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Influence of homodyne-based feedback control on the entropic uncertainty in open quantum system*

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For an open quantum system containing two qubits under homodyne-based feedback control, we investigate the dynamical behaviors of quantum-memory-assisted entropic uncertainty. Moreover, we analyze the influence of feedback modes and coefficients on the entropic uncertainty. Numerical investigations show that the memory qubit should be placed in a non-dissipative channel if the single dissipative channel condition can be chosen, which helps reduce the entropic uncertainty of the system. For the homodyne feedback control $F = \lambda\sigma_x$ (or $F = \lambda\sigma_y$), due to different roles of the entangled qubits A and B, when they are subject to feedback control with different feedback coefficients λ, the exchange of feedback coefficients will cause variations of the entropic uncertainty. When the feedback coefficient corresponding to the memory qubit B is larger ($\lambda_B > \lambda_A$), the steady value of the entropic uncertainty will be small, which is conducive to enhancing the robustness of the system. However, for the feedback control $F = \lambda\sigma_z$, the difference between the feedback coefficients has no effect on the steady values of the entropic uncertainty.

Keywords: homodyne-based feedback control, quantum-memory-assisted entropic uncertainties, memory qubit, open quantum system

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1. Introduction

The quantum uncertainty principle reveals a significant physical phenomenon, that the measurement outcomes of two incompatible observables cannot be predicted with an arbitrary precision simultaneously in a quantum frame, and was first proposed by Heisenberg in the case of position and momentum measurements.[1] With the development of quantum information theory, various versions of quantum uncertainty relations have been proposed in recent years.

In order to overcome the defects of the Heisenberg uncertainty relation, Hirschman *et al.* used Shannon entropy to measure the uncertainty in quantum mechanics, and first proposed the entropic uncertainty relation between position and momentum.[2] Bialynicki-Birula and Deutsch *et al.* extended it to any pair of non-commutative mechanical quantities and proposed a more general classical entropic uncertainty relation.[3,4] Later, Maassen and Uffink recast the uncertainty inequality in terms of Shannon entropy and gave a tighter uncertainty relation as follows:[5]

$$H(Q) + H(R) \geqslant -\log_2 c, \tag{1}$$

where $H(Q)\,(H(R))$ is the Shannon entropy of the probabilities of measurement outcomes of the observable $Q(R)$. The parameter $c = \max_{m,n} \left| \left\langle \psi_m^Q \middle| \psi_n^R \right\rangle \right|^2$ is the maximal overlap of the observables Q and R with $\left|\psi_m^Q\right\rangle$ and $\left|\psi_n^R\right\rangle$ being the eigenstates of Q and R, respectively. Obviously, the lower bound is independent of the state $|\psi\rangle$, which means that the uncertainty principle is an intrinsic nature of incompatible observables rather than that of the quantum state.

Recently, based on quantum entropy and quantum entanglement, Renes *et al.* put forward a new kind of quantum-memory-assisted entropic uncertainty relation (QMA-EUR) with the aid of a quantum memory qubit B that is entangled with the qubit A to be measured, i.e., the QMA-EUR is[6,7]

$$S(Q|B) + S(R|B) \geqslant S(A|B) - \log_2 c, \tag{2}$$

where $S(A|B) = S(\rho_{AB}) - S(\rho_B)$ is the conditional von Neumann entropy of ρ_{AB}, $S(\rho)$ is von Neumann entropy with $S(\rho) = -\mathrm{tr}(\rho\log_2\rho)$, $S(X|B)$ with $X \in (Q, R)$ is the conditional von Neumann entropy of the post-measurement state $\rho_{XB} = \sum_k \left(\left|\psi_k^X\right\rangle\left\langle\psi_k^X\right| \otimes I_B\right) \rho_{AB} \left(\left|\psi_k^X\right\rangle\left\langle\psi_k^X\right| \otimes I_B\right)$ after the quantum system A is measured by X, and I_B is an identity operator. Obviously, the lower bound of the entropic uncertainty is reduced whenever $S(A|B) < 0$.

In the QMA-EUR, since quantum memory provides quantum information, it breaks the original lower limit of the classical entropy uncertainty and can reduce or eliminate the uncertainty of the measurement results of two non-commutative mechanical quantities. Therefore, it is an improvement and supplement to the original uncertainty relationship. Recently, several experiments have confirmed the QMA-EUR,[8–10] and several tighter bounds for QMA-EUR have been put forward.[11,12] Owing to its appealing perfor-

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mance, the entropic uncertainty relation has attracted much attention. Some quantum operations of quantum states are proposed for stable systems, quantum correlation protection and the reduction of entropic uncertainty,[13–19] for example, the behavior of QMA-EUR under different noise, the relations between the QMA-EUR and teleportation and entanglement evidence,[20] and the influence of quantum discord and classical correlation on entropic uncertainty in the presence of quantum memory.[21,22] In addition, QMA-EUR has many potential applications in the field of quantum information science, including quantum metrology,[23] quantum transition,[24,25] quantum key distribution,[26] cryptography, and quantum randomness.[27,28]

Feedback control is the core of control theory, which can ensure a quantum system has a good control effect even under the influence of decoherence and perturbation.[29,30] Quantum feedback is based on the feedback of measurement results to alter the future dynamics of the system, and can be used to control decoherence and entanglement degradation. Measurement-based feedback is a basic quantum feedback method and is closely related to the quantum continuous measurement theory. For example, the measurement-based feedback can be used to control the solid-state qubits. Reference [31] obtained the statistical properties of a quantum field by measuring the output current of superconducting qubits, which provides an effective way of state monitoring. The direct (Markovian) feedback introduced by Wiseman and Milburn[32,33] has been widely applied by feeding back the measurement results to systems to modify the future dynamics of the system, such as suppressing decoherence,[34–36] improving the creation of steady state entanglement in open quantum systems,[37,38] and enhancing parameter-estimation precision.[39] Recently, researchers have combined feedback control with optimal control, Lyapunov control and other control methods, and proposed corresponding control strategies for the quantum correlation maintenance of single and double channels.[40,41]

Compared to direct measurement, homodyne measurement is more advantageous since it can detect properties related to the phase of the system. In this paper we consider the uncertainty with homodyne-based feedback and address how different feedback parameters and channels affect the uncertainty relations. We find that the memory qubit should be placed in a non-dissipative channel if the single dissipative channel condition can be chosen, which helps reduce the entropic uncertainty of the system.

2. Investigated model

We consider a system formed by two qubits A and B (without interaction) locally interacting respectively with a dissipative cavity, which is illustrated in Ref. [42]. Qubits A and B are initially entangled, and local quantum homodyne feedback is applied to qubit A or B. Since the two qubits are independent, we can first consider the dynamics of a single qubit in a dissipative cavity. The dynamical evolution of a quantum state under homodyne measurement is described by simply dropping the stochastic term

$$\begin{aligned}\frac{\mathrm{d}\rho^{\mathrm{S}}(t)}{\mathrm{d}t} &= -\mathrm{i}\left[\frac{1}{2}\omega\sigma_z+\frac{1}{2}\left(\sigma_+F+F\sigma_-\right),\rho^{\mathrm{S}}(t)\right]\\ &\quad +D[\sqrt{\gamma}\sigma_- -\mathrm{i}F]\rho^{\mathrm{S}}(t),\end{aligned} \tag{3}$$

where F is the feedback Hamiltonian. Without loss of generality, in this paper, three Hermitian operators $\lambda\sigma_x$, $\lambda\sigma_y$, and $\lambda\sigma_z$ are selected as the feedback operator to study the effects of different feedback types on the quantum entanglement and entropic uncertainty.

For convenience, we combine $\lambda\sigma_x$ with $\lambda\sigma_y$ and make

$$F=\lambda\left(\sigma_x\sin\alpha+\sigma_y\cos\alpha\right). \tag{4}$$

Obviously, $F=F_1=\lambda\sigma_x$ for $\alpha=\pi/2$, and $F=F_2=\lambda\sigma_y$ for $\alpha=0$.

When the environment is at zero temperature and the qubit is initially in a general composite state of its two levels, the single-qubit reduced density matrix $\rho^{\mathrm{S}}(t)$ in the qubit basis $\{|0\rangle,|1\rangle\}$ has the form

$$\rho^{\mathrm{S}}(t)=\begin{pmatrix}\rho^{\mathrm{S}}_{11}(t) & \rho^{\mathrm{S}}_{12}(t)\\ \rho^{\mathrm{S}}_{21}(t) & \rho^{\mathrm{S}}_{22}(t)\end{pmatrix}. \tag{5}$$

Under the feedback of $F=\lambda\left(\sigma_x\sin\alpha+\sigma_y\cos\alpha\right)$, we obtain the elements of the reduced density matrix $\rho^{\mathrm{S}}(t)$ as

$$\rho^{\mathrm{S}}_{11}(t)=\mu^{\mathrm{S}}\rho^{\mathrm{S}}_{11}(0)+\nu^{\mathrm{S}}\rho^{\mathrm{S}}_{22}(0), \tag{6}$$

$$\rho^{\mathrm{S}}_{22}(t)=\left(1-\mu^{\mathrm{S}}\right)\rho^{\mathrm{S}}_{11}(0)+\left(1-\nu^{\mathrm{S}}\right)\rho^{\mathrm{S}}_{22}(0), \tag{7}$$

$$\rho^{\mathrm{S}}_{12}(t)=\xi^{\mathrm{S}}\rho^{\mathrm{S}}_{12}(0)+\eta^{\mathrm{S}}\rho^{\mathrm{S}}_{21}(0), \tag{8}$$

$$\rho^{\mathrm{S}}_{21}(t)=\eta^{*S}\rho^{\mathrm{S}}_{12}(0)+\xi^{*S}\rho^{\mathrm{S}}_{21}(0), \tag{9}$$

$$\mu^{\mathrm{S}}=\frac{1}{\Gamma}\left[\lambda^2+\left(\lambda^2+2\sqrt{\gamma}\lambda\cos\alpha+\gamma\right)\mathrm{e}^{-\Gamma t}\right], \tag{10}$$

$$\nu^{\mathrm{S}}=\frac{1}{\Gamma}\lambda^2\left(1-e^{-\Gamma t}\right), \tag{11}$$

$$\xi^{\mathrm{S}}=\frac{1}{\Delta}\mathrm{e}^{-\frac{1}{2}\Gamma t}\left[\Delta\cosh\left(\frac{\Delta}{2}t\right)-\mathrm{i}2\lambda\sin\alpha\sinh\left(\frac{\Delta}{2}t\right)\right], \tag{12}$$

$$\eta^{\mathrm{S}}=-\frac{2}{\Delta}\lambda\,\mathrm{e}^{\mathrm{i}\alpha}\left(\sqrt{\gamma}+\lambda\,\mathrm{e}^{\mathrm{i}\alpha}\right)\mathrm{e}^{-\frac{1}{2}\Gamma t}\sinh\left(\frac{\Delta}{2}t\right), \tag{13}$$

with

$$\Gamma=2\lambda^2+2\sqrt{\gamma}\lambda\cos\alpha+\gamma, \tag{14}$$

$$\Delta=2\sqrt{\lambda^2(\lambda^2+2\sqrt{\gamma}\lambda\cos\alpha+\gamma)-(\omega+\lambda\sin\alpha)^2}. \tag{15}$$

The specific solution process is shown in Appendix A.

The evolution of the single-qubit reduced density matrix elements can be easily extended to the two-qubit system. Following the procedures presented in Ref. [43], we find that in

the standard product basis $\{|00\rangle\ |01\rangle\ |10\rangle\ |11\rangle\}$, the diagonal elements of the reduced density matrix $\rho(t)$ for the two-qubit system can be written as

$$\begin{aligned}\rho_{11}(t) &= \mu^{\mathrm{A}}\mu^{\mathrm{B}}\rho_{11}^{\mathrm{AB}}(0)+\mu^{\mathrm{A}}\nu^{\mathrm{B}}\rho_{22}^{\mathrm{AB}}(0)+\nu^{\mathrm{A}}\mu^{\mathrm{B}}\rho_{33}^{\mathrm{AB}}(0)\\ &\quad+\nu^{\mathrm{A}}\nu^{\mathrm{B}}\rho_{44}^{\mathrm{AB}}(0), \qquad (16)\\ \rho_{22}(t) &= \mu^{\mathrm{A}}(1-\mu^{\mathrm{B}})\rho_{11}^{\mathrm{AB}}(0)+\mu^{\mathrm{A}}(1-\nu^{\mathrm{B}})\rho_{22}^{\mathrm{AB}}(0)\\ &\quad+\nu^{\mathrm{A}}(1-\mu^{\mathrm{B}})\rho_{33}^{\mathrm{AB}}(0)+\nu^{\mathrm{A}}(1-\nu^{\mathrm{B}})\rho_{44}^{\mathrm{AB}}(0), \qquad (17)\\ \rho_{33}(t) &= (1-\mu^{\mathrm{A}})\mu^{\mathrm{B}}\rho_{11}^{\mathrm{AB}}(0)+(1-\mu^{\mathrm{A}})\nu^{\mathrm{B}}\rho_{22}^{\mathrm{AB}}(0)\\ &\quad+(1-\nu^{\mathrm{A}})\mu^{\mathrm{B}}\rho_{33}^{\mathrm{AB}}(0)+(1-\nu^{\mathrm{A}})\nu^{\mathrm{B}}\rho_{44}^{\mathrm{AB}}(0), \qquad (18)\\ \rho_{44}(t) &= (1-\mu^{\mathrm{A}})(1-\mu^{\mathrm{B}})\rho_{11}^{\mathrm{AB}}+(1-\mu^{\mathrm{A}})(1-\nu^{\mathrm{B}})\rho_{22}^{\mathrm{AB}}(0)\\ &\quad+(1-\nu^{\mathrm{A}})(1-\mu^{\mathrm{B}})\rho_{33}^{\mathrm{AB}}(0)\\ &\quad+(1-\nu^{\mathrm{A}})(1-\nu^{\mathrm{B}})\rho_{44}^{\mathrm{AB}}(0), \qquad (19)\end{aligned}$$

and the nondiagonal elements are

$$\begin{aligned}\rho_{12}(t) &= \mu^{\mathrm{A}}\xi^{\mathrm{B}}\rho_{12}^{\mathrm{AB}}(0)+\mu^{\mathrm{A}}\eta^{\mathrm{B}}\rho_{21}^{\mathrm{AB}}(0)+\nu^{\mathrm{A}}\xi^{\mathrm{B}}\rho_{34}^{\mathrm{AB}}(0)\\ &\quad+\nu^{\mathrm{A}}\eta^{\mathrm{B}}\rho_{43}^{\mathrm{AB}}(0), \qquad (20)\\ \rho_{13}(t) &= \xi^{\mathrm{A}}\mu^{\mathrm{B}}\rho_{13}^{\mathrm{AB}}(0)+\xi^{\mathrm{A}}\nu^{\mathrm{B}}\rho_{24}^{\mathrm{AB}}(0)+\eta^{\mathrm{A}}\mu^{\mathrm{B}}\rho_{31}^{\mathrm{AB}}(0)\\ &\quad+\eta^{\mathrm{A}}\nu^{\mathrm{B}}\rho_{42}^{\mathrm{AB}}(0), \qquad (21)\\ \rho_{14}(t) &= \xi^{\mathrm{A}}\xi^{\mathrm{B}}\rho_{14}^{\mathrm{AB}}(0)+\xi^{\mathrm{A}}\eta^{\mathrm{B}}\rho_{23}^{\mathrm{AB}}(0)+\eta^{\mathrm{A}}\xi^{\mathrm{B}}\rho_{32}^{\mathrm{AB}}(0)\\ &\quad+\eta^{\mathrm{A}}\eta^{\mathrm{B}}\rho_{41}^{\mathrm{AB}}(0), \qquad (22)\\ \rho_{23}(t) &= \xi^{\mathrm{A}}\eta^{*B}\rho_{14}^{\mathrm{AB}}(0)+\xi^{\mathrm{A}}\xi^{*B}\rho_{23}^{\mathrm{AB}}(0)+\eta^{\mathrm{A}}\eta^{*B}\rho_{32}^{\mathrm{AB}}(0)\\ &\quad+\eta^{\mathrm{A}}\xi^{*B}\rho_{41}^{\mathrm{AB}}(0), \qquad (23)\\ \rho_{24}(t) &= \xi^{\mathrm{A}}(1-\mu^{\mathrm{B}})\rho_{13}^{\mathrm{AB}}(0)+\xi^{\mathrm{A}}(1-\nu^{\mathrm{B}})\rho_{24}^{\mathrm{AB}}(0)\\ &\quad+\eta^{\mathrm{A}}(1-\mu^{\mathrm{B}})\rho_{31}^{\mathrm{AB}}(0)+\eta^{\mathrm{A}}(1-\nu^{\mathrm{B}})\rho_{42}^{\mathrm{AB}}(0), \qquad (24)\\ \rho_{34}(t) &= (1-\mu^{\mathrm{A}})\xi^{\mathrm{B}}\rho_{12}^{\mathrm{AB}}(0)+(1-\mu^{\mathrm{A}})\eta^{\mathrm{B}}\rho_{21}^{\mathrm{AB}}(0)\\ &\quad+(1-\nu^{\mathrm{A}})\xi^{\mathrm{B}}\rho_{34}^{\mathrm{AB}}(0)+(1-\nu^{\mathrm{A}})\eta^{\mathrm{B}}\rho_{43}^{\mathrm{AB}}(0), \qquad (25)\end{aligned}$$

and $\rho_{jk}(t)=\rho_{kj}^{*}(t)$.

3. Results

Suppose that the observed qubit A and quantum memory qubit B are initially prepared in X-state, and let A and B independently pass through the noisy channels. For a two-qubit system which is described by the density operator, in the standard computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the density operator can be given by

$$\rho_{\mathrm{AB}}=\begin{pmatrix}\rho_{11} & 0 & 0 & \rho_{14}\\ 0 & \rho_{22} & \rho_{23} & 0\\ 0 & \rho_{32} & \rho_{33} & 0\\ \rho_{41} & 0 & 0 & \rho_{44}\end{pmatrix}. \qquad (26)$$

For convenience, based on Eq. (26), we define the right-hand side of Eq. (2) as[44]

$$EB=S(A|B)-\log_2 c=S(\rho_{\mathrm{AB}})-S(\rho_{\mathrm{B}})-\log_2 c, \qquad (27)$$

and the left-hand side of Eq. (2) as

$$EU=S(\rho_{\sigma_x B})+S(\rho_{\sigma_z B})-2S(\rho_B). \qquad (28)$$

As stated earlier, EU measures the accuracy of the measurement results. Thus, the higher the accuracy is, the smaller EU is. EB is the lower limit of the entropic uncertainty, which can be used to measure the quality of an uncertainty relation.

In this paper, we investigate the effect of feedback control on the QMA-EUR. We choose Pauli observables $Q=\sigma_x$ and $R=\sigma_z$ as the incompatibility to discuss in three cases. In the first case, we assume that qubit A undergoes a local noisy environment, and quantum feedback control is applied to it, while qubit B is free from environment noise. In the second case, qubit B experiences a noisy environment and quantum feedback control, but qubit A is independent of a noisy environment. The third case is that qubit A and B are respectively located in an independent environment and suffer local quantum feedback controls which are identical to each other.

In the following simulations, we assume the initial states of the qubits as:

$$\begin{aligned}\rho_{\psi}(r,\theta) &= |\psi(\theta)\rangle\langle\psi(\theta)|, |\psi(\theta)\rangle\\ &= \cos\theta\,|00\rangle+\sin\theta\,|11\rangle. \qquad (29)\end{aligned}$$

3.1. The feedback control with $F_1=\lambda\sigma_x$

In this section we study the case of

$$F_1=\lambda\sigma_x. \qquad (30)$$

The density matrix can be exactly solved by substituting $\alpha=\pi/2$ into Eqs. (6)–(25). In order to show the evolution of the entropic uncertainty under feedback control, we perform numerical investigation under different conditions, such as a single dissipative channel and double dissipative channels. Figures 1–3 show typical numerical investigation results.

In Fig. 1, we present the time evolution of the entropic uncertainty under quantum feedback control $F_1=\lambda\sigma_x$ with the feedback coefficient λ. It can be clearly seen that for the model studied in this paper, under homodyne measurement feedback control, the entropic uncertainty of the system increases for open quantum systems with single dissipative channel or with double dissipative channels. The physical reasons can be simply explained as follows. To some extent, the homodyne measurement interferes with the quantum state and causes decoherence of the quantum system, thus increasing the uncertainty of the system. In addition, it is easy to know from Fig. 1 that although the feedback effect leads to the increase of the entropic uncertainty, with the increase of feedback coefficient λ, the entropic uncertainty is more likely to approach a steady value, which is conducive to improving the stability of the system. Further numerical investigations show that increasing the feedback coefficient λ will increase the steady value of entropic uncertainty. Therefore, from both feedback control and

system robustness, the feedback coefficient λ should take a smaller value.

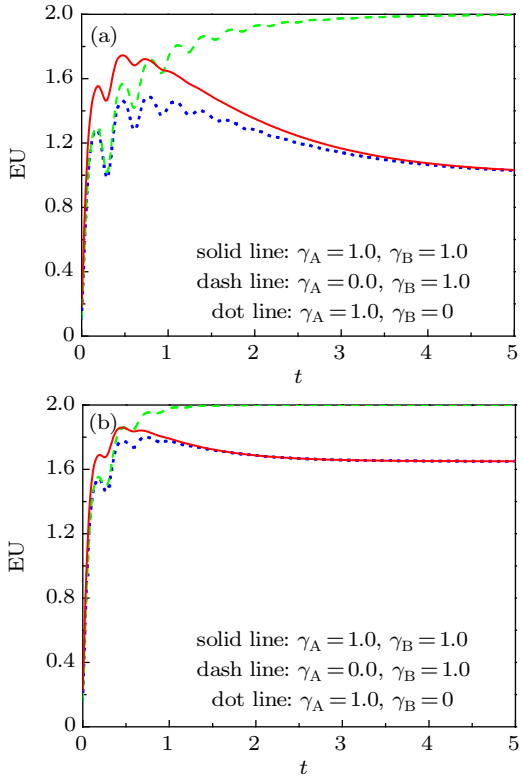


Fig. 1. The influence of feedback on entropic uncertainty with $\theta = \pi/4$. Under the influence of homodyne feedback control, the entropic uncertainty of an open quantum system, with single dissipative channel or double dissipative channels, will increase (a) without feedback: $\lambda_A = \lambda_B = 0$; (b) with feedback: $\lambda_A = \lambda_B = 0.5$.

In Figs. 2 and 3, we present evolutions of the entropic uncertainty as a function of time t and feedback coefficient λ under different situations. Figure 2 describes the influence of a single-sided dissipative channel on the entropic uncertainty under the feedback control $F_1 = \lambda\sigma_x$. By comparing Fig. 2(a) and 2(b), we can see that, under exactly the same feedback condition, when qubit B serves as the memory qubit and is placed in the channel without dissipation, the entropic uncertainty first increases to a maximum value, and then decreases to a smaller steady value. Inversely, if the memory qubit B is placed in a dissipative channel, the entropic uncertainty will be significantly increased. Obviously, the greater the entropic uncertainty relation is, the greater the uncertainty of the measurement results of two non-commuting observables will be, which will affect its application in quantum information technology. At present, since people do not have a deep understanding of the physical mechanism of the influence of quantum measurement, we cannot properly understand the physical cause of the difference. Nevertheless, in quantum engineering applications, previous results tell us that, if possible, memory qubits should be placed in a non-dissipative channel.

Figure 3 presents the time evolution of entropic uncertainty under the feedback $F_1 = \lambda\sigma_x$, where A and B are in identical local dissipative channels. Compared with the single-sided channel (see Fig. 2(a)), when the corresponding parameters are the same, the increase of dissipative channel enhances the environmental decoherence effect, leading to a significant increase in the entropic uncertainty in the initial stage of evolution. However, there is no obvious difference in the steady value.

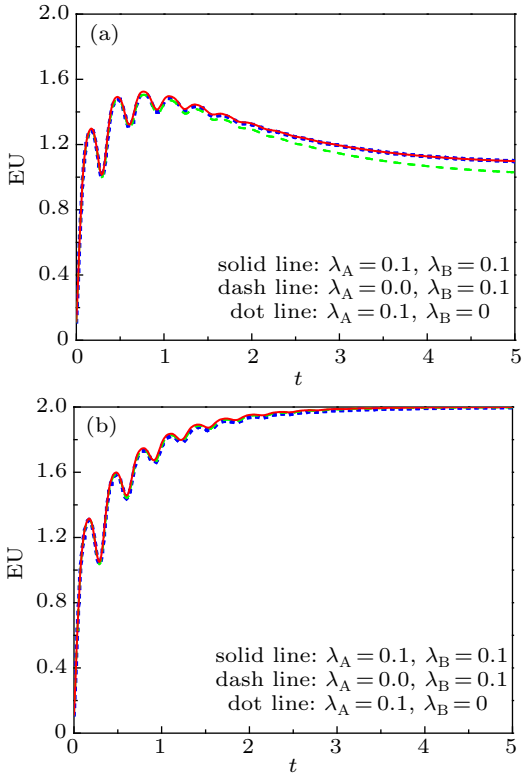


Fig. 2. Dynamics of the entropic uncertainty as a function of time t in the single-sided dissipative channel by homodyne-based feedback control $F_1 = \lambda\sigma_x$. (a) Channel A is a dissipative channel with $\gamma_A = 1$, channel B is an ideal noise free channel. (b) Channel A is an ideal noise free channel, channel B is a dissipative channel with $\gamma_B = 1$.

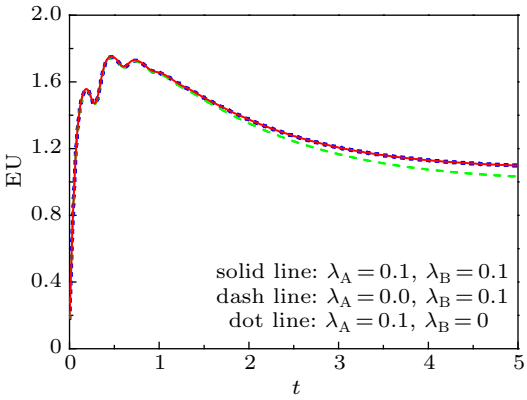


Fig. 3. Dynamics of the entropic uncertainty as a function of time t in the identical local dissipative channels with $\gamma_A = \gamma_B = 1.0$ by homodyne-based feedback control $F_1 = \lambda\sigma_x$.

3.2. The feedback control with $F_2 = \lambda\sigma_y$

In this section, we suppose that the feedback control is

$$F_2 = \lambda\sigma_y. \tag{31}$$

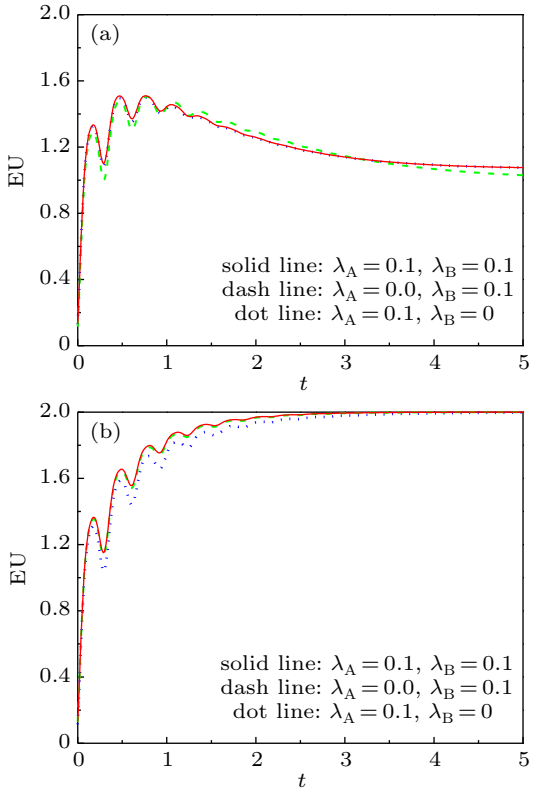


Fig. 4. Dynamics of the entropic uncertainty as a function of time t in single-sided dissipative channel by homodyne-based feedback control $F_2 = \lambda\sigma_y$. (a) Channel A is a dissipative channel with $\gamma_A = 1$, channel B is an ideal noise free channel. (b) Channel A is an ideal noise free channel, channel B is a dissipative channel with $\gamma_B = 1$.

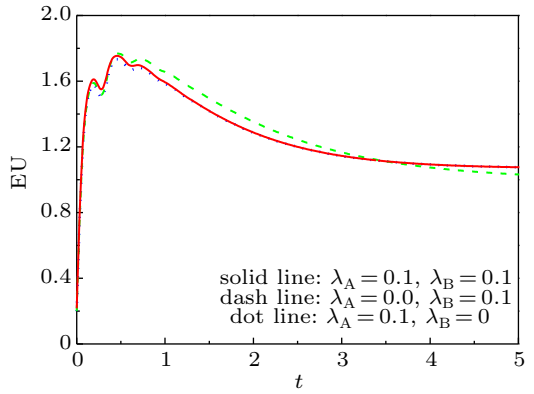


Fig. 5. Dynamics of the entropic uncertainty as a function of time t in the identical local dissipative channels with $\gamma_A = \gamma_B = 1.0$ by homodyne-based feedback control $F_2 = \lambda\sigma_y$.

Substituting $\alpha = 0$ into Eqs. (6)–(25), we obtain the entropic uncertainty as a function of t and the feedback coefficient λ.

Comparing Fig. 2 with Fig. 4, and Fig. 3 with Fig. 5 under the same conditions, we find that the homodyne-based feedback control $F_2 = \lambda\sigma_y$ has almost the same effect on the entropic uncertainty with $F_1 = \lambda\sigma_x$. In summary, under the homodyne-based feedback control $F_1 = \lambda\sigma_x$ or $F_2 = \lambda\sigma_y$, when only qubit A is in a dissipative environment or both qubits A and B interact independently with individual environments, the entropic uncertainty firstly increases with time, then turns down and tends to a steady value. It is similar to the dynamical behavior of QMA-EUR discussed in Ref. [45], where qubits A and B are in the depolarizing channel, and no feedback effect exists. In contrast, when only the memory qubit B is in a dissipative environment, the entropic uncertainty monotonically increases and tends to a steady value, which is similar to the dynamical behavior of QMA-EUR, where qubits A and B are in the phase damping channel, and no feedback effect exists.

3.3. The feedback with $F_3 = \lambda\sigma_z$

As a comparison, the homodyne-based feedback control

$$F_3 = \lambda\sigma_z \tag{32}$$

is investigated below. The solution process is shown in Appendix B.

The dynamical behavior of QMA-EUR is shown in Fig. 6. Unlike with the feedback control $F_1 = \lambda\sigma_x$ or $F_2 = \lambda\sigma_y$, we find that, under the homodyne-based feedback control $F_3 = \lambda\sigma_z$, there is no significant difference in the entropic uncertainty either in the single-sided dissipative channel (with or without dissipation in the channel where the memory qubit is located) or the two-sided dissipative channels. The feedback coefficient λ also has no obvious influence on the entropic uncertainty.

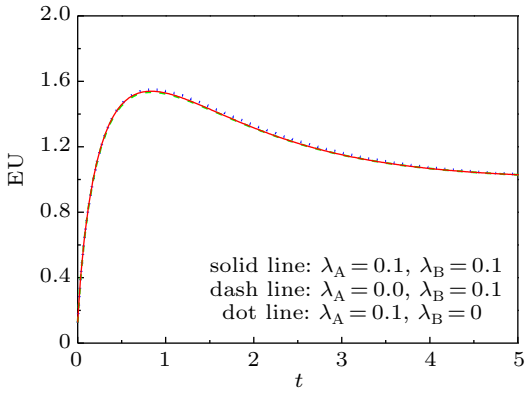


Fig. 6. Dynamics of the entropic uncertainty as a function of time t in the identical local dissipative channels with $\gamma_A = \gamma_B = 1.0$ and $F = \lambda\sigma_z$.

4. Influence of the difference between feedback parameters

The above results show that entangled qubits A and B play different roles in the uncertainty relation. Thus, under the single-sided dissipative channel, the entropic uncertainty of the system is inconsistent for the dissipative or non-dissipative channel where the memory qubit is located. Our further study also indicates that it is precisely because of the different roles played by A and B that the difference of feedback on them will lead to the difference of entropic uncertainty in the initial stage of dynamic evolution. Figure 7 exhibits the influence of difference between feedback coefficients on the entropic uncertainty. Clearly, from Fig. 7, we find that under $F_1 = \lambda\sigma_x$ or

$F_2 = \lambda\sigma_y$, when the feedback coefficient of the memory qubit is larger ($\lambda_B > \lambda_A$), the steady value of the entropic uncertainty is small; the steady value increases with λ_B. In addition, for $\lambda_B > \lambda_A$, the greater the difference of feedback coefficient, the greater the difference of entropic uncertainty in the initial stage, but it has no influence on the steady value (no figures are given). Under the feedback control $F_3 = \lambda\sigma_z$, however, a difference in feedback coefficient will only cause a difference in entropic uncertainty in the initial stage, and have no effect on the steady value of entropic uncertainty.

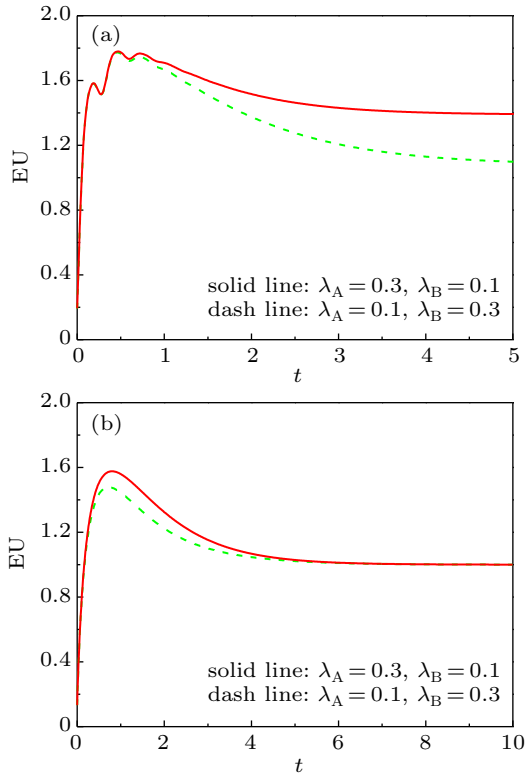


Fig. 7. The influence of difference between feedback coefficients on the entropic uncertainty with $\gamma_A = \gamma_B = 1.0$. Qubits A and B are in identical local dissipative channels. (a) $F_1 = \lambda\sigma_x$. (b) $F_3 = \lambda\sigma_z$.

5. Conclusion

In summary, we have studied the effect of homodyne-based feedback control on the QMA-EUR in different dissipative environments. (i) In order to reduce the entropic uncertainty of the system as much as possible and improve the robustness of the system against decoherence, memory qubits should be placed in the non-dissipative channel when the situation of a single-sided dissipative channel can be selected. (ii) Due to different roles played by entangled qubits A and B, when the homodyne-based feedback control is $F = \lambda\sigma_x$ (or $F = \lambda\sigma_y$) and the control quantities applied to the two qubits are different (namely the feedback coefficients are different), then it is more conducive to reducing the entropic uncertainty of the system when the feedback received by memory qubit B is larger. (iii) Considering both the feedback control and the robustness of the system, the feedback coefficient λ should be as small as possible when the homodyne-based feedback control is adopted.

Appendix A

In what follows, we derive the reduced density matrix $\rho^S(t)$ of the single-qubit system under $F = \lambda\,(\sigma_x \sin\alpha + \sigma_y \cos\alpha)$. Substituting Eq. (4) into Eq. (3) yields the following formulae:

$$\dot{\rho}^S_{11}(t) = -(\lambda^2 + 2\sqrt{\gamma}\lambda\cos\alpha + \gamma)\rho^S_{11}(t) + \lambda^2\rho^S_{22}(t), \tag{A1}$$

$$\dot{\rho}^S_{11}(t) = (\lambda^2 + 2\sqrt{\gamma}\lambda\cos\alpha + \gamma)\rho^S_{11}(t) - \lambda^2\rho^S_{22}(t), \tag{A2}$$

$$\dot{\rho}^S_{12}(t) = -\frac{1}{2}\left[\mathrm{i}2(\lambda\sin\alpha+\omega)+2\lambda^2+2\sqrt{\gamma}\lambda\cos\alpha+\gamma\right]\rho^S_{12}(t) - \lambda\,\mathrm{e}^{\mathrm{i}\alpha}\left(\sqrt{\gamma}+\lambda\,\mathrm{e}^{\mathrm{i}\alpha}\right)\rho^S_{21}(t), \tag{A3}$$

$$\dot{\rho}^S_{21}(t) = -\lambda\,\mathrm{e}^{-\mathrm{i}\alpha}\left(\sqrt{\gamma}+\lambda\,\mathrm{e}^{-\mathrm{i}\alpha}\right)\rho^S_{12}(t) + \frac{1}{2}\left[\mathrm{i}2(\lambda\sin\alpha+\omega)-2\lambda^2-2\sqrt{\gamma}\lambda\cos\alpha-\gamma\right]\rho^S_{21}(t). \tag{A4}$$

We see that equations (A1)–(A4) are separated into two independent equation groups. For Eqs. (A1) and (A2), they satisfy the following matrix form:

$$\begin{pmatrix}\dot{\rho}^S_{11}(t)\\ \dot{\rho}^S_{22}(t)\end{pmatrix} = \begin{pmatrix}-(\lambda^2+2\sqrt{\gamma}\lambda\cos\alpha+\gamma) & \lambda^2\\ \lambda^2 & (\lambda^2+2\sqrt{\gamma}\lambda\cos\alpha+\gamma)\end{pmatrix} \times \begin{pmatrix}\rho^S_{11}(t)\\ \rho^S_{22}(t)\end{pmatrix} = M_1\begin{pmatrix}\rho^S_{11}(t)\\ \rho^S_{22}(t)\end{pmatrix}. \tag{A5}$$

According to the eigenvectors and eigenvalues of the matrix M, the solution to Eq. (A5) can be obtained; see Eqs. (6) and (7).

Similarly, we can give the solutions of Eqs. (A3) and (A4).

Appendix B

Inserting Eq. (32) into Eq. (4), the master equation of the density matrix may be expressed in terms of its component forms

$$\dot{\rho}^S_{11}(t) = -\gamma\rho^S_{11}(t) + \mathrm{i}\frac{(\sqrt{\gamma}-1)\lambda}{2}\left(\rho^S_{12}(t) - \rho^S_{21}(t)\right), \tag{B1}$$

$$\dot{\rho}^S_{22}(t) = \gamma\rho^S_{11}(t) - \mathrm{i}\frac{(\sqrt{\gamma}-1)\lambda}{2}\left(\rho^S_{12}(t) - \rho^S_{21}(t)\right), \tag{B2}$$

$$\dot{\rho}^S_{12}(t) = -\mathrm{i}\frac{\lambda}{2}(1+3\sqrt{\gamma})\rho^S_{11}(t) + \mathrm{i}\frac{\lambda}{2}(1-\sqrt{\gamma})\rho^S_{22}(t) - \frac{1}{2}\left(\gamma+4\lambda^2\right)\rho^S_{12}(t), \tag{B3}$$

$$\dot{\rho}^S_{21}(t) = \mathrm{i}\frac{\lambda}{2}(1+3\sqrt{\gamma})\rho^S_{11}(t) - \mathrm{i}\frac{\lambda}{2}(1-\sqrt{\gamma})\rho^S_{22}(t) - \frac{1}{2}\left(\gamma+4\lambda^2\right)\rho^S_{21}(t). \tag{B4}$$

When $\gamma = 1$, we see that equations (B1)–(B4) can be directly obtained by an integral. The analytical solutions of Eq. (4) can be expressed as

$$\rho_{11}^{\mathrm{S}}(t) = \mathrm{e}^{-t}\rho_{11}^{\mathrm{S}}, \tag{B5}$$

$$\rho_{22}^{\mathrm{S}}(t) = (1-\mathrm{e}^{-t})\rho_{11}^{\mathrm{S}} + \rho_{22}^{\mathrm{S}}(0), \tag{B6}$$

$$\rho_{12}^{\mathrm{S}}(t) = \mathrm{e}^{-(2\lambda^2+0.5)t}\rho_{12}^{\mathrm{S}} + \mathrm{i}\frac{4\lambda}{4\lambda^2-1} \times \left[\mathrm{e}^{-(2\lambda^2+0.5)t} - \mathrm{e}^{-t}\right]\rho_{11}^{\mathrm{S}}(0), \tag{B7}$$

$$\rho_{21}^{\mathrm{S}}(t) = \left(\rho_{12}^{\mathrm{S}}(t)\right)^{*}. \tag{B8}$$

When $\gamma \neq 1$, for Eqs. (B1)–(B4), we first transform them into the form of a matrix. Then, similar to Appendix A, the solutions can be given by the eigenvectors and eigenvalues of the matrix.

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