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Dipole–dipole interactions enhance non-Markovianity and protect information against dissipation*

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Preserving non-Markovianity and quantum entanglement from decoherence effect is of theoretical and practical significance in the quantum information processing technologies. In this context, we study a system S that is initially correlated with an ancilla A, which interacts with the environment E via an amplitude damping channel. We also consider dipole–dipole interactions (DDIs) between the system and ancilla, which are responsible for strong correlations. We investigate the impact of DDIs and detuning on the non-Markovianity and information exchange in different environments. We show that DDIs are not only better than detuning at protecting the information (without destroying the memory effect) but also induce memory by causing a transition from Markovian to non-Markovian dynamics. In contrast, although detuning also protects the information, it causes a transition from non-Markovian to the Markovian dynamics. In addition, we demonstrate that the non-Markovianity grows with increasing DDI strength and diminishes with increasing detuning. We also show that the effects of negative detuning and DDIs can cancel out each other, causing a certain loss of coherence and information.

Keywords: decoherence, non-Markovianity, dipole–dipole interactions, entanglement

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1. Introduction

Unwanted interaction between an open quantum system and its environment causes a detrimental effect called decoherence.[1,2] A variety of schemes have been proposed to overcome this problem, such as the quantum Zeno effect (QZE),[3] dynamical decoupling,[4–7] weak measurement and quantum measurement reversal.[8] However, previous theoretical studies have found that dipole–dipole interactions (DDIs) also help to reduce decoherence, in both two-atom[9] and three-atom[10] systems interacting with independent environments, protecting entanglement in both the Markovian and non-Markovian regimes. Non-Markovianity itself also helps to protect entanglement,[11] and similarly, various methods have been proposed for generating as well as preserving entanglement against decoherence.[12–15] It has been found experimentally that, in the presence of DDIs, entanglement is sensitive to the number of trapped ions relative to the distance between them.[16] Zhang *et al.*[17] have reported that the DDIs show similar behavior to a positive detuning and exhibit the opposite role of negative detuning.

In cavity quantum electrodynamics,[18] the strong coupling regime (SCR), where the photons emitted by an atom can be reabsorbed and re-emitted, is a clear indication of information being exchanged between the system and environment,[19] implying that the process is non-Markovian (reversible process), and hence that the system can recover part of the lost coherence and information.[20,21] In recent decades, there has been increasing interest in the non-Markovian (reversible) nature of quantum processes,[22–27] which has been found to be a vital resource in applications such as detecting the environment characteristic properties,[28] continuous-variable quantum key distribution,[29–31] quantum metrology,[32] and steady-state entanglement.[33] Several studies have used the most common and significant measures of information flow to quantify the degree of non-Markovianity.[34–39] To study the DDIs further experimentally, highly excited Rydberg atoms with large principal quantum numbers ($n \gg 1$) and strong DDIs are a good choice.[40] Currently, both Markovian and non-Markovian dynamics have been studied experimentally using ultra-cold Rydberg atoms,[41] and the transition from Markovian to non-Markovian dynamics has been realized experimentally in all-optical frameworks.[42–44] In addition, Yuan *et al.*[45] have observed the transition from Markovian to non-Markovian dynamics caused by DDIs for a qubit coupled to a Bose–Einstein

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condensate (BEC) reservoir characterized by a pure dephasing spin-boson model.

In the recent schemes,[9,17] the authors limited their studies just to protect the entanglement via DDIs and detuning but in this study, we examine especially their impacts on the non-Markovianity (memory) and will see how they cause a transition from the Markovian to non-Markovian dynamics and *vice versa*. In detail, we consider a decoherence scheme,[46,47] where the system S is initially coupled to the ancilla A, interacting with the environment E via an amplitude-damping channel (ADC). We also go one step further and assume the realistic scenario where there are DDIs between the system S and ancilla A, while the system S and the environment E exchange information without direct interaction. Note that, during the processing of information, the survival of memory and entanglement plays an important role in the application of quantum technologies. In this regard, we have two parameters that can be used to adjust the system–environment coupling strength which controls the decoherence and dissipation. The first is the DDI strength, which increases the coupling between the system S and ancilla A, and the second is the detuning, which decreases the coupling between ancilla A and environment E.[48] Further, we analyze the role of DDIs and detuning, using the concepts of both accessible information $J_{SE}^{\leftarrow}$[49] and entanglement of formation (EOF) E_{SA} to quantify the exchange of information between the system and environment. We show that entanglement sudden death (ESD) can be safely avoided and entanglement can be maintained for a long time without impacting the memory effect by properly tuning the DDIs in the non-Markovian regime. In our model, even weak DDIs ($\eta > 0.3$) are enough to cause a transition from Markovian to non-Markovian dynamics, and the degree of non-Markovianity (which we calculate numerically) is better than the case mentioned in Ref. [45]. In addition, comparing DDIs with detuning reveals that DDIs in our model provide better protection than detuning, and the proposals of any previous study.[9,10] We also analyze how negative detuning and DDI can cancel out the effects of each other. Ultimately, we hope that our model will be easier to realize in the laboratory than those of previous studies, due to its use of weaker DDIs.

The paper is organized as follows. In Section 2, we discuss the general behavior of Markovian and non-Markovian dynamics and also introduce the measures of non-Markovianity, such as accessible information $J_{SE}^{\leftarrow}$, EOF E_{SA}. In Section 3, we present our example and its solutions in the presence of DDI. In Section 4, we present our results and discuss our findings. Finally, in Section 5 we present our conclusions.

2. Non-Markovianity measure via information flow

Generally, the quantum process's dynamical map is described by a time-local master equation[50] of the form

$$\frac{\partial \rho(t)}{\partial t} = \mathcal{L}\rho(t). \tag{1}$$

Here, the Lindblad superoperator $\mathcal{L}$ is defined as

$$\mathcal{L}\rho = -\mathrm{i}[H,\rho] + \sum_j \gamma_j \left[F_j \rho F_j^\dagger - \frac{1}{2}\{F_j^\dagger F_j, \rho\}\right],$$

where H denotes the system Hamiltonian, (γ_j's), F_j's are the time-independent decay rates and Lindblad operators describing the effect of noise on the system, respectively. For positive γ_j's, the dynamical map for Eq. (1) can be written as $\Lambda(t,0) = \exp[\mathcal{L}t]$, satisfying the divisibility condition for $t > 0$

$$\Lambda(t_1 + t_2, 0) = \Lambda(t_1, 0)\Lambda(t_2, 0), \tag{2}$$

for all positive t_1, t_2. This type of quantum dynamics is called as a conventional Markovian process. In time-dependent Markovian processes, the Hamiltonian H, noise operator F_j, and decay rates γ_j are explicitly time-dependent. If $\gamma_j(t) \geq 0$ throughout the system's evolution, then the dynamical map can be expressed in terms of a time-ordered exponential as $\Lambda(t,0) = T\exp[\int_0^t \mathcal{L}(\acute{t})\,\mathrm{d}\acute{t}]$, which transforms the state at time 0 to that at any other time t. Such dynamical maps have the fundamental property of satisfying divisibility condition. In such a scenario, the completely positive and trace-preserving (CPTP) map $\Lambda(t_2,0)$ can be written as the composition of two other CPTP maps

$$\Lambda(t_2,0) = \Lambda(t_2,t_1)\Lambda(t_1,0), \tag{3}$$

where $\Lambda(t_2,t_1) = T\exp[\int_{t_1}^{t_2} \mathcal{L}(\acute{t})\,\mathrm{d}\acute{t}]$, for all $t_{1,2} \geq 0$. We also note that the value of $\gamma_j(t)$ may be temporarily negative during the system's dynamics, in which the quantum dynamical map $\Lambda(t_2,t_1)$ is indeterminate, which is no longer a CPTP map, and also violates the composition law for divisibility[51,52] given by Eq. (3).

2.1. Accessible information

Here, we discuss a well-known quantity that can extract the maximum amount of classical information about the system S by locally observing the environment E called accessible information,[49] is given by

$$J_{SE}^{\leftarrow} = \max_{\{\Gamma_j^E\}} \left[S(\rho_S) - \sum_j p_j S\left(\rho_S^j \mid \Gamma_j^E\right)\right], \tag{4}$$

where $S(\rho_S) = -\mathrm{Tr}(\rho_S \log_2 \rho_S)$ is interpreted as the von Neumann entropy of the reduced density operator of the system S and $\{\Gamma_j^E\}$ is a general projective measurement acting on the

environment's qubit E. The remaining state of the system S after measurement is given by $\rho_S^j = \mathrm{Tr}_E\left[(I^S \otimes \Gamma_j^E)\rho_{SE}\right]/p_j$, with probability $p_j = \mathrm{Tr}\left[(I^S \otimes \Gamma_j^E)\rho_{SE}\right]$ in the subsystem E. Hence, any deviation of the accessible information from monotonically increasing behavior is a clear sign of non-Markovianity. Here, we demonstrate that, in the presence of DDIs, the dynamics remain non-Markovian for a long time and the system S exchanges some information with the environment E over time.

Non-Markovianity can be defined in terms of information flow via the accessible information. As we assume a zero-temperature reservoir, initially in a pure state, the composite tripartite state SAE always remain pure, and the Koashi–Winter relation[53] for the corresponding state can be characterized as

$$E_{SA} = S(\rho_S) - J_{SE}^{\leftarrow}. \tag{5}$$

Here, E_{SA} represents the EOF shared by the system S and the ancilla A, which can be defined for the two-qubits case as

$$E(\rho_{SA}(t)) = h\left(\frac{1+\sqrt{1-C^2(\rho_{SA}(t))}}{2}\right), \tag{6}$$

where

$$h(y) = -y\log_2 y - (1-y)\log_2(1-y), \tag{7}$$

and

$$C(\rho_{SA}(t)) = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}$$

defines the concurrence,[54] in decreasing order of the eigenvalues λ_i, of the product matrix $\rho_{SA}\tilde{\rho}_{SA}$. In addition, $\tilde{\rho}_{SA} = (\sigma^y \otimes \sigma^y)\rho_{SA}^*(\sigma^y \otimes \sigma^y)$, where ρ_{SA}^* is the complex conjugate of ρ_{SA} and σ^y is the Pauli spin operator. As we mentioned earlier, the system S and environment E do not interact directly, so the state ρ_S is time-invariant throughout the dynamics after tracing over the subsystem A. Therefore, after taking the time derivative of Eq. (5), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}E_{SA} = -\frac{\mathrm{d}}{\mathrm{d}t}J_{SE}^{\leftarrow}. \tag{8}$$

This relation shows how information exchange occurs between the system S and environment E over time. It clearly indicates that any temporary increase in E_{SA}, during the open-system dynamics, leads to a temporary decrease in $J_{SE}^{\leftarrow}$ and *vice versa*. Thus, any temporary decrease in $J_{SE}^{\leftarrow}$ is a signal of divisibility violation and a direct consequence of non-Markovianity. However, for some non-divisible processes, E_{SA} may still decay monotonically. In that scenario, we can also use the entanglement-based measure of non-Markovianity criterion which can be expressed as

$$\mathcal{N}(\Lambda) = \max_{\rho_{SA}(0)} \int_{\mathrm{d}/\mathrm{d}t E_{SA}>0} \frac{\mathrm{d}}{\mathrm{d}t}E_{SA}(t)\,\mathrm{d}t, \tag{9}$$

where the maximum is taken over all possible pure initial states of the bipartite system SA.

3. Example

Here, as shown in Fig. 1, we consider a system S that is initially entangled with an ancilla A, namely a two-level quantum system interacting with a zero temperature bosonic reservoir, represented by a collection of harmonic oscillators. We also consider the realistic scenario where there are DDIs between the system S and ancilla A, while the reservoir only influences the composite system SA via an ADC between A and E.

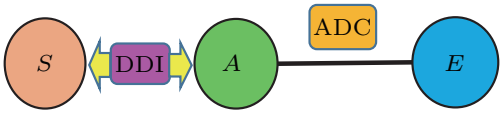


Fig. 1. A system S entangled with an ancilla A, and also including DDIs between them, while the ancilla A interacts with the environment E via an amplitude-damping channel (ADC).

The Hamiltonian that describes the entire composite system SAE is given by

$$\begin{aligned} H_{SAE} &= H_{SA}^{dd} + H_{AE}, \\ H_{SA}^{dd} &= \eta\left(\sigma_+^S\sigma_-^A + \sigma_+^A\sigma_-^S\right), \\ H_{AE} &= \omega_0\sigma_+^A\sigma_-^A + \sum_k \omega_k b_k^\dagger b_k + \left(\sigma_+^A\chi + \sigma_-^A\chi^\dagger\right), \end{aligned} \tag{10}$$

where $\chi = \sum_k g_k b_k$ and $\sigma_\pm^i (i = S, A)$ denote the qubits raising and lowering operators, respectively, with transition frequency of $\omega_S = \omega_A = \omega_0$. The environment annihilation (creation) operators are represented by b_k ($b_k^\dagger$), respectively, with mode frequencies ω_k. For continuous distribution of reservoir modes, the discrete coupling constant must be $\sum_k |g_k|^2 \to \int_{-\infty}^{\infty} J(\omega)\,\mathrm{d}\omega$, where $J(\omega) = \gamma_0\lambda^2/2\pi[(\omega_0-\omega-\Delta)^2+\lambda^2]$ is the Lorentzian spectral density. Here, Δ is the detuning of ω_0 and the cavity central frequency, while λ defines the spectral width of the coupling; Both λ and γ_0 are connected to the reservoir's correlation time τ_B and the system's relaxation time τ_R via $\tau_\mathrm{B} \approx 1/\lambda$ and $\tau_\mathrm{R} \approx 1/\gamma_0$, respectively. The static part of dipole–dipole interactions between system S and ancilla A is proportional to $\eta = [\boldsymbol{d}_i \cdot \boldsymbol{d}_j - 3(\boldsymbol{d}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{d}_j \cdot \boldsymbol{r}_{ij})/r_{ij}^2]r_{ij}^{-3}$, where $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$ defines the relative position and $\boldsymbol{d}_{i,j}$ with $(i = S,\ j = A)$ are the electric dipole moment of their respective dipoles. The weak-coupling regime, where the dynamics could be Markovian, corresponds to $\tau_\mathrm{R} >> 2\tau_\mathrm{B}$. In our scheme, the bipartite state SA is initially in a pure state, and the environment only interacts with the ancilla A, to gain information about the system S. Furthermore, we assume that the system S and ancilla A are maximally-entangled, interacting with a vacuum zero temperature bosonic reservoir which can be expressed as

$$|\Psi(0)\rangle_{SAE} = \sqrt{\frac{1}{2}}\left(|10\rangle + |01\rangle\right)_{SA}|0\rangle_E. \tag{11}$$

The interaction between A and E induce ADC, that can be described by a unitary operator U as, $U|0_A\rangle|0_E\rangle = |0_A\rangle|0_E\rangle$ and $U|1_A\rangle|0_E\rangle = \sqrt{1-p(t)}\,|1_A\rangle|0_E\rangle + \sqrt{p(t)}\,|0_A\rangle|1_E\rangle$. In the operator sum representation the state dynamics can be formulated as, $\rho_{SAE}(t) = \sum_{i=1}^{2} K_i(t)\rho_{SAE}(0)K_i^{\dagger}(t)$, where $K_i(t)$ represent the corresponding time-dependent Kraus operator satisfying the relation $\sum_{i=1}^{2} K_i^{\dagger}(t)K_i(t) = I$ for all time t,

$$\begin{aligned} K_1(t) &= |0\rangle\langle 0| + \sqrt{1-p(t)}|1\rangle\langle 1|, \\ K_2(t) &= \sqrt{p(t)}|0\rangle\langle 1|. \end{aligned} \tag{12}$$

Here, $p(t)$ is the damping parameter which satisfies the relation

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\sqrt{1-p(t)} \\ &= -\int_0^t \left(f(t-t')\sqrt{1-p(t')} + \eta^2\sqrt{1-p(t')} \right) \mathrm{d}t'. \end{aligned} \tag{13}$$

With the help of reservoir correlation function

$$\begin{aligned} f(t-t') &= \int_{-\infty}^{\infty} \mathrm{d}\omega_k J(\omega_k)\, \mathrm{e}^{-\mathrm{i}(\omega_k-\omega_0)(t-t')} \\ &= \frac{\gamma_0\lambda}{2}\, \mathrm{e}^{-(\lambda-\mathrm{i}\Delta)(t-t')}, \end{aligned} \tag{14}$$

and pseudomode approach,[9] we can easily calculate the damping parameter $p(t)$ as

$$p(t) = 1 - \left| \mathscr{L}^{-1}\left\{ \frac{(s+\lambda-\mathrm{i}\Delta)(s-\mathrm{i}\eta)}{(s+\lambda-\mathrm{i}\Delta)(s^2+\eta^2)+\gamma_0\lambda s/2} \right\} \right|^2, \tag{15}$$

where $\mathscr{L}^{-1}$ is the inverse Laplace transform and η (Δ) are the strength of DDI (detuning), respectively. In our case the time-dependent decay rate can be calculated as $\gamma(t) = -2\Re\{(\mathrm{d}/\mathrm{d}t(\sqrt{1-p(t)})/\sqrt{1-p(t)}\}$. The whole density state can be written as

$$\begin{aligned} &\rho_{SAE}(t) \\ &= \frac{1}{2}\Big[|100\rangle\langle 100| + |1-p(t)|\,|010\rangle\langle 010| \\ &\quad + |p(t)|\,|001\rangle\langle 001| + \Big(\sqrt{p(t)(1-p(t))}\,|010\rangle\langle 001| \\ &\quad + \sqrt{1-p(t)}|010\rangle\langle 100| + \sqrt{p(t)}\,|001\rangle\langle 100| + \mathrm{H.c} \Big) \Big]. \end{aligned} \tag{16}$$

Here, H.c represents the Hermitian conjugate of the last three terms. Now we trace out the environment E and the ancilla A, one by one, respectively, their corresponding reduced density states take the form

$$\begin{aligned} &\rho_{SA}(t) = \mathrm{Tr}_E(\rho_{SAE}(t)) \\ &= \frac{1}{2}\left[|\Phi_{SA}(t)\rangle\langle\Phi_{SA}(t)| + |p(t)|\,|00\rangle\langle 00| \right], \end{aligned} \tag{17}$$

$$\begin{aligned} &\rho_{SE}(t) = \mathrm{Tr}_A(\rho_{SAE}(t)) \\ &= \frac{1}{2}\left[|\Phi_{SE}(t)\rangle\langle\Phi_{SE}(t)| + |1-p(t)|\,|00\rangle\langle 00| \right], \end{aligned} \tag{18}$$

where $|\Phi_{SA}(t)\rangle = |10\rangle + \sqrt{1-p(t)}|01\rangle$, and $|\Phi_{SE}(t)\rangle = |10\rangle + \sqrt{p(t)}|01\rangle$, respectively.

4. Results and discussion

In this section, we study the effect of DDIs on the correlations dynamics in terms of EOF $E_{SA}(t)$ and accessible information $J_{SE}^{\leftarrow}(t)$ as quantifiers. In the absence of DDIs, figure 2(a) demonstrates the dynamics in the non-Markovian regime,[38] where we can observe that whenever E_{SA} increases temporarily, a decrease in $J_{SE}^{\leftarrow}(t)$ leads to a negative decay rate $\gamma(t)$, as shown in the inset of Fig. 2(a). In this case, the dynamical map is non-divisible, indicating a non-Markovian (reversible) quantum process. Next, we show that introducing DDIs between the system S and ancilla A, not only avoid ESD in the non-Markovian regime but also protect the information for a very long time. In Figs. 2(b) and 2(c) the DDI strengths are $\eta = 0.5, 1$, respectively, which is still very weak relative to the strength, as reported in Refs. [9] and [10]. In our case, these plots show that information is exchanged between the system S and environment E much more rapidly. We can also observe (insets of Figs. 2(b) and 2(c)) that $\gamma(t)$ oscillates in the same way throughout the whole dynamics and is periodically negative, implying the quantum processes are non-divisible.

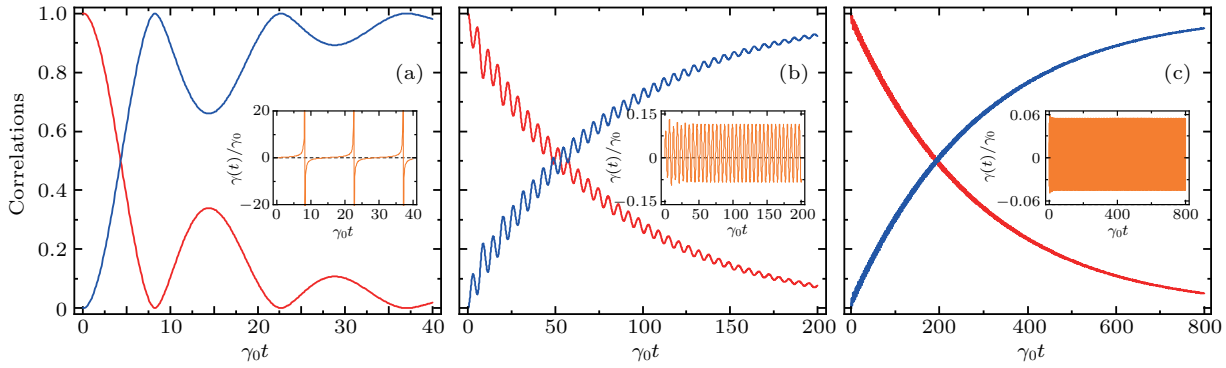


Fig. 2. Accessible information $J_{SE}^{\leftarrow}(t)$ (blue line), the entanglement of formation $E_{SA}(t)$ (red line), for (a) $\eta = 0$, (b) $\eta = 0.5$, (c) $\eta = 1$, respectively, with $\Delta = 0$, and in the insets, the decay rate $\gamma(t)/\gamma_0$ (orange line) as a function of the dimensionless quantity $\gamma_0 t$, in the non-Markovian regime where $\lambda/\gamma_0 = 0.1$. These plots demonstrate how information can be protected for a very long time without affecting non-Markovianity.

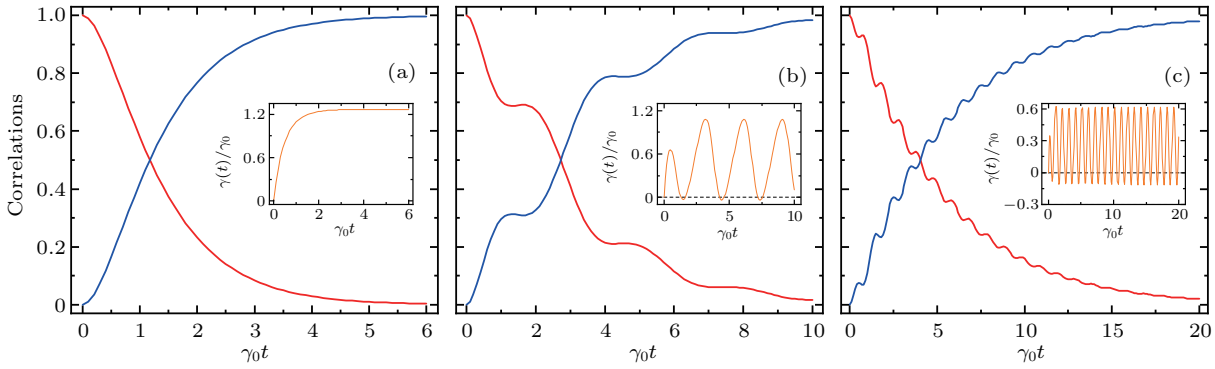


Fig. 3. Accessible information $J_{SE}^{\leftarrow}(t)$ (blue line), the entanglement of formation $E_{SA}(t)$ (red line), for (a) $\eta = 0$, (b) $\eta = 1$, (c) $\eta = 3$, respectively, with $\Delta = 0$, and in the insets, the decay rate $\gamma(t)/\gamma_0$ (orange line) as a function of the dimensionless quantity $\gamma_0 t$, in the Markovian regime where $\lambda/\gamma_0 = 3$. Here, panel (a) shows the monotonicity of information flow in the absence of DDIs, while panels (b) and (c) show non-monotonic behavior, revealing how the dynamics change when DDIs are introduced in the Markovian regime.

Similarly, figure 3(a) represents the dynamics for zero DDIs in Markovian regime,[38] where we can notice that E_{SA} decays monotonically in this case, increasing $J_{SE}^{\leftarrow}(t)$ and leading to $\gamma(t)$ positive throughout the evolution, as shown in the inset of Fig. 3(a). As long as $\gamma(t)$ remains positive, the dynamical map is divisible, indicating a Markovian quantum process. If we introduce DDIs in this Markovian regime, we observed that they disturb this monotonic behavior, depending on the DDI strength: figure 3(b) shows that the behavior of E_{SA} and $J_{SE}^{\leftarrow}(t)$ is no longer monotonic. This change, and the oscillating behavior of $\gamma(t)$ which is periodically negative throughout the dynamics, as shown in the inset of Fig. 3(b) is a clear indication of memory. It is important to note that, initially Markovian environment shows non-Markovian dynamics with the introduction of weak DDIs. We also demonstrate that the degree of non-Markovianity directly depends on the DDI strength. Increasing this causes information to be exchanged between the system S and environment E much more rapidly, protecting the information for a little longer, as we can see in Fig. 3(c). It is thus clear that, increasing the DDI strength in the strong-coupling regime causes the system and environment to exchange information faster without affecting its memory, whereas introducing weak DDIs causes a transition from Markovian (memory-less) to non-Markovian (with memory) dynamics. This is one of the main consequences of DDIs in the Markovian regime.

We also investigate the DDIs ability to protect entanglement, measured in terms of the concurrence. Recently, the authors in Refs. [9] and [10] have investigated using strong DDIs to prevent decoherence, whereas we have found that weak DDIs are sufficient to reduce entanglement degradation in our scheme. Figure 4 compares the behaviors of both DDIs and detuning in the non-Markovian regime, for different values of η (Fig. 4(a)) and Δ (Fig. 4(b)). From this we can conclude that DDIs are better than detuning at protecting entanglement in both regimes, but stronger DDIs are needed for protection in the Markovian regime than in the non-Markovian one. In addition, figure 4(a) shows that DDIs can protect entanglement without destroying the memory effect. We can observe oscillations here for all DDI strengths, indicating that information is being exchanged between the system and environment, as also shown in the inset of Figs. 2(b) and 2(c). In contrast, protection through detuning leads to a loss of coherence, which in turn causes a transition from non-Markovian to Markovian dynamics. Although both DDIs and detuning provide less protection in the Markovian regime compared with the non-Markovian one, the important point here is the oscillation present in the DDI case, which represents a transition from Markovian to non-Markovian dynamics. Based on our present model, we prefer DDIs over detuning for two reasons: they provide better protection, preserving non-Markovianity in the strong-coupling (non-Markovian) regime, and induces memory in the weak-coupling (Markovian) case.

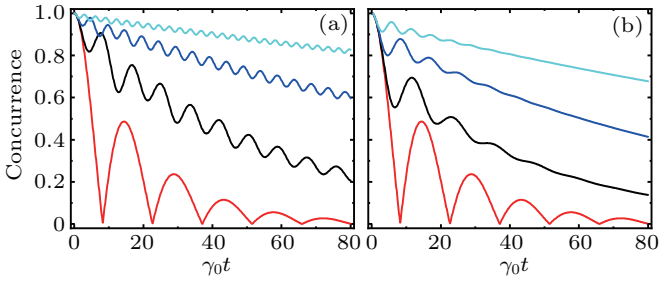


Fig. 4. Concurrence $C(\rho_{SA}(t))$ as a function of dimensionless quantity $\gamma_0 t$ for (a) DDI values of $\eta = 0, 0.3, 0.6, 1$, with $\Delta = 0$ and (b) detuning $\Delta = 0, 0.3, 0.6, 1$, with $\eta = 0$, corresponding to solid red, black, blue, cyan lines, respectively, in the non-Markovian regime where $\lambda/\gamma_0 = 0.1$. This comparison demonstrates that DDIs are better than detuning at protecting entanglement against dissipation.

Non-Markovianity enables information to return from the environment to the system. Figure 5 compares its behavior in the presence of DDIs and detuning. DDIs increase the coupling between the system and ancilla, whereas detuning reduces the coupling between the ancilla and environment, so although they both have the ability to protect information against dissipation but in case of non-Markovianity both have opposite behavior. Here, we use Eq. (9) to calculate numerically the degree of non-Markovianity for both DDIs and detuning over the

whole time evolution. This shows that the non-Markovianity in the non-Markovian regime increases with DDI strength and decreases with detuning, corresponding to the red and blue dotted curves, respectively, in Fig. 5(a). Initially, the non-Markovianity is zero in the Markovian regime (Fig. 5(b)) but it arises with increasing the strength of DDI, representing a transition from a divisible to a non-divisible quantum map. We are able to achieve a much higher degree of non-Markovianity during this transition than as showed in Ref. [45]. From this, it is clear that increasing the coupling between the system and ancilla can protect the information and increase non-Markovianity while decreasing the coupling between the ancilla and environment can also protect the information but degrade the non-Markovianity.

As we have discussed earlier, DDIs show behavior opposite to that of negative detuning, although both are internal effects and can be used to prevent decoherence. Next, we investigate how negative detuning can be used to cancel out the effect of DDIs in the non-Markovian regime. Figure 6(a) shows that adjusting the strengths of both DDI and negative detuning to be equal but opposite can cause their effects to cancel out. We can also observe some non-periodicity for both weak DDIs and negative detuning, which can be eliminated by increasing their strengths in the same way. If we compare Fig. 6(a) ($\eta = -\Delta = 4$) with Fig. 2(a) ($\eta = \Delta = 0$), we can see that although the figures are not identical, they do show the same behavior. Detuning causes a loss of non-Markovianity while protecting the information, whereas DDIs maintain it. Similarly, when DDIs and negative detuning are combined, they can cancel out each other, some coherence and information are lost, as can be seen from the amplitudes and numbers of peaks shown in Fig. 6(a). Further, intersection points of curves along time-axis clarify that these effects cannot cancel each other completely. In the non-Markovian dynamics, which possesses memory mean, future states depend on past states, introducing a weak detuning can eliminate this behavior. In Fig. 6(b) ($\eta = 0, \Delta = 0.5$), there is an initial oscillatory phase, representing non-Markovian behavior, which later on reduces to monotonic behavior, indicating Markovian dynamics. The inset of Fig. 6(b) also reveals that there is a transition at $\gamma_0 t = 26.58$, from a non-divisible to a divisible quantum process. The transition time also depends on the detuning strength. In summary, we conclude that DDIs cause a transition from Markovian to non-Markovian dynamics, while detuning can cause a slow and gradual shift from non-Markovian to Markovian dynamics, depending on the strength. We hope that this DDI trait will play a role in the quantum electrodynamics of atom–cavity systems against dissipation.

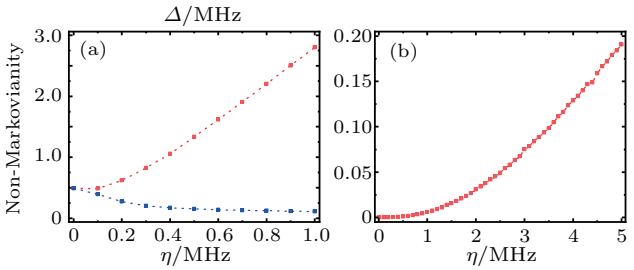


Fig. 5. Non-Markovianity $\mathcal{N}(\rho_{SA})$ as a function of DDI (η) (red curve) and detuning (Δ) (blue curve) throughout the whole time evolution for (a) non-Markovian regime with $\lambda/\gamma_0 = 0.1$ and (b) Markovian regime with $\lambda/\gamma_0 = 3$. These demonstrate that non-Markovianity increases with DDI strength in both regimes, and decreases with increasing detuning in the non-Markovian regime.

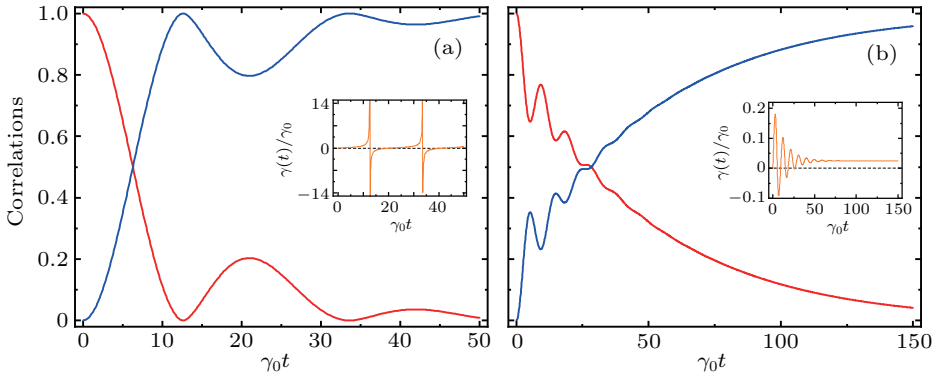


Fig. 6. Accessible information $J^{\leftarrow}_{SE}(t)$ (blue line), the entanglement of formation $E_{SA}(t)$ (red line) for (a) $\eta = -\Delta = 4$, (b) $\eta = 0$, $\Delta = 0.5$, respectively, and in the insets, the decay rate $\gamma(t)/\gamma_0$ (orange line) as a function of dimensionless quantity $\gamma_0 t$ in the non-Markovian regime where $\lambda/\gamma_0 = 0.1$. Here, panel (a) demonstrates that the effects of DDIs and negative detuning can cancel each other out, while panel (b) illustrates the transition from non-Markovian to Markovian dynamics caused by detuning while it protects the information.

5. Conclusion

In this article, we have analyzed the decoherence scheme in the presence of DDI and detuning, in both Markovian and non-Markovian environments. Especially, we have considered a system S that is initially coupled to an ancilla A, where the environment E can gain information about the system S through ancilla A, without disturbing the system S directly. We have shown that, in the non-Markovian regime, even weak DDIs are sufficient to avoid ESD and enable entanglement to be maintained for a very long time. In order to prevent en-

tanglement degradation, we need strong DDIs in the Markovian regime, but weak DDIs are sufficient to cause a transition from Markovian to non-Markovian dynamics. We have also found that the degree of non-Markovianity increases with DDI strength, and decreases with increasing detuning. It is also important to note that DDIs are more effective than detuning, both in protecting against decoherence and producing memory in the Markovian regime, whereas detuning causes a loss of memory (while still protecting the information) in the non-Markovian regime. In addition, we have found that the effects of DDIs and negative detuning can cancel out each other, with a certain loss of coherence and information. We believe that these results will be useful in understanding the role of DDIs and detuning to prevent decoherence, and prove to be helpful when realizing many future quantum protocols in practice.

References

[1] Lidar D A, Chuang I L and Whaley K B 1998 *Phys. Rev. Lett.* **81** 2594
[2] Shor P W 1995 *Phys. Rev. A* **52** R2493
[3] Maniscalco S, Francica F, Zaffino R L, Lo Gullo N and Plastina F 2008 *Phys. Rev. Lett.* **100** 090503
[4] Du J, Rong X, Zhao N, Wang Y, Yang J and Liu R B 2009 *Nature* **461** 1265
[5] Singh H, Arvind and Dorai K 2018 *Phys. Rev. A* **97** 022302
[6] Peng S J, Xu X K, Xu K B, Huang P, Wang P F, Kong X, Rong X, Shi F Z, Duan C K and Du J F 2018 *Sci. Bull.* **63** 336
[7] Gong B, Tu T, Zhou Z Q, Zhu X Y, Li C F and Guo G C 2017 *Sci. Rep.* **7** 18030
[8] Wang S C, Yu Z W, Zou W J and Wang X B 2014 *Phys. Rev. A* **89** 022318
[9] Li Y, Zhou J and Guo H 2009 *Phys. Rev. A* **79** 012309
[10] Khan S and Jan M 2016 *Int. J. Theor. Phys.* **55** 1515
[11] Rosario A, Massoni E and De Zela F 2012 *J. Phys. B: At. Mol. Opt. Phys.* **45** 095501
[12] Huang L Y and Fang M F 2010 *Chin. Phys. B* **19** 090318
[13] Liao X P, Fang M F, Fang J S and Zhu Q Q 2014 *Chin. Phys. B* **23** 020304
[14] Yang G, Lian B W and Nie M 2016 *Chin. Phys. B* **25** 080310
[15] Golkar S and Tavassoly M K 2018 *Chin. Phys. B* **27** 040303
[16] Yazdanpanah N and Tavassoly M K 2016 *J. Opt. Soc. Am. B* **33** 382
[17] Zhang Y Q, Tan L and Barker P 2014 *Phys. Rev. A* **89** 043838
[18] Miller R, Northup T E, Birnbaum K M, Boca A, Boozer A D and Kimble H J 2005 *J. Phys. B: At. Mol. Opt. Phys.* **38** S551
[19] Thompson R J, Rempe G and Kimble H J 1992 *Phys. Rev. Lett.* **68** 1132
[20] Bellomo B, Lo Franco R and Compagno G 2007 *Phys. Rev. Lett.* **99** 160502
[21] Franco R L, Bellomo B, Maniscalco S and Compagno G 2013 *Int. J. Mod. Phys. B* **27** 1345053
[22] Piilo J, Maniscalco S, Härkönen K and Suominen K A 2008 *Phys. Rev. Lett.* **100** 180402
[23] Tang J S, Li C F, Li Y L, Zou X B, Guo G C, Breuer H P, Laine E M and Piilo J 2012 *Europhys. Lett.* **97** 10002
[24] Xu J S, Sun K, Li C F, Xu X Y, Guo G C, Andersson E, Franco R L and Compagno G 2013 *Nat. Commun.* **4** 2851
[25] Fanchini F F, Karpat G, Castelano L K and Rossatto D Z 2013 *Phys. Rev. A* **88** 012105
[26] Chruściński D and Maniscalco S 2014 *Phys. Rev. Lett.* **112** 120404
[27] Addis C, Bylicka B, Chruściński D and Maniscalco S 2014 *Phys. Rev. A* **90** 052103
[28] Smirne A, Cialdi S, Anelli G, Paris M G A and Vacchini B 2013 *Phys. Rev. A* **88** 012108
[29] Vasile R, Olivares S, Paris M A and Maniscalco S 2011 *Phys. Rev. A* **83** 042321
[30] Tan Y G and Liu Q 2016 *Chin. Phys. Lett.* **33** 090303
[31] Zhang S L, Jin C H, Guo J S, Shi J H, Zou X B and Guo G C 2016 *Chin. Phys. Lett.* **33** 120302
[32] Chin A W, Huelga S F and Plenio M B 2012 *Phys. Rev. Lett.* **109** 233601
[33] Huelga S F, Rivas A and Plenio M B 2012 *Phys. Rev. Lett.* **108** 160402
[34] Wolf M M, Eisert J, Cubitt T S and Cirac J I 2008 *Phys. Rev. Lett.* **101** 150402
[35] Breuer H P, Laine E M and Piilo J 2009 *Phys. Rev. Lett.* **103** 210401
[36] Rivas A, Huelga S F and Plenio M B 2010 *Phys. Rev. Lett.* **105** 050403
[37] Luo S, Fu S and Song H 2012 *Phys. Rev. A* **86** 044101
[38] Fanchini F F, Karpat G, Çakmak B, Castelano L K, Aguilar G H, Farías O J, Walborn S P, Ribeiro P H S and de Oliveira M C 2014 *Phys. Rev. Lett.* **112** 210402
[39] Haseli S, Karpat G, Salimi S, Khorashad A S, Fanchini F F, Çakmak B, Aguilar G H, Walborn S P and Ribeiro P H S 2014 *Phys. Rev. A* **90** 052118
[40] Ravets S, Labuhn H, Barredo D, Béguin L, Lahaye T and Browaeys A 2014 *Nat. Phys.* **10** 914
[41] Genkin M, Schönleber D, Wüster S and Eisfeld A 2016 *J. Phys. B: At. Mol. Opt. Phys.* **49** 134001
[42] Liu B H, Li L, Huang Y F, Li C F, Guo G C, Laine E M, Breuer H P and Piilo J 2011 *Nat. Phys.* **7** 931
[43] Smirne A, Brivio D, Cialdi S, Vacchini B and Paris M G A 2011 *Phys. Rev. A* **84** 032112
[44] Bernardes N K, Cuevas A, Orieux A, Monken C, Mataloni P, Sciarrino F and Santos M F 2015 *Sci. Rep.* **5** 17520
[45] Yuan J B, Xing H J, Kuang L M and Yi S 2017 *Phys. Rev. A* **95** 033610
[46] Zurek W H 1981 *Phys. Rev. D* **24** 1516
[47] Zurek W H 2003 *Rev. Mod. Phys.* **75** 715
[48] Li J G, Zou J and Shao B 2010 *Phys. Rev. A* **81** 062124
[49] Henderson L and Vedral V 2001 *J. Phys. A: Math. Gen.* **34** 6899
[50] Gorini V, Kossakowski A and Sudarshan E C G 1976 *J. Math. Phys.* **17** 821
[51] Laine E M, Piilo J and Breuer H P 2010 *Phys. Rev. A* **81** 062115
[52] Breuer H P 2012 *J. Phys. B: At. Mol. Opt. Phys.* **45** 154001
[53] Koashi M and Winter A 2004 *Phys. Rev. A* **69** 022309
[54] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245