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Explosive synchronization of multi-layer frequency-weighted coupled complex systems*

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Synchronization is a phenomenon that is ubiquitous in engineering and natural ecosystems. The study of explosive synchronization on a single-layer network gives the critical transition coupling strength that causes explosive synchronization. However, no significant findings have been made on multi-layer complex networks. This paper proposes a frequency-weighted Kuramoto model on a two-layer network and the critical coupling strength of explosive synchronization is obtained by both theoretical analysis and numerical validation. It is found that the critical value is affected by the interaction strength between layers and the number of network oscillators. The explosive synchronization will be hindered by enhancing the interaction and promoted by increasing the number of network oscillators. Our results have importance across a range of engineering and biological research fields.

Keywords: explosive synchronization, Kuramoto model, complex systems

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1. Introduction

In recent years, complex system science has become a popular area of scientific research. Many phenomena in complex systems can be abstracted into complex networks for research, such as the operation of organisms, the cascading failures of power grid, and the blocking of Internet.[1,2] However, the interactions between the constituent elements and dynamic behaviors of complex systems are complicated, especially when they involve biology, sociology, computer science, and statistical physics.[3–5] Synchronization is one of the important dynamic behaviors, and has attracted a great deal of attention, especially the mechanisms generating synchronization and the control methods have become a major focus of research.[6,7] At present, several oscillator models are proposed to study synchronization, such as Kuramoto model, FitzHugh–Nagumo model, and Lorenz oscillator model.[8–11] In particular, the Kuramoto model is a classical phase model composed of kuramoto oscillators, which plays an important role in investigating the synchronization behavior of networks. Therefore, the corresponding analysis methods have also been proposed, such as the mean field method,[12] self-consistent analysis,[13] Ott–Antonsen ansatz, etc.[14,15]

Many researchers have found that the phase transition process of synchronous is a continuous process to synchronization.[16–19] However, this phenomenon was challenged in the scale-free network composed of Kuramoto oscillators, where the phase transition process was proved to be a discontinuous process when the natural frequencies of the Kuramoto oscillators and the degrees of the nodes are positively correlated. This abrupt transition from an incoherent state to a synchronous state in the complex network is named explosive synchronization (ES),[20] and it has opened up a whole new field of research on synchronization. Li *et al.*[21] studied the influence of network topology on ES, and found that ES occurs when the degree of node is positively correlated with the natural frequency. Leyva *et al.*[22] researched the ES on the complex network model with weights. Recently, Zhang *et al.*[23] proposed a single-layer frequency-weighted model of Kuramoto oscillators and observed the ES in general complex networks. They obtained the critical coupling strength $\lambda_c = 2$ for ES through mean field method and self-consistent theory, and extended the investigation of ES from the scale-free networks to the general complex networks. With the intensive investigation of ES and improvements of the Kuramoto model, many factors affecting ES have been considered, such as frustration, external force, pacemaker, and frequency distribution.[24–28]

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Although many related works on ES have been published, these works are all based on single-layer networks. In recent years, the research of synchronization on complex network has gradually shifted from the single-layer network to the multi-layer network. The interaction between layers should be considered because one network always interacts with other networks, such as the interdependence of client system and server system in computer network;[29] and the interaction control of communication network and power grid in electric power infrastructure.[30] Some related works of synchronization in multi-layer networks have been reported.[31–35] Among them, Su *et al.*[32] found that the strength of the inter-layer affects ES in the two-layer co-evolutionary networks. Zhang *et al.*[33] observed the phenomenon that the synchronous process changes from the first order phase transition to the second order phase transition in multi-layer networks. However, these works on ES in multi-layer networks are mainly investigated through numerical verification, while no rigorous theoretical analysis has been given. Therefore, the theoretical research of ES is challenged in multi-layer networks of Kuramoto oscillators, and how to build an appropriate Kuramoto model is a significant topic. In this work, we present a multi-layer frequency-weighted coupled Kuramoto model based on two-layer networks, in which two subnetworks are connected one-to-one. A rigorous theoretical analysis by mean field method and self-consistent theory is provided to obtain the critical coupling strength of ES in the two-layer networks. We analyze the influence of ES between layers and verify the theoretical results by numerical simulation.

This paper is organized as follows. In Section 2, we propose the frequency-weighted Kuramoto model of two-layer networks and obtain the critical coupling strength of ES by mean field and self-consistent methods. In Section 3, numerical simulations are provided for verifying the correctness of the theoretical results. Finally, we give the discussions and conclusions in Section 4.

2. Model and related analysis

2.1. The frequency-weighted Kuramoto model of multi-layer networks

For a frequency-weighted model of N Kuramoto phase oscillators on a single layer network, the evolution of each oscillator follows[23]

$$\dot{\theta}_i = \omega_i + \frac{\lambda\,|\omega_i|}{k_i}\sum_{i=1}^{N} A_{ij}\sin(\theta_j-\theta_i), \tag{1}$$

where $i=1,\ldots,N$, θ_i and θ_j represent the instant phases of oscillators i and j, respectively. λ is the coupling strength of oscillators and ω_i is the natural frequency of the i-th oscillator. $k_i=\sum_{j=1}^{N}A_{ij}$ is the oscillator's degree, and A_{ij} is the element of adjacency matrix $\boldsymbol{A}$, hence $A_{ij}=1$ when the oscillators i and j are connected and $A_{ij}=0$, otherwise. To measure the synchronization motion, we use a fully order parameter R defined as[21]

$$R\mathrm{e}^{\mathrm{i}\Psi} = \frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\theta_j}, \tag{2}$$

where ψ denotes the network's average phase, and $R\subseteq[0,1]$. $R=1$ denotes the fully synchronized state and $R=0$ is the incoherent solution.

We consider a frequency-weighted Kuramoto model on a two-layer one-to-one connected complex system, as shown in Fig. 1. For a two-layer frequency-weighted coupled complex network composed of $2N$ Kuramoto oscillators, each layer of the network contains N oscillators, and the dynamic behavior of the oscillators can be described by its phase $\theta_i(t)$. Each oscillator follows the dynamic equation and is described by

$$\begin{cases} \dot{\theta}_{i,1} = \omega_{i,1} + \dfrac{\lambda\,|\omega_{i,1}|}{k_{i,1}+h}\sum_{j=1}^{2N} A_{ij}\sin(\theta_{j,1}-\theta_{i,1}), \\ \dot{\theta}_{i,2} = \omega_{i,2} + \dfrac{\lambda\,|\omega_{i,2}|}{k_{i,2}+h}\sum_{j=1}^{2N} A_{ij}\sin(\theta_{j,2}-\theta_{i,2}), \end{cases} \tag{3}$$

where subscripts 1 and 2 denote the first layer and the second layer, respectively, $i=1,\ldots,N$, and h stands for the interaction strength of inter-layer. A_{ij} is the element of adjacency matrix $\boldsymbol{A}$ on the multi-layer networks given as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 & h\boldsymbol{I} \\ h\boldsymbol{I} & \boldsymbol{A}_2 \end{bmatrix}, \tag{4}$$

where $\boldsymbol{I}$ is the identity matrix that represents the one-to-one interaction between the two layers, and $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are the adjacency matrices of the first layer network and the second layer network, respectively.

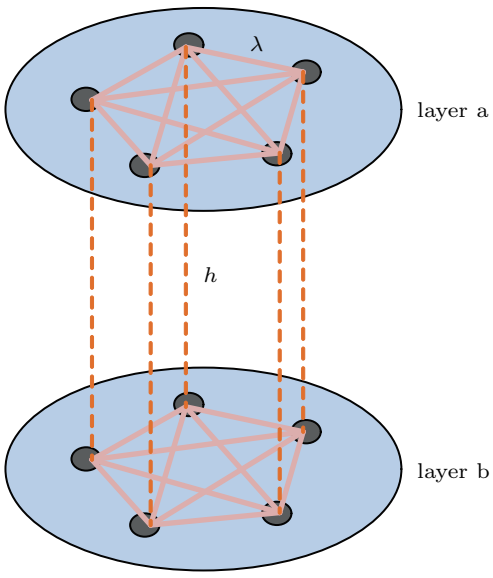


Fig. 1. The topology diagram of a two-layered network composed of fully-connected networks. λ represents the intra-layer coupling strength and h denotes the inter-layer interaction strength.

2.2. Related analysis

To simplify the research, we consider the multi-layer networks composed of the globally connected networks, and the natural frequency distributions of the two-layer network oscillators are the same symmetric bell curve distribution $g(\omega)$. Therefore, we only need to observe the dynamic evolution of one of the network oscillators and equation (3) can be rewritten as

$$\dot{\theta}_i = \omega_i + \frac{\lambda|\omega_i|}{N+h}\left[\sum_{i=1}^{N}\sin(\theta_j-\theta_i)+h\sin(\theta_i'-\theta_i)\right], \quad i=1,\ldots,N, \tag{5}$$

where θ_i' stands for the instant phase of the other layer network's oscillator i, and the degree of each oscillator $k_i \approx N$ for the globally connected network. Through Eq. (2), we have the imaginary equation

$$R\sin(\Psi-\theta_i) = \frac{1}{N}\sum_{i=1}^{N}\sin(\theta_j-\theta_i). \tag{6}$$

Through self-consistent theoretical analysis and mean field theory, we can substitute Eq. (6) into Eq. (5) and obtain the mean field form

$$\dot{\theta}_i = \omega_i + \frac{\lambda N|\omega_i|}{N+h}\left[R\sin(\Psi-\theta_i)+\frac{h}{N}\sin(\theta_i'-\theta_i)\right], \quad i=1,\ldots,N. \tag{7}$$

$\psi(t)=\psi(0)+\langle\omega\rangle t$ can be obtained through rotational coordinate transformation, where $\langle\omega\rangle$ is the average of the oscillators' natural frequencies. When the natural frequency follows a symmetric distribution, we have $\langle\omega\rangle=0$ and the phase difference between the instant phase and the average phase $\Delta\theta_i=\theta_i-\psi$, equation (7) can then be rewritten as

$$\Delta\dot{\theta}_i = \omega_i - \frac{\lambda N|\omega_i|}{N+h}\left[R\sin(\Delta\theta_i)-\frac{h}{N}\sin(\theta_i'-\theta_i)\right], \quad i=1,\ldots,N. \tag{8}$$

When the system reaches the complete synchronized state, we have $\Delta\theta_i=0$ and $\theta_i'-\theta_i\approx 0$, we can then solve the phase of the oscillator phase lock

$$\Delta\theta_i = \begin{cases} \arcsin\left(\dfrac{N+h}{N\lambda R}\right), & \omega_i>0, \\ \arcsin\left(-\dfrac{N+h}{N\lambda R}\right), & \omega_i<0. \end{cases} \tag{9}$$

Through Eq. (9), we find that $\Delta\theta_i$ only depends on the sign of ω_i but not on its value, $\Delta\theta_i$ will gradually approach to 0 with increasing coupling strength λ. In this case, from Eq. (2) we can obtain

$$R = \frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\Delta\theta_j} = \frac{1}{2}(\mathrm{e}^{\mathrm{i}\Delta\theta_+}+\mathrm{e}^{\mathrm{i}\Delta\theta_-}), \tag{10}$$

where $\mathrm{e}^{\mathrm{i}\Delta\theta_+}$ and $\mathrm{e}^{\mathrm{i}\Delta\theta_-}$ represent the synchronized phases when the natural frequency is positive or negative, respectively. Substituting Eq. (9) into Eq. (10), we have

$$R = \frac{1}{2}(\cos\Delta\theta_+ + \cos\Delta\theta_-) = \frac{\sqrt{(\lambda NR)^2-(N+h)^2}}{\lambda NR}. \tag{11}$$

Then, we can obtain the unary quadratic equation about the order parameter R

$$R^4 - R^2 + \frac{(N+h)^2}{N^2\lambda^2} = 0. \tag{12}$$

Through the stability analysis, we have the range of the coupling strength $\lambda \geqslant 2(1+h/N)$ when equation (12) has a nonzero solution and obtain the critical coupling strength

$$\lambda_{\mathrm{c}} = 2\left(1+\frac{h}{N}\right). \tag{13}$$

Observing Eq. (13), we can find that λ_{c} of ES is related to the interaction strength h of inter-layer and the number of oscillators N of the network. $\lambda_{\mathrm{c}}=2$ is the ES threshold of the frequency-weighted coupled model in a single layer network when $h=0$. In addition, from Eq. (12) we have

$$R = \sqrt{\frac{\lambda+\sqrt{\lambda^2-4(1+h/N)^2}}{2\lambda}}. \tag{14}$$

Equation (14) implies that ES exists when $\lambda \geqslant 2(1+h/N)$, and if the $g(\omega)$ is symmetric, then the synchronous behavior of the oscillators has no correlation with the specific distribution of the natural frequency.

3. Numerical simulation results

To verify the correctness of the theoretical results, we simulate the synchronization behavior of the two-layer frequency-weighted coupled complex system. The two-layer complex network is composed of the globally connected networks, and the natural frequency distribution follows the Lorentzian distribution $g(\omega)=(1/\pi)[\sigma/(\omega-\omega_0)^2+\sigma^2]$ with $\omega_0=0$ and $\sigma^2=1$. The initial phases of the oscillators follow a random uniform distribution in the range $[0,2\pi]$. To track the phase transition to synchronization in the two-layer networks, we use the order parameters R_1 and R_2, which are given by

$$R_1\mathrm{e}^{\mathrm{i}\Psi_1} = \frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\theta_{j,1}}, \quad R_2\mathrm{e}^{\mathrm{i}\Psi_2} = \frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\theta_{j,2}}. \tag{15}$$

First, we increase coupling strength λ adiabatically from λ_0 to $\lambda_0+n\delta\lambda$ with an increment $\delta\lambda=0.02$. Second, we decrease coupling strength λ adiabatically from $\lambda_0+n\delta\lambda$ to

λ_0 in steps of $\delta\lambda$. We compute the stationary values of R_1 and R_2 with $\lambda_0, \lambda_0+\delta\lambda, \ldots, \lambda_0+n\delta\lambda$ for both the increasing (forward phase transition) and decreasing λ (backward phase transition) directions. We eliminate the initial phases before each variation of $\delta\lambda$ and integrate the whole system long time enough by utilizing the fourth-order Runge–Kutta method to reach the stationary state.

3.1. The influence of inter-layer interaction strength on explosive synchronization of two-layer networks

Figures 2(a)–2(d) show the dependence of R and λ when the inter-layer interaction strength h is 0, 20, 40, and 60, respectively, where the red-solid line represents the forward phase transition process of increasing λ and the blue hyphenated line stands for the backward process of decreasing λ. R_{1F}, R_{1B} and R_{2F}, R_{2B} represent the synchronous order parameters of the forward and backward processes on the first and the second layers, respectively. In the case of $N = 200$, we observe that there are two abrupt transitions at λ_{cF} for the forward phase transition (red line) and λ_{cB} for the backward phase transition (blue line) in each panel of Fig. 2. The two abrupt transitions take place at different values of λ and the whole synchronization diagram has a significant hysteresis phenomenon. This discontinuous phase transition indicates that the ES appears in the two-layer network. In addition, the critical coupling strength λ_c of backward phase transition is 1.96, 2.18, 2.40, and 2.58 in Figs. 2(a)–2(d), it is demonstrated that the increase of the inter-layer interaction strength h will prevent the appearance of ES on the multi-layer network.

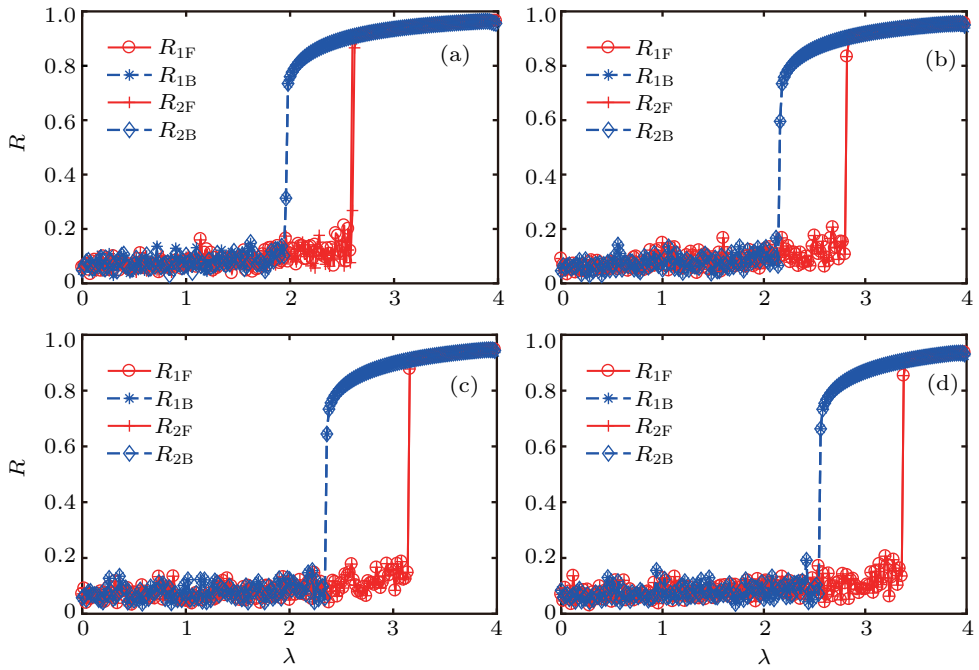


Fig. 2. Synchronous phase transition diagrams of order parameters for different inter-layer interaction strengths ((a) $h = 0$, (b) $h = 20$, (c) $h = 30$, (d) $h = 60$) when $N = 200$.

To emphasize the theoretical results, we show the relationship between the critical coupling strength λ_c of backward phase transition and the interaction strength h of inter-layer in Fig. 3 when $N = 200$. The red-solid line is the theoretical results, and the blue circles represent the simulation results of the critical coupling strength λ_c obtained when the inter-layer interaction strength h is changed from 0 to 100 in steps of 10. When ES appears, we can find that the simulation results match with the theoretical ones and λ_c changes linearly with the increase of h. Furthermore, we can also observe that the error of λ_c between the theoretical and experimental results is very small in Table 1. This indicates that the simulation results verify the theoretical analysis.

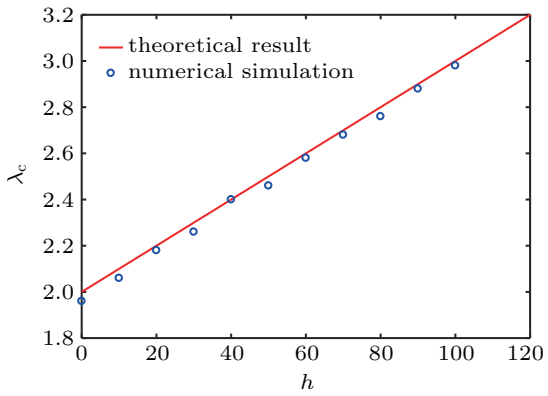


Fig. 3. Comparison of theoretical results and numerical simulations of critical coupling strength variation with inter-layer interaction strength.

Table 1. Error γ between the theoretical λ_{cT} and experimental λ_{cE} with different inter-layer interaction strengths.

h	λ_{cT}	λ_{cE}	$\gamma = \lvert\lambda_{cT} - \lambda_{cE}\rvert$
0	2.00	1.96	0.04
10	2.10	2.06	0.04
20	2.20	2.18	0.02
30	2.30	2.36	0.04
40	2.40	2.40	0.00
50	2.50	2.56	0.04
60	2.60	2.58	0.02
70	2.70	2.68	0.02
80	2.80	2.76	0.04
90	2.90	2.88	0.02
100	3.00	2.98	0.02

3.2. The influence of number of network oscillators on explosive synchronization of two-layer networks

Through the previous theoretical analysis, we can also find that the critical coupling strength is affected by the number of network oscillators N. To further analyze the evolution of the oscillator's collective behavior, we compute the oscillator's effective frequency of one layer along the backward process as $\omega_i^{\text{eff}} = \int_{\tau}^{\tau+T} \dot{\theta}_i(\tau)\mathrm{d}\tau$ with $T \gg 1$ by following Ref. [20]. We take the case that the number of network oscillators N is 300, 500, 700, and 900 when the inter-layer interaction strength h is 100 as examples and focus on the backward continuation in λ. Figures 4(a)–4(d) show the result.

In Figs. 4(a)–4(d), we can observe that all the ω_i^{eff} of the network oscillators will retain the average frequency of $\langle\omega\rangle = (1/N)\sum_{j=1}^{N}\omega_i$ (since the natural frequency ω_i of the network oscillators follows a symmetric Lorentzian distribution, $\langle\omega\rangle \approx 0$ in Fig. 4) when $\lambda > \lambda_c$ and then they suddenly jump to their natural frequencies at the critical coupling λ_c. Moreover, we also notice the emergence of ES when λ is 2.64, 2.40, 2.26, and 2.20 in Figs. 4(a)–4(d), respectively. It is shown that increasing the number of network oscillators N can incite the appearance of ES when h is constant.

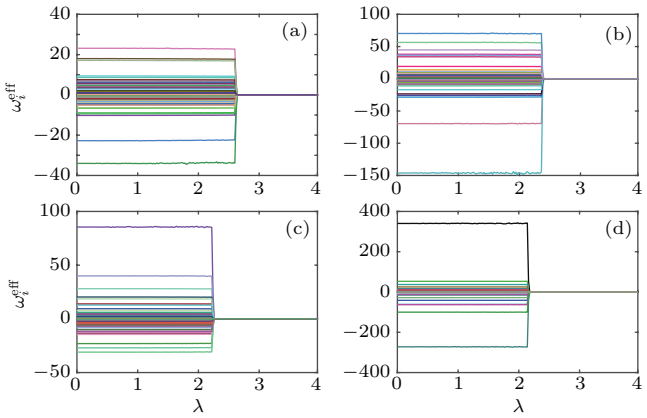


Fig. 4. Synchronous phase transition diagram of effective frequency variation with coupling strength of backward phase transition for different network oscillator numbers ((a) $N = 300$, (b) $N = 500$, (c) $N = 700$, (d) $N = 900$) when $h = 100$.

Similarly, to verify that the simulation results are consistent with the theoretical results, we give the relationship between λ_c of backward process and N when $h = 100$ in Fig. 5. The red-solid line is the theoretical results and the blue dot stands for the simulation results obtained when the number of network oscillators is changed from 200 to 1000 in steps of 100. From Fig. 5, we find that λ_c inversely decreases with the increase of N and the simulation results match the theoretical results very well. In addition, the similar results are also demonstrated in Table 2. These simulation results verify the correctness of the theoretical results.

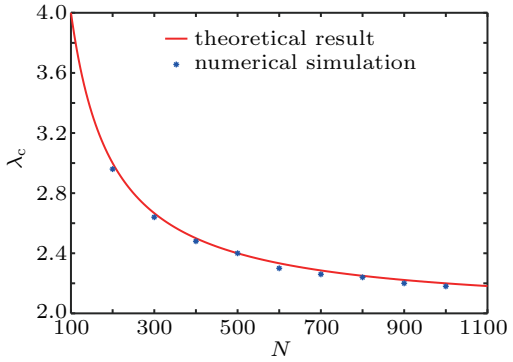


Fig. 5. Comparison of theoretical results and numerical simulations of critical coupling strength variation with network oscillator number.

Table 2. Error γ between the theoretical λ_{cT} and experimental λ_{cE} with different numbers of network oscillators.

N	λ_{cT}	λ_{cE}	$\gamma = \lvert\lambda_{cT} - \lambda_{cE}\rvert$
200	3.00	2.96	0.04
300	2.67	2.64	0.03
400	2.50	2.48	0.02
500	2.40	2.40	0.00
600	2.33	2.30	0.03
700	2.29	2.26	0.03
800	2.25	2.24	0.01
900	2.22	2.20	0.02
1000	2.20	2.18	0.02

4. Conclusion

Critical coupling strength is one of the key parameters that determinate the ES in multi-layer complex networks where the interaction between layers plays an important role. In this paper, we propose a frequency-weighted coupled model of Kuramoto oscillators and investigate the ES in the multi-layer networks. Through mean field method and rigorous self-consistent theoretical analysis, a critical coupling strength that ES appears on the multi-layer networks has been given. It is demonstrated that the critical coupling strength is determined by the inter-layer interaction strength and the number of network oscillators. Enhancing the interaction strength between layers will hinder the generation of ES and increasing the number of network oscillators will promote it. These conclusions are supported by both theoretical analysis and the matching numerical simulation results. It is a very significant research

and has impacted on the generation and control of multi-layer complex systems in the real-world.

In this paper, we only analyze the multi-layer networks composed of fully connected networks. What happens if the topology of networks is non-fully connected, and what is the influence of node degree on the ES are left as open questions. Given that the impact of node degree and topology on ES in multi-layer networks remain unsolved, we will study them in our further work.

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