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Dynamical properties of ultracold Bose atomic gases in one-dimensional optical lattices created by two schemes*

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An optical lattice could be produced either by splitting an input light (splitting scheme) or by reflecting the input light by a mirror (retro-reflected scheme). We study quantum dynamical properties of an atomic Bose–Einstein condensate (BEC) in the two schemes. Adopting a mean field theory and neglecting collision interactions between atoms, we find that the momentum and spatial distributions of BEC are always symmetric in the splitting scheme which, however, are asymmetric in the retro-reflected scheme. The reason for this difference is due to the local field effect. Furthermore, we propose an effective method to avoid asymmetric diffraction.

Keywords: optical lattice, Bose–Einstein condensate, local field effect

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1. Introduction

Optical lattices are interference fringes of counter-propagating coherent laser beams which have been used as a versatile tool in the fields of laser cooling,[1–3] molecular synthesis,[4,5] precision spectroscopy,[6,7] atomic clocks,[8–10] and quantum simulation of condensed matter physics.[11–13] Moving optical lattices can be used to generate matter-wave bright solitons[14] and to test the Galilean invariance in the spin-orbit-coupled systems.[15] Tilted optical lattices are used for precision measurement of gravity[16] and the fine structure constant,[17] observing quantum tunneling,[18,19] and exploring topological phase transitions.[20] Periodically driven optical lattices can generate artificial gauge fields[21,22] and can also be a powerful tool for discovering novel quantum phases of many-body systems.[23,24] Moreover, bichromatic optical lattices can lead to the formation of localized states in quantum gases[25,26] and can also simulate relativistic wave equation predictions.[27]

Generally, a one-dimensional optical lattice can be created by splitting one optical field with a beam splitter (BS)[28–32] (see Fig. 1(a), named a splitting scheme) or by reflecting an incident optical field with a mirror[33–38] (see Fig. 1(b), named a retro-reflected scheme). The latter is the most common way in the experiments of atomic beam diffraction. So far, the atomic dynamics between these two schemes are considered to be no difference. Because in the above research, the optical lattice is regarded as a perfect periodic potential, in other words, the two counter-propagating laser beams far detuned from the atomic resonance will not be affected by atomic gases. However, recent studies have shown that the atomic motion will produce a back influence on the light propagation through the local modulation of the density-dependent refractive index even in large detuned conditions. This process is called the local field effect (LFE) which results in asymmetric diffraction of Bose–Einstein condensates (BEC) in standing wave optical fields,[39] polaritonic solitons of BEC in optical lattices,[40] self-structuring in optomechanical systems,[41] photon bubbles in ultracold matter,[42] and spontaneous crystallization of ultracold atoms coupling with two incoherent counter-propagating lights.[43]

In this paper, our theoretical results show that when the LFE is significant, atomic dynamics in an optical lattice of the two schemes are different. Especially, when the lattice is formed by the retro-reflected scheme, asymmetric momentum distribution of BEC could be induced. This asymmetric diffraction phenomenon, which appeared in an experiment two decades ago,[44] will reduce the accuracy of precision measurement by BEC diffraction. We propose a simple and effective method to avoid the asymmetric diffraction induced by the LFE.

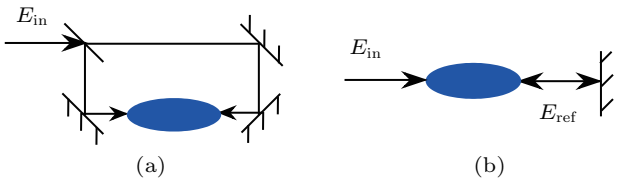


Fig. 1. Two schemes for the formation of optical lattices in BEC diffraction experiments. (a) Splitting scheme. (b) Retro-reflected scheme.

2. Theoretical method

Both the splitting and retro-reflected schemes can be described by the theoretical model below, as shown in Fig. 2. An

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elongated two-level atomic BEC is irradiated by two far off-resonant counter-propagating coherent optical fields, $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$. Outside the condensate, $\boldsymbol{E}_3$ and $\boldsymbol{E}_4$ are scattered optical fields. The BEC is treated within the mean-field approximation, and the condensate wave function $\Psi(\boldsymbol{r},t)$ satisfies

$$\begin{aligned} &\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) \\ &= \left[-\frac{\hbar^2\nabla^2}{2m}+\frac{|\boldsymbol{d}\cdot\mathcal{E}(\boldsymbol{r},t)|^2}{\hbar\Delta}+g|\Psi(\boldsymbol{r},t)|^2\right]\Psi(\boldsymbol{r},t), \end{aligned} \quad (1)$$

where $\Psi(\boldsymbol{r},t)$ is subjected to the normalization condition $\int_{-\infty}^{+\infty}\mathrm{d}\boldsymbol{r}\,|\Psi(\boldsymbol{r},t)|^2=1$, $\hbar$ is the reduced Planck constant, $\mathcal{E}(\boldsymbol{r},t)$ describes the total optical field within the condensate, $\boldsymbol{d}$ is the dipole matrix element between the ground and excited states, $\Delta=\omega_\mathrm{L}-\omega_\mathrm{a}$ is the detuning of the laser frequency ω_L from the atomic resonance frequency ω_a, and g is the effective s-wave atom–atom interaction strength.

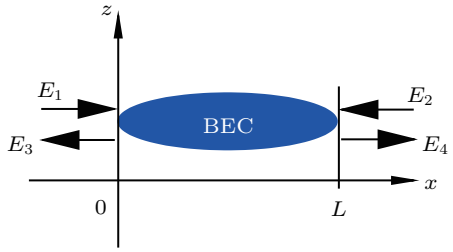


Fig. 2. Light fields incident (E_1 and E_2) and scattered (E_3 and E_4) from the two sides of the BEC. On the boundary, the total optical field and its first derivative are continuous.

The propagation of the optical field which can be written as $\mathcal{E}(\boldsymbol{r},t)=\boldsymbol{E}(\boldsymbol{r},t)\,\mathrm{e}^{-\mathrm{i}\omega_\mathrm{L}t}$ will be influenced by the refractive index of the condensate[45]

$$n(\boldsymbol{r},t)=\left[1-\frac{\mathrm{d}^2N}{\varepsilon_0\hbar\Delta}|\Psi(\boldsymbol{r},t)|^2\right]^{1/2}, \quad (2)$$

where N is the atom number. Obviously, the refractive index is modified by dynamical evolution of the wave function.

As the light transit time through the condensate is negligible compared to the time scale of the atomic center-of-mass motion, we can assume that the envelope of the optical field adiabatically follows the matter wave and is governed by the Helmholtz equation[43]

$$\nabla^2\boldsymbol{E}(\boldsymbol{r},t)+k_\mathrm{L}^2n^2(\boldsymbol{r},t)\,\boldsymbol{E}(\boldsymbol{r},t)=0, \quad (3)$$

where k_L is the wave number of the incident optical field.

We consider a wide transverse width of the laser beams and the matter wave, such that the radial distribution can be considered uniform. Therefore, equations (1)–(3) could be simplified to one dimension directly. We assume that a linearly polarized light, $\boldsymbol{E}(\boldsymbol{r},t)=E(x,t)\,\hat{e}_z$, propagates along the x-axis and a Gaussian atomic wave function distributes in the axial direction initially, which can be written as

$$\Psi(x,0)=\frac{1}{w_\perp\sqrt{w_x}\sqrt[4]{\pi^3}}\exp\left(-\frac{x^2}{2w_x^2}\right), \quad (4)$$

where $w_\perp$ characterizes the radial width. Furthermore, we assume the axial width parameter w_x of the atomic gas is much less than the length L of the considered spatial region. Thus, on the boundary, the wavefunction vanishes, *i.e.*, $\Psi(0,t)=\Psi(L,t)=0$. The incident and scattered optical fields can be written as $E_1=A_1\exp(\mathrm{i}k_\mathrm{L}x)$, $E_2=A_2\exp(-\mathrm{i}k_\mathrm{L}x)$, $E_3=A_3\exp(-\mathrm{i}k_\mathrm{L}x)$, $E_4=A_4\exp(\mathrm{i}k_\mathrm{L}x)$, and satisfy the continuous boundary conditions

$$\begin{aligned} E_1(0^-,t)+E_3(0^-,t)&=E(0^+,t), \\ E_2(L^+,t)+E_4(L^+,t)&=E(L^-,t), \end{aligned} \quad (5)$$

and

$$\begin{aligned} \frac{\partial}{\partial x}E_1(0^-,t)+\frac{\partial}{\partial x}E_3(0^-,t)&=\frac{\partial}{\partial x}E(0^+,t), \\ \frac{\partial}{\partial x}E_2(L^+,t)+\frac{\partial}{\partial x}E_4(L^+,t)&=\frac{\partial}{\partial x}E(L^-,t), \end{aligned} \quad (6)$$

where the superscript $+$ ($-$) denotes the right (left) limit.

In the numerical simulation, the atom species are Rb^{87} with the ground state $5^2\mathrm{S}_{1/2}$ and the excited state $5^2\mathrm{P}_{3/2}$. We set $w_x=50$ μm, $w_\perp=10$ μm, $\Delta=-2$ GHz, and the incident light field is continuous wave. In the splitting scheme, $A_1=A_2$ for the 50:50 BS, whereas in the retro-reflected scheme, E_4 turns into E_2 after reflected by the mirror, then we have $A_2=\sqrt{R}\exp(\mathrm{i}\theta)A_4$, where R is the reflectivity of the mirror and θ is the phase shift. In order to simplify the calculation, we suppose both the loss and the phase shift of the laser are zero after reflected by the mirror, i.e., $R=1$, $\theta=0$, and $A_2=A_4$. Moreover, we neglect collisions between atoms ($g=0$) to highlight the LFE, which can be realized by the Feshbach resonance technique.[46] We use the modified coupled-wave theory[47] to further analyze the optical field and use the transfer matrix method to solve the propagation of the light fields numerically, as we have applied in a previous work.[39]

3. Numerical results

First, we show the momentum distribution of the condensate $\Phi(p,t)=\int_{-\infty}^{+\infty}\Psi(x,t)\exp(\mathrm{i}pt/\hbar)\,\mathrm{d}x/\sqrt{2\pi}$ in Fig. 3, the potential depth of the optical lattice V_0 is $8E_\mathrm{R}$, where E_R is the recoil kinetic energy. Figure 3 illustrates that atomic dynamics depends on how the lattice is formed. In the splitting scheme, the momentum distribution is always symmetric (red solid lines). In contrast, in the retro-reflected scheme, the momentum distribution is asymmetric (black bars). When the intensity of the incident optical field increases, more momentum components arise, and the difference of the momentum distributions between the two schemes is also obvious, as shown in Fig. 4, where $V_0=40E_\mathrm{R}$.

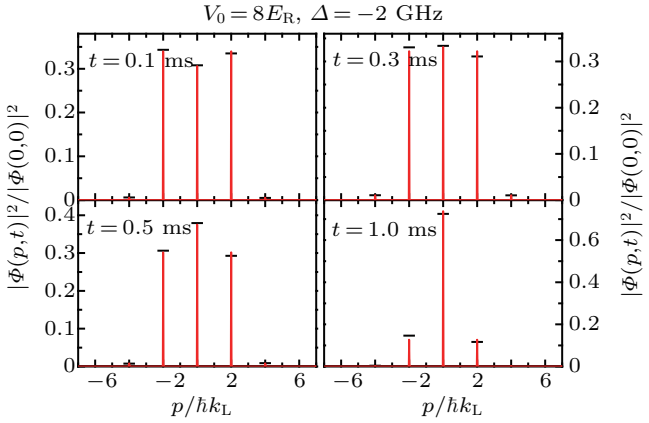


Fig. 3. The momentum distributions of BEC in the splitting scheme (red solid lines) and the retro-reflected scheme (black bars). The potential depth of the optical lattice is $8E_R$.

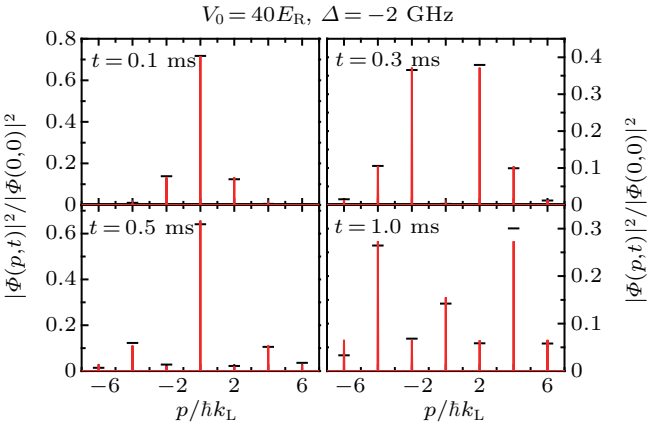


Fig. 4. The momentum distributions of BEC in the splitting scheme (red solid lines) and the retro-reflected scheme (black bars). The potential depth of the optical lattice is $40E_R$.

In the previous work,[39] we have proved that two counter-propagating coherent optical fields with different intensities will give rise to asymmetric momentum distribution of BEC because of the LFE. Therefore, we infer that the intensities I_{in} and I_{ref} of the light fields E_1 and E_2 are imbalanced in the retro-reflected scheme. We plot the ratio I_{ref}/I_{in} over time in Fig. 5 for shallow and deep optical lattices.

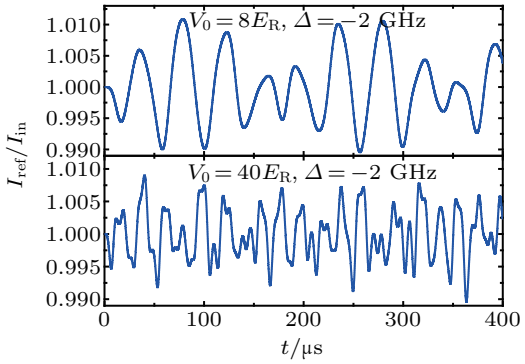


Fig. 5. Ratio of the reflected light intensity to the incident light intensity in the retro-reflected scheme. When the incident light is weak ($V_0 = 8E_R$), a beat-like signal appears in the reflected light. When the incident light is strong ($V_0 = 40E_R$), the reflected light signal becomes irregular.

Figure 5 shows that the reflected light can be weaker or stronger than the incident light over time. Therefore, in the retro-reflected scheme, the BEC is asymmetrically irradiated by the light fields, which leads to an asymmetric momentum distribution. This phenomenon could be explained with the LFE as follows. Due to the interference of the momentum components in the BEC, the refractive index of the condensate has almost a periodic spatial structure. This photonic structure could store optical energy, thus the transmitted light intensity is lowered. However, the evolution of the momentum components changes the interference pattern and the refractive index of the condensate, so the stored energy could be released, thus the transmitted light intensity could be higher than the incident optical intensity. Furthermore, interference information of the condensate could be carried by the transmitted light signal, *i.e.*, I_{ref}, due to the LFE. When the number of the momentum components is small, the corresponding interference pattern is simple, so that the transmitted light signal has a beat-like shape. However, when the number of the momentum components of BEC increases significantly, the corresponding interference pattern becomes very complicated, so that the transmitted light signal becomes irregular.

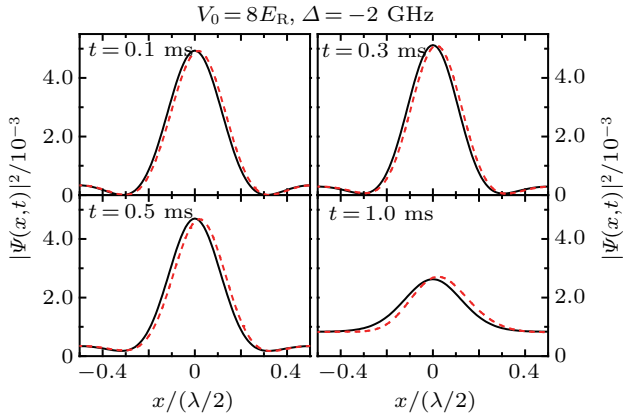


Fig. 6. The spatial distributions of the BEC in a shallow lattice potential well ($V_0 = 8E_R$) within the splitting scheme (black solid line) and the retro-reflected scheme (red dashed line).

In order to fully understand the asymmetric momentum distribution, we further investigate the spatial distributions of the atomic density in Figs. 6 and 7. It is clear that the spatial density distributions of the condensate in the optical lattices created by a BS (black solid lines) and by a mirror (red dashed lines) are different. The central peak of the BEC wave packet is always at the origin in the splitting scheme, which is shifted in the retro-reflected scheme. Comparing the results obtained when the incident light is weak (Fig. 6) and strong (Fig. 7), we find that there is no significant change in the shift of the central peak of the wave packet. This phenomenon indicates the LFE, which is an intrinsic property of the light–atom system, is not affected by the intensity of the incident light. However, the evolution of the BEC wave packet is directly affected by the incident light intensity. The spatial distribution of BEC is

the result of interference of its momentum components. When the incident light is strong, the evolution of the condensate is more intense (Fig. 7), correspondingly, there are more momentum components (Fig. 4). This result is consistent with the previous conclusion of the transmitted light signals in the retro-reflected scheme (Fig. 5).

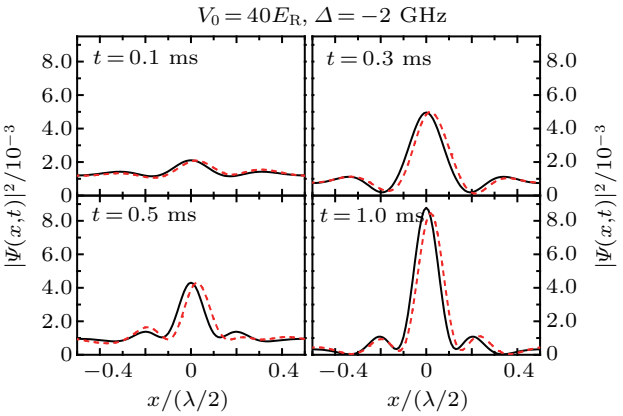


Fig. 7. The spatial distributions of the BEC in a deep lattice potential well ($V_0 = 40E_R$) within the splitting scheme (black solid line) and the retro-reflected scheme (red dashed line).

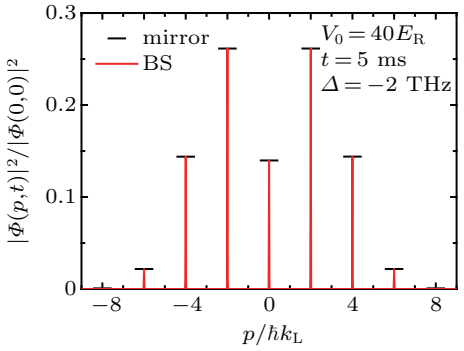


Fig. 8. The momentum distributions of BEC in the splitting scheme (red solid lines) and the retro-reflected scheme (black bars) for the detuning $\Delta = -2$ THz. The potential depth of the optical lattice is $40E_R$.

In experiments, it is easier to produce an optical lattice by adding a mirror, rather than using a BS. However, asymmetric diffraction in the retro-reflected scheme caused by the LFE may reduce the accuracy of some precision measurement experiments, e.g., degradation of the contrast of atomic interferometers. In this case, ways should be made to weaken the LFE. The LFE originates from the coupling of light and atoms, which can be quantified by the dimensionless constant ζ defined as[43]

$$\zeta = -\frac{d^2}{\varepsilon_0 \hbar \Delta} n \frac{w_x}{\lambda_L}, \tag{7}$$

where n is the atomic number density of BEC, λ_L is the wavelength of the incoming laser beams. In our numerical simulation above, $\zeta \sim 10^{-4}$. Therefore, in ultracold atomic gases, LFE should be taken into account when $\zeta > 10^{-4}$ and matter–wave diffraction fringes are formed. As can be seen from Eq. (7), the LFE can be weakened by reducing the number of atoms. However, too few atoms will make the experimental observations unsatisfactory. Therefore, the LFE can be avoided by increasing the detuning. Using an ultra-far off-resonant laser field (Fig. 8, $\Delta = -2$ THz, $\zeta \sim 10^{-7}$), we find that the momentum distributions of the two schemes for creating optical lattices have no difference and are always symmetric.

4. Conclusion

We have studied the dynamics of BEC in optical lattices created by two different schemes. In the splitting scheme, the diffraction of BEC is always symmetric. Whereas in the retro-reflected scheme, the diffraction of BEC is asymmetric. The reason for this difference is that the LFE results in imbalanced intensities of the incident and the reflected optical fields in the retro-reflected scheme. We propose that increasing the detuning between light and atoms is an effective way to avoid the LFE.

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