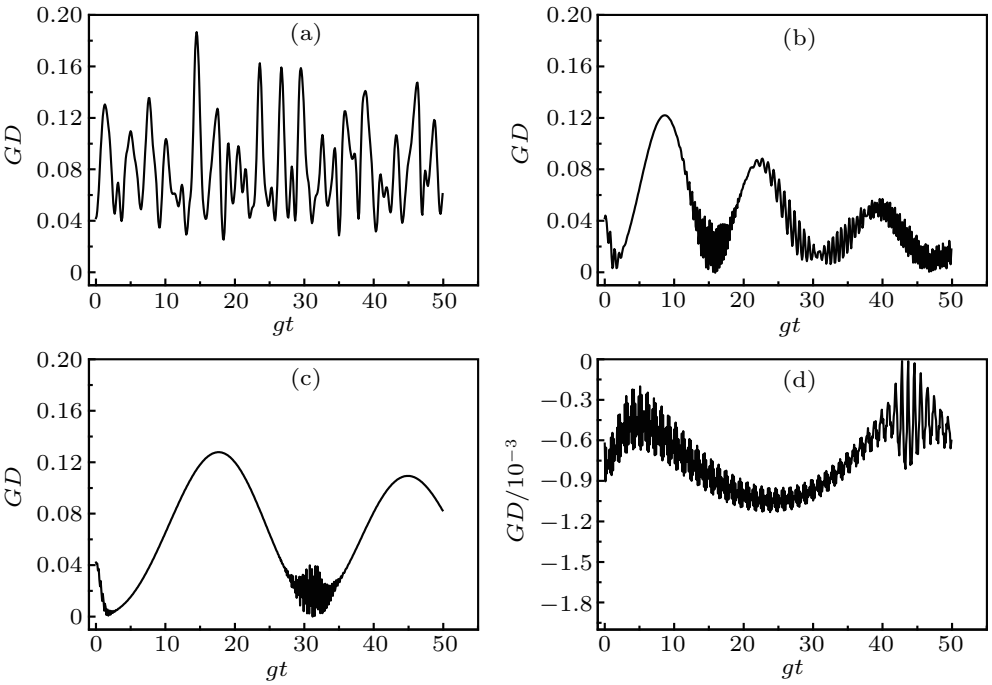


Fig. 2. The $GD(t)$ for $P = 0.25$ against the scaled time gt. In panels (a)–(d), $|\alpha| = 1, 5, 10, 15$, respectively.

In summary, it can be seen from Figs. 1–3 that there are quantum correlations between the two qubits, when two qubits are respectively initially in the ground state and the mixed state, but there is no quantum correlation when the two qubits initially in the maximum mixed state. When two qubits are initially in the ground state, the quantum correlation degree is the maximum; when two qubits are initially in the maximum mixed state, the quantum correlation degree is the minimum; when two qubits are initially in the mixed state, the quantum correlation degree is between the above two. When $|\alpha| = 15$, the time evolution of the GQD of three different qubits shows the similar phenomenon, and the quantum correlation is a small negative value. The appearance of this phenomenon is believed to be caused by the relatively large intensity of light field. In general, the intensity of the light field interacting with atoms is relatively weak. The present theory for quantum correlation may be inappropriate when the intensity of light field is increased at $|\alpha| = 15$. Therefore, it can be seen that the light field parameters can regulate the degree of quantum correlation between two qubits.

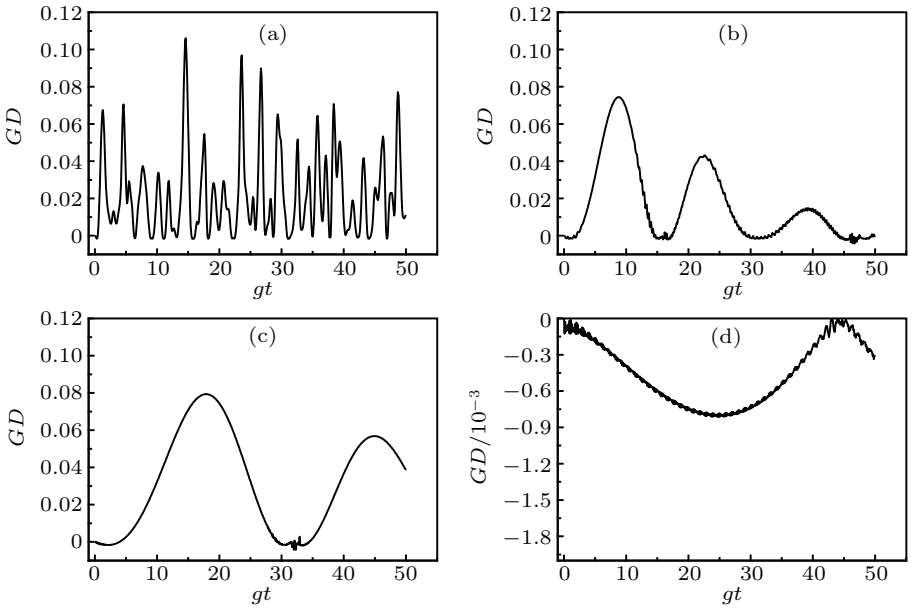


Fig. 3. The $GD(t)$ for $P = 0.5$ against the scaled time gt. In panels (a)–(d), $|\alpha| = 1, 5, 10, 15$, respectively.

3.2. Quantum entanglement of two qubits

On the other hand, the negative eigenvalues of partial transposition of density matrix is a measurement of the degree of entanglement between the two subsystems. Also known as the negativity. The defined measurement by Peres[2] is

$$E(t) = -2\sum_{i=1}\mu_i^-. \tag{9}$$

Here μ_i^- is a negative eigenvalue of $\hat{\rho}^{\mathrm{T}}(t)$. $E(t)$ denotes the two qubits in an entanglement state, when $0 < E(t) < 1$. $E(t) = 1$ denotes the two qubits in the maximum entangled state, and $E(t) = 0$ denotes the two qubits being separated.

Here, the parameter selection is the same as in Subsection 3.1, the time evolution of $E(t)$ is shown in Figs. 4–6, respectively.

As shown in Figs. 4–6, when the mean photon number of the light field increases, the time evolution characteristics of the quantum entanglement $E(t)$ are different when the two qubits are in three different states. When the average number of photons is large, the quantum entanglement between the two qubits is small as shown in Figs. 4(d)–6(d). When the average photon number takes a certain value, the phenomena of sudden death and generation of entanglement appear as shown in Figs. 5(a)–5(c) and Figs. 6(a)–6(c). At the same time, we also see that the quantum entanglement between the two qubits appears after a finite evolution time in three different states. Again, these phenomena show that the existence of quantum correlations does not necessarily mean the existence of quantum entanglement.

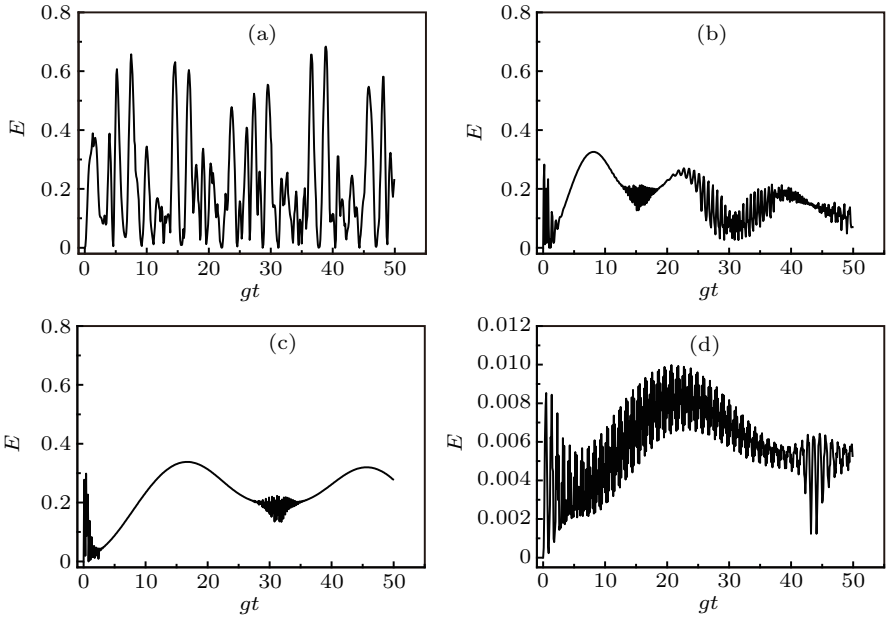


Fig. 4. The $E(t)$ for $P = 0$ against the scaled time gt. In panels (a)–(d), $|\alpha| = 1, 5, 10, 15$, respectively.

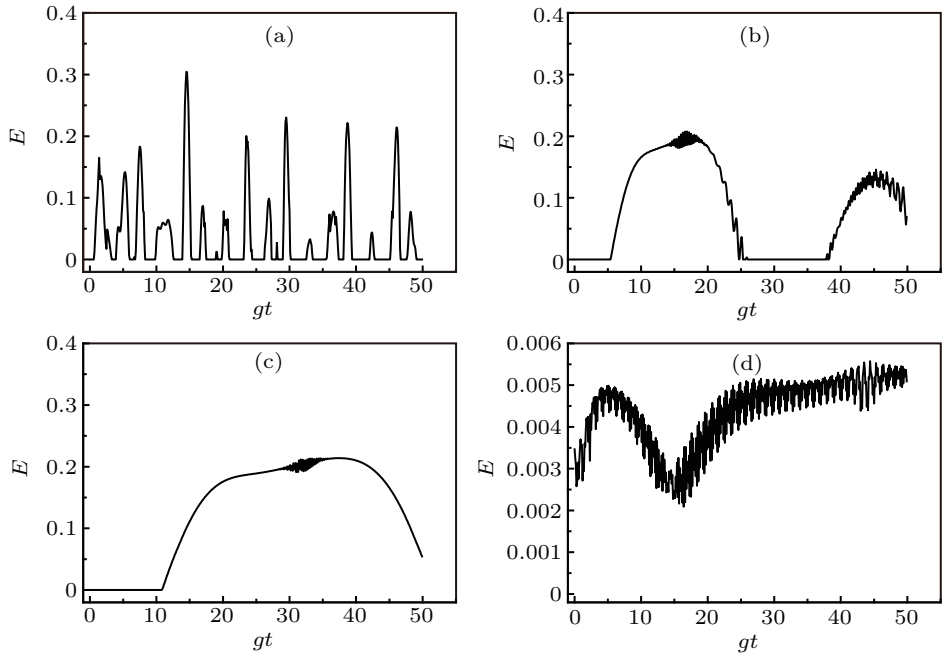


Fig. 5. The $E(t)$ for $P = 0.25$ against the scaled time gt. In panels (a)–(d), $|\alpha| = 1, 5, 10, 15$, respectively.

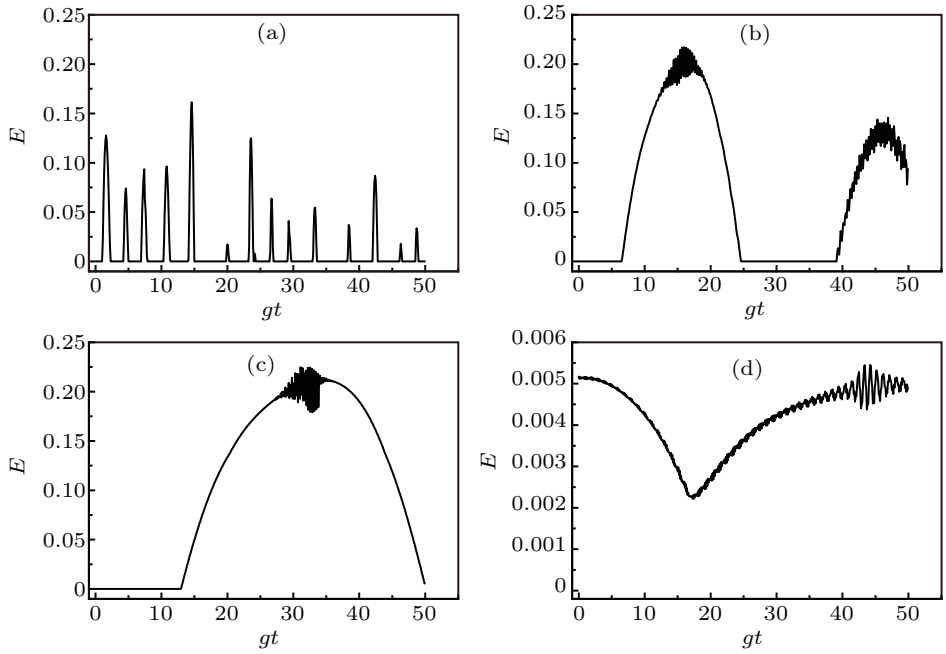


Fig. 6. The $E(t)$ for $P = 0.5$ against the scaled time gt. In panels (a)–(d), $|\alpha| = 1, 5, 10, 15$, respectively.

4. Conclusions

In this paper, quantum correlation and quantum entanglement evolution properties of the coherent light field interacting with the two qubits that are initially prepared in a separable and mixed state have been studied in details. The influence of the mean photon number and the atomic distribution probabilities on the temporal evolution of the quantum correlation and quantum entanglement between two qubits are discussed, respectively. The results show that the evolving curve of the quantum correlation and quantum entanglement can be changed by the mean photon number, and the phenomena of negative quantum correlation and entanglement sudden death and generation are presented.

Appendix A

The elements of reduced density matrix (7) are as follows:

$$\rho_{11} = \sum_{n=0}^{\infty} F_n F_n^* P^2 C(n+1,t)C(n+1,t)C(n+1,t)C(n+1,t) + \sum_{n=0}^{\infty} F_{n+1} F_{n+1}^* P(1-P)C(n+1,t)S(n+1,t)C(n+1,t)S(n+1)$$

$$+\sum_{n=0}^{\infty}F_{n+1}F_{n+1}^{*}P(1-P)C(n+2,t)S(n+1,t)C(n+2,t)S(n+1,t)$$
$$+\sum_{n=0}^{\infty}F_{n+2}F_{n+2}^{*}(1-P)^{2}S(n+2,t)S(n+1,t)S(n+2,t)S(n+1,t), \tag{A1}$$

$$\rho_{12}=\mathrm{i}\sum_{n=1}^{\infty}P^{2}F_{n}F_{n-1}^{*}C(n+1,t)C(n+1,t)C(n+1,t)S(n,t)-\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n+1}F_{n}^{*}C(n+1,t)S(n+1,t)C(n,t)C(n+1,t)$$
$$+\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n+1}F_{n}^{*}C(n+2,t)S(n+1,t)S(n+1,t)S(n+1,t)$$
$$-\mathrm{i}\sum_{n=0}^{\infty}(1-P)^{2}F_{n+2}F_{n+1}^{*}S(n+2,t)S(n+1,t)C(n+1,t)S(n+1,t), \tag{A2}$$

$$\rho_{13}=\mathrm{i}\sum_{n=1}^{\infty}P^{2}F_{n}F_{n-1}^{*}C(n+1,t)C(n+1,t)C(n,t)S(n,t)+\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n+1}F_{n}^{*}S(n+1,t)C(n+1,t)S(n,t)S(n,t)$$
$$-\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n+1}F_{n}^{*}C(n+2,t)S(n+1,t)C(n+1,t)C(n,t)$$
$$-\mathrm{i}\sum_{n=0}^{\infty}(1-P)^{2}F_{n+2}F_{n+1}^{*}S(n+2,t)S(n+1,t)C(n,t)S(n+1,t), \tag{A3}$$

$$\rho_{14}=-\sum_{n=2}^{\infty}P^{2}F_{n}F_{n-2}^{*}C(n+1,t)C(n+1,t)S(n-1,t)S(n,t)+\sum_{n=1}^{\infty}P(1-P)F_{n+1}F_{n-1}^{*}C(n+1,t)S(n+1,t)C(n-1,t)S(n,t)$$
$$+\sum_{n=1}^{\infty}P(1-P)F_{n+1}F_{n-1}^{*}C(n+2,t)S(n+1,t)S(n,t)C(n,t)-\sum_{n=0}^{\infty}(1-P)^{2}F_{n+2}F_{n}^{*}S(n+2,t)S(n+1,t)C(n,t)C(n,t), \tag{A4}$$

$$\rho_{21}=-\mathrm{i}\sum_{n=1}^{\infty}P^{2}F_{n-1}F_{n}^{*}C(n+1,t)S(n,t)C(n+1,t)C(n+1,t)+\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n+1}^{*}C(n+1,t)C(n,t)S(n+1,t)C(n+1,t)$$
$$-\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n+1}^{*}S(n+1,t)S(n+1,t)S(n+1,t)C(n+2,t)$$
$$+\mathrm{i}\sum_{n=0}^{\infty}(1-P)^{2}F_{n+1}F_{n+2}^{*}C(n+1,t)S(n+1,t)S(n+2,t)S(n+1,t), \tag{A5}$$

$$\rho_{22}=\sum_{n=1}^{\infty}P^{2}F_{n-1}F_{n-1}^{*}C(n+1,t)S(n,t)S(n,t)C(n+1,t)+\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n}^{*}C(n+1,t)C(n,t)C(n+1,t)C(n,t)$$
$$+\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n}^{*}S(n+1,t)S(n+1,t)S(n+1,t)S(n+1,t)$$
$$+\sum_{n=0}^{\infty}(1-P)^{2}F_{n+1}F_{n+1}^{*}C(n+1,t)S(n+1,t)S(n+1,t)C(n+1,t), \tag{A6}$$

$$\rho_{23}=\sum_{n=1}^{\infty}P^{2}F_{n-1}F_{n-1}^{*}C(n+1,t)S(n,t)S(n,t)C(n,t)-\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n}^{*}C(n+1,t)C(n,t)S(n,t)S(n,t)$$
$$-\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n}^{*}S(n+1,t)S(n+1,t)C(n+1,t)C(n,t)+\sum_{n=0}^{\infty}(1-P)^{2}F_{n+1}F_{n+1}^{*}C(n+1,t)S(n+1,t)S(n+1,t)C(n,t), \tag{A7}$$

$$\rho_{24}=\mathrm{i}\sum_{n=2}^{\infty}P^{2}F_{n-1}F_{n-2}^{*}C(n,t)S(n,t)S(n-1,t)S(n,t)+\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_{n}F_{n-1}^{*}C(n+1,t)C(n,t)C(n-1,t)S(n,t)$$
$$-\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_{n}F_{n-1}^{*}S(n,t)S(n+1,t)C(n,t)S(n,t)-\mathrm{i}\sum_{n=0}^{\infty}(1-P)^{2}F_{n+1}F_{n}^{*}C(n+1,t)S(n+1,t)C(n,t)C(n,t), \tag{A8}$$

$$\rho_{31}=-\mathrm{i}\sum_{n=1}^{\infty}P^{2}F_{n-1}F_{n}^{*}C(n,t)S(n,t)C(n+1,t)C(n+1,t)-\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n+1}^{*}S(n,t)S(n,t)C(n+1,t)S(n+1,t)$$
$$+\mathrm{i}\sum_{n=0}^{\infty}P(1-P)F_{n}F_{n+1}^{*}C(n,t)C(n+1,t)C(n+2,t)S(n+1,t)$$
$$+\mathrm{i}\sum_{n=0}^{\infty}(1-P)^{2}F_{n+1}F_{n+2}^{*}C(n,t)S(n+1,t)S(n+2,t)S(n+1,t), \tag{A9}$$

$$\rho_{32}=\sum_{n=1}^{\infty}P^2F_{n-1}F_{n-1}^*C(n,t)S(n,t)C(n+1,t)S(n,t)-\sum_{n=0}^{\infty}P(1-P)F_nF_n^*S(n,t)S(n,t)C(n+1,t)C(n,t)$$
$$-\sum_{n=0}^{\infty}P(1-P)F_nF_n^*C(n,t)C(n+1,t)S(n+1,t)S(n+1,t)+\sum_{n=0}^{\infty}(1-P)^2F_{n+1}F_{n+1}^*C(n,t)S(n+1,t)S(n+1,t)C(n+1,t), \tag{A10}$$

$$\rho_{33}=\sum_{n=1}^{\infty}P^2F_{n-1}F_{n-1}^*C(n,t)S(n,t)C(n,t)S(n,t)+\sum_{n=0}^{\infty}P(1-P)F_nF_n^*S(n,t)S(n,t)S(n,t)S(n,t)$$
$$+\sum_{n=0}^{\infty}P(1-P)F_nF_n^*C(n,t)C(n+1,t)C(n,t)C(n+1,t)+\sum_{n=0}^{\infty}(1-P)^2F_{n+1}F_{n+1}^*C(n,t)S(n+1,t)S(n+1,t)C(n,t), \tag{A11}$$

$$\rho_{34}=\mathrm{i}\sum_{n=2}^{\infty}P^2F_{n-1}F_{n-2}^*C(n,t)S(n,t)S(n-1,t)S(n,t)-\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_nF_{n-1}^*S(n,t)S(n,t)S(n,t)C(n-1,t)$$
$$+\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_nF_{n-1}^*C(n,t)C(n+1,t)C(n,t)S(n,t)-\mathrm{i}\sum_{n=0}^{\infty}(1-P)^2F_{n+1}F_n^*C(n,t)S(n+1,t)C(n,t)C(n,t), \tag{A12}$$

$$\rho_{41}=-\sum_{n=2}^{\infty}P^2F_{n-2}F_n^*S(n,t)S(n-1,t)C(n+1,t)C(n+1,t)+\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_{n+1}^*S(n-1,t)S(n-1,t)S(n,t)C(n,t)$$
$$+\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_{n+1}^*C(n,t)S(n,t)C(n+2,t)S(n+1,t)-\sum_{n=0}^{\infty}(1-P)^2F_nF_{n+2}^*C(n,t)C(n,t)S(n+2,t)S(n+1,t), \tag{A13}$$

$$\rho_{42}=-\mathrm{i}\sum_{n=2}^{\infty}P^2F_{n-2}F_{n-1}^*S(n,t)S(n-1,t)C(n+1,t)S(n,t)-\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_n^*S(n,t)C(n-1,t)C(n+1,t)C(n,t)$$
$$+\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_n^*C(n,t)S(n,t)S(n+1,t)S(n+1,t)+\mathrm{i}\sum_{n=0}^{\infty}(1-P)^2F_nF_{n+1}^*C(n,t)C(n,t)S(n+1,t)C(n+1,t), \tag{A14}$$

$$\rho_{43}=-\mathrm{i}\sum_{n=2}^{\infty}P^2F_{n-2}F_{n-1}^*S(n,t)S(n-1,t)S(n,t)C(n,t)+\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_n^*S(n,t)C(n-1,t)S(n,t)S(n,t)$$
$$-\mathrm{i}\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_n^*C(n,t)S(n,t)C(n,t)C(n+1,t)+\mathrm{i}\sum_{n=0}^{\infty}(1-P)^2F_nF_{n+1}^*C(n,t)C(n,t)C(n,t)S(n+1,t), \tag{A15}$$

$$\rho_{44}=\sum_{n=2}^{\infty}P^2F_{n-2}F_{n-2}^*S(n,t)S(n-1,t)S(n-1,t)S(n,t)+\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_{n-1}^*S(n,t)C(n-1,t)S(n,t)C(n-1,t)$$
$$+\sum_{n=1}^{\infty}P(1-P)F_{n-1}F_{n-1}^*C(n,t)S(n,t)C(n,t)S(n,t)+\sum_{n=0}^{\infty}(1-P)^2F_nF_n^*C(n,t)C(n,t)C(n,t)C(n,t), \tag{A16}$$

where

$$C(n,t)=\cos(\sqrt{n}gt),\quad C(n-1,t)=\cos(\sqrt{n-1}gt),$$
$$C(n+1,t)=\cos(\sqrt{n+1}gt),\quad C(n+2,t)=\cos(\sqrt{n+2}gt),$$
$$S(n,t)=\sin(\sqrt{n}gt),\quad S(n-1,t)=\sin(\sqrt{n-1}gt),$$
$$S(n+1,t)=\sin(\sqrt{n+1}gt),\quad S(n+2,t)=\sin(\sqrt{n+2}gt). \tag{A17}$$

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridy: Cambridy University Press)
[2] Peres A 1996 *Phys. Rev. Lett.* **77** 1413
[3] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[4] Horodecki R, Horodecki P, Horodecki M, *et al.* 2009 *Rev. Mod. Phys.* **81** 865
[5] Lu C Y, Yang T and Pan J W 2009 *Phys. Rev. Lett.* **103** 020501
[6] Pan J W, Chen Z B, Lu C Y, *et al.* 2012 *Rev. Mod. Phys.* **84** 777
[7] Yao X C, Wang T X, Chen H Z, *et al.* 2012 *Nature* **482** 489
[8] Ollivier H and Zurek W H 2001 *Phys. Rev. Lett.* **88** 017901
[9] Chen L, Chitambar E, Modi K and Vacanti G 2011 *Phys. Rev. A* **83** 020101
[10] Li Bo, Wang Z X and Fei S M 2011 *Phys. Rev. A* **83** 022321
[11] Fanchini F F, Werlang T, Brasil C A, Arruda L G E and Caldeira A O 2010 *Phys. Rev. A* **81** 052107
[12] Ali M, Ran A R P and Alber G 2010 *Phys. Rev. A* **81** 042105
[13] Wang B, Xu Z Y, Chen Z Q and Feng M 2010 *Phys. Rev. A* **81** 014101
[14] Liu D, Zhao X and Long G L 2010 *Commun. Theor. Phys.* **54** 825
[15] Rulli C C and Sarandy M S 2011 *Phys. Rev. A* **84** 042109
[16] Cao Y, Li H and Long G L 2013 *Chin. Sci. Bull.* **58** 48
[17] Dakic B, Vedral V and Brukner C 2010 *Phys. Rev. Lett.* **105** 190502
[18] Werlang T and Rigolin G 2010 *Phys. Rev. A* **81** 044101
[19] Xu P, Wu T and Ye L 2015 *Int. J. Theor. Phys.* **54** 1958
[20] Qin W and Guo J L 2015 *Int. J. Theor. Phys.* **54** 2386
[21] Liu T K, Tao Y, Shan C J and Liu J B 2016 *Int. J. Theor. Phys.* **55** 4219
[22] Liu T K, Zhang K L, Tao Y, Shan C J and Liu J B 2016 *Chin. Phys. B* **25** 070304
[23] Tavis M and Cummings F W 1968 *Phys. Rev.* **170** 379