ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理 Chinese Physics B

Pyramid scheme model for consumption rebate frauds

Yong Shi(石勇), Bo Li(李博), Wen Long(龙文)

Citation: Chin. Phys. B . 2019, 28(7): 078901. **doi:** 10.1088/1674-1056/28/7/078901
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Coordinated chaos control of urban expressway based on synchronization of complex networks

Ming-bao Pang(庞明宝), Yu-man Huang(黄玉满)
Chin. Phys. B . 2018, 27(11): 118902. **doi:** 10.1088/1674-1056/27/11/118902

Dominant phase-advanced driving analysis of self-sustained oscillations in biological networks

Zhi-gang Zheng(郑志刚), Yu Qian(钱郁)
Chin. Phys. B . 2018, 27(1): 018901. **doi:** 10.1088/1674-1056/27/1/018901

An improved genetic algorithm with dynamic topology

Kai-Quan Cai(蔡开泉), Yan-Wu Tang(唐焱武), Xue-Jun Zhang(张学军), Xiang-Min Guan(管祥民)
Chin. Phys. B . 2016, 25(12): 128904. **doi:** 10.1088/1674-1056/25/12/128904

Subtle role of latency for information diffusion in online social networks

Fei Xiong(熊菲), Xi-Meng Wang(王夕萌), Jun-Jun Cheng(程军军)
Chin. Phys. B . 2016, 25(10): 108904. **doi:** 10.1088/1674-1056/25/10/108904

Universal relation for transport in non-sparse complex networks

Wang Yan, Yang Xiao-Rong
Chin. Phys. B . 2015, 24(11): 118902. **doi:** 10.1088/1674-1056/24/11/118902

Pyramid scheme model for consumption rebate frauds*

Yong Shi(石勇)[1,2,3], Bo Li(李博)[1,2,3], and Wen Long(龙文)[1,2,3,†]

[1] School of Economics and Management, University of Chinese Academy of Sciences, Beijing 100190, China
[2] Research Center on Fictitious Economy and Data Science, Chinese Academy of Sciences, Beijing 100190, China
[3] Key Laboratory of Big Data Mining and Knowledge Management, Chinese Academy of Sciences, Beijing 100190, China

(Received 15 March 2019; revised manuscript received 17 April 2019; published online 12 June 2019)

There are various types of pyramid schemes that have inflicted or are inflicting losses on many people in the world. We propose a pyramid scheme model which has the principal characters of many pyramid schemes that have appeared in recent years: promising high returns, rewarding the participants for recruiting the next generation of participants, and the organizer takes all of the money away when they find that the money from the new participants is not enough to pay the previous participants interest and rewards. We assume that the pyramid scheme is carried out in the tree network, Erdös–Réney (ER) random network, Strogatz–Watts (SW) small-world network, or Barabasi–Albert (BA) scale-free network. We then give the analytical results of the generations that the pyramid scheme can last in these cases. We also use our model to analyze a pyramid scheme in the real world and we find that the connections between participants in the pyramid scheme may constitute a SW small-world network.

Keywords: pyramid scheme, complex networks, small-world networks

PACS: 89.65.Gh, 64.60.aq **DOI:** 10.1088/1674-1056/28/7/078901

1. Introduction

r

On June 18, 2017, China Central Television's (CCTV's) 'Focus' program, which is one of the most-watched shows on CCTV, exposed a so-called consumption rebate platform that was named "RenRenGongYi" which was actually a pyramid scheme.[1] The alleged operation pattern of RenRenGongYi was to let the franchisees give a fixed proportion of the customers' consumption to the platform to form a fund pool, such as 24%, 12%, or 6%. The platform then returned the money to the customers and franchisees by instalments. However, most transactions were fabricated in RenRenGongYi, and the platform in fact lured the participants by promising high returns to invest in the fund pool and rewarded the participants who attracted the next generation of participants. Most of the franchisees were fictitious because most transactions were fabricated, and the participants of the pyramid scheme were mainly consumers. For example, if a participant fabricated a 100 yuan consumption (the participant was both the franchisee and the consumer) and gave 24 (or 12 or 6) yuan to the platform, then he/she could gradually get a consumption rebate of nearly 100 yuan, which was more than 4 (or 8 or 16) times the principal. In less than a month, 5267 franchisees and 48505 consumers were involved in the platform. From the time that the project officially opened on December 1, 2016 to its crash at the end of the month, the amount absorbed reached 1 billion yuan.

Besides RenRenGongYi, there are many more platforms of this form and Chinese government has warned about the risk of consumption rebate platforms.[2] There are many other forms of pyramid schemes, such as IGOFX[3] that originated in Malaysia and MMM[4] that originated in Russia.

Pyramid schemes are different from ordinary Ponzi schemes, which are named after the eponymous fraudster Charles Ponzi, though in both Ponzi and pyramid schemes, existing investors are paid by the money from new investors.[5] In a Ponzi scheme, the participants believe they are actually earning returns from their investment. While in a pyramid scheme, the participants are aware that they are earning money by finding new participants—they become part of the scheme.

Pyramid schemes and Ponzi schemes have been researched from many different perspectives. Joseph Gastwirth proposed a probability model of a pyramid scheme and concluded that the vast majority of participants have less than a 10% chance of recouping their initial investment.[6] Stimulated by the Madoff investment scandal in 2008, Marc Artzrouni put forward a first order linear differential equation to describe Ponzi schemes.[7] The model that was developed by Marc Artzrouni depends on the following parameters: a promised but unrealistic interest rate, the actual realized nominal interest rate, the rate at which new deposits are accumulated, and the withdrawal rate. Marc Artzrouni gave the conditions on these parameters for the Ponzi scheme to be solvent or to collapse. The model was fitted to data available on Charles Ponzi's 1920 eponymous scheme and illustrated with a philanthropic version of the scheme. Tyler Moore *et al*. conducted an empirical analysis of these high yield investment programs but did not put forward a mathematical model.[8] A

*Project supported by the National Natural Science Foundation of China (Grant Nos. 71771204 and 91546201).
†Corresponding author. E-mail: longwen@ucas.edu.cn
© 2019 Chinese Physical Society and IOP Publishing Ltd

high yield investment program (HYIP) is considered to be an online Ponzi scheme because it pays outrageous levels of interest using money from new investors. In contrast from the traditional Ponzi schemes, there are many sophisticated investors who understand the fraud but who hope to profit by joining early, while the investors cannot withdraw their money at any time. Peihua Fu *et al*. researched some problems when the Ponzi scheme diffuses in complex networks.[9,10]

Since the introduction of random networks, small-world networks, and scale-free networks, complex networks have attracted great attention from researchers in various fields, such as management and statistical physics. Researches show that many natural and social phenomena have small-world or scale-free characteristics. At present, complex networks have been successfully applied to improve transportation networks,[11,12] analyze innovative networks,[13] research the spread of infectious diseases and rumors.[14,15]

Most of the existing literature focuses on the research of Ponzi schemes, including the spread of Ponzi schemes in complex networks, while the research on pyramid schemes is relatively scarce. To the best of our knowledge, for the pyramid schemes of consumption rebate type, no scholar has put forward a model at present. To understand and explain the operation mechanism and characteristics of the pyramid schemes of consumption rebate type, then provide ideas for monitoring this kind of pyramid schemes, and offer the basis for further research, we propose a pyramid scheme model which has the principal characters of many pyramid schemes appeared in recent years: promising high returns, rewarding the participants, recruiting the next generation of participants, and the organizer will take all the money away when he finds the money from the new participants is not enough to pay the previous participants interest and rewards. We assume that the pyramid scheme carries on in the tree network, Erdös–Réney (ER) random network, Strogatz–Watts (SW) small-world network, or Barabasi–Albert (BA) scale-free network, respectively, then give the analytical results of how many generations the pyramid scheme can last in these cases. We also use our model to analyze a pyramid scheme in the real world and we find the connections between participants in the pyramid scheme may constitute a SW small-world network.

This paper is organized as follows. In Section 2, we briefly introduce the tree network, the random network, the small-world network, and the scale-free network. In Section 3, we propose our pyramid scheme model. In Section 4, we analyze a pyramid scheme in real world. Some discussions and conclusions are given in Section 5.

2. Networks

2.1. Tree network

Tree networks are connected acyclic graphs. The word tree suggests branching out from a root and never completing a cycle. Tree networks are hierarchical, and each node can have an arbitrary number of child nodes. Trees as graphs have many applications, especially in data storage, searching, and communication.[16]

2.2. Random network

Random network, also known as stochastic network or stochastic graph, refers to a complex network created by stochastic process. The most typical random network is the ER model proposed by Paul Erdös and Alfred Réney.[17] The ER model is based on a natural construction method: suppose there are n nodes, and assume that the possibility of connection between each pair of nodes is constant $0 < p < 1$. The network constructed in this way is an ER model network. Scientists first used this model to explain real-life networks.

2.3. Small-world network

The original model of small-world was first proposed by Watts and Strogatz, and it is the most classical model of small-world network, which is called SW small-world network.[18] The SW small-world network model can be constructed as follows: take a one-dimensional lattice of L vertices with connections or bonds between nearest neighbors and periodic boundary conditions (the lattice is a ring), then go through each of the bonds in turn and independently with some probability ϕ "rewiring" it. Rewiring in this context means shifting one end of the bond to a new vertex chosen uniformly at random from the whole lattice, with the exception that no two vertices can have more than one bond running between them and no vertex can be connected by a bond to itself. The most striking feature of small-world networks is that most nodes are not neighbors of one another, but the neighbors of any given node are likely to be neighbors of each other and most nodes can be reached from every other node by a small number of hops or steps. It has been found that many networks in real life have the small-world property, such as social networks,[19] the connections of neural networks,[18] and the bond structure of long macromolecules in the chemical.[20]

2.4. Scale-free network

A scale-free network is a network whose degree distribution follows a power law, at least asymptotically. The first model of scale-free network was proposed by Barabasi and Albert, which is called BA scale-free network.[21] The BA model describes a growing open system starting from a group of core nodes, new nodes are constantly added to the system. The two basic assumptions of the BA scale-free network model are

as follows: (i) from m_0 nodes, a new node is added at each time step, and m nodes from the m_0 nodes are selected to be connected to the new node ($m \leq m_0$); (ii) the probability Π_i that the new node is connected to an existing node i satisfies $\Pi_i = k_i / \sum_{j=1}^{N-1} k_j$, where k_i denotes the degree of the node i and N denotes the number of nodes. In this way, when added enough new nodes, the network generated by the model will reach a stable evolution state, and then the degree distribution follows the power law distribution. In Ref. [22], it was shown that the degree distribution of many networks in real world is approximate or exact obedience to the power law distribution.

3. The model

3.1. Assumptions

We consider a simple pyramid scheme that meets the basic features of many pyramid schemes in the real world, especially the consumption rebate platforms. First, it has an organizer that attracts participants through promising a high rate of return compared to the normal interest rate. Besides the promising return, any participant will be rewarded by the organizer with a proportion of the total investment of the participants he or she directly attracted, thus the early participants will be motivated enough to recruit the next-generation participants and the next-generation participants will do the same thing in order to get more returns. Secondly, we assume all the participants at current generation are recruited by the participants at the upper generation, and the organizer pays the participants at the previous generations the interests and rewards when all possible participants at current generation have joined in the scheme. The third assumption is that the organizer will take all the money away when he finds the money from the new participants is not enough to pay the previous participants interest and rewards. To simplify the model, we also assume all the participants invest the same amount of money and invest only once. Figure 1 is a schematic diagram of the pyramid scheme, which has one organizer and two generations of participants.

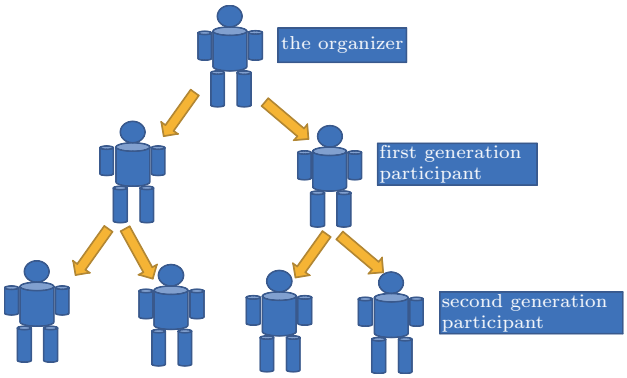


Fig. 1. A schematic diagram of pyramid scheme. From top to bottom are the organizer, the first generation participants, and the second generation participants.

Based on these assumptions, we discuss the pyramid scheme spreads in the tree network, random network, small world network, and scale-free network below.

3.2. Tree network case

If the pyramid scheme expands in the form of tree network that has a constant branching coefficient α and the root node of the tree network represents the organiser, we can simply write the number of participants at the g-th generation as $n_1\alpha^{g-1}$ and the total amount of money entering the pyramid scheme at the g-th generation as $mn_1\alpha^{g-1}$, where n_1 is the number of participants at the first generation and m is the amount of money that every participant invests. For simplification, we assume $n_1 = \alpha$ and $m = 1$. We suppose the number of all potential participants is N in this case. Removing the interest and rewards, the relationship between the net inflow of money M of the pyramid scheme and the generation g when all possible participants at the g-th generation have joined in the scheme can be given by

$$M(g) = \alpha^g - r_0 \sum_{i=1}^{g-1} \alpha^i - r_1 \alpha^g, \tag{1}$$

where r_0 is the promised rate of return of the organizer, and r_1 is the ratio of the money rewarded to a participant to the total investment of the participants he or she directly recruited. Normally in real pyramid scheme cases, r_0 and r_1 are between 0% and 50%. The first term of Eq. (1) represents the investment of all the participants, the second term represents the interest paid to the participants before the generation g, and the third term represents the rewards paid to the recruiters of participants at the g-th generation. Notice that in our pyramid scheme, the participants at the g-th generation are all recruited by the participants at the $(g-1)$-th generation.

The second term of Eq. (1) is the sum of geometric sequences, after summing them up, equation (1) can be rewritten as

$$M(g) = \frac{\alpha}{\alpha - 1}[(1 - r_1)\alpha^g - (1 + r_0 - r_1)\alpha^{g-1} + r_0]. \tag{2}$$

Through Eq. (2) we can find that if the branching coefficient α satisfies the condition

$$\alpha \geq \frac{1 - r_1 + r_0}{1 - r_1}, \tag{3}$$

the inflow of money $M(g)$ of the pyramid scheme is always positive, so the pyramid scheme will continue forever under the circumstances.

However, the potential participants are limited to N and the pyramid scheme will stop eventually. The maximum generation G of the pyramid scheme is given by

$$G_{\mathrm{TR}} = \lfloor \log_\alpha \left(\frac{N\alpha - N + \alpha}{\alpha} \right) \rfloor + 1, \tag{4}$$

where $\lfloor x \rfloor$ is the integer part of x. At the G-th generation, all the potential participants have joined the pyramid scheme, and

the organizer will take away all the money and not pay the interest and rewards any more. We can write the final income of the pyramid as

$$R_{\rm p} = N - r_0 \sum_{i=1}^{G-2} (G-i-1)\alpha^i - r_1 \sum_{i=2}^{G-1} \alpha^i, \tag{5}$$

and the income of the participants at the i-th generation is

$$R_i = \begin{cases} r_0(G-i-1)\alpha^i + r_1\alpha^{i+1} - \alpha^i, & \text{for } 1 \le i \le G-2, \\ -\alpha^i, & \text{for } G-2 < i \le G. \end{cases} \tag{6}$$

Figure 2(a) shows the analytical result and the simulative result of maximum generation $G_{\rm ER}$ when the branching coefficient α changes, and we take the parameters $N = 10000$, $r_0 = 0.1$, $r_1 = 0.1$. Figure 2(b) shows the analytical result and the simulative result of maximum generation $G_{\rm ER}$ when the number of possible participants N changes, and we take the parameters $\alpha = 4$, $r_0 = 0.1$, $r_1 = 0.1$. Figure 2 illustrates intuitively that in the tree network case, if other conditions of the pyramid scheme remain unchanged, the larger the branch coefficient—that is, the more new participants each person recruits—, the fewer generations the pyramid scheme can last. Meanwhile, when other conditions remain unchanged, the larger the number of potential participants, the more generations the pyramid scheme can sustain, but every new generation needs more participants and this growth of new participants is exponential.

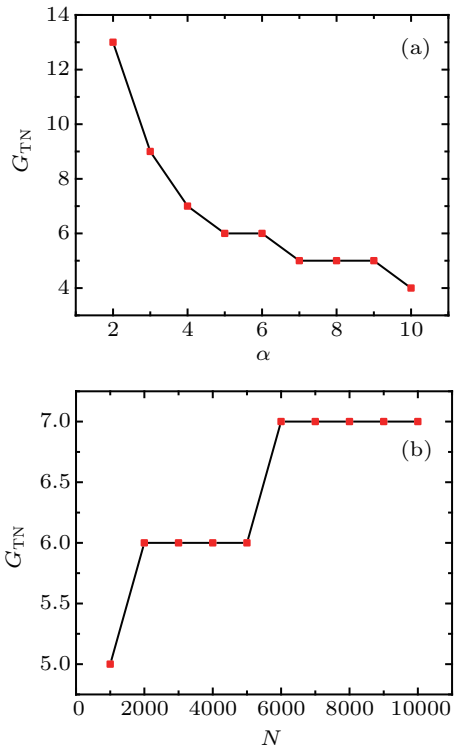


Fig. 2. (a) The analytical result and the simulative result of maximum generation $G_{\rm TN}$ when the branching coefficient α changes. We take the parameters $N = 10000$, $r_0 = 0.1$, $r_1 = 0.1$. (b) The analytical result and the simulative result of maximum generation $G_{\rm TN}$ when the number of possible participants N changes. We take the parameters $\alpha = 4$, $r_0 = 0.1$, $r_1 = 0.1$.

3.3. Random network case

If the pyramid scheme takes place in an ER random network that has an average degree k and N nodes, we assume the organizer is a random node in the network and other nodes represent the potential participants. The organizer recruits the potential participants nearest to him as the first generation participants, and the first generation participants recruit the potential participants nearest to them as the second generation participants, and so on. So the generation of any participant in the pyramid scheme is given by the shortest path length to the node representing the organizer. Reference [23] has given the approximate analytical results for the distribution of shortest path lengths in ER random networks, the number of nodes at the i-th generation is about k^i if $i\log_N k < 1$ and all the nodes are included in the pyramid scheme if $i\log_N k > 1$. Therefore, the pyramid scheme in the ER random network is approximate to the case in the tree network above and the difference is the branching coefficient α should be replaced by the average degree k.

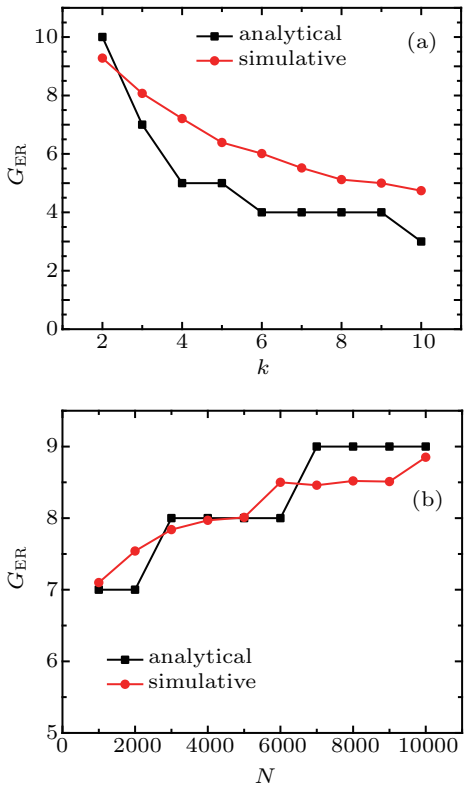


Fig. 3. (a) The analytical result and the simulative result of maximum generation $G_{\rm ER}$ when the average degree k changes. We take the parameters $N = 1000$, $r_0 = 0.1$, $r_1 = 0.1$. (b) The analytical result and the simulative result of maximum generation $G_{\rm ER}$ when the number of possible participants N changes. We take the parameters $k = 4$, $r_0 = 0.1$, $r_1 = 0.1$. The simulative results are averaged after 100 simulations.

First, like the case in the tree network, r_0, r_1, and k should satisfy the following condition:

$$k \ge \frac{1 - r_1 + r_0}{1 - r_1}. \tag{7}$$

The approximate maximum generation G of the pyramid scheme in this case is given by

$$G_{\mathrm{ER}} \approx \lfloor 1/\log_N k \rfloor + 1. \tag{8}$$

In addition, we can also write the approximate expressions of the organiser's and participants' income which have the same form of Eq. (6), which we omit here. Figure 3(a) shows the analytical result and the simulative result of maximum generation G_{ER} when the average degree k changes, and we take the parameters $N = 1000$, $r_0 = 0.1$, $r_1 = 0.1$. Figure 3(b) shows the analytical result and the simulative result of maximum generation G_{ER} when the number of possible participants N changes, and we take the parameters $k = 4$, $r_0 = 0.1$, $r_1 = 0.1$. The simulative results in the figures are averaged after 100 simulations. From Fig. 3, we can find that in the ER random network case, the relationship between maximum generation G_{ER} and mean degree k, and the relationship between G_{ER} and N are similar to those in the tree network case, where the mean degree k represents the amount of participants that each participant can recruit averagely. We can also find that within the range of parameters we have chosen, the analytical results and simulative results are very close.

3.4. Small world network case

Now we consider the pyramid scheme carries out in an SW small-world network, to some extent this case is similar to the case in the ER random network. We also randomly choose a node as the organizer, other nodes represent the potential participants, and r_0, r_1 represent the interest rate and reward ratio, respectively. The generation of any participant in the pyramid scheme is the shortest path length to the node representing the organizer. Reference [24] pointed out that the number of nodes increases exponentially with the average length of the shortest path when the nodes are infinite. The approximate surface area of a sphere of radius r on the SW small-world network can be given by[24]

$$A(r) = 2\mathrm{e}^{4r/\xi}, \tag{9}$$

where $\xi = 1/\phi k$, and ϕ is the rewiring probability and k is the degree of the corresponding rule graph.

Changing r to g, we can obtain the approximate number of participants at the g-th generation. Because of the exponential form of $A(g)$, we can deal with this case just like in the cases of tree network and ER random network. The branching coefficient α should be replaced by $\mathrm{e}^{4/\xi}$, and the following condition should be satisfied:

$$\mathrm{e}^{4/\xi} \geq \frac{1 - r_1 + r_0}{1 - r_1}. \tag{10}$$

If the nodes are finite, then the number of nodes reaches the maximum when the distance from the node to the organizer is near the average length of the shortest path. If the distance is greater than the average length of the shortest path, the number of nodes quickly reduces to 0, so it can be approximately considered that the maximum generation G is close to the average length of the shortest path. The average path length $\bar{d}$ of the SW small-world network is given by[24]

$$\bar{l}_{\mathrm{SW}} \approx \frac{N}{K} f(\phi K N), \tag{11}$$

where

$$f(u) = \begin{cases} 1/4, & \text{if } u \to 0, \\ \ln u/u, & \text{if } u \to \infty. \end{cases} \tag{12}$$

The number of nodes with the average shortest path length to the node representing the organizer is the largest. So we can infer the maximum generation G_{SW} of the pyramid scheme is given by[25]

$$G_{\mathrm{SW}} \approx \lfloor \bar{l}_{\mathrm{SW}} \rfloor + 1. \tag{13}$$

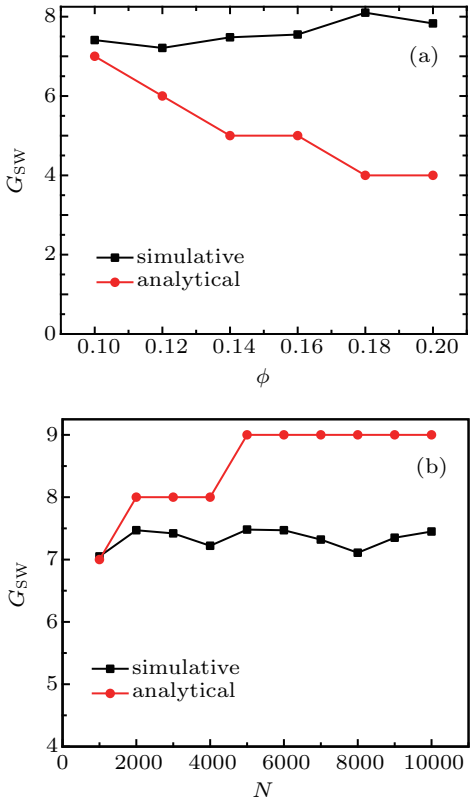


Fig. 4. (a) The analytical result and the simulative result of maximum generation G_{SW} when the possible participants ϕ changes. We take the parameters $N = 1000$, $K = 3$, $r_0 = 0.2$, $r_1 = 0.2$. (b) The analytical result and the simulative result of maximum generation G_{SW} when the number of possible participants N changes. We take the parameters $K = 3$, $\phi = 0.1$, $r_0 = 0.2$, $r_1 = 0.2$. The simulative results are averaged after 100 simulations.

In the simulation, we find that the values of r_0 and r_1 are very important. Generally speaking, the greater the values of r_0 and r_1 satisfying Eq. (10) are, the closer the simulation results and numerical results are. This happens because when the values of r_0 and r_1 are larger, the pyramid scheme can easily terminate when the number of generations exceeds the

average shortest path length. Figure 4(a) shows the analytical result and the simulative result of maximum generation G_{SW} when the possible participants ϕ changes, and we take the parameters $N = 1000$, $K = 3$, $r_0 = 0.2$, $r_1 = 0.2$. Figure 4(b) shows the analytical result and the simulative result of maximum generation G_{SW} when the number of possible participants N changes, and we take the parameters $K = 3$, $\phi = 0.1$, $r_0 = 0.2$, $r_1 = 0.2$. The simulative results in the figures are averaged after 100 simulations. In Fig. 4, we find that within the range of parameters we selected, the maximum generation G_{SW} of the pyramid scheme is not very sensitive to the reconnection probability ϕ and the potential participants, and the analytical results are basically in accordance with the simulative results.

3.5. Scale-free network case

If the pyramid scheme expands in a BA scale-free network, similar to the cases in ER random network and SW small-world network above, then we also randomly choose a node as the organizer and and other nodes represent the potential participants. The organizer recruits participants and the participants recruit the next generation participants through the network connections. To ensure positive inflows, the following condition must be satisfied:

$$(1-r_1)n(g+1) \geq r_0 \sum_{i=1}^{g} n(g), \tag{14}$$

where $n(g)$ represents the number of participants at the g-th generation, and $n(g+1)$ represents the number of participants at the $(g+1)$-th generation. The distribution of shortest path length approximates the normal distribution and the position corresponding to the highest point of the normal distribution is the average shortest path length.[26] The average path length $\bar{s}$ of the BA scale-free network is given by[27]

$$\bar{l}_{BA} \approx \frac{\ln N}{\ln \ln N}. \tag{15}$$

Before the peak, the number of participants per generation grows faster than the exponential growth. But after that, the number of participants per generation declines rapidly, so the condition can no longer be satisfied. So we can infer that the maximum generation G_{BA} is close to the average shortest path length, and is given by

$$G_{BA} \approx \lfloor \bar{l}_{BA} \rfloor + 1. \tag{16}$$

Figure 5 shows the analytical result and the simulative result of maximum generation G_{BA} when the number of possible participants N changes. We taken the parameters $r_0 = 0.2$, $r_1 = 0.2$, and the simulative result is averaged after 100 simulations.We can find that in scale-free networks, the maximum generation G_{BA} is not very sensitive to the potential participants in Fig. 5. The analytical results can basically reflect this characteristic.

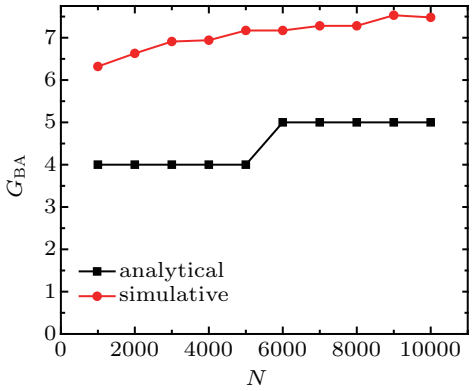


Fig. 5. The analytical result and the simulative result of maximum generation G_{BA} when the number of possible participants N changes. We take the parameters $r_0 = 0.2$, $r_1 = 0.2$. The simulative result is averaged after 100 simulations.

4. A pyramid scheme in real world

Although real cases of pyramid scheme are easy to find in news reports, there are few cases that give details of the number of people involved and the pyramid generations. Usually, when the organizer of the pyramid scheme disappears, the participants with loss will report the case to the police, who will then investigate the case. On July 23, 2018, China news network Guangzhou Station reported a pyramid scheme that had 75663 account numbers and 46 generations, and the pyramid scheme had amassed 76 million yuan in three months.[28] This is the same type of pyramid scheme as described in the introduction. Using the analysis in Section 3, we assume that the pyramid scheme carries on the tree network, ER random network, SW small-world network, and BA scale-free network. We then verify which network can describe the pyramid scheme in the real world well. We assume that one account number represents a participant.

If this real pyramid scheme expands in a tree network, we can calculate the tree network' branching coefficient $\alpha \approx 1.28$. This means on average, less than two participants are recruited by each participant. However, we cannot know more about the connections between the participants, except the branching coefficient.

If this real pyramid scheme spreads in an ER Random network, we can calculate the average degree $k \approx 1.28$ through Eq. (8). So each node is connected to 1.28 nodes on average, and the connection probability in the ER random network is less than $1.28/75663 \approx 1.7 \times 10^{-5}$, which is very small, then isolated nodes and nodes with degree 1 easily o appear in the network. Although this case is similar to that of tree network, the branching coefficient in the random network is not stable and it is easy to end the pyramid scheme if equation (3) is not satisfied (the minimum of the formula $(1-r_1+r_0)/(1-r_1)$ is greater than 1). So we think the pyramid scheme can hardly happen in the ER random network.

If this real pyramid scheme carries out in a BA scale-free network, through the analysis and simulation, we find that developing to 46 generations needs far more than 75663 participants. Therefore, the connections between participants are impossible to form a BA scale-free network.

If this real pyramid scheme takes place in a SW small-world network and accords to all our assumptions, then we could find a simulative result to fit the result of the real pyramid scheme. The parameters we select are $N = 100000$, $\phi = 0.02$, $K = 4$, $r_0 = 0.1$, $r_1 = 0.1$, and each participant invests 23500 yuan. The simulative pyramid scheme has 74652 participants, and develops to 46 generations, and the fund pool of the pyramid is about 76 million yuan. The simulation results are in good agreement with the real pyramid scheme. Figures 6(a) and 6(b) show the cumulative number of participants $N_{\rm cum}$, the number of participants $N_{\rm g}$ in each generation, and the cumulative money $M_{\rm cum}$ changing over generation g in the simulative pyramid scheme.

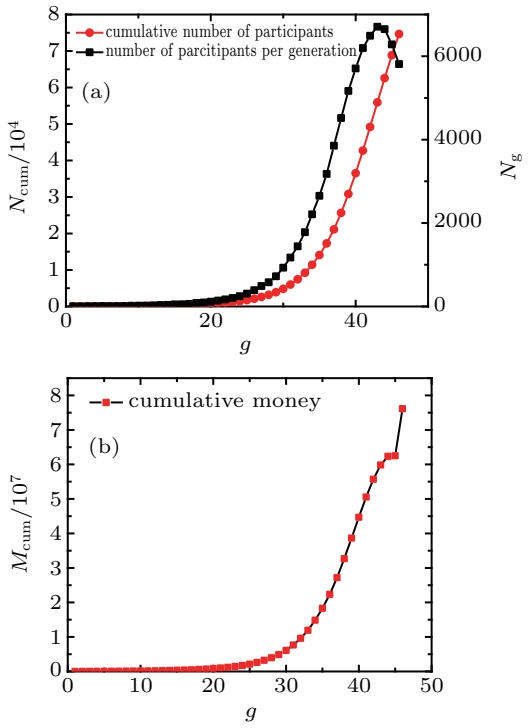


Fig. 6. The cumulative number of participants $N_{\rm cum}$, the number of participants $N_{\rm g}$ in each generation, and the cumulative money $M_{\rm cum}$ changing over generation g in the simulatve pyramid scheme. This is one simulative result in the SW small-world case, the parameters we select are $N = 100000$, $\phi = 0.02$, r_0=0.1, $r_1 = 0.1$, and each participant invests 23500 yuan.

Figure 6 shows that, the number of participants and the amount of accumulated money of the pyramid scheme grow slowly in the initial stage and explosively in the later stage. Once the growth rate slows down, the amount of the pyramid scheme's accumulated money will soon reach a peak and the organizer will escape. The probability of reconnection in simulation is 0.02, which can be understood according to the actual situation and means that participants tend to recruit new participants from familiar people. In fact, according to our investigation and many news reports, such pyramid frauds always arise in small cities and most of the participants recruit new participants from their familiar people. As the generations go on, the network constituted by all participants has the properties of small world: agglomeration and having some flocks, which is similar to the interpersonal network. Although our model has been simplified and approximated, it is enlightening to explain the real case.

Through this simulation analysis, we can speculate that the connections between participants in the real case may constitute a SW small-world network.

5. Conclusion

In summary, we have proposed a pyramid scheme model that has the principal characteristics of many of the pyramid schemes that have appeared in recent years: promising high returns and rewarding the participants attracting the next generation of participants. Assuming that the pyramid scheme spreads in the tree network, ER random network, SW small-world network and BA scale-free network, we give the conditions for the continuity of the pyramid scheme, and the analytical results of how many generations the pyramid scheme can last if the organizer of the pyramid scheme takes all the money away when he/she finds the new money is not enough to pay interest and incentives. We also use our model to analyst a pyramid scheme in the real world and the result displays that the connections of participants in the pyramid may have the feature of small world.

Our work is helpful to understand the operation mechanism and characteristics of the pyramid schemes of consumption rebate type. Our model may be able to apply to some current illegal high-interest loans, if these illegal projects promise a high interest rate and reward the investors who encourage others to invest in such projects but the money accumulated is not actually invested in any real projects. Our work shows that the pyramid schemes of consumption rebate type are not easy to be detected by the supervision because of the small amount of funds and the small number of participants accumulated in the initial stage. After the rapid growth of funds and participants, it often comes to the end of this kind of pyramid frauds, and the organizers have often already fled. So for regulators, it is better to nip such platforms in the bud to avoid any more people suffering a loss. In addition, to some extent, our work provides some basis for further study of such frauds. For example, we will further consider how the participants' beliefs about always having enough new participants affect the operation of these frauds.

References

[1] http://tv.cctv.com/2017/06/18/VIDE8gtfpFkpiBv2QYnNuGIF170618.shtml [2019-3-15]

[2] http://www.mps.gov.cn/n2253534/n2253543/c6108362/content.html [2019-3-15]

[3] https://news.china.com/news100/11038989/20170707/30931598_all.html [2019-3-15]

[4] https://en.wikipedia.org/wiki/MMM_(Ponzi_scheme_company) [2019-3-15]

[5] http://www.51voa.com/VOA_Special_English/pyramid-vs-ponzi-69196.html [2019-3-15]

[6] Gastwirth J L 1977 *Am. Stat.* **31** 79

[7] Artzrouni M 2009 *Math. Soc. Sci.* **58** 190

[8] Moore T, Han J and Clayton R 2012 *Financial Cryptography and Data Security-16th International Conference*, Februray 27–March 2, 2012, Kralendijk, Bonaire, p. 41

[9] Zhu A, Fu P, Zhang Q and Chen Z 2017 *Physica A* **479** 128

[10] Fu P, Zhu A, Ni H, Zhao X and Li X 2017 *Physica A* **490** S0378437117308087

[11] Háznagy A, Fi I, London A and Nemeth T 2015 *International Conference on Models and Technologies for Intelligent Transportation Systems*, June 3–5, 2015, Budapest, Hungary, p. 371

[12] Liu J, Xiong Q Y, Wang K and Shi W R 2015 *Chin. Phys. B* **24** 076401

[13] Brantle T F and Fallah M 2007 *PICMET '07 - 2007 Portland International Conference on Management of Engineering and Technology*, August 5–9, 2007, Portland, OR, USA, p. 540

[14] Pastor-Satorras R and Vespignani A 2001 *Phys. Rev. Lett.* **86** 3200

[15] Moreno Y, Nekovee M and Pacheco A F 2004 *Phys. Rev. E* **69** 066130

[16] West D B 2001 *Networks* **30** 73

[17] Erdös P and Rényi A 2011 *Trans. Am. Math. Soc.* **286** 257

[18] Watts D J and Strogatz S H 1998 *Nature* **393** 440

[19] Grossman J W 2002 *Phys. Today* **31** 74

[20] Santos V M L D, Moreira F G D and Longo R L 2004 *Chem. Phys. Lett.* **390** 157

[21] Barabasi A L and Albert R 1999 *Science* **286** 509

[22] Albert R and Barabasi A L 2001 *Rev. Mod. Phys.* **74** 12

[23] Katzav E, Nitzan M, ben-Avraham D, Krapivsky P, Khn R, Ross N and Biham O 2015 *Europhys. Lett.* **111** 26006

[24] Newman M E and Watts D J 1999 *Phys. Rev. E* **60** 7332

[25] Barrat A and Weigt M 1999 *Eur. Phys. J. B* **13** 547

[26] Ventrella A V, Piro G and Grieco L A 2018 *IEEE Syst. J.* **12** 3869

[27] Reuven C and Shlomo H 2003 *Phys. Rev. Lett.* **90** 058701

[28] http://www.gd.chinanews.com/2018/2018-07-24/2/397998.shtml [2019-3-15]