ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理
Chinese Physics B

The evolution of cooperation in public good game with deposit

Xian-Jia Wang(王先甲), Wen-Man Chen(陈文嫚)

Citation: Chin. Phys. B . 2019, 28(8): 080201 . **doi:** 10.1088/1674-1056/28/8/080201
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Evolutionary game dynamics of combining the Moran and imitation processes

Xian-Jia Wang(王先甲), Cui-Ling Gu(顾翠伶), Shao-Jie Lv(吕少杰), Ji Quan(全吉)

Chin. Phys. B . 2019, 28(2): 020203 . **doi:** 10.1088/1674-1056/28/2/020203

Effects of the planarity and heterogeneity of networks on evolutionary two-player games

Xu-Sheng Liu(刘旭升), Zhi-Xi Wu(吴枝喜), Jian-Yue Guan(关剑月)

Chin. Phys. B . 2018, 27(12): 120203 . **doi:** 10.1088/1674-1056/27/12/120203

Stochastic evolutionary public goods game with first and second order costly punishments in finite populations

Ji Quan(全吉), Yu-Qing Chu(储育青), Wei Liu(刘伟), Xian-Jia Wang(王先甲), Xiu-Kang Yang(阳修康)

Chin. Phys. B . 2018, 27(6): 060203 . **doi:** 10.1088/1674-1056/27/6/060203

On fairness, full cooperation, and quantum game with incomplete information

Zhen-Zhou Lei(雷振州), Bo-Yang Liu(刘博阳), Ying Yi(易英), Hong-Yi Dai(戴宏毅), Ming Zhang(张明)

Chin. Phys. B . 2018, 27(3): 030202 . **doi:** 10.1088/1674-1056/27/3/030202

Improving the secrecy rate by turning foes to allies: An auction scheme

Ma Ya-Yan, Wang Bao-Yun

Chin. Phys. B . 2015, 24(9): 090209 . **doi:** 10.1088/1674-1056/24/9/090209

The evolution of cooperation in public good game with deposit*

Xian-Jia Wang(王先甲)[1,2] and Wen-Man Chen(陈文嫚)[1,†]

[1] *Economics and Management School, Wuhan University, Wuhan* 430070, *China*
[2] *Institute of System and Engineer, Wuhan University, Wuhan* 430070, *China*

(Received 5 May 2019; revised manuscript received 6 June 2019; published online 25 July 2019)

The emergence of cooperation still remains a fundamental conundrum in the social and behavior sciences. We introduce a new mechanism, deposit mechanism, into theoretical model to explore how this mechanism promotes cooperation in a well-mixed population. Firstly, we extend the common binary-strategy combination of cooperation and defection in public good game by adding a third strategy, namely, deposit cooperation. The players with deposit cooperation strategy pay a deposit in advance to obtain the benefits of public good at a lower contributions compared with the players with cooperation strategy, when the provision of public good is successful. Then, we explore the evolution of cooperation in the public good game with deposit by means of the replicator dynamics. Theoretical computations and stimulations show that the deposit mechanism can promote cooperation in a well-mixed population, and the numbers of equilibrium point are determined by variables of public good game. On the one hand, when the coexistence of cooperators and defectors is the stable equilibrium point in the evolutionary system, increasing the threshold of public good and adopting the weak altruism way for share benefits can enhance the level of cooperation in the population. On the other hand, if the coexistence of deposit cooperators and defectors is the stable equilibrium point, it is effective to promote the deposit cooperation by lowering the values of discount and deposit, and raising the threshold of public good.

Keywords: evolutionary game theory, cooperation, deposit, weak altruism, discount rate

PACS: 02.50.Le, 05.45.Pq **DOI:** 10.1088/1674-1056/28/8/080201

1. Introduction

Theoretically, defectors will outperform cooperators within a well-mixed group. However, the emergence of cooperation in a group is widely observed by social and biological scientists. Until recently, the understanding of the emergence of cooperation still remains a fundamental conundrum in social and behavior sciences.[1–6] The conundrum is a so-called social dilemma and has been extensively investigated as the public good game (PGG) in an enormous body of theoretical studies.[7–9] In the PGG, cooperators confer benefits on other group members with some contributions to the public good, while defectors exploit the benefits without any contributions. From the perspective of evolution, a defector can obtain a higher individual payoff compared with a cooperator in the PGG. As a result, defection is regarded to be a rational strategy. Therefore, the natural selection process will drive the elimination of cooperation within a population. In order to resist defection and promote cooperation, many effective mechanisms have been put forward in evolutionary game studies, such as kin selection,[10] direct reciprocity,[11–13] group selection, reputation,[14–16] reward,[17–20] punishment,[21–28] spatial structure,[29–41] and so on.[42–49]

To further reveal the underlying reasons of the emergence of cooperation in the PGG, in this paper, we explore a new mechanism, *i.e.*, deposit mechanism, which is originated from our real life. In the realistic world, people often prefer to get a product or service at a relatively lower price. Therefore, many advertising strategies are invented by companies to promote the selling. One effective way is to provide some discounts under particular conditions. For example, it is widespread that a discount can be obtained by the consumers if they are willing to pay a deposit in advance.[50] Intuitively, obtaining a public good at a lower contributions might motivate players to choose a cooperation strategy in the PGG. Nevertheless, it is still unclear whether such a deposit mechanism can elicit cooperation from the theoretical view. Considering the role of deposit played in the promotion of cooperation, it is valuable to investigate how such a new mechanism can affect the evolutionary dynamics of cooperation, and whether it can resist defection and promote cooperation.

In a theoretical model of the PGG with a deposit mechanism constructed in this paper, three different strategies are proposed:[51] cooperators (C), deposit cooperators (DC), and defectors (D). For clarity, we divide each round of one-shot game into three stages and different types of players adopt different behaviors in different stages. Specifically, in the first stage, DC-players pay a deposit; in the second stage, DC- and C-players contribute to the public good, but defectors contribute nothing; in the third stage, the benefits of provided

*Project supported by the National Natural Science Foundation of China (Grant Nos. 71871171, 71871173, and 71701076).
†Corresponding author. E-mail: wenmanchen@whu.edu.cn
© 2019 Chinese Physical Society and IOP Publishing Ltd

public good will be allocated among players in the one-shot game. In this paper, we assume that the public good will be provided only when the total contributions reach a certain threshold which is the minimum contributions to the provision of the public good. So if the total contributions provided by DC- and C-players are less than the threshold, the provision of the public good is failure. In this case, the one-shot game will end, and the total contributions other than deposit will be returned to DC- and C-players in the second stage. Otherwise, the public good can be provided, and the benefits of the public good will be shared among group members in the third stage. In addition, we consider two types of benefit-sharing ways, which differ on whether or not a contributor in the PGG may oneself benefit, namely "weak altruism" and "strong altruism". Moreover, we also consider the contributions' time value, which is different from the previous work.[52] In view of the aforementioned description, we explore the evolution of cooperation in the PGG with deposit by means of the replicator dynamics.[53,54]

2. Theoretical model

Consider the PGG with deposit mechanism in an infinitely large and well-mixed population. For clarity, we divide each one-shot round game with this mechanism into three stages. In stage 1, n players are randomly selected from the well-mixed population and then form a group (where $n \geq 2$). All players in the group are required to decide whether to pay a deposit or not. The players paying a deposit will provide c_1 resources in this stage and must contribute to the public good in next stage. In stage 2, the players who do not pay a deposit need to make a decision whether to contribute to the public good. Cooperators who decide to contribute will provide c resources and defectors contribute nothing. In addition, the players paying a deposit in stage 1 will be given a $(1-k)$ $(0<k<1)$ discount on c so that they only need to provide $kc-c_1$ resources in stage 2. If the amount of the integrated contributions $kcn_{\mathrm{DC}}+cn_{\mathrm{C}}$, where n_{DC} and n_{C} are the numbers of the players who pay the deposit in stage 1 and players who contribute c in stage 2, are less than the threshold T ($T<kcn$), an one-shot round game will end. As a result, the resources contributed in stage 2 will be returned intact to those who contribute, but the deposit will not be refunded to the players who pay the deposit. On the contrary, if the integrated contributions $kcn_{\mathrm{DC}}+cn_{\mathrm{C}}$ reach T, the public good will be successfully provided and the benefits of the public good will be shared among group members in stage 3. Furthermore, the benefits of the public good are distributed in the following different way: one is "weak altruism (W)", in the case of which the benefits of it will be equally shared among all n players including themselves; the other is "strong altruism (S)", in the case of which the benefits will be shared equally among $n-1$ other co-players except themselves. Combined with the above-mentioned description, we use Fig. 1 to describe the whole process of each one-shot public good game with deposit mechanism. To explore the evolution of cooperation, we consider three different strategies: deposit cooperators (DC) who pay c_1 deposit in stage 1 and contribute $kc-c_1$ $(kc>c_1)$ in stage 2, cooperators (C) who only contribute c in stage 2, and defectors (D) who neither pay deposit in stage 1 nor contribute in stage 2. In this paper, each round of one-shot public good game has multiple stages. In fact, it is true that money has a time cost in real life. In order to be closer to the reality, here we introduce a discount rate into the theoretical model and suppose that discount rate for each stage is δ $(0<\delta<1)$. Therefore, when the integrated contributions $kcn_{\mathrm{DC}}+cn_{\mathrm{C}}$ reach the threshold T, the public good will be provided and all group members can obtain benefit P_i^j in stage 3, where $i=\mathrm{C},\mathrm{DC},\mathrm{D}$; $j=\mathrm{S},\mathrm{W}$. In the case of W and S, the benefits of each player can be calculated as follows respectively:

$$
\begin{aligned}
&P_{\mathrm{C}}^{\mathrm{W}}=P_{\mathrm{DC}}^{\mathrm{W}}=P_{\mathrm{D}}^{\mathrm{W}}=\frac{r\left(kcn_{\mathrm{DC}}+cn_{\mathrm{C}}\right)}{n},\\
&P_{\mathrm{C}}^{\mathrm{S}}=\frac{r\left[kcn_{\mathrm{DC}}+c\left(n_{\mathrm{C}}-1\right)\right]}{n-1},\\
&P_{\mathrm{DC}}^{\mathrm{S}}=\frac{r\left[kc\left(n_{\mathrm{DC}}-1\right)+cn_{\mathrm{C}}\right]}{n-1},\\
&P_{\mathrm{D}}^{\mathrm{S}}=\frac{r\left(kcn_{\mathrm{DC}}+cn_{\mathrm{C}}\right)}{n-1},
\end{aligned}
$$

where r is the return rate of the integrated contributions. Conversely, when $kcn_{\mathrm{DC}}+cn_{\mathrm{C}}<T$ which means the provision of public good is failure, the contributions given by C- and DC-players in stage 2 will be returned intact to them but the deposit c_1 will not be refunded to DC-players. In this case, the integrated contributions in stage 2 $kcn_{\mathrm{DC}}+cn_{\mathrm{C}}$ are multiplied by discount rate δ and then distributed to players who have contributed to public good, if any. So DC- and C-players can receive benefits Q_i^j and the benefits can be calculated as follows in W and S cases respectively,

$$
\begin{aligned}
&Q_{\mathrm{C}}^{\mathrm{W}}=Q_{\mathrm{DC}}^{\mathrm{W}}=\frac{\delta cn_{\mathrm{C}}+\delta\left(kc-c_1\right)n_{\mathrm{DC}}}{n_{\mathrm{C}}+n_{\mathrm{DC}}},\\
&Q_{\mathrm{D}}^{\mathrm{W}}=0,\\
&Q_{\mathrm{C}}^{\mathrm{S}}=\frac{\delta c\left(n_{\mathrm{C}}-1\right)+\delta\left(kc-c_1\right)n_{\mathrm{DC}}}{n_{\mathrm{C}}+n_{\mathrm{DC}}-1},\\
&Q_{\mathrm{DC}}^{\mathrm{S}}=\frac{\delta cn_{\mathrm{C}}+\delta\left(kc-c_1\right)\left(n_{\mathrm{DC}}-1\right)}{n_{\mathrm{C}}+n_{\mathrm{DC}}-1},\\
&Q_{\mathrm{D}}^{\mathrm{S}}=0.
\end{aligned}
$$

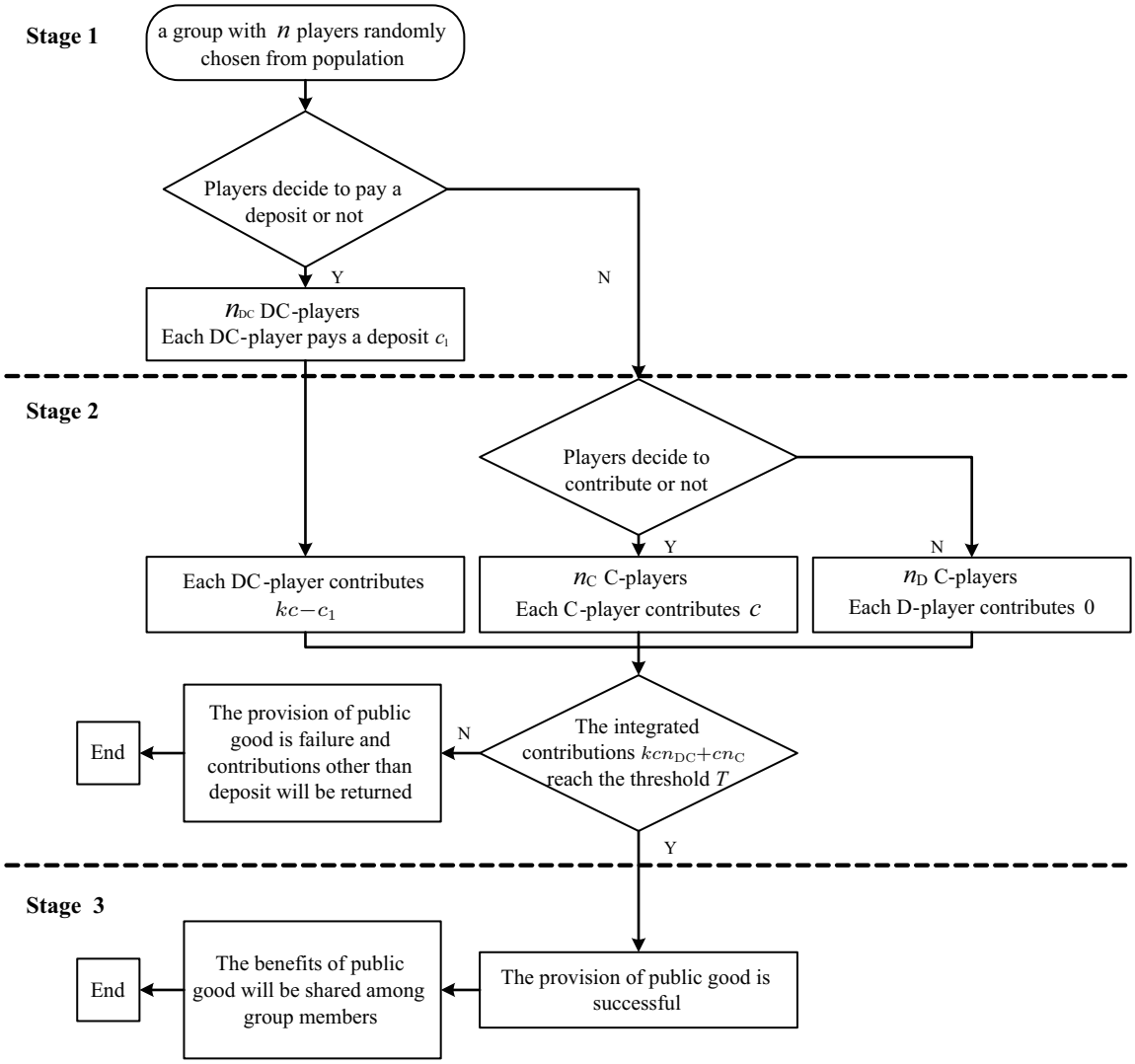


Fig. 1. The specific process of each one-shot round game with deposit mechanism. In stage 1, the randomly chosen n players have to decide whether to contribute to the public good. In stage 2, all players are divided into three different types, namely DC-player, C-player and D-player according to the behavior they adopt in stage 1 and 2. In stage 3, the benefits of the provided public good are allocated among all players.

For simplicity, let $\Omega = \{(n_C, n_{DC}, n_D) \,|n_C + n_{DC} + n_D = n, n_C \geq 0, n_{Dc} \geq 0, n_D \geq 0\}$ be the set that is the list of all possible numbers of three different types' players in a group with n players, $\Omega_1 = \{(n_C, n_{DC}, n_D) \,|kcn_{DC} + cn_C \geq T\}$ be the set that the list of the numbers of three types' players that can successfully provide public good, and $\Omega_0 = \{(n_C, n_{DC}, n_D) \,|kcn_{DC} + cn_C < T\}$ represent the set that is the list of the numbers of three types' players that cannot provide public good. Also, the three sets satisfy condition $\Omega = \Omega_1 + \Omega_0$. To investigate how the set Ω_1 varies with the threshold T, three different sceneries are considered. Presented results in Fig. 2 demonstrate that the set Ω_1 becomes smaller when the threshold T of public good increases from 6 to 12 from Figs. 2(a)–2(c). This tendency indicates that the higher the threshold T of public good, the more difficult the successful provision of the public good. So according to the above description, the cost of each player with different strategies can be written as follows:

$$\sigma_C = \begin{cases} \sigma_C^0 = 0, & (n_C, n_{DC}, n_D) \in \Omega_0, \\ \sigma_C^1 = (1+\delta)c, & (n_C, n_{DC}, n_D) \in \Omega_1, \end{cases}$$

$$\sigma_{DC} = \begin{cases} \sigma_{DC}^0 = (1+\delta)c_1, & (n_C, n_{DC}, n_D) \in \Omega_0, \\ \sigma_{DC}^1 = (1+\delta)(kc-c_1) + (1+\delta)^2 c_1, & (n_C, n_{DC}, n_D) \in \Omega_1, \end{cases}$$

$$\sigma_D = \begin{cases} \sigma_D^0 = 0, & (n_C, n_{DC}, n_D) \in \Omega_0, \\ \sigma_D^1 = 0, & (n_C, n_{DC}, n_D) \in \Omega_1. \end{cases}$$

In reality, the cost of a DC-player will be less than that of a D-player when the provision of public good is successful, so that

$\sigma_{\rm DC}^1 < \sigma_{\rm C}^1$, then $c_1 < c(1-k)/\delta$. Therefore, the payoffs of C-, DC-, and D-players can be written respectively as follows:

$$\pi_i = \begin{cases} \pi_i^0 = \theta\left(P_i^{\rm W} - \sigma_i\right) + (1-\theta)\left(P_i^{\rm S} - \sigma_i\right), & (n_{\rm C}, n_{\rm DC}, n_{\rm D}) \in \Omega_0, \\ \pi_i^1 = \theta\left(Q_i^{\rm W} - \sigma_i\right) + (1-\theta)\left(Q_i^{\rm S} - \sigma_i\right), & (n_c, n_{\rm DC}, n_{\rm D}) \in \Omega_1, \end{cases}$$

where $\theta = \begin{cases} 1, & \text{the case of W,} \\ 0, & \text{the case of S,} \end{cases}$ $i = {\rm C}, {\rm DC}$, and D.

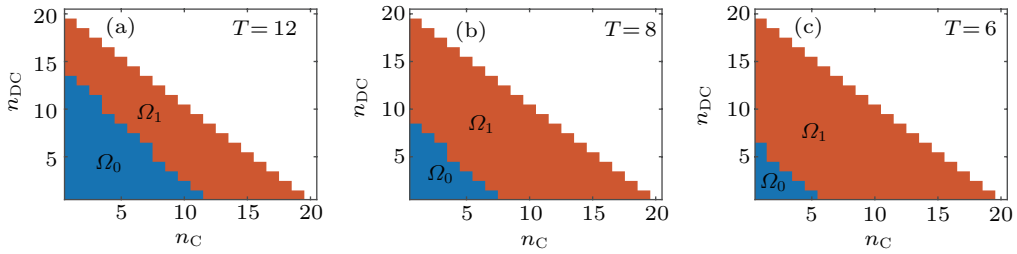


Fig. 2. The changes of the magnitude of set Ω_1 and Ω_0 with increasing the threshold T. The red area represents the magnitude of set Ω_0, while the blue area represents the magnitude of set Ω_1. The higher the threshold T of public good, the larger the magnitude of the red area.

Next, we apply replicator equations in analyzing the evolutionary dynamics of three strategies in this model. We denote the frequencies of C-, DC-, and D-players in the infinite population with x, y, and z respectively $(x+y+z=1, x, y, z \geq 0)$. The expected payoff values for C-, DC-, and D-players are expressed as $f_{\rm C}$, $f_{\rm DC}$, and $f_{\rm D}$ respectively and can be calculated as

$$f_i = \sum_{(n_{\rm C}, n_{\rm DC}, n_{\rm D}) \in \Omega_1} \binom{n-1}{n_{\rm C}} \binom{n-n_{\rm C}-1}{n_{\rm DC}} x^{n_{\rm C}} y^{n_{\rm DC}} z^{n_{\rm D}} \pi_i^1 + \sum_{(n_{\rm C}, n_{\rm DC}, n_{\rm D}) \in \Omega_0} \binom{n-1}{n_{\rm C}} \binom{n-n_{\rm C}-1}{n_{\rm DC}} x^{n_{\rm C}} y^{n_{\rm DC}} z^{n_{\rm D}} \pi_i^0,$$

where $\binom{n-1}{n_{\rm C}} \binom{n-n_{\rm C}-1}{n_{\rm DC}}$ means the probability that $n-1$ co-players consist of n_i ($i = {\rm C}, {\rm DC}$, and D) i-players. Accordingly the average payoff of the population can be written as $\overline{f} = x f_{\rm C} + y f_{\rm DC} + z f_{\rm D}$. Accordingly, the replicator equations of these three strategies in the well-mixed population are respectively given as

$$\begin{aligned} \frac{{\rm d}x}{{\rm d}t} &= x\left(f_{\rm C} - \bar{f}\right), \\ \frac{{\rm d}y}{{\rm d}t} &= y\left(f_{\rm DC} - \bar{f}\right), \\ \frac{{\rm d}z}{{\rm d}t} &= z\left(f_{\rm D} - \bar{f}\right). \end{aligned}$$

3. Dynamics

In this part, we will comprehensively explore how the evolutionary dynamics of C, DC, and D strategies are influenced by the strengths of variables and find out the stable equilibrium points in dynamical system. Within the three-strategy environment, the evolutionary dynamics take place in the state space $S_3 = \{(x,y,z) | x,y,z \geq 0, x+y+z=1\}$. Indeed, the three homogeneous states in which all players of the population are C-players $(x=1)$, DC-players $(y=1)$, and D-players $(z=1)$ correspond to three vertices of the simplex S_3.

For given N, T, θ, r, c which are parameters of the PGG, it can be observed that the number of equilibria in the evolutionary system is determined by parameters k, c_1, and δ. When the values of parameters k, c_1, and δ satisfy the condition $c(1-k)/\delta > c_1 > \delta k c/1+2\delta$, there will only exist one stable equilibrium point in the dynamic system. However, there will appear two stable equilibrium points in the evolutionary dynamics in the case of $c_1 \leq \delta k c/1+2\delta$ and $c_1 < c(1-k)/\delta$ (see Appendix A for detail). In what follows, we consider the evolutionary dynamics of these two typical cases under different values of parameters and discuss the resulting game dynamics one by one.

3.1. Case 1: $c(1-k)/\delta > c_1 > \delta k c/1+2\delta$

In this case, according to preliminary numerical simulations, presented results in Figs. 3(a)–3(f) show that there is no interior equilibrium point and all interior orbits in the simplex S_3 converge to the stable equilibrium point $(\bar{x}, 0, 1-\bar{x})$ on the edge C–D, irrespective of the initial condition. The direction of evolution on the edge C–DC is from C to DC so that C-players always prevail in the competition between C-players and D-players, and from DC to D on the edge DC–D so that D-players dominate DC-players in the competition between them. But the edge C–D of S_3 is separated into two segments by the stable equilibrium point $(\bar{x}, 0, 1-\bar{x})$, and both orbits of the two segments converge to the equilibrium point. Moreover, all the three vertices of the simplex S_3 corresponding to full C-players $(x=1)$, DC-players $(y=1)$, and D-players $(z=1)$ are

unstable equilibrium points in the evolutionary system. Thus, random drift and occasional invasion of the missing D-player will eventually bring the population to the stable state, which is the coexistence of cooperation and defection.

Next, we study how the values of parameters T and θ affect the equilibrium point $(\bar{x}, 0, 1-\bar{x})$ on the edge C–D. As shown in Fig. 3, for a fixed value of T, $\bar{x}$ slightly increases when θ changes from 0 to 1. For a high $T = 12$, $\bar{x}$ shows an upward trend, increasing from 0.49 to 0.5 when θ changes from 0 to 1. Besides, for $T = 8$ or $T = 6$, $\bar{x}$ rises from 0.28 to 0.3, or from 0.13 to 0.18 respectively when θ increases from 0 to 1. Furthermore, for a fixed value of θ , $\bar{x}$ increases when T increases. For $\theta = 0$ or 1, when T increases from 6 to 12, $\bar{x}$ significantly rises from 0.2 to 0.9 or from 0.19 to 0.49 respectively. Therefore, both adopting the "weak altruism" benefit-sharing way and increasing the threshold of public good are effective to enhance the level cooperation in the well-mixed population.

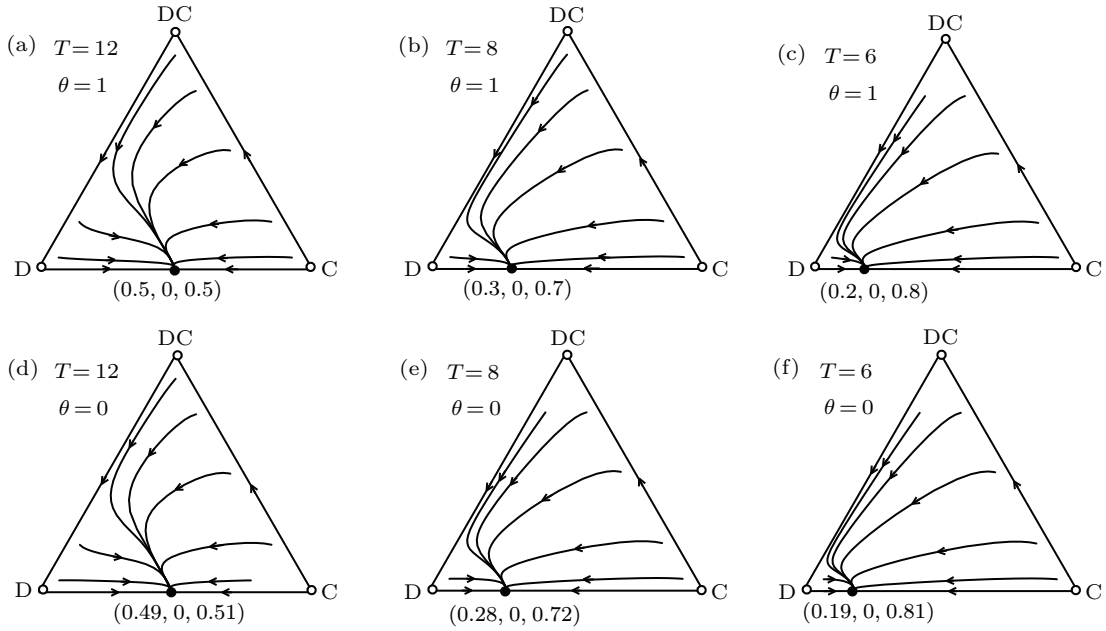


Fig. 3. The evolutionary dynamics of the three strategies of T and θ in the public good game. Filled cycles in S_3 denote stable equilibria and open cycles denote unstable equilibria. All orbits in the system converge to stable state which is a coexisting state of C-players and D-players. For given T values, the fraction of C-players in the stable state on the C–D edge increases as θ increases. Moreover, for given θ values the fraction of C-players decreases as T increases. Other parameter values are $N = 20$, $\theta = 1$, $k = 0.8$, $c = 1$, $c_1 = 0.2$, and $\delta = 0.3$ respectively.

3.2. Case 2: $c_1 \leq \min\{\delta kc/1+2\delta, c(1-k)/\delta\}$

In this case, there will appear two stable equilibrium points in the evolutionary dynamics system. One of the two stable equilibrium points is on the edge C–D and the other on the edge DC–D. There also does not exist any interior equilibrium point and all interior orbits in the simplex S_3 converge to the stable equilibrium point on the edge C–D (Figs. 4(a)–4(f)). The direction of evolution on the edge C–DC is from C to DC so that C-players prevail in the competition between C-players and D-players. But the edge C–D of S_3 is separated into two segments by the stable equilibrium point $(\bar{x}, 0, 1-\bar{x})$ on this edge, and all of the orbits of this edge converge to the stable equilibrium point $(\bar{x}, 0, 1-\bar{x})$. Similarly, the edge DC–D of S_3 is separated into two segments by the stable equilibrium point $(\bar{y}, 0, 1-\bar{y})$ on the edge DC–D, all orbits of this edge converge to the stable equilibrium point $(\bar{y}, 0, 1-\bar{y})$. Besides, the three vertices of the simplex S_3 corresponding to full C-players $(x = 1)$, DC-players $(y = 1)$, and D-players $(z = 1)$ in the population are unstable equilibrium points in the evolutionary system. Thus, random drift and occasional invasion of the missing D-player will eventually bring the population to the stable equilibrium points, and which one stable state will be converged to mainly depends on the initial state of the population. Due to the irrelevance between the dynamics of the equilibrium point $(\bar{y}, 0, 1-\bar{y})$ on the edge DC–D and θ, it is not necessary to further analyze the influence of θ on the evolutionary dynamics (see Appendix A). In this part, we further analyze the specifically evolutionary dynamics for different values of parameters T, c_1, and k. As depicted in Fig. 4, for fixed values of c_1 and k, we note that both $\bar{x}$ and $\bar{y}$ increase when T rises from 6 to 12. For example, for $c_1 = 0.15$, $k = 0.8$, $\bar{x}$ and $\bar{y}$ increase from 0.23 to 0.53 and from 0.25 to 0.58 respectively when T rises from 6 to 12. Besides, for fixed values of T and k, we observe that $\bar{x}$ does not change with the increase of c_1, but $\bar{y}$ decreases when c_1 rises from 0.1 to 0.15. For example, for $T = 12$, $k = 0.8$ or $T = 12$, $k = 0.9$, $\bar{y}$ decreases from 0.62 to 0.58 or 0.57 to 0.54 respectively when c_1 rises from 0.1 to 0.15. Furthermore, for fixed value of parameters c_1 and T, we can observe from Fig. 2 that $\bar{x}$ remains

unchanged with the increase of k, but $\bar{y}$ decreases when k rises from 0.8 to 0.9. For example, for $T = 12$, $c_1 = 0.1$ or $T = 12$, $c_1 = 0.15$, $\bar{y}$ decreases from 0.62 to 0.57 or 0.58 to 0.54 respectively when k rises from 0.8 to 0.9. In conclusion, to elicit deposit cooperation in the large population, it is effective to raise the threshold of public good and lower k and c_1.

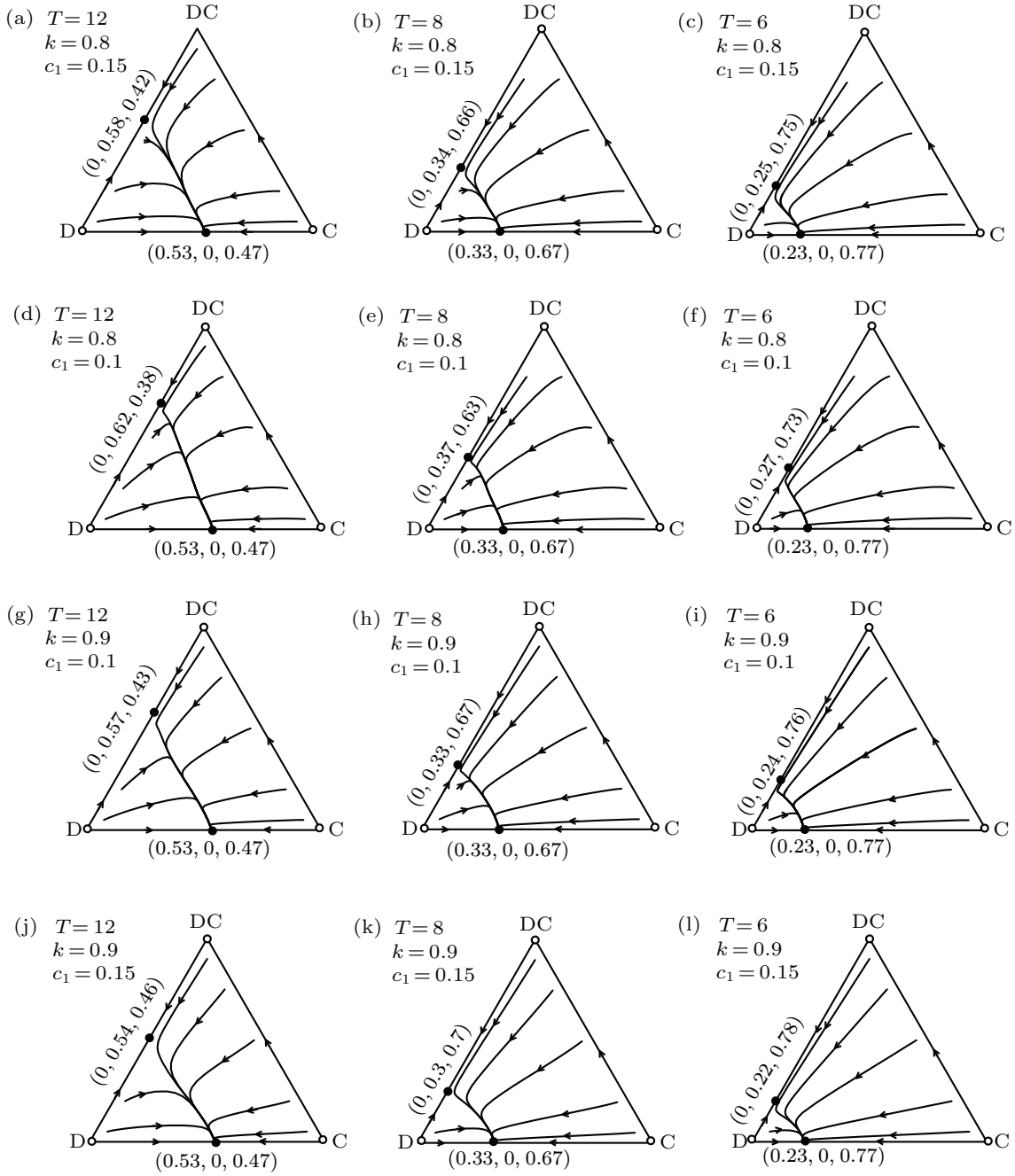


Fig. 4. Evolutionary dynamics of C, DC, and D strategies for different values of T. Nodes C, DC, and D of the simplex denote homogeneous population of C-players, DC-players, and D-players respectively. And open cycles represent unstable equilibrium points and filled cycles means stable points in these pictures. For given T values and c_1 values, the fraction of DC-players in the stable state on the D–DC edge decreases as k increases. For given T values and k values, the fraction of DC-players and C-players increase as T increases respectively. Besides, for given values of k and c_1, the larger the value of T, the larger the fraction of C-players on the edge of C–D edge and DC-players in the edge of DC–D edge respectively. For given T values, the fraction of C-players in the stable state on the D–C edge remain unchanged irrespective of the values of k and c_1. Other parameters values are $N = 20$, $\theta = 1$, $c = 1$, and $\delta = 0.3$ respectively.

4. Conclusion

It is inevitable that conflict often appears between contributors and freeloaders in public goods interactions. So far, many theoretical works have revealed that many mechanisms can maintain sufficiently high levels of cooperation. To further explore the emergence and maintenance of cooperation in a population, in this paper, we have introduced the deposit cooperation strategy with which players need to pay a deposit before the PGG. Based on the evolutionary game theoretical models, we have studied the evolutionary dynamics in the in-

finitely well-mixed population with a new deposit mechanism, by focusing particularly on the role of deposit played in the evolutionary dynamics of cooperation. In order to be closer to reality, this paper has also considered the value of contributions and introduced two different benefit-sharing ways. In the view of above-mentioned discussion, we have demonstrated that a stable coexistence state of cooperators and defectors can appear when $c(1-k)/\delta > c_1 > \delta kc/1+2\delta$. Besides, two stable coexistence states can appear, one is the coexistence of cooperator and defectors, the other is the coexistence of deposit-cooperators and defectors when $c_1 \leq \delta kc/1+2\delta$ and $c_1 < c(1-k)/\delta$. On the one hand, we also have found that adopting the "weak altruism" benefit-sharing way and increasing the threshold of public good T are effective to enhance the level cooperation in the well-mixed population. Interestingly, the level of cooperation is independent of k and c_1. On the other hand, the increase of the threshold T of public good and the reduction of discount k and deposit c_1 are also conducive to enhance the level of deposit cooperation in the well-mixed population. But we also have observed that the level of deposit cooperation within a population has no correlation with the benefit-sharing ways.

Appendix A: Appendix heading

When the provision of public good is failure, the payoffs of C, DC, and D is given as respectively

$$\begin{aligned}
\pi_{\mathrm{C}}^{0} &= \frac{(n_{\mathrm{C}}+n_{\mathrm{DC}}-\theta)\,\delta\,[cn_{\mathrm{C}}+(kc-c_1)\,n_{\mathrm{DC}}]}{(n_{\mathrm{C}}+n_{\mathrm{DC}})\,(n_{\mathrm{C}}+n_{\mathrm{DC}}-1)} \\
&\quad - \frac{(1-\theta)\delta c}{n_{\mathrm{C}}+n_{\mathrm{DC}}-1}, \\
\pi_{\mathrm{DC}}^{0} &= \frac{(n_{\mathrm{C}}+n_{\mathrm{DC}}-\theta)\,\delta\,[cn_{\mathrm{C}}+(kc-c_1)\,n_{\mathrm{Dc}}]}{(n_{\mathrm{C}}+n_{\mathrm{DC}})\,(n_{\mathrm{C}}+n_{\mathrm{DC}}-1)} \\
&\quad - \frac{(1-\theta)\delta\,(kc-c_1)}{n_{\mathrm{C}}+n_{\mathrm{DC}}-1} - (1+\delta)c_1, \\
\pi_{\mathrm{D}}^{0} &= 0.
\end{aligned}$$

On the other hand, when $kcn_{\mathrm{DC}}+cn_{\mathrm{C}} \geq T$, the payoffs of C, DC, and D is given respectively as

$$\begin{aligned}
\pi_{\mathrm{C}}^{1} &= \frac{(n-\theta)rc\,(kn_{\mathrm{DC}}+n_{\mathrm{C}})}{n(n-1)} - \frac{(1-\theta)rc}{n-1} - (1+\delta)c, \\
\pi_{\mathrm{DC}}^{1} &= \frac{(n-\theta)rc\,(kn_{\mathrm{DC}}+n_{\mathrm{C}})}{n(n-1)} - \frac{(1-\theta)rkc}{n-1} \\
&\quad - (1+\delta)\,(kc-c_1) - (1+\delta)^2 c_1, \\
\pi_{\mathrm{D}}^{1} &= \frac{(n-\theta)rc\,(kn_{\mathrm{DC}}+n_{\mathrm{C}})}{n(n-1)}.
\end{aligned}$$

Here, we discuss the competition of one strategy over another which mainly depends on the expected payoffs difference between strategies.

A1: On C–DC edge

The edge C–DC of S_3 represents that there only exist C-players and DC-players in the well-mixed population. In this case, all the players will contribute to public good and it is certain to successfully provide the public good no matter how many C- or DC-players are within the population. So, $\Omega = \Omega_1 = \{(n_{\mathrm{C}}, n_{\mathrm{DC}}, 0)\,|n_{\mathrm{C}}+n_{\mathrm{DC}} = n, x+y=1, z=0\}$ and $\Omega_0 = \emptyset$. For any given x and y, we can determine the fitness of C- and DC-type players respectively as follows:

$$\begin{aligned}
f_{\mathrm{C}} &= \sum_{\Omega_1} \binom{n-1}{n_{\mathrm{C}}} x^{n_{\mathrm{C}}} y^{n-n_{\mathrm{C}}-1} \pi_{\mathrm{C}}^{1} \\
&= \sum_{n_{\mathrm{C}}=0}^{n-1} \binom{n-1}{n_{\mathrm{C}}} x^{n_{\mathrm{C}}} (1-x)^{n-n_{\mathrm{C}}-1} \pi_{\mathrm{C}}^{1} \\
&= \sum_{n_{\mathrm{C}}=0}^{n-1} \binom{n-1}{n_{\mathrm{C}}} x^{n_{\mathrm{C}}} (1-x)^{n-n_{\mathrm{C}}-1} \frac{(n-\theta)rc\,(kn_{\mathrm{DC}}+n_{\mathrm{C}})}{n(n-1)} \\
&\quad - \frac{(1-\theta)rc}{n-1} - (1+\delta)c, \\
f_{\mathrm{DC}} &= \sum_{\Omega_1} \binom{n-1}{n_{\mathrm{C}}} x^{n_{\mathrm{C}}} y^{n-n_{\mathrm{C}}-1} \pi_{\mathrm{DC}}^{1} \\
&= \sum_{n_{\mathrm{C}}=0}^{n-1} \binom{n-1}{n_{\mathrm{C}}} x^{n_{\mathrm{C}}} (1-x)^{n-n} c^{-1} \pi_{\mathrm{DC}}^{1} \\
&= \sum_{n_{\mathrm{C}}=0}^{n-1} \binom{n-1}{n_{\mathrm{C}}} x^{n_{\mathrm{C}}} (1-x)^{n-n_{\mathrm{C}}-1} \frac{(n-\theta)rc\,(kn_{\mathrm{DC}}+n_{\mathrm{C}})}{n(n-1)} \\
&\quad - \frac{(1-\theta)rkc}{n-1} - (1+\delta)\,(kc-c_1) - (1+\delta)^2 c_1.
\end{aligned}$$

Because $\pi_{\mathrm{C}}^{1} < \pi_{\mathrm{DC}}^{1}$ in the both S and W cases, so $f_{\mathrm{C}} - f_{\mathrm{DC}} = \sum_{n_{\mathrm{C}}=0}^{n-1} \binom{n-1}{n_c} x^{n_{\mathrm{C}}} (1-x)^{n-n_{\mathrm{C}}-1} (\pi_{\mathrm{C}}^{1} - \pi_{\mathrm{DC}}^{1}) < 0$.

Next the replicator equations of C-players can be given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x\left(f_{\mathrm{C}} - \overline{f}\right) = x(1-x)\,(f_{\mathrm{C}} - f_{\mathrm{DC}})\,.$$

Here, let $\mathrm{d}x/\mathrm{d}t = 0$, we can obtain $x=0$, $x=1$ and so that $(x,y,z) = (0,1,0)$ and $(1,0,0)$ are the equilibrium points of the evolutionary system. To find out whether the two equilibrium points are stable or not, we define $M_1(\mathrm{x}) = \mathrm{d}x/\mathrm{d}t = 0$ and get $x=0$ or $x=1$. It is not difficult to verify that $M_1'(0) = f_{\mathrm{C}} - f_{\mathrm{DC}} > 0$ and $M_1'(1) = -(f_{\mathrm{C}} - f_{\mathrm{DC}}) > 0$. Herein, we can conclude that both $(0,1,0)$ and $(1,0,0)$ are unstable equilibrium points in the dynamic system in the both case of W and S.

A2: On C–D edge

The edge C–D of S_3 indicates that the well-mixed population consists of C- and D-players ($y=0$ and $x+z=1$), and the public good will be provided only when $cn_{\mathrm{C}} \geq T$. Here we define $m_1 = \lceil T/c \rceil$, and give a simple judgment: $n_{\mathrm{C}} \geq m_1$

leads to the successful provision of public good, but $n_C < m_1$ represents the failure of provision of public good. Accordingly the formation of Ω_1 and Ω_0 can be given respectively by

$$\Omega_1 = \{(n_C, 0, n_D) | n_C + n_D = n, n_C \geq m_1, x + z = 1, y = 0\},$$
$$\Omega_0 = \{(n_C, 0, n_D) | n_C + n_D = n, n_C < m_1, x + z = 1, y = 0\}.$$

For any given x and z, we can determine the difference of the fitness of C- and D-type players as follows:

$$\begin{aligned} f_C - f_D &= \sum_{\Omega_1} \binom{n-1}{n_C} x^{n_C} z^{n-n_C-1} \left(\pi_C^1 - \pi_D^1\right) \\ &\quad + \sum_{\Omega_0} \binom{n-1}{n_C} x^{n_C} z^{n-n_C-1} \left(\pi_C^0 - \pi_D^0\right) \\ &= \sum_{n_C=0}^{m_1} \binom{n-1}{n_C} x^{n_C} (1-x)^{n-n_C-1} \delta c \\ &\quad - \sum_{n_C=m_1+1}^{n-1} \binom{n-1}{n_C} x^{n_C} (1-x)^{n-n_C-1} \\ &\quad \times \left[\frac{(1-\theta)rc}{n-1} + (1+\delta)c\right]. \end{aligned}$$

When C-players compete with D-players in the well-mixed population, the replicator equation of C-players can be written as follows:

$$\frac{dx}{dt} = x\left(f_C - \overline{f}\right) = x(1-x)\left(f_C - f_D\right).$$

We let $\emptyset(x) = f_C - f_D$, we can get

$$\begin{aligned} \frac{d\emptyset(x)}{dx} &= -\left[\frac{(1-\theta)rc}{n-1} + (1+2\delta)c\right] \\ &\quad \times \sum_{n_C=m_1}^{n-1} \binom{n-1}{n_C} \left[n_C x^{(n_C-1)} (1-x)^{n-n_C-1}\right. \\ &\quad \left. - (n-n_C-1) x^{n_C} (1-x)^{n-n_C-2}\right] < 0, \\ &\quad x \in (0,1). \end{aligned}$$

Then we can get $\lim_{x\to 0} \phi(x) > 0$ and $\lim_{x\to 1} \emptyset(x) < 0$ so there is only one interior root on the edge C–D. When $\emptyset(\overline{x}) = f_C - f_D = 0$ we have

$$\begin{aligned} &\frac{\delta c(n-1)}{(1-\theta)rc + (n-1)(1+2\delta)c} \\ &= \sum_{n_C=m_1}^{n-1} \binom{n-1}{n_C} \overline{x}^{n_C} (1-\overline{x})^{n-n_C-1}. \end{aligned}$$

So that $(x,y,z) = (1,0,0)$, $(0,0,1)$, and $(\overline{x}, 0, 1-\overline{x})$ are equilibrium points of the evolutionary system. Here we let $M_2(x) = dx/dt = 0$ and then, we get $x = 0, x = 1$ or $x = \overline{x}$. Actually it is not difficult to verify that $M_2'(0) = f_C - f_D > 0$, $M_2'(1) = -(f_C - f_D) > 0$, $M_2'(\overline{x}) = \overline{x}(1-\overline{x}) d\emptyset(x)/dx < 0$. Herein, we can conclude that $x = 0$ and $x = 1$ are unstable equilibrium point, whereas $x = \overline{x}$ is a stable equilibrium point in both case of W and S.

A3: On DC–D edge

The edge DC–D of S_3 represents that the well-mixed population consists of DC- and D-players ($x = 0$ and $y + z = 1$), so the public good will be provided only when $kcn_{DC} \geq T$. Here we define $m_2 = \lceil T/kc \rceil$ and can give a simple judgment: $n_{DC} \geq m_2$ leads to the successful provision of public good game to the population, but $n_{DC} < m_2$ represents the failure of provision of public good. Accordingly the formation of Ω_1 and Ω_0 can be given by

$$\Omega_1 = \{(0, n_{DC}, n_D) | n_{DC} + n_D = n, n_{DC} \geq m_2, y + z = 1, x = 0\},$$
$$\Omega_0 = \{(0, n_{DC}, n_D) | n_{DC} + n_D = n, n_{DC} < m_2, y + z = 1, x = 0\}.$$

Next the replicator equations of C-players can be given by,

$$\frac{dy}{dt} = y(1-y)\left(f_{DC} - f_D\right).$$

For any given y and z, we can determine the difference of the fitness of DC- and D-type players as follows:

$$\begin{aligned} f_{DC} - f_D &= \sum_{\Omega_1} \binom{n-1}{n_{DC}} y^{n_{DC}} (1-y)^{n-n_{DC}-1} \left(\pi_{DC}^1 - \pi_D^1\right) \\ &\quad + \sum_{\Omega_0} \binom{n-1}{n_{DC}} y^{n_{DC}} y^{n-n_{DC}-1} \left(\pi_{DC}^0 - \pi_D^0\right) \\ &= \sum_{\Omega_1} \binom{n-1}{n_{DC}} y^{n_{DC}} (1-y)^{n-n_{DC}-1} \left(\pi_{DC}^1 - \pi_D^1\right) \\ &\quad + \sum_{\Omega_0} \binom{n-1}{n_{DC}} y^{n_{DC}} y^{n-n_{DC}-1} \pi_{DC}^0 \\ &= \delta\left(kc - 2c_1\right) - c_1 - \left[(1+2\delta)\left(kc - c_1\right) + \delta(1+\delta)c_1\right] \\ &\quad \times \sum_{n_C=m_2}^{n-1} \binom{n-1}{n_C} y^{n_{DC}} (1-y)^{n-n_{DC}-1}. \end{aligned}$$

When $\pi_{DC}^0 < \pi_D^0 = 0$, we have $c_1 > \delta kc/1+2\delta$ and $f_{DC} - f_D < 0$, so we can say that there is no interior equilibrium point on the edge DC–D.

However, when $\pi_{DC}^0 > \pi_D^0$, we have $0 < c_1 < \delta kc/1+2\delta$. We let $\varphi(x) = f_C - f_D$, we can obtain

$$\begin{aligned} \frac{d\varphi(y)}{dy} &= -\left[\frac{(1-\theta)rc}{n-1} + (1+2\delta)c\right] \sum_{n_C=m_1}^{n-1} \binom{n-1}{n_C} \\ &\quad \times \left[n_C y^{(n_C-1)} (1-y)^{n-n_C-1}\right. \\ &\quad \left. - (n-n_C-1) y^{n_C} (1-y)^{n-n_C-2}\right] < 0, \quad y \in (0,1). \end{aligned}$$

Then we can get $\lim_{y\to 0} \varphi(y) > 0$ and $\lim_{y\to 1} \varphi(y) < 0$, so there is only one interior root on the edge C–D. In this case, when $\varphi(\overline{y}) = f_{DC} - f_D = 0$, we can obtain

$$\sum_{n_c=m_2}^{n-1} \binom{n-1}{n_C} \overline{y}^{n_{DC}} (1-\overline{y})^{n-n_{DC}-1}$$

$$= \frac{\delta(kc-2c_1)-c_1}{(1+2\delta)(kc-c_1)+\delta(1+\delta)c_1}.$$

Let $M_3(\mathrm{x}) = \mathrm{d}y/\mathrm{d}t = 0$ and we gain $y = 0$, $y = 1$ or $y = \bar{y}$. Actually it is not difficult to verify that $M_3'(0) = f_\mathrm{C} - f_\mathrm{D} > 0$, $M_3'(1) = -(f_\mathrm{C} - f_\mathrm{D}) > 0, M_3'(\bar{y}) = \bar{y}(1-\bar{y})\mathrm{d}\varphi(x)/\mathrm{d}x < 0$. Herein, we can conclude that $y = 0$ and $y = 1$ are unstable equilibrium points, whereas $y = \bar{y}$ is a stable equilibrium point in both cases of W and S.

Acknowledgment

We thank anonymous reviewers and Jia Liu, Jinhua Zhao and Dezhuang Hu for insightful discussions.

References

[1] Christie M R, Mcnickle G G, French R A and Michael S B 2018 *Proc. Natl. Acad. Sci. USA* **115** 4441
[2] Colman A M 2006 *Nature* **440** 744
[3] Martinezvaquero L A, Han T A, Pereira L M and Lenaerts T 2017 *Sci. Rep.* **7** 2478
[4] Axelrod R, Hamilton W D 1981 *Science* **211** 1390
[5] Du P, Xu C and Zhang W 2015 *Chin. Phys. Lett.* **32** 058901
[6] Wei L Y, Cui Y F and Li D Y 2018 *Acta Phys. Sin.* **67** 190201 (in Chinese)
[7] Martinezvaquero L L A, Han T A, Pereira L M and Lenaerts T 2017 *Sci. Rep.* **7** 2478
[8] Ulrich F 2019 *Biol. Lett.* **15** 0143
[9] Li Y M, Du W B, Yang P, Wu T H, Wu D P and Perc M 2019 *IEEE Internet Things J.* **6** 1866
[10] Barker J L, Bronstein J L, Friesen M L,Jones E I, Reeve H K, Zink A G and Frederickson M E 2017 *Evolution* **71** 814
[11] Trivers R L 1971 *Q. Rev. Biol.* **46** 35
[12] Righi S and Takacs K 2018 *Sci. Rep.* **8** 11149
[13] M T, Jg F and C R 2016 *Philos. Trans. R. Soc. Lond. Ser. B-Biol. Sci.* **371** 20150472
[14] Manfred M, Dirk S and Krambeck H J 2002 *Nature* **415** 424
[15] Christoph H 2010 *J. Theor. Biol.* **267** 22
[16] Santos F P, Santos F C and Pacheco J M 2018 *Nature* **555** 242
[17] Sasaki T and Uchida S 2014 *Biol. Lett.* **10** 20130903
[18] Sasaki T and Unemi T 2011 *J. Theor. Biol.* **287** 109
[19] Forsyth P A I and Hauert C 2011 *J. Math. Biol.* **63** 109
[20] Andreoni J, Harbaugh W and Vesterlund L 2003 *Am. Econ. Rev.* **93** 893
[21] Boyd R and Richerson P J 1992 *Ethol. Sociobiol.* **13** 171
[22] Li X, Jusup M, Wang Z, Li H, Lei S, Boris P, Eugene S, Shlomo H and Stefano B 2018 *Proc. Natl. Acad. Sci. USA* **115** 30
[23] Yang H X and Chen X 2018 *Appl. Math. Comput.* **316** 460
[24] Boyd R, Gintis H and Bowles S 2010 *Science* **328** 617
[25] James H F 2005 *Proc. Natl. Acad. Sci. USA* **102** 7047
[26] Ohdaira T 2017 *Chaos, Solitons and Fractals* **95** 77
[27] Geng Y, Shen C, Hu K and Shi L 2018 *Physica A* **503** 540
[28] Yu T, Chen S H and Li H 2016 *J. Econ. Interact. Coord.* **11** 313
[29] Rand D G, Nowak M A, Fowler J H and Christakis N A 2014 *Proc. Natl. Acad. Sci. USA* **111** 17093
[30] Nowak M A and May R M 1992 *Nature* **359** 826
[31] Wang X, Lv S and Quan J 2017 *Physica A* **482** 286
[32] Wang X, Zhang L, Du X and Sun Y 2017 *Nat. Comput.* **16** 99
[33] Xie F, Shi J and Lin J 2017 *PLoS One* **12** e182524
[34] Melamed D and Simpson B 2016 *Soc. Networks* **45** 32
[35] Huang Z G, Wang S J, Xu X J and Wang Y H 2008 *Europhys. Lett.* **81** 28001
[36] Zhao L, Ye X J, Huang Z G, Sun J T, Yang L, Do Y and Wang Y H 2010 *J. Stat. Mech.-Theory Exp.* **2010** P08009
[37] Perc M and Szolnoki A 2008 *Phys. Rev. E* **77** 011904
[38] Du W B, Cao X B, Hu M B and Wang W X 2009 *Europhys. Lett.* **87** 60004
[39] Li Y M, Zhang J and Perc M 2017 *Appl. Math. Comput.* **320** 437
[40] Chen C Q, Dai Q L, Han W C and Yang J Z 2017 *Chin. Phys. Lett.* **34** 028901
[41] Tao S Y, Cui M Z, Dai Q L and Yang J Z 2014 *Chin. Phys. Lett.* **31** 110201
[42] Jianlei Z, Chunyan Z and Ming C 2015 *Sci. Rep.* **5** 9098
[43] Fletcher J A and Zwick M 2007 *J. Theor. Biol.* **245** 26
[44] Li Y and Ye H 2018 *Appl. Math. Comput.* **320** 621
[45] Shimura H and Nakamaru M 2018 *J. Theor. Biol.* **451** 46
[46] Connelly B D, Dickinson K J, Hammarlund S P and Kerr B 2016 *Evol. Ecol.* **30** 267
[47] Huck S, Leutgeb J and Oprea R 2017 *Nat. Commun.* **8** 15147
[48] Cassese D 2018 *Appl. Netw. Sci.* **3** 29
[49] Zhang W, Li Y S and Xu C 2015 *Chin. Phys. Lett.* **32** 118901
[50] Lopez G R and Levinton J S 1987 *Q. Rev. Biol.* **62** 235
[51] Fletcher J A and Doebeli M 2009 *Proc. R. Soc. Lond. Ser. B-Biol. Sci.* **276** 13
[52] Huang Z G, Wu Z X, Wu A C, Yang L and Wang Y H 2008 *Europhys. Lett.* **84** 50008
[53] Novak S, Chatterjee K and Nowak M A 2013 *J. Theor. Biol.* **334** 26
[54] Ohtsuki H and Nowak M A 2006 *J. Theor. Biol.* **243** 86