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Uniformity principle of temperature difference field in heat transfer optimization*

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The uniformity principle of temperature difference field is very useful in heat exchanger analyses and optimizations. In this paper, we analyze some other heat transfer optimization problems in the thermal management system of spacecrafts, including the cooling of thermal components, the one-stream series-wound heat exchanger network, the volume-to-point heat conduction problem, and the radiative heat transfer optimization problem, and have found that the uniformity principle of temperature difference field also holds. When the design objectives under the given constraints are achieved, the distributions of the temperature difference fields are uniform. The principle reflects the characteristic of the distribution of potential in the heat transfer optimization problems. It is also shown that the principle is consistent with the entransy theory. Therefore, although the principle is intuitive and phenomenological, the entransy theory can be the physical basis of the principle.

Keywords: heat transfer optimization, uniformity principle of temperature difference field, equipartition of potential, entransy theory

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1. Introduction

Heat exchangers are very important and widely used in thermal engineering. For the analyses and design of heat exchangers, Guo *et al.*[1,2] proposed a parameter, the uniformity factor of temperature difference field. For a two-stream heat exchanger, its definition is[1–3]

$$f = \frac{\int_A |T_1 - T_2|\,\mathrm{d}A}{\sqrt{A\int_A (T_1 - T_2)^2\,\mathrm{d}A}} = \frac{\int_A |\Delta T|\,\mathrm{d}A}{\sqrt{A\int_A (\Delta T)^2\,\mathrm{d}A}}, \tag{1}$$

where T_1 and T_2 are the local temperatures of the streams, ΔT is the local temperature difference, A is the heat transfer area, and $f \in [0,1]$. When the number of heat transfer units (NTU) and the ratio of the heat capacity flow rates are fixed, larger uniformity factor of temperature difference field leads to larger heat exchanger effectiveness. Especially, ΔT = const when $f = 1$. This is the uniformity principle of temperature difference field, which has been verified with many cases.[1–8] For instance, Guo *et al.*[1] showed the validity of the principle with theoretical analyses and experiments in two-stream heat exchangers. Gao *et al.*[5] proved the principle directly in multi-pressure condenser. Fu and Zhang[8] gave a rigorous proof for the principle with deductive method.

The principle finds many applications in engineering because it is very convenient for designing and optimizing heat exchangers. For instance, if some local temperature differences in one heat exchanger are obviously higher than other places, the principle tells us that our technical design should decrease the local temperature differences to increase the heat exchanger effectiveness. However, it should be pointed out that the principle is not from the viewpoint of the irreversibility of heat transfer, but is intuitive and phenomenological. Therefore, the physical basis of the principle should be investigated. In the entransy theory,[9] it has been proved that the concept of entransy dissipation can describe the irreversibility of heat transfer.[10,11] With this theory, Song *et al.*[6] analyzed one-dimensional two-stream and three-stream heat exchangers and proved that the principle holds when the heat transfer rate or the entransy dissipation rate is fixed. In the analyses of two-stream heat exchangers, Cheng *et al.*[4] set up the mathematical relationship between the uniformity factor of temperature difference field and the heat exchanger effectiveness with the help of the entransy theory, and found that larger uniformity factor of temperature difference field always corresponds to smaller entransy-dissipation-based thermal resistance. Therefore, larger uniformity factor of temperature difference field means better heat transfer, and the entransy theory can be the physical basis of the intuitive and phenomenological principle.

Furthermore, it is obvious that the application scope of the principle is limited because it can only be used to analyze and optimize heat exchangers. In this paper, we try to extend the application scope of the principle, and analyze the applicability of the principle to the heat transfer processes in the thermal management system of spacecrafts. The cooling problem of

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thermal components, the one-stream heat exchanger networks, the volume-to-point heat conduction optimization for cooling electronic devices, and the radiative heat transfer optimization are analyzed. The physical basis of the principle in the new application cases is also discussed.

2. Cooling of thermal components

In the thermal management system of spacecrafts, the cooling of thermal components may be simplified to the steady heat transfer system shown in Fig. 1. There are n thermal components from which heat is released to the environment at temperature T_0. For the i-th thermal component, the released heat flow rate is Q_i, and the thermal conductance between the thermal component and the environment is K_i. First, we can assume that the released heat flow rates are fixed, and the limiting condition is

$$K = \sum_{i=1}^{n} K_i = \text{const}, \tag{2}$$

where K is the total thermal conductance. In this case, the distribution of the total thermal conductance is optimized to obtain the minimum average temperature of the thermal components weighted by released heat flow rate

$$T = \frac{\sum_{i=1}^{n} Q_i T_i}{\sum_{i=1}^{n} Q_i}. \tag{3}$$

Considering the energy conservation, we have

$$T_i = \frac{Q_i}{K_i} + T_0. \tag{4}$$

Therefore

$$T = \frac{\sum_{i=1}^{n} Q_i (Q_i K_i + T_0)}{\sum_{i=1}^{n} Q_i} = \frac{\sum_{i=1}^{n} Q_i^2 / K_i}{\sum_{i=1}^{n} Q_i} + T_0. \tag{5}$$

To find the minimum value of Eq. (5) with the limitation condition of Eq. (2), a function can be set up

$$F_1 = \frac{\sum_{i=1}^{n} Q_i^2 / K_i}{\sum_{i=1}^{n} Q_i} + T_0 + \lambda_1 \left(\sum_{i=1}^{n} K_i - K \right), \tag{6}$$

where λ_1 is the Lagrange multiplier. The variations that lead to the minimum average temperature should satisfy

$$\frac{\partial F_1}{\partial K_i} = -\frac{Q_i^2}{K_i^2 \sum_{i=1}^{n} Q_i} + \lambda_1 = 0. \tag{7}$$

Then, we can obtain that

$$K_i = \frac{Q_i}{\sqrt{\lambda_1 \sum_{i=1}^{n} Q_i}}. \tag{8}$$

Substituting Eq. (8) into Eq. (2) gives

$$\lambda_1 = \frac{\sum_{i=1}^{n} Q_i}{K^2}. \tag{9}$$

The distribution of the total thermal conductance is obtained

$$K_i = \frac{Q_i}{\sum_{i=1}^{n} Q_i} K. \tag{10}$$

With Eq. (10), the heat transfer temperature difference of the i-th thermal components can also be obtained

$$\Delta T_i = T_i - T_0 = \frac{Q_i}{K_i} = \frac{\sum_{i=1}^{n} Q_i}{K} = \text{const}. \tag{11}$$

When the heat transfer of the system is optimized, it can be seen that the heat transfer temperature differences of thermal components equal each other.

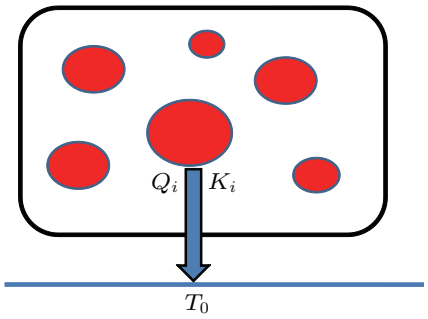


Fig. 1. A thermal system with n thermal components.

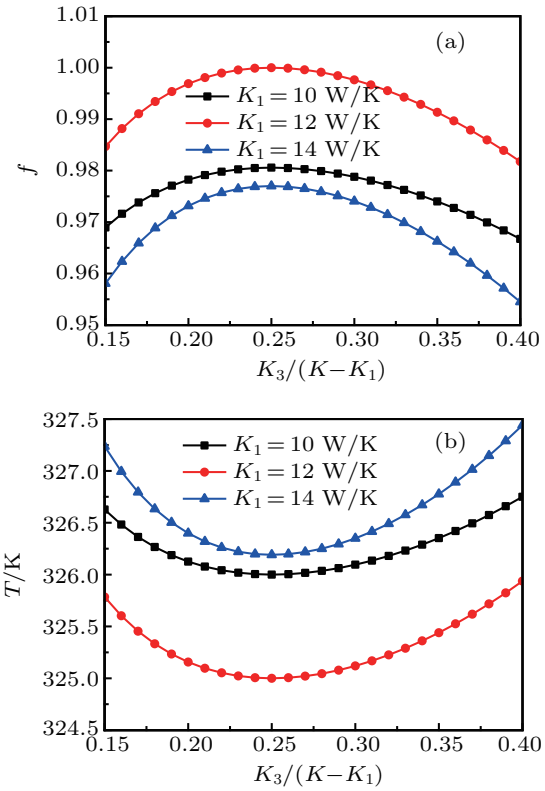


Fig. 2. Variations of the average temperature of the thermal components weighted by released heat flow rate and the uniformity factor of temperature difference field with the distribution of the total thermal conductance. (a) Variation of the uniformity factor of temperature difference field. (b) Variation of the average temperature of the thermal components.

As below, a numerical example is presented. Assume that there are three thermal components in the system, the heat transfer rates $Q_1 = 300$ W, $Q_2 = 150$ W, and $Q_3 = 50$ W, the

total thermal conductance $K = 20$ W/K and the temperature $T_0 = 300$ K. For the system, the uniformity factor of temperature difference field can be defined as

$$f = \frac{\sum_{i=1}^{3} K_i (T_i - T_0)}{\sqrt{\left(\sum_{i=1}^{3} K_i\right) \sum_{i=1}^{3} K_i (T_i - T_0)^2}}. \tag{12}$$

In this case, the variations of the average temperature of the thermal components weighted by released heat flow rate and the uniformity factor of temperature difference field with the distribution of the total thermal conductance are calculated and shown in Fig. 2. The results show that the uniformity factor of temperature difference field and the average temperature have opposite variation tendencies. Especially, when the thermal conductances $K_1 = 12$ W/K, $K_2 = 6$ W/K, and $K_3 = 2$ W/K, the average temperature reaches the minimum value, 325 K, and the uniformity factor of temperature difference field reaches the maximum value, 1. Therefore, the theoretical analyses have been verified by the numerical results.

Second, we can assume that the thermal conductance of each thermal component is given, and the limiting condition is

$$Q = \sum_{i=1}^{n} Q_i = \text{const}, \tag{13}$$

where Q is the total released heat flow rate. In this case, the distribution of the total heat flow rate can also be optimized to obtain the minimum value of Eq. (5). We can also set up a function for this problem

$$F_2 = \frac{\sum_{i=1}^{n} Q_i^2 / K_i}{Q} + T_0 + \lambda_2 \left(\sum_{i=1}^{n} Q_i - Q\right), \tag{14}$$

where λ_2 is the Lagrange multiplier. The optimized distribution of the total released heat flow rate should satisfy

$$\frac{\partial F_2}{\partial Q_i} = \frac{2Q_i}{QK_i} + \lambda_2 = 0. \tag{15}$$

Then, we can obtain that

$$Q_i = -\frac{1}{2}\lambda_2 Q K_i. \tag{16}$$

Substituting Eq. (16) into Eq. (13) gives

$$\lambda_2 = -\frac{2}{\sum_{i=1}^{n} K_i}. \tag{17}$$

So, we have

$$Q_i = \frac{Q}{\sum_{i=1}^{n} K_i} K_i. \tag{18}$$

With Eq. (18), we can also have

$$\Delta T_i = T_i - T_0 = \frac{Q_i}{K_i} = \frac{Q}{\sum_{i=1}^{n} K_i} = \text{const}. \tag{19}$$

The heat transfer temperature differences of thermal components also equal each other when the optimal heat transfer is obtained.

As below, a numerical example can also be presented. We can also assume that there are three thermal components, the thermal conductance $K_1 = 6$ W/K, $K_2 = 3$ W/K, and $K_3 = 1$ W/K, the total heat transfer rate $Q = 600$ W, and the temperature $T_0 = 300$ K. In this case, the variations of the average temperature of the thermal components weighted by released heat flow rate and the uniformity factor of temperature difference field with the distribution of the total released heat flow rate are calculated and shown in Fig. 3. It is very clear that the uniformity factor of temperature difference field and the average temperature have opposite variation tendencies. When the heat transfer rates $Q_1 = 360$ W, $Q_2 = 180$ W, and $Q_3 = 60$ W, the results show that the average temperature reaches the minimum value, 360 K, and the uniformity factor of temperature difference field reaches the maximum value, 1. The theoretical analyses have also been verified by the numerical results.

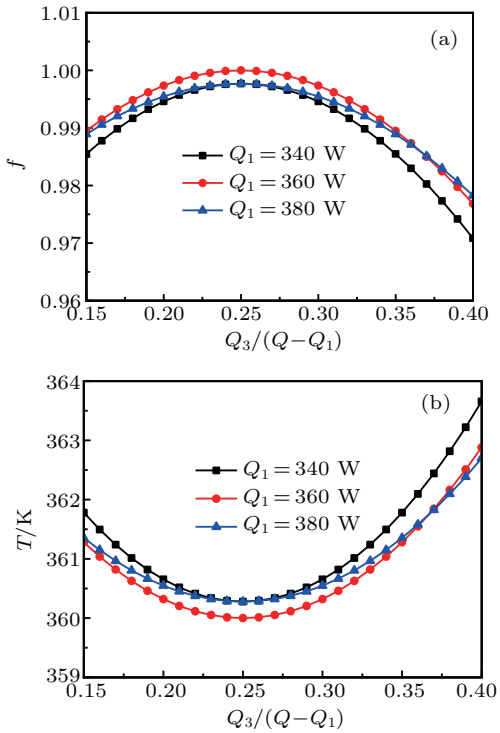


Fig. 3. Variations of the average temperature of the thermal components weighted by released heat flow rate and the uniformity factor of temperature difference field with the distribution of the total released heat flow rate. (a) Variation of the uniformity factor of temperature difference field. (b) Variation of the average temperature of the thermal components.

It can be seen that the theoretical and numerical optimization results of the cases above all lead to uniform heat transfer temperature difference fields. Therefore, the uniformity principle of temperature difference field holds in these heat transfer optimization problems.

3. One-stream series-wound heat exchanger networks

Heat exchanger networks are widely used in the thermal management system of spacecrafts. As shown in Fig. 4, we

can analyze a simple one-stream series-wound heat exchanger network. In the network, the n thermal components are cooled by the cold stream with mass flow rate m and inlet temperature $T_{\rm in}$. The specific heat capacity and the outlet temperature of the stream are c and $T_{\rm out}$, and the released heat flow rate and the corresponding heat transfer area of the i-th thermal component are Q_i and A_i, respectively. We assume that the limiting conditions of the system are Eq. (13) and

$$A = \sum_{i=1}^{n} A_i = \text{const}, \tag{20}$$

where A is the total heat transfer area. In this case, the distributions of the total heat transfer area and the total released heat flow rate are optimized at the same time to obtain the minimum value of Eq. (3).

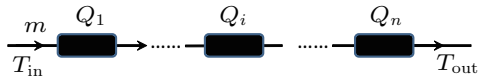


Fig. 4. A one-stream series-wound heat exchanger network.[13]

For the i-th thermal component, we can assume that[12,13]

$$k_i = km^{\alpha}, \tag{21}$$

where k_i is the heat transfer coefficient, k is a constant, the value of α is between 0.5 and 0.8 (0.5 and 0.8 correspond to the laminar flow and the turbulent flow of the stream, respectively). Therefore, we have

$$\begin{aligned} Q_i &= cm\left(T_i - T_{{\rm in}-i}\right)\left[1-\exp\left(-\frac{k_i A_i}{cm}\right)\right] \\ &= cm\left(T_i - T_{{\rm in}-i}\right)\left[1-\exp\left(-\frac{km^{\alpha-1}}{c}A_i\right)\right], \end{aligned} \tag{22}$$

where T_{in-i} is the inlet temperature of the stream for the i-th thermal component

$$T_{{\rm in}-i} = T_{\rm in} + \frac{\sum_{j=1}^{i-1} Q_j}{cm}. \tag{23}$$

Then, we can obtain that

$$T_i = \frac{Q_i}{cm\left[1-\exp\left(-km^{\alpha-1}A_i/c\right)\right]} + T_{\rm in} + \frac{\sum_{j=1}^{i-1} Q_j}{cm}. \tag{24}$$

Based on the entransy theory,[9] we have proved that the minimization of Eq. (3) is equivalent to the entransy dissipation minimization.[13] Therefore, we can find the optimal distributions of the total heat transfer area and the total released heat flow rate that lead to the entransy dissipation minimization below. For the i-th thermal component, the entransy dissipation rate is

$$\begin{aligned} G_{{\rm dis}-i} &= G_{{\rm in}-i} - G_{{\rm out}-i} \\ &= \left(Q_i T_i + \frac{1}{2}cmT_{{\rm in}-i}^2\right) - \frac{1}{2}cmT_{{\rm out}-i}^2 \\ &= Q_i T_i + \frac{1}{2}cm\left(T_{\rm in} + \frac{\sum_{j=1}^{i-1} Q_j}{cm}\right)^2 \\ &\quad - \frac{1}{2}cm\left(T_{\rm in} + \frac{\sum_{j=1}^{i} Q_j}{cm}\right)^2 \\ &= \frac{Q_i^2}{cm}\left[\frac{1}{1-\exp\left(-km^{\alpha-1}A_i/c\right)} - \frac{1}{2}\right], \end{aligned} \tag{25}$$

where $G_{{\rm in}-i}$ and $G_{{\rm out}-i}$ are the inlet entransy flow rate and outlet entransy flow rate in the heat transfer process of the i-th thermal component, and $T_{{\rm out}-i}$ is the outlet temperature of the stream. The entransy dissipation rate of the system is

$$\begin{aligned} G_{\rm dis} &= \sum_{i=1}^{n} Q_i T_i + \frac{1}{2}cmT_{\rm in}^2 - \frac{1}{2}cmT_{\rm out}^2 = \sum_{i=1}^{n} G_{{\rm dis}-i} \\ &= \sum_{i=1}^{n} \frac{Q_i^2}{cm}\left[\frac{1}{1-\exp\left(-km^{\alpha-1}A_i/c\right)} - \frac{1}{2}\right]. \end{aligned} \tag{26}$$

To find the minimum value of Eq. (26) with the limiting condition expressed by Eqs. (13) and (20), a function can be set up here

$$F_3 = G_{\rm dis} + \lambda_3\left(\sum_{i=1}^{n} A_i - A\right) + \lambda_4\left(\sum_{i=1}^{n} Q_i - Q\right), \tag{27}$$

where λ_3 and λ_4 are the Lagrange multipliers. The optimal distribution of the total heat transfer area and the total released heat flow rate should satisfy

$$\frac{\partial F_3}{\partial Q_i} = \frac{\partial F_3}{\partial A_i} = \frac{\partial F_3}{\partial \lambda_3} = \frac{\partial F_3}{\partial \lambda_4} = 0. \tag{28}$$

So, the optimal distribution of the total released heat flow rate gives

$$\frac{Q_i}{cm}\left[\frac{1}{1-\exp\left(-km^{\alpha-1}A_i/c\right)} - \frac{1}{2}\right] = -\frac{\lambda_4}{2} = \text{const}. \tag{29}$$

The entransy dissipation rate of the i-th thermal component can be rewritten as

$$G_{{\rm dis}-i} = Q_i \Delta T_i, \tag{30}$$

where ΔT_i is the equivalent heat transfer temperature difference of the thermal component. Considering Eqs. (25) and (29), we can obtain that

$$\begin{aligned} \Delta T_i &= \frac{G_{{\rm dis}-i}}{Q_i} = \frac{Q_i}{cm}\left[\frac{1}{1-\exp\left(-km^{\alpha-1}A_i/c\right)} - \frac{1}{2}\right] = -\frac{\lambda_4}{2} \\ &= \text{const}. \end{aligned} \tag{31}$$

From the derivation above, it can be seen that equation (31) also holds when the heat transfer areas of the thermal components are fixed and only the distribution of the total released heat flow rate is optimized. If the one-stream series-wound heat exchange network is treated as a heat exchanger, the temperature difference field is uniform. Therefore, the uniformity principle of temperature difference field is also tenable.

Here, a numerical example can also be presented. Assume that there are three thermal components, the heat capacity flow rate $cm = 100$ W/K, the heat transfer coefficient k, the

mass flow rate m and the heat capacity c satisfy $km^{\alpha-1}/c = 2\ \mathrm{m}^{-2}$, the heat transfer areas $A_1 = 0.5\ \mathrm{m}^2$, $A_2 = 0.4\ \mathrm{m}^2$, and $A_3 = 0.1\ \mathrm{m}^2$, the inlet temperature of the stream $T_{\mathrm{in}} = 300$ K, and the total heat transfer rate $Q = 1000$ W. The values of the parameters above are given according to the relevant values in the thermal control systems of spacecrafts.[12] In this case, the uniformity factor of temperature difference field can be defined as

$$f = \frac{\sum_{i=1}^{3} A_i \Delta T_i}{\sqrt{\left(\sum_{i=1}^{3} A_i\right) \sum_{i=1}^{3} A_i \Delta T_i^2}}. \tag{32}$$

The variations of average temperature of the thermal components weighted by released heat flow rate and the uniformity factor of temperature difference field with the distribution of the total released heat flow rate are calculated and shown in Fig. 5. When the distribution of the total released heat flow changes, the average temperature of the thermal components and the uniformity factor of temperature difference field also have opposite variation tendencies. Obviously, the numerical results can verify the theoretical analyses above.

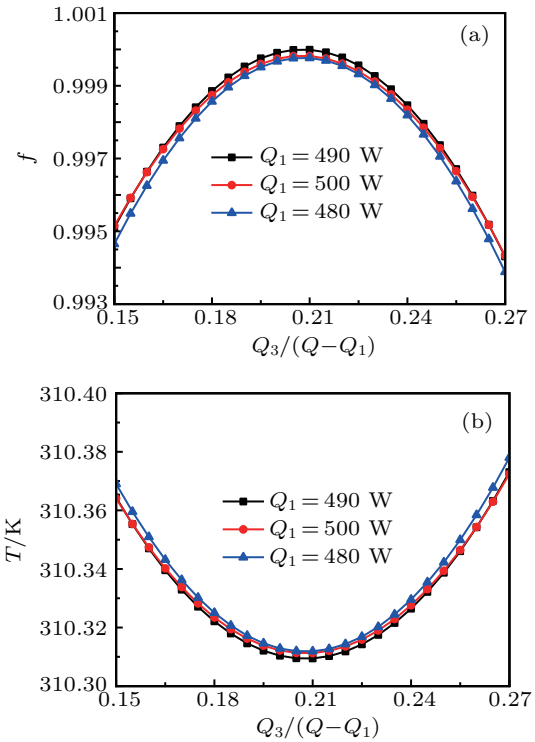


Fig. 5. Variations of the average temperature of the thermal components weighted by released heat flow rate and the uniformity factor of temperature difference field with the distribution of the total released heat flow rate for the one-stream heat exchanger network. (a) Variation of the uniformity factor of temperature difference field. (b) Variation of the average temperature of the thermal components.

4. Volume-to-point heat conduction problem

In the thermal management system of spacecrafts, there are many electronic devices that should be cooled, and the cooling problem may be simplified to be the volume-to-point heat conduction problem. As shown in Fig. 6, there is an internal heat source in the two-dimensional region, and the generated heat can only be transferred to the environment through a "point" boundary with temperature T_0 on one boundary. In order to decrease the average temperature of the region weighted by heat flow, a certain amount of high thermal conductivity material can be inserted in the region. As the amount of the high thermal conductivity material is fixed, we can find an optimal distribution of the limited high thermal conductivity material to minimize the average temperature of the heated region

$$T = \frac{\int_V q_{\mathrm{L}} T_{\mathrm{L}} \mathrm{d}V}{\int_V q_{\mathrm{L}} \mathrm{d}V}, \tag{33}$$

where V is the volume, q_L is the local inner heat source, and T_L is the local temperature. It can be seen the average temperature weighted by heat flow equals the volume average temperature when the inner heat source is distributed uniformly.

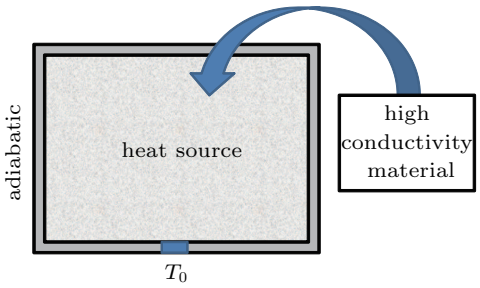


Fig. 6. Volume-to-point heat conduction problem.[9,14]

The limiting condition of the problem is

$$K_{\mathrm{c}} = \int_V k_{\mathrm{c}} \mathrm{d}V = \mathrm{const}, \tag{34}$$

$$Q = \int_V k_{\mathrm{c}} \left|\nabla T_{\mathrm{L}}\right| \mathrm{d}V = \mathrm{const}, \tag{35}$$

where K_{c} is a constant, k_{c} is the local thermal conductivity, and Q is the total heat transfer rate.

With the entransy theory,[9] we can find that the entransy balance of the system is

$$G_{\mathrm{dis}} = G_{\mathrm{in}} - G_{\mathrm{out}} = \int_V q_{\mathrm{L}} T_{\mathrm{L}} \mathrm{d}V - \left(\int_V q_{\mathrm{L}} \mathrm{d}V\right) T_0, \tag{36}$$

where G_{dis} is the entransy dissipation rate, and G_{in} and G_{out} are the entransy flow rates into and out of the system, respectively. As the temperature T_0 and the generated heat in the system are fixed, it can be seen that the minimization of Eq. (36) is equivalent to the entransy dissipation minimization.

In heat conduction, the entransy dissipation rate can be expressed as[9,14]

$$G_{\mathrm{dis}} = \int_V k_{\mathrm{c}} \left(\nabla T_{\mathrm{L}}\right)^2 \mathrm{d}V. \tag{37}$$

Therefore, to find the minimum value of Eq. (37) with the limiting condition expressed by Eq. (35), we can set up a function

$$F_4 = G_{\mathrm{dis}} + \lambda_5 \left(\int_V k_{\mathrm{c}} \left|\nabla T_{\mathrm{L}}\right| \mathrm{d}V - Q\right)$$

$$= \int_V \left[k_c (\nabla T_L)^2 + \lambda_5 k_c |\nabla T_L|\right] dV - \lambda_5 Q, \quad (38)$$

where λ_5 is the Lagrange multiplier. The optimal distribution of the thermal conductivity should satisfy

$$\frac{\partial F_4}{\partial (|\nabla T_L|)} = \frac{\partial F_4}{\partial \lambda_5} = 2k_c |\nabla T_L| + \lambda_5 k_c = \int_V k_c |\nabla T_L| dV - Q = 0. \quad (39)$$

Therefore, we have[9,14]

$$|\nabla T_L| = -\frac{\lambda_5}{2}. \quad (40)$$

Then, with Eqs. (34) and (39), we can obtain that

$$\lambda_5 = -\frac{2Q}{\int_V k_c dV} = -\frac{2Q}{K_c}. \quad (41)$$

Obviously, there is

$$|\nabla T_L| = \frac{Q}{K_c} = \text{const}. \quad (42)$$

It can be seen that the temperature gradient field is uniform. Obviously, the local temperature gradient is the expression of the temperature difference in the micro-body. From this point of view, the uniformity principle of temperature difference field also holds here. As the volume-to-point heat conduction problem has been analyzed many times, we do not give any numerical example here.

5. Radiative heat transfer

The optimization analyses of radiative heat transfer processes also widely exist in the thermal management system of spacecrafts. Assume that there are n thermal components on a flat plate, and the thermal components release heat flow into the environment at temperature T_0 only through radiative heat transfer. For the i-th thermal component, the temperature is T_i, the released heat flow rate is Q_i, the heat transfer area is A_i, and the thermal emissivity is ε_i, respectively. In radiative heat transfer, the driving potential is σT^4,[15] where σ is the Stefan-Boltzmann constant. Therefore, we can find the optimal design that leads to the minimum equivalent thermal potential weighted by released heat flow rate

$$U = \frac{\sum_{i=1}^n Q_i U_i}{\sum_{i=1}^n Q_i} = \frac{\sum_{i=1}^n \sigma Q_i T_i^4}{\sum_{i=1}^n Q_i}, \quad (43)$$

where U_i is the thermal potential of the i-th thermal component.

First, we assume that the limiting condition is

$$E = \sum_{i=1}^n E_i = \sum_{i=1}^n (\varepsilon_i A_i) = \text{const}. \quad (44)$$

In this case, we have

$$Q_i = E_i (U_i - U_0), \quad (45)$$

where U_0 is the thermal potential of the environment, which equals σT_0^4. So,

$$U_i = \frac{Q_i}{E_i} + U_0, \quad (46)$$

and

$$U = \frac{\sum_{i=1}^n Q_i (Q_i/E_i + U_0)}{\sum_{i=1}^n Q_i} = \frac{\sum_{i=1}^n Q_i^2/E_i}{\sum_{i=1}^n Q_i} + U_0. \quad (47)$$

To find the minimum value of Eq. (47) with the limiting condition expressed by Eq. (44), we can set up a function

$$F_5 = \frac{\sum_{i=1}^n Q_i^2/E_i}{\sum_{i=1}^n Q_i} + U_0 + \lambda_6 \left(\sum_{i=1}^n E_i - E\right), \quad (48)$$

The optimal distribution of E_i should satisfy

$$\frac{\partial F_5}{\partial E_i} = \frac{\partial F_5}{\partial \lambda_6} = \frac{Q_i^2}{\sum_{i=1}^n Q_i}\left(-\frac{1}{E_i^2}\right) + \lambda_6 = \sum_{i=1}^n E_i - E = 0. \quad (49)$$

where λ_6 is the Lagrange multiplier. Then, we can obtain that

$$E_i = \frac{Q_i}{\sum_{i=1}^n Q_i} E, \quad (50)$$

$$U_i = \frac{\sum_{i=1}^n Q_i}{E} + U_0 = \text{const}. \quad (51)$$

Considering the definition of thermal potential, we can see that the temperatures and the heat transfer temperature differences of the thermal components are all the same. Obviously, the uniformity principle of temperature difference field is also tenable here.

Furthermore, we can analyze a case in which the values of E_i are fixed, and the limiting condition is expressed by Eq. (13). In this case, we can also set up a function

$$F_6 = \frac{\sum_{i=1}^n Q_i^2/E_i}{Q} + U_0 + \lambda_7 \left(\sum_{i=1}^n Q_i - Q\right), \quad (52)$$

where λ_7 is the Lagrange multiplier. The optimal distribution of the total released heat flow rate should satisfy

$$\frac{\partial F_6}{\partial Q_i} = \frac{\partial F_6}{\partial \lambda_7} = \frac{2Q_i}{QE_i} + \lambda_7 = \sum_{i=1}^n Q_i - Q = 0. \quad (53)$$

Therefore, we can obtain that

$$Q_i = \frac{E_i}{\sum_{i=1}^n E_i} Q, \quad (54)$$

$$U_i = \frac{Q}{\sum_{i=1}^n E_i} + U_0 = \text{const}. \quad (55)$$

In this case, the thermal potentials, the temperatures, and the heat transfer temperature differences of the thermal components are also distributed uniformly. Hence, the uniformity principle of temperature difference field also holds here.

For the two optimization problems discussed above, it can be seen that the mathematical expressions are very similar to those in Section 2, and the variation tendencies of the variables are also similar to each other. Therefore, we do not present numerical examples for the optimization problems of radiative heat transfer, either.

6. Discussion

6.1. Consistence between the uniformity principle of temperature difference field and the entransy theory

From the analyses above, it can be seen that the derivations are related to the entransy theory. As below, the consistence between the uniformity principle of temperature difference field and the entransy theory is analyzed and discussed.

In the analyses of one-dimensional two-stream and three-stream heat exchangers, Song *et al.*[6] proved that a uniform temperature difference field would lead to the maximum heat transfer rate with the prescribed entransy dissipation rate and the minimum entransy dissipation rate with the prescribed heat transfer rate, respectively. For two-stream heat exchangers, Cheng *et al.*[4] proved that

$$f = \frac{1}{\sqrt{KR}}, \tag{56}$$

where K is the thermal conductance, and R is the entransy-dissipation-based thermal resistance. As the heat transfer units are fixed, K should be constant. It is very obvious that smaller entransy-dissipation-based thermal resistance always leads to larger uniformity factor of temperature difference field. With the entransy theory, the relationship between the uniformity factor of temperature difference field and the heat exchanger effectiveness was obtained[4]

$$\chi = \left[NTU^{-1} f^{-2} + \frac{1}{2}\left(1 + C^*\right)\right]^{-1}, \tag{57}$$

where χ is the heat exchanger effectiveness, and C^* is the ratio of the minimum heat capacity flow rate to the maximum heat capacity flow rate. With Eq. (57), the uniformity principle of temperature difference field for two-stream heat exchangers has been directly proved.

In the analyses from Section 2 to Section 5, it can be seen that the total heat transfer flow rates are all fixed. According to the entransy theory, the entransy dissipation minimization leads to the minimum heat transfer temperature difference.

For the system shown in Fig. 1, the entransy dissipation rate is

$$G_{\mathrm{dis}} = \sum_{i=1}^{n} Q_i T_i - \left(\sum_{i=1}^{n} Q_i\right) T_0. \tag{58}$$

As the total released heat flow rate is fixed, it can be seen that the minimizations of Eqs. (3) and (58) are consistent with each other. For the systems shown in Figs. 4 and 6, the consistence between the optimization objective and the entransy dissipation minimization has already been proved.[13,14] For the radiative heat transfer analyzed in Section 5, the entransy dissipation rate is

$$G_{\mathrm{dis}} = \sum_{i=1}^{n} Q_i U_i - \left(\sum_{i=1}^{n} Q_i\right) U_0, \tag{59}$$

It is also very clear that the entransy dissipation minimization gives the minimum value of Eq. (43). When the optimization objectives in these systems are obtained, we have shown that the heat transfer temperature differences are uniform. Therefore, the uniformity principle of temperature difference field and the entransy theory are consistent in these systems.

It is clear that the uniformity principle of temperature difference field works for the application cases in the thermal management system of spacecrafts in this paper, and a more uniform temperature difference field leads to better heat transfer. The principle is also very intuitive and phenomenological. Therefore, in the design of the thermal management system in spacecrafts, if we find that the temperature difference field is not uniform, we can intuitively conclude that the heat transfer is not optimal, and then we can change the relevant parameters to increase the uniformity of the temperature difference field. Correspondingly, the heat transfer performance of the system can be improved. Hence, the uniformity principle of temperature difference field can be very useful in engineering. Furthermore, as the principle is consistent with the entransy theory, which profoundly reveals the physical nature of heat transfer and is widely used in many thermal systems,[9–43] the entransy theory can work as the physical basis of the principle and provide the corresponding explanations of the physical mechanisms.

6.2. Balance of driving potential in heat transfer optimization

In nature, flows are driven by the corresponding potentials. For instance, the potentials of fluid flow, electrical current, conductive heat transfer, and radiative heat transfer are pressure, voltage, temperature, and radiative thermal potential, respectively. In the optimizations of these flows, the distributions of the corresponding potentials are also adjusted to achieve the optimization objectives. As below, the uniformity principle of temperature difference field in heat transfer optimization is discussed from this viewpoint.

From a certain point of view, we can say that a heat transfer system is composed of many subsystems which are located at different positions of the whole system. With specific constraints, the change of the heat transfer process in any subsystem can not only affect the heat transfer performance of the heat transfer process itself, but also affect the performance of the whole system. Therefore, the heat transfer optimization under given constraints and objectives can be treated as a result of competing and balancing the heat transfer processes of the subsystems. At the same time, the competition and balance of the heat transfer processes in the subsystems can obviously lead to the change of the distribution of the driving potential. Here, the uniformity principle of temperature difference field just reflects the equipartition of potential in heat transfer.

In two-stream heat exchangers, if the heat exchanger is divided into many parts along the direction of fluid flow, it can be seen that the heat transfer processes of the parts are connected

in series, and the heat flows are transferred into the fluid at different potentials. In this case, the competing and balancing of the heat transfer processes lead to a uniform potential difference field. In the one-stream series-wound heat exchanger network, the thermal components are also connected in series, and the results also show that the potential differences are uniform.

On the other hand, in the systems analyzed in Sections 2 and 5, the heat flows in each system are transferred into the same cold boundary, so we can say that the thermal components are connected in parallel. In these cases, the equipartition of potential can even give uniform potentials for each thermal component, and the potentials of the thermal components are the same. When the components are connected in parallel, we can further analyze a case shown in Fig. 7. In the system, the distribution of the mass flow rate of the stream to the branches can also be optimized, so we have the third limiting condition

$$m = \sum_{i=1}^{n} m_i = \text{const}, \tag{60}$$

where m_i is the mass flow rate of the i-th branch.

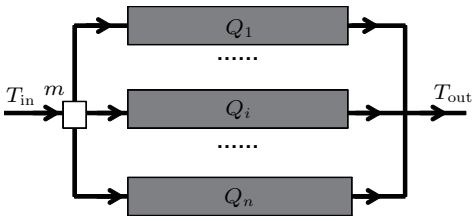


Fig. 7. A one-stream parallel heat exchanger network.[12,13]

For the i-th thermal component, there is

$$\begin{aligned} Q_i &= cm_i(T_i - T_{\text{in}})\left[1 - \exp\left(-\frac{k_i A_i}{cm_i}\right)\right] \\ &= cm_i(T_i - T_{\text{in}})\left[1 - \exp\left(-\frac{km_i^{\alpha-1}}{c}A_i\right)\right]. \end{aligned} \tag{61}$$

So, we have

$$T_i = \frac{Q_i}{cm_i\left[1 - \exp\left(-km_i^{\alpha-1}A_i/c\right)\right]} + T_{\text{in}}, \tag{62}$$

$$T = \frac{\sum_{i=1}^{n} Q_i\left(\dfrac{Q_i}{cm_i\left[1 - \exp\left(-km_i^{\alpha-1}A_i/c\right)\right]} + T_{\text{in}}\right)}{Q}. \tag{63}$$

To find the minimum value of Eq. (63) with the limiting conditions expressed by Eqs. (13), (20), and (60), a function can also be set up

$$\begin{aligned} F_7 = {} & T + \lambda_8\left(\sum_{i=1}^{n} A_i - A\right) + \lambda_9\left(\sum_{i=1}^{n} Q_i - Q\right) \\ & + \lambda_{10}\left(\sum_{i=1}^{n} m_i - m\right), \end{aligned} \tag{64}$$

where λ_8, λ_9, and λ_{10} are the Lagrange multipliers. The optimal distribution of the mass flow rate, the total heat transfer area, and the total released heat flow rate should satisfy

$$\frac{\partial F_4}{\partial Q_i} = \frac{\partial F_4}{\partial A_i} = \frac{\partial F_4}{\partial m_i} = \frac{\partial F_4}{\partial \lambda_8} = \frac{\partial F_4}{\partial \lambda_9} = \frac{\partial F_4}{\partial \lambda_{10}} = 0. \tag{65}$$

Therefore, the optimal distribution of the total released heat flow rate leads to

$$\frac{Q_i}{cm_i\left[1 - \exp\left(-km_i^{\alpha-1}A_i/c\right)\right]} = -\frac{\lambda_9 + T_{\text{in}}}{2} = \text{const}. \tag{66}$$

Considering Eq. (62), we can see that the temperatures of the thermal components equal each other. Obviously, the competing and balancing of the heat transfer processes in the branches lead to a uniform temperature distribution for the thermal components.

In heat transfer optimization, when the constraints, the objectives, or the analyzed system change, maybe the distribution of potential can show some other characteristics, and the uniformity principle of temperature difference field may not hold any more. We should not assume that any theory is omnipotent and always correct.[38] However, when we pay attention to the distribution of potential with the consideration of the competing and balancing of the local heat transfer processes in the subsystems, the uniformity principle of temperature difference field can be obtained in some cases. This is still important and interesting. It is advised that we can analyze the characteristics of the distribution of potential in other heat transfer optimization problems.

7. Conclusions

In this paper, it is found that the uniformity principle of temperature difference field holds not only for heat exchanger optimization, but also for some other heat transfer optimization problems in the thermal management system of spacecrafts, including the cooling of thermal components, the optimization of a one-stream series-wound heat exchanger network, the volume-to-point heat conduction optimization problem, and the radiative heat transfer optimization. Numerical examples are also presented, and the results have verified the theoretical analyses.

The uniformity principle of temperature difference field is intuitive and phenomenological, and reflects the characteristic of the distribution of potential in the heat transfer optimization cases with specific constraints and design objectives. However, it is shown that the uniformity principle of temperature difference field is consistent with the entransy theory. Therefore, the entransy theory can work as the physical basis of the principle and provide the corresponding explanations of the physical mechanisms.

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