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Impurity-induced Shiba bound state in the BCS–BEC crossover regime of two-dimensional Fermi superfluid*

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For a two-dimensional ultra-cold Fermi superfluid with an effective static magnetic impurity, we theoretically investigated the variation of the Yu–Shiba–Rusinov (YSR) bound state in the Bardeen–Cooper–Schrieffer (BCS) to Bose–Einstein condensation (BEC) crossover regime. Within the framework of mean-field theory, analytical results of the YSR bound state energy were obtained as a function of the interaction parameters. First, when the background Fermi superfluid system stays in the weakly interacting BCS regime, we found that the YSR bound state energy is linearly dependent on the gap parameter with its coefficient slightly different from previous results. Second, we discovered re-entrance phenomena for the YSR state and an upper bound of the strength of the interaction between the paired atoms. By carefully analyzing the bound state energy as a function of the interaction parameters, we obtained a phase diagram showing the existence of the YSR state. Finally, we concluded that the re-entrance phenomena and the critical point can be easily experimentally detected through measurement of radio-frequency spectroscopy and density of states using current experimental techniques.

Keywords: ultra-cold quantum gases, BCS–BEC crossover, magnetic impurity, Yu–Shiba–Rusinov (YSR) state

PACS: 05.30.Fk, 37.10.Jk, 03.75.Hh **DOI:** 10.1088/1674-1056/28/7/070501

1. Introduction

Experimental realizations of Fermi degeneracy and Fermi superfluid in ultra-cold atom systems have provided more opportunities for us to deepen our understanding of many important problems in condensed matter physics,[1,2] e.g., the topological superfluid phase.[3,4] In particular, interactions between different spin states can be tuned by the technique of Feshbach resonance.[5,6] Therefore, the system can change from weakly the interacting Bardeen–Cooper–Schrieffer (BCS) regime with loosely overlapping Cooper pairs to the strongly interacting Bose–Einstein condensation (BEC) regime with tightly bound molecules (BCS–BEC crossover).[7] Another ground-breaking development in ultra-cold experiments is the realization of the interplay between magnetism and superconductivity. The first scheme is the realization of spin-correlations by trapping the ultra-cold fermion system in an optical lattice at a very low temperature, and can be described by the well-know Hubbard model.[8–10] This brings us to a stage where the interplay between superconductivity and magnetism can now be investigated systematically in a comparatively clean system compared with the conventional solid material systems. It is also believed that this system may shed new light on the underlying physics of high-temperature superconductivity. The second scheme is the generation of effective magnetic impurities in clean Fermi superfluid systems.

Magnetic impurities always attenuate superconductivity[11] by the formation of the in-gap Yu–Shiba–Rusinov (YSR) bound states.[12–16] These YSR states can also be used as a local probe to detect the pairing symmetry of the background superconductors by measuring the local density of states (DOS) using scanning tunneling microscopy.[17–22] Moreover, theoretical calculations have discovered that when a chain of periodically arranged magnetic impurities is doped on the surface of a superconductor, the system should exhibit Majorana states[23–25] crucial for topological quantum computing.[26] Recently, there have been suggestions that the YSR state is also useful for detecting the topological states of matter.[27–29]

In ultra-cold experiments, impurities can be introduced into the system using laser speckles or quasi-periodic lattices.[30] Furthermore, a single impurity in cold atom systems can be manipulated using off-resonant laser light or by a third species of atoms interacting differently with the two paired atoms.[31–33] Based on these experimental progresses, many theoretical calculations have been performed regarding the YSR state. Vernier *et al.* calculated the YSR bound state energy for a single magnetic impurity immersed in a three-dimensional (3D) Fermi superfluid, and discussed the experimental detection of these YSR states by measuring the radio frequency (RF) spectrum for both under-sea and in-gap YSR states.[34] Yan *et al.* investigated the effect of spin–orbit coupling on the YSR bound state energy of a two-dimensional (2D) Fermi superfluid and calculated the DOS for a finite

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but small concentration of magnetic impurities.[35] By numerically solving the Bogoliubov–de Gennes equations for a 2D Fermi superfluid trapped in a harmonic potential, Hu *et al.* found a universal in-gap bound state induced by either nonmagnetic or magnetic impurities in the strong scattering limit.[36] In order to detect the YSR bound state unambiguously, a modified spatially resolved RF spectroscopy was proposed to measure the local DOS in Ref. [37].

Generally, the YSR bound state energy depends on two kinds of interaction parameters. The first one is the interaction between the paired atoms and magnetic impurities denoted by g_σ. The second one is the interaction between the paired atoms denoted by g. However, the YSR bound state energy implicitly depends on g. For small g, it can be shown that the YSR bound state energy linearly depends on the paring parameter, which is a function of the interaction parameter g. Previous investigations have been limited to the regime with g being very small. As far as we know, no calculations have been done considering the variation of the YSR bound state energy in the whole BCS–BEC crossover.

In this paper, we consider a static magnetic impurity immersed in a 2D Fermi superfluid. Within the framework of BCS mean-field theory, we use the T-matrix formalism to deal with the effect of the magnetic impurity on the single particle Green's function. From the T-matrix, we obtain the analytical results of the in-gap YSR bound state energy as a function of the interaction parameters. We discuss in detail the dependence of the YSR bound state energy on the two interaction parameters with special attention on the interaction between the paired atoms. Most interestingly, we find re-entrance phenomena regarding the existence of the YSR state and an upper critical value g_c beyond which the YSR states cease to exist.

The outline of the paper is as follows. In Section 2, we first introduce the model system and the Hamiltonian. Then the bare Green's function is obtained within the framework of BCS mean-field theory. Consequently, we obtain the T-matrix for the potential scattering of the paired atoms with a single magnetic impurity, which will be used to derive the YSR bound state energy. In Section 3, we obtain the analytical results of the YSR bound state energy and discuss its asymptotic behavior in the weakly interacting BCS regime. We also discuss the re-entrance phenomena and obtain the critical interaction parameter beyond which the YSR bound states cease to exist. Finally, we summarize our paper with future outlooks and propose possible experimental scenarios in Section 4.

2. Formalism

In this paper, we consider a feasible experimental scheme to realize a static magnetic impurity in a uniform 2D Fermi superfluid system. Following Ref. [34], we consider a third species of atoms interacting differently with the paired atoms (denoted by spin up ($\uparrow$) and spin down ($\downarrow$)). The impurity atom is very heavy compared with the pairing atoms. Therefore, it can be considered as a static magnetic impurity. The scattering between the paired atoms and the magnetic impurity reduces to a potential scattering problem. The system can be described by $H = H_0 + H_\mathrm{I}$, where

$$H_0 = \int \mathrm{d}\boldsymbol{r}\phi_\sigma^\dagger(\boldsymbol{r})\left(-\frac{\nabla^2}{2m}-\mu\right)\phi_\sigma(\boldsymbol{r}) - g\int \mathrm{d}\boldsymbol{r}\phi_\uparrow^\dagger(\boldsymbol{r})\phi_\downarrow^\dagger(\boldsymbol{r})\phi_\downarrow(\boldsymbol{r})\phi_\uparrow(\boldsymbol{r})$$

is the pairing Hamiltonian, with $\phi_\sigma(\boldsymbol{r})$ and $\phi_\sigma^\dagger(\boldsymbol{r})$ denoting the annihilation and creation field operators, respectively. m is the mass of the paired Fermi atoms, and g denotes the strength of the contact interaction between the paired atoms. $\hbar = 1$ throughout this paper. The interaction Hamiltonian between the paired atoms and the impurity atom is

$$H_\mathrm{I} = g_\sigma \int \mathrm{d}\boldsymbol{r}\phi_\sigma^\dagger(\boldsymbol{r})\Psi^\dagger(\boldsymbol{r})\Psi(\boldsymbol{r})\phi_\sigma(\boldsymbol{r}), \tag{1}$$

where $\Psi^\dagger(\boldsymbol{r})$ and $\Psi(\boldsymbol{r})$ denote the creation and annihilation field operators of the impurity atom, respectively, and g_σ denotes the strength of the interaction between the paired atoms of spin state σ and the impurity atom. Supposing the impurity atom is localized at $\boldsymbol{r} = 0$ without loss of generality, we can ignore the dynamics of the impurity atom and reduce H_I to

$$H_\mathrm{I} = g_\sigma \int \mathrm{d}\boldsymbol{r}\delta(\boldsymbol{r})\phi_\sigma^\dagger(\boldsymbol{r})\phi_\sigma(\boldsymbol{r}) = g_\sigma\phi_\sigma^\dagger(0)\phi_\sigma(0). \tag{2}$$

Equations (1) and (2) are our starting points in the investigation of the YSR bound state around the single static magnetic impurity. In order to understand the general structure of the T-matrix formalism that will be used to determine the YSR state, we start from the Schrödinger's equation $H\psi = E\psi$, which can be rewritten as

$$(E - H_0)\psi(\boldsymbol{r}) = \delta(\boldsymbol{r})U\psi(\boldsymbol{0}), \tag{3}$$

with the spinor $\psi(\boldsymbol{r}) = \left[\phi_\uparrow(\boldsymbol{r}), \phi_\downarrow(\boldsymbol{r})\right]^\mathrm{T}$ and $U = \mathrm{diag}\left(g_\uparrow, g_\downarrow\right)$. The general solution of the above equation can be easily obtained in the momentum space, and the final results at the origin are given by

$$\left[U^{-1} - \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^2}G^0(E,\boldsymbol{p})\right]\psi(\boldsymbol{0}) = 0,$$

where $G^0(E,\boldsymbol{p}) = (E - H_0)^{-1}$ is the Green's function of the clean system and

$$T^{-1}(E) = U^{-1} - \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^2}G^0(E,\boldsymbol{p}) \tag{4}$$

is just the T-matrix introduced in Refs. [34] and [38] by which the bound state energy of the YSR state can be given by

$$\det T^{-1}(E) = 0. \tag{5}$$

In the general situation, H_0 is not integrable, and therefore $G^0(E,\boldsymbol{p})$ can be analytically obtained only within a proper approximation strategy. In the context of Fermi pairing and superfluid systems, the BCS mean-field theory has been proven to be of great success at the low-temperature regime. Although the mean-field theory has been introduced in detail in previous investigations, it is given briefly in the next section for self-consistency.

3. BCS mean-field theory

The BCS mean-field theory is most conveniently given in momentum space where H_0 can be written as $H_0 = \sum_{\boldsymbol{k}} \xi_{\boldsymbol{k}} C^{\dagger}_{\boldsymbol{k}\sigma} C_{\boldsymbol{k}\sigma} - g\sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} C^{\dagger}_{\boldsymbol{k}\uparrow} C^{\dagger}_{-\boldsymbol{k}+\boldsymbol{q}\downarrow} C_{-\boldsymbol{k}'+\boldsymbol{q}\downarrow} C_{\boldsymbol{k}'\uparrow}$, with the kinetic energy term being $\xi_{\boldsymbol{k}} = \frac{\boldsymbol{k}^2}{2m} - \mu$ and $C^{\dagger}_{\boldsymbol{k}\sigma(\sigma=\uparrow,\downarrow)}$ ($C_{\boldsymbol{k}\sigma(\sigma=\uparrow,\downarrow)}$) being the creation (annihilation) operators. Within the mean-field theory, the interaction term can be decoupled and H_0 can be approximated with the familiar BCS mean-field Hamiltonian $H_0 \simeq H_{\rm BCS} = \sum_{\boldsymbol{k}} \xi_{\boldsymbol{k}} C^{\dagger}_{\boldsymbol{k}\sigma} C_{\boldsymbol{k}\sigma} - \Delta\sum_k (C^{\dagger}_k C^{\dagger}_{-k} + C_{-k} C_k) + V \mid \Delta \mid^2/g$, where Δ is the order parameter or the superconductor gap, and V denotes the size of the system. Since BCS quasi-particles involve coherent mixing of particles and holes, we can use Nambu's four-dimensional spinors to rewrite the Hamiltonian $H_{\rm BCS}$ into matrix form as $H_{\rm BCS} = \frac{1}{2}\sum_k \psi^{\dagger}_k H_{\rm BdG} \psi_k + \sum_k \xi_k + V \mid \Delta \mid^2/g$, with $\psi_k = (C_{k\uparrow}, C_{k\downarrow}, C^{\dagger}_{-k\uparrow}, C^{\dagger}_{-k\downarrow})^{\rm T}$ being Nambu's four-dimensional spinors, and $H_{\rm BdG}$ being

$$H_{\rm BdG} = \begin{bmatrix} \xi_k & 0 & 0 & -\Delta \\ 0 & \xi_k & \Delta & 0 \\ 0 & \Delta & -\xi_k & 0 \\ -\Delta & 0 & 0 & -\xi_k \end{bmatrix}. \tag{6}$$

From $\det(E_{\boldsymbol{k}} - H_{\rm BdG}) = 0$, the excitation spectrum can be obtained as $E_{\boldsymbol{k}} = \pm\sqrt{\xi_{\boldsymbol{k}}^2 + \Delta^2}$. The ground-state free energy is given by $E_{\rm g} = V\Delta^2/g + \frac{1}{2}\sum_k (\xi_k - E_k)$, from which we can write down the thermodynamic potential as $\Omega(T=0) = E_{\rm g}$. Then, the gap and number equations can be derived by using $\partial\Omega/\partial\Delta = 0$ and $\partial\Omega/\partial\mu = -N$, and the final results are given by

$$\frac{1}{g} = \frac{1}{V}\sum_{\boldsymbol{k}} \frac{1}{E_{\boldsymbol{k}}}, \tag{7}$$

$$N = \sum_{\boldsymbol{k}} \left(1 - \frac{\xi_{\boldsymbol{k}}}{E_{\boldsymbol{k}}}\right). \tag{8}$$

Obtaining the chemical potential and order parameter from the self-consistent solution of the above two equations, the Green's function for the clean superfluid system can be given by $G^0(E,\boldsymbol{k}) = (E - H_0)^{-1}$, and the final results are

$$G^0(E,\boldsymbol{k}) = \frac{1}{E^2 - E_{\boldsymbol{k}}^2} \begin{bmatrix} E+\xi_{\boldsymbol{k}} & 0 & 0 & -\Delta \\ 0 & E+\xi_{\boldsymbol{k}} & \Delta & 0 \\ 0 & \Delta & E-\xi_{\boldsymbol{k}} & 0 \\ -\Delta & 0 & 0 & E-\xi_{\boldsymbol{k}} \end{bmatrix}. \tag{9}$$

With the definition of Nambu's four-dimensional spinors, the bare interaction matrix should be written as

$$U^{-1} = \begin{bmatrix} g_{\uparrow}^{-1} & 0 & 0 & 0 \\ 0 & g_{\downarrow}^{-1} & 0 & 0 \\ 0 & 0 & -g_{\uparrow}^{-1} & 0 \\ 0 & 0 & 0 & -g_{\downarrow}^{-1} \end{bmatrix}. \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (4), we can obtain the analytical form of the T-matrix, and the bound state energy of the YSR state can be given by solving Eq. (5).

The T-matrix was used in Ref. [34] to discuss both the under-sea and YSR bound states in a 3D Fermi superfluid. Our discussion in the 2D case follows their strategy with the focus only on the YSR bound state.

4. Results and discussion

In this paper, an effective magnetic impurity is produced by introducing a third species of atoms interacting differently with the paired atoms with opposite spins. Both the interactions between the two paired atoms and the interaction between the paired atoms and the impurity atom are of the short-ranged contact type. As already known, this type of interaction leads to divergences in the momentum integrals in Eq. (7) as well as in Eq. (4). These divergences can be eliminated via the following relations:[34,39]

$$g^{-1} = \sum_{\boldsymbol{k}} \frac{1}{2\varepsilon_{\boldsymbol{k}} + \varepsilon_{\rm b}}, \tag{11}$$

$$g_{\uparrow,\downarrow}^{-1} = -\sum_{\boldsymbol{k}} \frac{1}{\varepsilon_{\boldsymbol{k}} + \varepsilon_{{\rm b}_{\uparrow,\downarrow}}}. \tag{12}$$

Instead of the bare contact interaction parameter, we now use $\varepsilon_{\rm b}$ to denote the interaction strength between the paired atoms and $\varepsilon_{{\rm b}_{\uparrow,\downarrow}}$ to denote the interaction between the impurity atom and different paired atoms. By tuning through a Feshbach resonance from a BCS side, $\varepsilon_{\rm b}$ increases from zero and becomes large in the BEC limit.

First, with the help of Eq. (11), the momentum integrals in Eqs. (7) and (8) can be obtained explicitly. By analytically solving the gap equation (7) and the number equation (8), we obtain the chemical potential μ and order parameter Δ as functions of $\varepsilon_{\rm b}$ as follows:

$$\begin{aligned} \mu/E_{\rm F} &= 1 - \frac{\varepsilon_{\rm b}}{2E_{\rm F}}, \\ \Delta/E_{\rm F} &= \sqrt{2\varepsilon_{\rm b}/E_{\rm F}}\,. \end{aligned} \tag{13}$$

The above results are the self-consistent solutions of the mean-field number and gap equations corresponding to Eqs. (7) and (8), respectively. Previous investigations have shown that the mean-field theory is sufficient for qualitatively capturing the correct physics, especially at zero temperature. However, at finite temperature, one needs to include the pair fluctuations to get the correct physics in the strong coupling regime.[40–44]

Moreover, in two dimensions, fluctuations become more important than in the 3D case, as demonstrated in Ref. [45]. In this paper, we focus on the YSR bound state formed by coherent superfluid fermions and a single static and localized magnetic impurity. Since it is a potential scattering problem, thermal fluctuation does not change the results. At zero temperature, the fluctuation does not have any important effect on the superfluid properties of the background system. Our interests are concentrated on the formation and existence of the YSR state formed by the paired fermions and single magnetic impurity. Therefore, we believe that the mean-field theory is enough for our current purpose in this paper.

Second, following Ref. [34], we can reduce the four-dimensional Nambu space of $G^0(E,\boldsymbol{k})$ to a pair of 2D subspaces. These two subspaces are connected to each other through particle–hole symmetry. For our purpose of calculating the bound state energy of the YSR state, we only focus on one subspace, say, the first one for example. Consequently, $G^0(E,\boldsymbol{k})$ and U^{-1} become

$$G^0(E,\boldsymbol{k}) = \frac{1}{E^2-E_{\boldsymbol{k}}^2}\begin{bmatrix} E+\xi_{\boldsymbol{k}} & -\Delta \\ -\Delta & E-\xi_{\boldsymbol{k}} \end{bmatrix}, \tag{14}$$

$$U^{-1} = \begin{bmatrix} g_\uparrow^{-1} & 0 \\ 0 & -g_\downarrow^{-1} \end{bmatrix}. \tag{15}$$

Finally, after performing the momentum integral in Eq. (4), we analytically obtain $T^{-1}(E)$ as

$$T^{-1}(E) = \frac{m}{2\pi}\begin{bmatrix} b_\uparrow + \dfrac{E\theta}{\sqrt{\Delta^2-E^2}} - \ln\dfrac{\sqrt{\Delta^2-E^2+\mu^2}}{E_{\rm F}} & -\dfrac{\theta\Delta}{\sqrt{\Delta^2-E^2}} \\ -\dfrac{\theta\Delta}{\sqrt{\Delta^2-E^2}} & -b_\downarrow + \dfrac{E\theta}{\sqrt{\Delta^2-E^2}} + \ln\dfrac{\sqrt{\Delta^2-E^2+\mu^2}}{E_{\rm F}} \end{bmatrix}, \tag{16}$$

where

$$\theta = \frac{\pi}{2} + \arctan\frac{\mu}{\sqrt{\Delta^2-E^2}},$$

and $b_{\uparrow,\downarrow} = \ln\varepsilon_{\rm b_{\uparrow,\downarrow}}/E_{\rm F}$ with the Fermi energy $E_{\rm F} = k_{\rm F}^2/2m$, and $k_{\rm F} = \sqrt{2\pi N/V}$.

In the weakly interacting BCS limit where $\varepsilon_{\rm b}$ is small, it is easily seen from Eq. (13) that the order parameter $\Delta = \sqrt{2E_{\rm F}\varepsilon_{\rm b}} \sim 0$ and the chemical potential $\mu \sim E_{\rm F}$ is large compared with the order parameter. Furthermore, in the deep BCS regime, the bound state energy is known to be linearly dependent on the order parameter, and thus is small compared with the chemical potential. Under these conditions, we have

$$\theta = \frac{\pi}{2} + \arctan\frac{\mu}{\sqrt{\Delta^2-E^2}} \simeq \pi,$$
$$\ln\frac{\sqrt{\Delta^2-E^2+\mu^2}}{E_{\rm F}} \simeq 0.$$

By combining these conclusions, the T-matrix can be reduced to

$$T^{-1} \approx \frac{m}{2\pi}\begin{bmatrix} b_\uparrow + \dfrac{E\pi}{\sqrt{\Delta^2-E^2}} & -\dfrac{\pi\Delta}{\sqrt{\Delta^2-E^2}} \\ -\dfrac{\pi\Delta}{\sqrt{\Delta^2-E^2}} & -b_\downarrow + \dfrac{E\pi}{\sqrt{\Delta^2-E^2}} \end{bmatrix},$$

and the energy of the two YSR bound states is given from $\det T^{-1} = 0$, with the final result being

$$E = \pm\Delta\sqrt{\frac{\left(\pi^2+b_\uparrow b_\downarrow\right)^2}{\pi^2\left(b_\uparrow-b_\downarrow\right)^2+\left(\pi^2+b_\uparrow b_\downarrow\right)^2}}. \tag{17}$$

Equation (17) agrees with previous results[34,36] that the bound state energy of the YSR state is linearly dependent on the order parameter; it is also consistent with our assumption that the bound state energy is small compared with the chemical potential in the BCS limit. Furthermore, as long as $b_\uparrow \neq b_\downarrow$, which means the impurity atom plays the role of a magnetic impurity, the YSR state always exists. Finally, we note that equation (17) is slightly different from the previous results[34,35] for the dependence of the coefficient on the interaction parameters $b_{\uparrow,\downarrow}$. Based on the fact that our result is obtained from the exact treatment of the T-matrix within BCS theory, we believe that our result is more reliable than previous ones.[34,35]

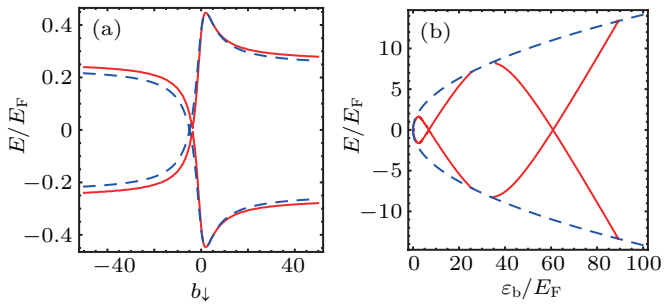


Fig. 1. Bound state energy E as a function of interaction parameters (a) $b_\downarrow$ and (b) $\varepsilon_{\rm b}$. In panel (a), the red solid line is the numerical solution of Eq. (5) with the inverse T-matrix being given by Eq. (16), and the blue dashed line is the plot of the approximate solution of Eq. (17). The parameters are chosen as $\varepsilon_{\rm b} = 0.1E_{\rm F}$ and $b_\uparrow = 2.0$. In panel (b), the red solid line represents the energy of the YSR bound state obtained from the numerical solution of Eq. (5), and the blue dashed line is the order parameter (Δ), which acts as an indicator of the range of the energy of the YSR bound state. The parameters are chosen to be $b_\uparrow = 3.5$ and $b_\downarrow = 1.0$. $b_{\uparrow,\downarrow}$ are the interaction parameters between the paired atoms and impurity atom, and $\varepsilon_{\rm b}$ denotes the strength of the interaction between the paired atoms defined in the main text.

For general cases, we numerically solve $\det T^{-1} = 0$ to obtain the bound state energy as a function of the three interaction parameters ε_b and $b_{\uparrow,\downarrow}$. In the weakly interacting BCS limit, our numerical result agrees well with analytical result Eq. (17) for sufficiently small ε_b, as can be seen from Fig. 1(a). The bound state energy as a function of ε_b is shown in Fig. 1(b). One can see that the bound state energy shows an non-monotonic behavior with increasing ε_b. In particular, as ε_b approaches a critical point ε_c, the bound state energy touches the order parameter line, which means that the YSR state ceases to exist for $\varepsilon_b \geq \varepsilon_c$. The critical value ε_c can be obtained by the following condition:

$$\det T^{-1}(E = \Delta, \varepsilon_c) = 0. \tag{18}$$

The numerical results are presented in Fig. 2, from which we find that the YSR state shows more than one re-entrance character; this is also seen in Fig. 1(b). It is also worth mentioning that there is always an upper critical ε_c solution for any $b_{\uparrow,\downarrow}$ parameters, beyond which the YSR state ceases to exist, as indicated by the two red points in Fig. 1.

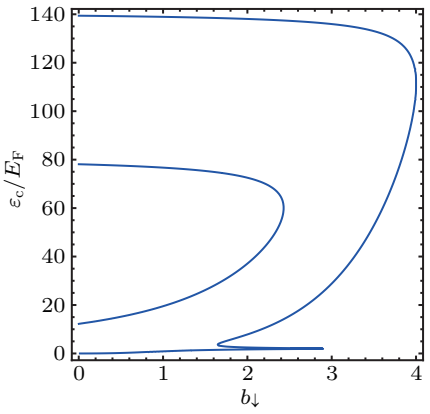


Fig. 2. Critical interaction parameter ε_b as a function of $b_\downarrow$ for $b_\uparrow = 4.0$ obtained by numerically solving Eq. (18).

The basic mechanism behind this behavior is that for a strong enough interaction, the system enters the BEC regime and becomes more difficult to be destroyed by the magnetic impurity. However it cannot be considered as a benchmark of the critical point separating the BCS from the BEC regime. Generally speaking, one can use the chemical potential as an indicator to determine when the system enters the BEC regime ($\mu < 0$). However, as can be seen from the upper line in Fig. 2, the largest critical value ε_c is much bigger than $2E_F$, at which $\mu = 0$.

5. Conclusion and outlook

We investigated the variation of the YSR bound state in BCS–BEC crossover for a single magnetic impurity immersed in a 2D Fermi superfluid. The uniform 2D case provides us with a unique scenario where most of the momentum integrals can be done analytically. Based on these analytical results, we found that in the weakly interacting BCS limit, the bound state energy is linearly dependent on the pairing order parameter with its coefficient slightly different from that in previous investigations.[34,35] More interestingly, we found more than one re-entrance point of the YSR state. Finally, we discovered that there is always an upper bound for the strength of the interaction between the paired atoms. Although these phenomena are obtained in a uniform 2D case, we believe that these conclusions should be valid in general cases, which will be investigated in future work.

For the experimental detection of these interesting behaviors regarding the YSR state, there are two main experimental proposals. One is using RF spectroscopy to probe the spectrum of the YSR state around the magnetic impurity.[34] Another feasible plan is given in Ref. [37], where the authors proposed a modified RF spectroscopy to measure the local DOS, resembling the scanning tunneling microscopy used widely in solid state systems.

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