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Dynamics of entropic uncertainty for three types of three-level atomic systems under the random telegraph noise*

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We study the dynamics of the entropic uncertainty for three types of three-level atomic systems coupled to an environment modeled by random matrices. The results show that the entropic uncertainty in the Ξ-type atomic system is lower than that in the V-type atomic system which is exactly the same as that in the Λ-type atomic system. In addition, the effect of relative coupling strength on entropic uncertainty is opposite in Markov region and non-Markov region, and the influence of a common environment and independent environments in Markov region and non-Markov region is also opposite. One can reduce the entropic uncertainty by decreasing relative coupling strength or placing the system in two separate environments in the Markov case. In the non-Markov case, the entropic uncertainty can be reduced by increasing the relative coupling strength or by placing the system in a common environment.

Keywords: entropic uncertainty, three-level atom, random telegraph noise

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1. Introduction

The nature of the quantum world is inherently unpredictable. By far at the most famous statement of unpredictability lies the Heisenberg uncertainty principle about position and momentum, which was first introduced in 1927.[1] In fact, for arbitrary observable R and S, there is a uncertainty relation which is shown by Robertson in 1929,[2]

$$\Delta R \cdot \Delta S \geq \frac{|\langle [R,S] \rangle|}{2}, \tag{1}$$

with the variance $\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$ (Q is an arbitrary observable). $\langle \cdot \rangle$ is the expectation of the observable in a quantum system ρ, and $[R,S]$ denotes the commutator. In particular, information theory offers a very versatile, abstract framework that allows us to formalize notions like uncertainty and unpredictability. In 1988, Maassen and Uffink proposed the entropic uncertainty relation based on information entropy.[3] It states that

$$H(R) + H(S) \geq \log_2 \frac{1}{c}, \tag{2}$$

where $H(X)$ is Shannon's entropy and c denotes the maximum overlap between any two eigenvectors of the observable R and S. One of the most important recent developments is the generalization of the uncertainty paradigm, which is the entropic uncertainty relation of quantum memory.[4,5] The quantum memory uncertainty relations can be understood through a guessing game.

Indeed, in 2010 Berta *et al.*[6,7] proved the following entropic uncertainty relation

$$H(X|B) + H(Z|B) \geq \log_2 \frac{1}{c} + H(A|B), \tag{3}$$

where the conditional entropy $H(X|B)$ is calculated on the classical-quantum state

$$\rho_{XB} = \sum_i (|\psi_i\rangle\langle\psi_i| \otimes I)\rho_{AB}(|\psi_i\rangle\langle\psi_i| \otimes I) \tag{4}$$

with $|\psi_i\rangle$ as the eigenstate of the observable X and similarly for $H(Z|B)$.

The uncertainty principle in the presence of memory is important for cryptographic applications,[8–10] quantum correlations,[11,12] and witnessing entanglement.[13–17] Furthermore, such uncertainty relations are also important for basic physics. For example, interferometry experiments and the quantum-to-classical transition. Recently, there have been many studies on the dynamics of entropic uncertainty in qubit system,[17–24] but there are few studies on the dynamics of entropic uncertainty in qutrit system.[25,26] Many studies have shown that qutirt system has more advantages than qubit system in quantum information processing.[27–31] For example, key distribution is more secure, quantum locality is much stronger. Therefore, it is meaningful and necessary to study the dynamics of entropic uncertainty in qutrit system.

The aim of of this paper is to investigate the dynamics of the entropic uncertainty for different types of three-level atomic systems under the random telegraph noise (RTN). Our

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choice for the noise is motivated by its relevance in systems of interest for quantum information processing[32–36] and by the fact that RTN is at the root of the $1/f$ noise affecting superconducting qubits.[37] The rest of this paper is arranged as follows. In Section 2, we present our physical model which consider atoms under the random telegraph noise. In Section 3, the analytic and the graphical result of the entropic uncertainty is investigated in Markovian and non-Markovian regimes. Finally, we summarize our work in Section 4.

2. Model

We consider two atoms are initially coupled to a random telegraph noise environment.[38–40] The Hilbert space of the atoms will be labeled by the sub-indices a and b. We assume that the dynamics in the whole Hilbert space is unitary, the Hamiltonian can be expressed in general as

$$H = H_a \otimes I_b + I_a \otimes H_b, \tag{5}$$

where I is the identity matrix of atoms, $H_{a(b)}$ is the single atomic Hamiltonian.

$$H_k = \omega_0 I_k + \gamma\chi_k(t)(|l_1\rangle\langle l_2| + |m_1\rangle\langle m_2| + \text{H.C.}), \quad k \in \{a,b\}, \tag{6}$$

where $|l_1\rangle$, $|l_2\rangle$, $|m_1\rangle$, and $|m_2\rangle$ represent the energy levels and they have different values for different types of atoms.

For the Ξ-type atomic system (In this atom, the transition $|0\rangle \rightleftharpoons |1\rangle$ and $|1\rangle \rightleftharpoons |2\rangle$ are allowed, whereas the transition $|0\rangle \rightleftharpoons |2\rangle$ is forbidden in the electric-dipole approximation), $l_1 = 0$, $l_2 = 1$, $m_1 = 1$, and $m_2 = 2$.

For the V-type atomic system (In this atom, the transition $|0\rangle \rightleftharpoons |1\rangle$ and $|0\rangle \rightleftharpoons |2\rangle$ are allowed, whereas the transition $|1\rangle \rightleftharpoons |2\rangle$ is forbidden in the electric-dipole approximation), $l_1 = 0$, $l_2 = 1$, $m_1 = 0$, and $m_2 = 2$.

For the Λ-type atomic system (In this atom, the transition $|0\rangle \rightleftharpoons |2\rangle$ and $|1\rangle \rightleftharpoons |2\rangle$ are allowed, whereas the transition $|0\rangle \rightleftharpoons |1\rangle$ is forbidden in the electric-dipole approximation), $l_1 = 0$, $l_2 = 2$, $m_1 = 1$, and $m_2 = 2$.

The first terms in Eq. (6) represent the free evolution of atomic system, and the second term provides the coupling of random telegraphic noise. ω_0 is the average energy frequency of an isolated atom. γ is the system–environment coupling constant. The density matrix of the atomic system for time t is given by

$$\rho(t) = \langle U(\{\chi\},t)\rho(0)U^{\dagger}(\{\chi\},t)\rangle_{\{\chi\}}, \tag{7}$$

where $U(\{\chi\},t)$ is the unitary time evolution operator for system, given by

$$U(\{\chi\},t) = U_a(\chi_a,t) \otimes U_b(\chi_b,t), \{\chi\} = \{\chi_a,\chi_b\} \tag{8}$$

with

$$U_a(\chi_a,t) = \exp\left[-\mathrm{i}\int_0^t \mathrm{d}\tau H_a(\tau) \otimes I_b\right],$$

the same as $U_b(\chi_b,t)$, assuming $\hbar = 1$; $\rho(0)$ is the initial state, and we consider it as the maximum entangled state $|\psi\rangle\langle\psi|$ with the following form:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle),$$

and $\langle\cdot\rangle_{\{\chi\}}$ represents the average for a random noisy environment. If both atoms are coupled to their respective environment called independent environments case. The two-atoms density matrix is given by the average over the random phase factors for independent environments case

$$\rho(t) = \langle\langle\rho(\xi_a(t),\xi_b(t))\rangle_{\xi_a}\rangle_{\xi_b}. \tag{9}$$

Else two atoms are coupled to a common environment ($\xi_a = \xi_b$), called a common environment case. The time-evolving state for a common environment case can be given by

$$\rho(t) = \langle\rho(\xi(t))\rangle_{\xi}, \tag{10}$$

with the random phase factors

$$\xi_k(t) = -g\int_0^t \chi_k(\tau)\mathrm{d}\tau, \quad k \in \{a,b\},$$

and the stochastic parameter $\chi(t)$ describing a fluctuator randomly flipping between the values ± 1 at rate λ. In this work, we focus on the paradigmatic noise, *e.g.*, the random telegraphic noise

$$\langle\cos(n\xi(t))\rangle = \begin{cases} \mathrm{e}^{-\lambda t}\left[\cosh(\delta_{n\gamma}t) + \dfrac{\lambda}{\delta_{n\gamma}t}\sinh(\delta_{n\gamma}t)\right], \\ \quad \text{for } \lambda > n\gamma, \\ \mathrm{e}^{-\lambda t}\left[\cos(\delta_{n\gamma}t) + \dfrac{\lambda}{\delta_{n\gamma}t}\sin(\delta_{n\gamma}t)\right], \\ \quad \text{for } \lambda < n\gamma, \end{cases} \tag{11}$$

$$\langle\sin(n\xi(t))\rangle = 0,$$

where $\delta_{n\gamma} = \sqrt{|\lambda^2 - (n\gamma)^2|}$. Here, the relative interaction strengths $g = \gamma/\lambda$ is introduced. Two different regimes of the decay of quantum correlations are identified by g, namely, the Markovian regime ($g < 1$: fast RTN) and the non-Markovian regime ($g > 1$: slow RTN).

In all cases, whether it is the independent environment or the common environment, the state evolution for different atomic systems has the following form.

(i) Ξ-type atomic system:

$$\rho_{\Xi}(t) = \frac{1}{24}\begin{pmatrix} A & 0 & B & 0 & C & 0 & B & 0 & A \\ 0 & G & 0 & G & 0 & G & 0 & G & 0 \\ B & 0 & D & 0 & E & 0 & D & 0 & B \\ 0 & G & 0 & G & 0 & G & 0 & G & 0 \\ C & 0 & E & 0 & F & 0 & E & 0 & C \\ 0 & G & 0 & G & 0 & G & 0 & G & 0 \\ B & 0 & D & 0 & E & 0 & D & 0 & B \\ 0 & G & 0 & G & 0 & G & 0 & G & 0 \\ A & 0 & B & 0 & C & 0 & B & 0 & A \end{pmatrix}; \tag{12}$$

(ii) V-type atomic system:

$$\rho_V(t)=\frac{1}{24}\begin{pmatrix} F & 0 & 0 & 0 & C & E & 0 & E & C \\ 0 & G & G & G & 0 & 0 & G & 0 & 0 \\ 0 & G & G & G & 0 & 0 & G & 0 & 0 \\ 0 & G & G & G & 0 & 0 & G & 0 & 0 \\ C & 0 & 0 & 0 & A & B & 0 & B & A \\ E & 0 & 0 & 0 & B & D & 0 & D & B \\ 0 & G & G & G & 0 & 0 & G & 0 & 0 \\ E & 0 & 0 & 0 & B & D & 0 & D & B \\ C & 0 & 0 & 0 & A & B & 0 & B & A \end{pmatrix}; \quad (13)$$

(iii) Λ-type atomic system:

$$\rho_\Lambda(t)=\frac{1}{24}\begin{pmatrix} A & B & 0 & B & A & 0 & 0 & 0 & C \\ B & D & 0 & D & B & 0 & 0 & 0 & E \\ 0 & 0 & G & 0 & 0 & G & G & G & 0 \\ B & D & 0 & D & B & 0 & 0 & 0 & E \\ A & B & 0 & B & A & 0 & 0 & 0 & C \\ 0 & 0 & G & 0 & 0 & G & G & G & 0 \\ 0 & 0 & G & 0 & 0 & G & G & G & 0 \\ 0 & 0 & G & 0 & 0 & G & G & G & 0 \\ C & E & 0 & E & C & 0 & 0 & 0 & F \end{pmatrix}. \quad (14)$$

For the independent environments case,

$$\begin{aligned} A &= 3+4\langle e^{i\xi_a(t)}\rangle\langle e^{i\xi_b(t)}\rangle+\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle, \\ B &= -1+\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle, \\ C &= 2(1+2\langle e^{i\xi_a(t)}\rangle\langle e^{i\xi_b(t)}\rangle+\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle), \\ D &= 3-4\langle e^{i\xi_a(t)}\rangle\langle e^{i\xi_b(t)}\rangle+\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle, \\ E &= 2(1-2\langle e^{i\xi_a(t)}\rangle\langle e^{i\xi_b(t)}\rangle+\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle), \\ F &= 4+4\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle, \\ G &= -2B. \end{aligned} \quad (15)$$

For the common environment case,

$$\begin{aligned} A &= 3+4\langle e^{i2\xi(t)}\rangle+\langle e^{i4\xi(t)}\rangle, \\ B &= -1+\langle e^{i4\xi(t)}\rangle, \\ C &= 2(1+2\langle e^{i2\xi(t)}\rangle+\langle e^{i4\xi(t)}\rangle), \\ D &= 3-4\langle e^{i2\xi(t)}\rangle+\langle e^{i4\xi(t)}\rangle, \\ E &= 2(1-2\langle e^{i2\xi(t)}\rangle+\langle e^{i4\xi(t)}\rangle), \\ F &= 4+4\langle e^{i4\xi(t)}\rangle, \\ G &= -2B. \end{aligned} \quad (16)$$

3. Results and discussion

In this section, we present the analytic of the entropic uncertainty for Ξ-type atomic system under random telegraphic noise and the graphical results for different atomic systems. In the previous section, we have given the formal solution of time-evolving state so we can obtain the conditional entropy after atom A was measured by Alice(S_x or S_z).

$$\begin{aligned} H(S_x|B) &= H(\rho_{S_xB})-H(\rho_B), \\ H(S_z|B) &= H(\rho_{S_zB})-H(\rho_B), \end{aligned} \quad (17)$$

where $H(\rho)$ is the von-Neumann entropy by $H(\rho)=-\sum_i \lambda_i \log_2 \lambda_i$ and $\rho_B=\mathrm{Tr}_A(\rho(t))$. In the Ξ-type atomic system, the analytic results are as follows:

i) Eigenvalues of ρ_{S_xB}: $\lambda_1^x=\lambda_2^x=\lambda_3^x=1/3$, and others are zero.

ii) Eigenvalues of ρ_{S_zB}:

$$\begin{aligned} \lambda_1^z &= 0 \\ \lambda_2^z &= \frac{1-\beta}{6} \\ \lambda_3^z &= \frac{1+\beta}{6} \\ \lambda_4^z &= \lambda_5^z = \frac{1-\beta}{12} \\ \lambda_6^z &= \lambda_7^z = \frac{1}{24}\left(-\sqrt{16\alpha^2+(\beta-1)^2}+\beta+3\right) \\ \lambda_8^z &= \lambda_9^z = \frac{1}{24}\left(\sqrt{16\alpha^2+(\beta-1)^2}+\beta+3\right), \end{aligned} \quad (18)$$

where $\alpha=\langle e^{i\xi_a(t)}\rangle\langle e^{i\xi_b(t)}\rangle$, $\beta=\langle e^{i2\xi_a(t)}\rangle\langle e^{i2\xi_b(t)}\rangle$ for the independent environments case, $\alpha=\langle e^{i2\xi(t)}\rangle$, $\beta=\langle e^{i4\xi(t)}\rangle$ for the common environments case.

iii) Eigenvalues of ρ_B: $\lambda_1=\lambda_2=\lambda_3=1/3$.

In this paper we assume $\xi_a(t)=\xi_b(t)$. Therefore, the entropic uncertainty can be calculated as follows:

$$U_L=-\sum_i \lambda_i^z \log_2 \lambda_i^z-\log_2 3. \quad (19)$$

The numerical results of the entropic uncertainty for the three atomic systems are analyzed and discussed below.

3.1. The Markovian regime

The dynamics of the entropic uncertainty for three types of atomic systems under the Markov RTN in both the independent environments case and the common environment case are shown in Fig. 1. In the Markovian regime, the entropic uncertainty of three types of atomic systems keep increasing with time in both the independent environments case and the common environment case. However, the entropic uncertainty in the Ξ-type atomic system is lower than that in the V-type and Λ-type atomic systems which can be explained by the analytic result, because in the Ξ-type atomic system, the entropic uncertainty of S_x is always zero under RTN. The evolution of the entropic uncertainty in the V-type atomic system is exactly the same as that in the Λ-type atomic system. One can see that the entropic uncertainty grows faster in the common environment case than that in the independent environments case when the relative interaction strength g is the same. In the independent environments case, the increase of entropic uncertainty is accelerated with the increase of relative interaction strength g, and the higher the relative coupling strength is, the greater the entropic uncertainty. The same is true in the common environment case.

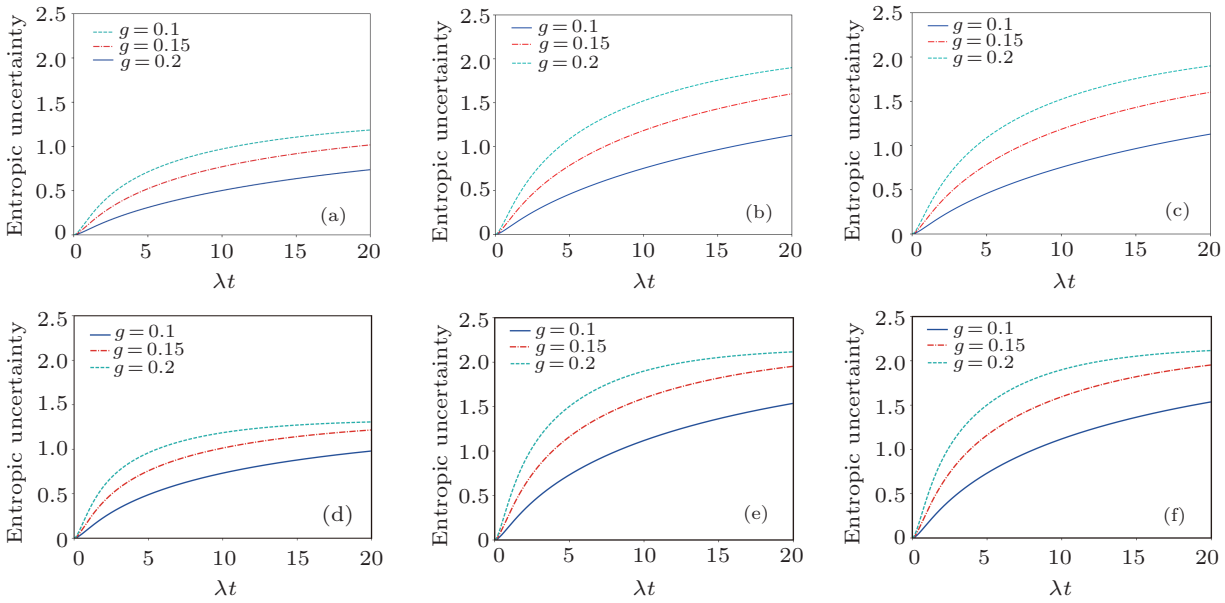


Fig. 1. The evolution of the entropic uncertainty for three types of atomic systems: (a) and (d) the Ξ-type atomic systems; (b) and (e) the V-type atomic systems; (c) and (f) the Λ-type atomic systems under the Markov RTN in both the independent environments case [(a), (b), (c)] and the common environments case [(d), (e), (f)] for fixed values of relative interaction strength g.

3.2. The non-Markovian regime

The numerical results of the non-Markovian RTN case are shown in Fig. 2. As expected from the non-Markovian nature of the noise, the entropic uncertainty for three types of atomic systems are oscillating function of time. Like the Markovian case, the entropic uncertainty in the Ξ-type atomic system is lower than that in the V-type and Λ-type atomic systems. The last two have exactly the same entropic uncertainty evolution. Oscillations of the entropic uncertainty become more and more prominent in the atoms system as the relative strength g increases. Contrary to the Markov's case, it is clearly that as the interaction intensity increases, the minimum value of entropic uncertainty decreases in both the independent environments case and the common environment case. In the non-Markovian regime, the entropic uncertainty in the common environment case is smaller than that in the independent environment case which is also different from the Markovian regime. This can be interpreted as the characteristic non-Markovianity of the information will lead to information being able to not only outflowing but also backflowing as strong coupling induces the fluctuations in the trace distance of the qutrit system. And the backflow of information is stronger in the common environment case.

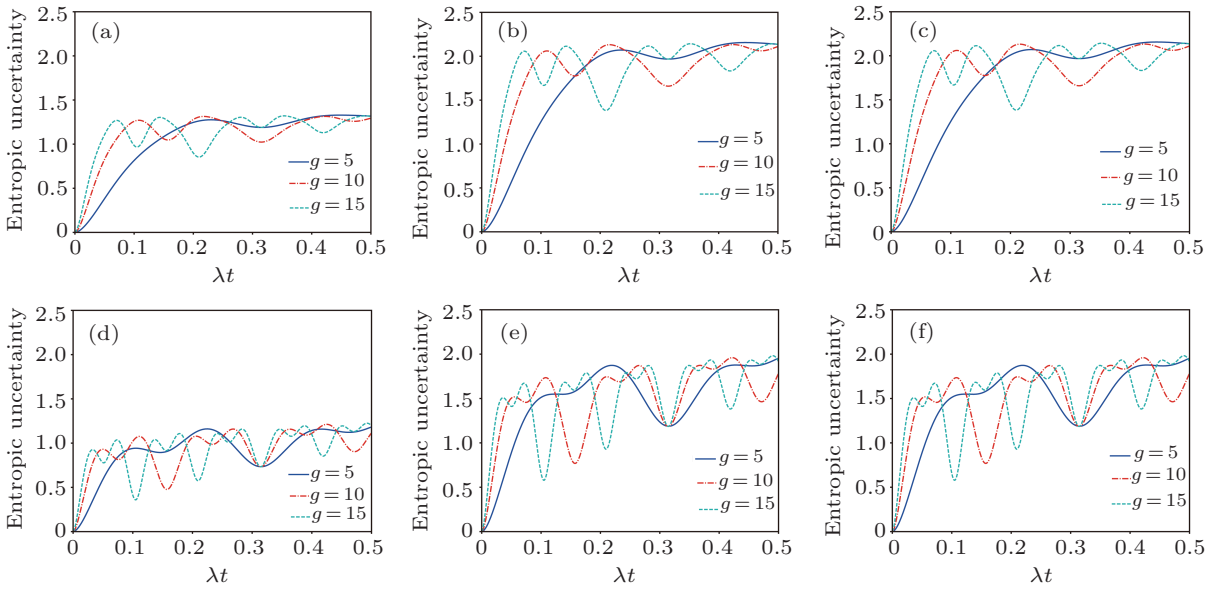


Fig. 2. The evolution of the entropic uncertainty for three types of atomic systems: (a) and (d) for the Ξ-type atomic systems; (b) and (e) for the V-type atomic systems; (c) and (f) for the Λ-type atomic systems under the non-Markovian RTN in both the independent environments case [(a), (b), (c)] and the common environments case [(d), (e), (f)] for fixed values of relative interaction strength g.

4. Conclusions

We have investigated the dynamics of the entropic uncertainty for three types of three-level atomic systems under the random telegraph noise (RTN). It is clearly seen that under the RTN case the entropic uncertainty in the Ξ-type atomic system is lower than that in the V-type and Λ-type atomic systems. Meanwhile, the evolution of the entropic uncertainty in the V-type atomoc system is exactly the same as that in the Λ-type atomic system. We have found that in the Markovian regime, two separate environments can help reduce entropic uncertainty, which can also be reduced by decreasing the relative coupling strength g. In the non-Markovian regime, a common environment is more favorable to keep the entropic uncertainty at a low value. And in this case, the relative coupling strength g plays an opposite role, increasing g can reduce the entropic uncertainty. These results enable us to select different operations for different conditions to reduce entropic uncertainty when dealing with quantum information problems.

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