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Macadam's theory in RGB laser display*

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We have developed Macadam's theory to deal with RGB laser display, which can well describe the color gamut display system varying with the laser bandwidth. By calculating the volume of Rösch–Macadam color solid of laser display system under the Rec.2020 standard, we can obtain that the volume of chromatic stereoscopic at 30-nm laser spectral linewidth is about 90% of that at 1 nm laser spectral linewidth, which is important in laser display system to trade off the color gamut and the suppression of laser speckles. Moreover, we can also calculate the color gamut volume with different primary numbers and different primary wavelengths.

Keywords: color gamut, laser display, optimal RGB value

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1. Introduction

Wide color gamut makes a display system capable to show real objects accurately. Recently, light sources such as quantum dot and monochromatic lasers[1–3] have been widely studied for handling wide-gamut standard of Rec.2020[4] to reproduce the natural object colors faithfully. RGB laser display has advantages of narrow spectrum width, high power, high conversion efficiency, and long lifetime. Until now, edge-emitting, laser diodes, diode-pumped solid-state lasers, vertical-cavity surface-emitting lasers (VCSELs), and fiber lasers have been utilized in laser displays.[3,5] The main issue of laser display is the speckle noise.[6–8] One of the methods to solve the speckle is to properly increase the spectral linewidth of the primary lasers to about 10 nm by LD laser array. However, both the spectral linewidth and center wavelength of the primary laser may influence color gamut. Thus, it is important to settle the relationship of the color gamut versus the spectrum width and center wavelength of primary lasers.

Macadam's theory[9,10] provides an algorithm to calculate color gamut of display systems. In 1935, based on works of Schrodinger[11] and Rösch,[12] Macadam proposed a stereoscopic color gamut algorithm by calculating optimal color. The optimal color was defined as the most saturation that an illuminant is able to reach under a given luminance value. Through calculating the coordinate of the optimal color, Macadam obtained the boundary of the color gamut. Different illuminants have different optimal colors. We have two usage for optimal color. One is to calculate color volume coverage ratio to an ideal or large color gamut such as Pointer's gamut[13] and Rec.2020. The other is to propose the color quality index for light sources by comparing the volume of their color gamut. In 2007, Francisco *et al.* calculated the stereoscopic color gamut of D_{65} and other standard light sources by calculating the optimal colors in CIELAB space.[14] However, the operation speed of Francisco's algorithm is comparatively slow and would cause problem in discontinuous or very peaked spectrum. Improvements have been made in Francisco's algorithm. Li *et al.* improved the efficiency and accuracy of the algorithm by using linear interpolation algorithm.[15] Masaoka ameliorated the algorithm by using trapezoid integral method.[16] Light source also belongs to display system and we may use this method to calculate its color gamut. Up to now, Francisco's algorithm still suffer problems in discontinuous or very peaked spectrum, which will limit its applications of the display system.

In this article, we propose an algorithm to calculate the color gamut of RGB display system, which can deal with any spectrum distribution including the case of discontinuous or very peaked spectrum.

2. Macadam's theory

2.1. Traditional Macadam's theory

In order to describe the problem of the traditional Macadam's theory in discontinuous or very peaked spectrum, for comparison, we use two kinds of three primary light sources (A and B) to calculate color gamut, light source A has a narrow spectral linewidth with all three primary colors, while light source B has a much wider spectral linewidth of one primary color.

The wavelength λ and intensity $E(\lambda)$ of the three primary colors follow Gaussian distribution:

$$E(\lambda) = E_0 * \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\lambda-\lambda_c)^2}{2\sigma^2}\right], \quad (1)$$

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where λ_c corresponds to the central wavelength of every primary color. According to Rec.2020 standard, the wavelengths of three primary colors are 467 nm, 532 nm, and 630 nm. E_0 serves as an intensity parameter of the light source. It is used to adjust the coordinate of white point. In order to make the white point close to the standard light source D_{65} point, we adjust the ratio among the three peak intensities to 1.35:1.05:1.0. σ serves as standard deviation. The relationship between σ and full spectral linewidth at half maximum (FWHM) is

$$\sigma = \frac{\text{FWHM}}{2\sqrt{2\ln 2}} = \frac{\text{FWHM}}{2.3548}. \tag{2}$$

Let FWHM of all three primary colors of light source A as 1 nm and let FWHM of red and blue of light source B as 1 nm and FWHM of green of light source B as 20 nm. The two types of light source have the same power value. Their distribution of wavelength and intensity are shown in Fig. 1.

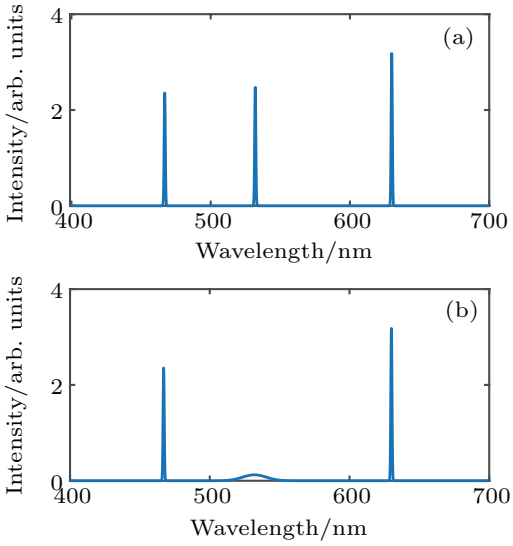


Fig. 1. The spectral intensity distributions of light source A (a) and light source B (b).

We try to use traditional Macadam's theory to calculate color gamut. It seems that light source A has a wider color gamut than that of light source B. But that was not the case according to traditional Macadam's theory.

Macadam considered only two types of the reflectance (transmittance) curve can obtain optimal colors. As shown in Fig. 2, type 1: the both ends of the reflectance curve are 0 and the central is 100% (010 form, Fig. 2(a)) and type 2: the central reflectance curve is 0 and both ends are 100% (101 form, Fig. 2(b)). In calculation, we set initial wavelength $\lambda_1 = 460$ nm and 468 nm, visual efficiency $R = 0.3$ and wavelength calculation accuracy as 0.1 nm.

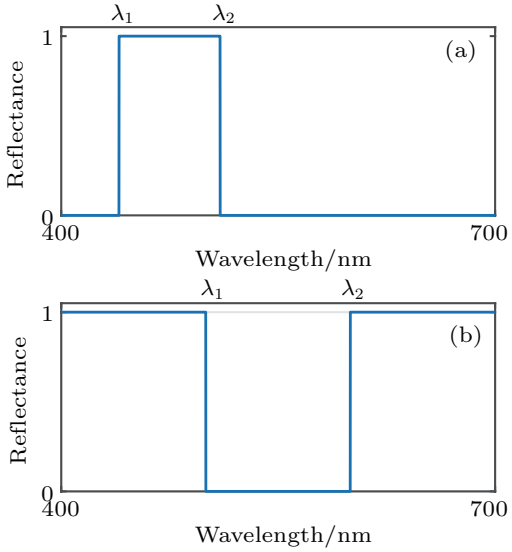


Fig. 2. Spectral reflectance (transmittance) distribution of type 1 (a) and type 2 (b) of Macadam limits.

In CIEXYZ color space, $y(\lambda)$ stands the relationship between wavelength and luminance. The luminance value of the optimal color can be expressed as:

$$Y_{\text{optimal}} = R \cdot \Sigma_{400}^{700} y(\lambda) \cdot E(\lambda), \tag{3}$$

$$Y_{\text{optimal}} = \Sigma_{\lambda_1}^{\lambda_2} y(\lambda) \cdot E(\lambda). \tag{4}$$

From Eq. (3) and Eq. (4), we can get the following expression:

$$\begin{aligned} &\Sigma_{400}^{\lambda_2} y(\lambda) \cdot E(\lambda) \\ &= R \cdot \Sigma_{400}^{700} y(\lambda) \cdot E(\lambda) + \Sigma_{400}^{\lambda_1} y(\lambda) \cdot E(\lambda), \end{aligned} \tag{5}$$

and λ_2 can be obtained from Eq. (5).

The others of the tristimulus value of optimal color can be written as:

$$X_{\text{optimal}} = \Sigma_{\lambda_1}^{\lambda_2} x(\lambda) \cdot E(\lambda), \tag{6}$$

$$Z_{\text{optimal}} = \Sigma_{\lambda_1}^{\lambda_2} z(\lambda) \cdot E(\lambda) \tag{7}$$

and the color coordinates are,

$$S = X_{\text{optimal}} + Y_{\text{optimal}} + Z_{\text{optimal}}, \tag{8}$$

$$(x, y)_{\text{optimal}} = \left(\frac{X_{\text{optimal}}}{S} \frac{Y_{\text{optimal}}}{S} \right). \tag{9}$$

The calculation results are shown in Table 1.

Table 1. Calculation results of Macadam limits under two wavelength distributions. Visual efficiency $R = 0.3$.

Source	λ_1/nm	Y_{optimal}	$\Sigma_{400}^{\lambda_1} y(\lambda) \cdot E(\lambda)$	$\Sigma_{400}^{\lambda_2} y(\lambda) \cdot E(\lambda)$	λ_2/nm	X_{optimal}	Z_{optimal}	x	y
A(1 nm)	460.0	10.2817	0	10.2817	531.795	7.4876	36.4698	0.1380	0.1896
	468.0	10.2817	1.9986	12.2803	531.889	2.1886	0.6716	0.1665	0.7824
B(20 nm)	460.0	10.1165	0	10.1165	529.817	6.8428	36.8043	0.1273	0.1882
	468.0	10.1165	1.9986	12.1151	531.694	1.5202	1.0137	0.1202	0.7997

We calculate the optimal color of two types of light source and show the result in Fig. 3.

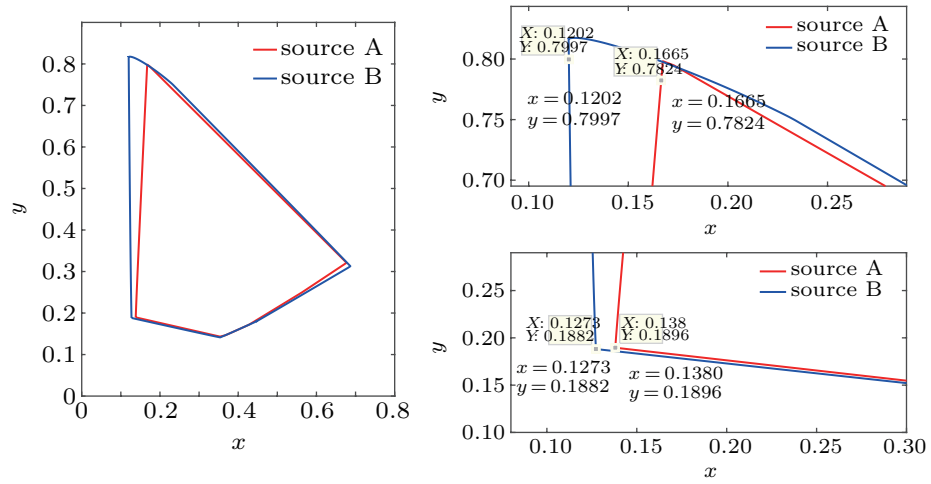


Fig. 3. Macadam limit of two wavelength distributions in XYZ color space

It can be seen from Fig. 3, the color gamut of light source A (spectrum width 1 nm) is actually smaller than that of light source B (spectrum width 20 nm). Obviously, this does not conform the actual situation. To find the reason, two pairs of wavelength of two light sources are list in Table 2.

According to λ_1, λ_2 from Table 2. We add the spectral reflectance curve in Fig. 1 and shown in Fig. 4.

Table 2. Wavelength pair of two light sources

Light source	λ_1/nm	λ_2/nm
A (1 nm)	460.0	531.795
	468.0	531.889
B (20 nm)	460.0	529.817
	468.0	531.694

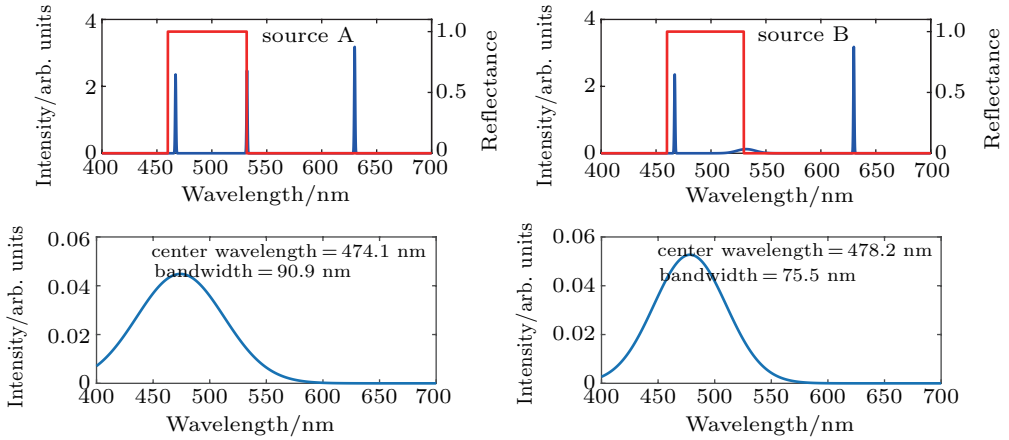


Fig. 4. Reflect spectrum (upper) and their Gauss equivalent spectrum (lower) of two starting wavelengths under two light sources, for $\lambda_1 = 460$ nm.

It can be seen from Fig. 4, the selected spectral linewidth of light source B is even narrower than that of light source A. To meet visual efficiency, Macadam's theory chose part of spectrum of one primary instead of the whole spectrum, which lead the Gauss equivalent spectrum width of wide spectrum narrower than that of narrow spectrum.

2.2. Our development Macadam's theory

To solve this problem, we define optimal RGB value to independently show the intensity of every primary color. It can be shown as

$$(I_r, I_g, I_b)_{\text{optimal}}.$$

The value range of it is [0,1] and they can be calculated by the ratio between the intensity of every primary color and the maximum intensity that every primary color can reach.

The development Macadam's theory, by substituting optimal RGB value for wavelength in traditional Macadam's theory, is suitable for RGB display system. The comparison between traditional Macadam's theory and developed Macadam's theory for the RGB display system is shown specifically in Table 3.

We sort primary colors in form of the ascending order of wavelength, calculate their luminance values separately and name them as Y_{Blue}, Y_{Green}, Y_{Red}. The relationship of the luminance and optimal RGB values are list in Table 4.

Table 3. Comparison between traditional Macadam's theory and Macadam's theory applicable to RGB display system.

Traditional Macadam's theory	Macadam's theory applicable to RGB display system
(i) Light source spectrum and visual efficiency is given. (ii) Use a series of wavelength interval to separate spectrum of each light source. (iii) Establish the relationship between luminance and wavelength. (iv) According to the relationship in (iii), calculate λ_1 and λ_2 under given visual efficiency of type 1 and type 2. (v) Use λ_1 and λ_2 to calculate color coordinates of the optimal color.	(I) Light source spectrum and visual efficiency is given. (II) Use optimal RGB value to separate spectrum of each light source. (III) Establish the relationship between luminance and optimal RGB value. (IV) According to the relationship in (III), calculate optimal RGB value under given visual efficiency of type 1 and type 2. (V) Use two pairs of optimal RGB value to calculate color coordinates of the optimal color.

Table 4. Conversion of luminance and optimal RGB values.

$Y < Y_{\text{Blue}}$	$I_{\text{r}} = 0, I_{\text{g}} = 0$ $I_{\text{b}} = Y/Y_{\text{Blue}}$
$Y = Y_{\text{Blue}}$	$I_{\text{r}} = 0, I_{\text{g}} = 0, I_{\text{b}} = 1$
$Y_{\text{Blue}} < Y < Y_{\text{Blue}} + Y_{\text{Green}}$	$I_{\text{r}} = 0$ $I_{\text{g}} = (Y - Y_{\text{Blue}})/Y_{\text{Green}}$ $I_{\text{b}} = 1$
$Y = Y_{\text{Blue}} + Y_{\text{Green}}$	$I_{\text{r}} = 0, I_{\text{g}} = 1, I_{\text{b}} = 1$
$Y > Y_{\text{Blue}} + Y_{\text{Green}}$	$I_{\text{r}} = (Y - Y_{\text{Blue}} - Y_{\text{Green}})/Y_{\text{Red}}$ $I_{\text{g}} = 1, I_{\text{b}} = 1$

3. Calculation process and results

3.1. Calculation process

Let light source A as an example to explain the calculation process of the algorithm:

We first calculate the luminance of three primaries.

$$Y_{\text{Blue}} = \Sigma_{464.8}^{469.2} y(\lambda) \cdot E(\lambda), \tag{10}$$

$$Y_{\text{Green}} = \Sigma_{529.8}^{534.2} y(\lambda) \cdot E(\lambda), \tag{11}$$

$$Y_{\text{Red}} = \Sigma_{627.8}^{632.2} y(\lambda) \cdot E(\lambda), \tag{12}$$

$$Y_{\text{optimal}} = R \cdot \Sigma_{400}^{700} y(\lambda) \cdot E(\lambda). \tag{13}$$

The X_{Red}, X_{Green}, X_{Blue}, Z_{Red}, Z_{Green}, and Z_{Blue} can be obtained similar to the calculations of Y_{Blue}, Y_{Green}, Y_{Red}.

We replace luminance Y_1 with λ_1 in traditional Macadam's theory, and set Y_1 as

$$Y_1 = 0.01 \cdot \Sigma_{400}^{700} y(\lambda) \cdot E(\lambda), \tag{14}$$

Y_2 value can be calculated below.

type 1

$$Y_2 = Y_1 + Y_{\text{optimal}}, \tag{15}$$

type 2

$$Y_2 = \left(\Sigma_{400}^{700} y(\lambda) \cdot E(\lambda) - Y_{\text{optimal}}\right) + Y_1. \tag{16}$$

The values of (I_{r_1}, I_{g_1}, I_{b_1}) and (I_{r_2}, I_{g_2}, I_{b_2}) can be obtained according to Table 4.

The value of optimal RGB

type 1

$$(I_{\text{r}}, I_{\text{g}}, I_{\text{b}})_{\text{optimal}} = \left(I_{\text{r}_2} - I_{\text{r}_1}, I_{\text{g}_2} - I_{\text{g}_1}, I_{\text{b}_2} - I_{\text{b}_1}\right), \tag{17}$$

type 2

$$(I_{\text{r}}, I_{\text{g}}, I_{\text{b}})_{\text{optimal}} = (1 - I_{\text{r}_2} + I_{\text{r}_1} 1 - I_{\text{g}_2} + I_{\text{g}_1} 1 - I_{\text{b}_2} + I_{\text{b}_1}). \tag{18}$$

The tristimulus values of the optimal color

$$X_{\text{optimal}} = (I_{\text{b}})_{\text{optimal}} \cdot X_{\text{Blue}} + (I_{\text{g}})_{\text{optimal}} \cdot X_{\text{Green}} + (I_{\text{r}})_{\text{optimal}} \cdot X_{\text{Red}}, \tag{19}$$

$$Z_{\text{optimal}} = (I_{\text{b}})_{\text{optimal}} \cdot Z_{\text{Blue}} + (I_{\text{g}})_{\text{optimal}} \cdot Z_{\text{Green}} + (I_{\text{r}})_{\text{optimal}} \cdot Z_{\text{Red}}. \tag{20}$$

3.2. Results and discussions

We can obtain $(x, y)_{\text{optimal}}$ by Eqs. (8) and (9) and draw trajectory of the optimal color in Fig. 5.

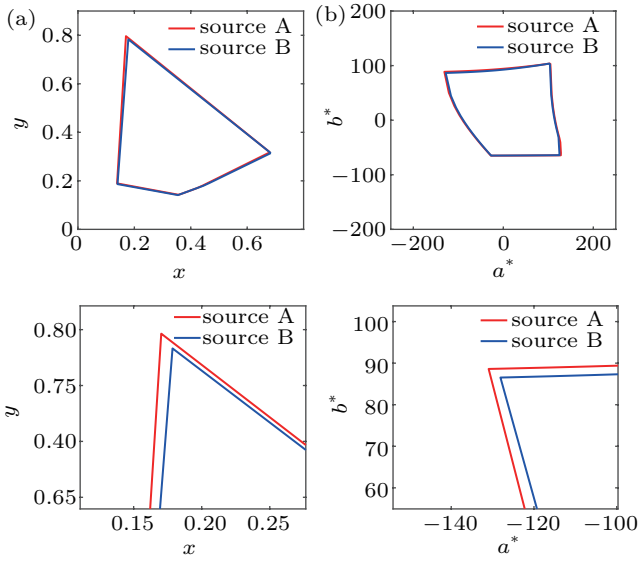


Fig. 5. Macadam limit of two wavelength distributions in CIEXYZ (a) and CIELAB (b) color space based on three primary color settings.

It can be seen in Fig. 5, the color gamut of light source A is wider than that of light source B, which conforms the actual situation. That is, this algorithm is able to restore color gamut of RGB display system precisely.

Furthermore, we calculate the boundary of optimal color by altering the spectral linewidth of all three primary colors at the same time under visual efficiency $R = 0.3$, as shown in Fig. 6. The color gamut coverage decreases when spectrum width increases, which is satisfied with our common knowledge.

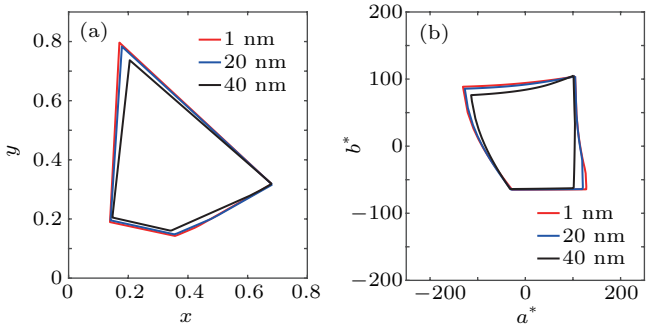


Fig. 6. Relationship between optimal color and spectral linewidth in CIEXYZ (a) and CIELAB (b) color space.

We change the visual efficiency value and set L relevant to visual efficiency as ordinate and get the relationship between visual efficiency and luminance L:[14,17]

$$L = \begin{cases} 903.3R & \text{if } R \leqslant 0.008856, \\ 116R^{1/3} - 16 & \text{if } R > 0.008856. \end{cases} \tag{21}$$

In CIELAB space, we draw the color solid of RGB display system with different spectral linewidths, and shown in Fig. 7.

The volume of chromatic stereoscopic under different spectral linewidths is listed in Table 5.

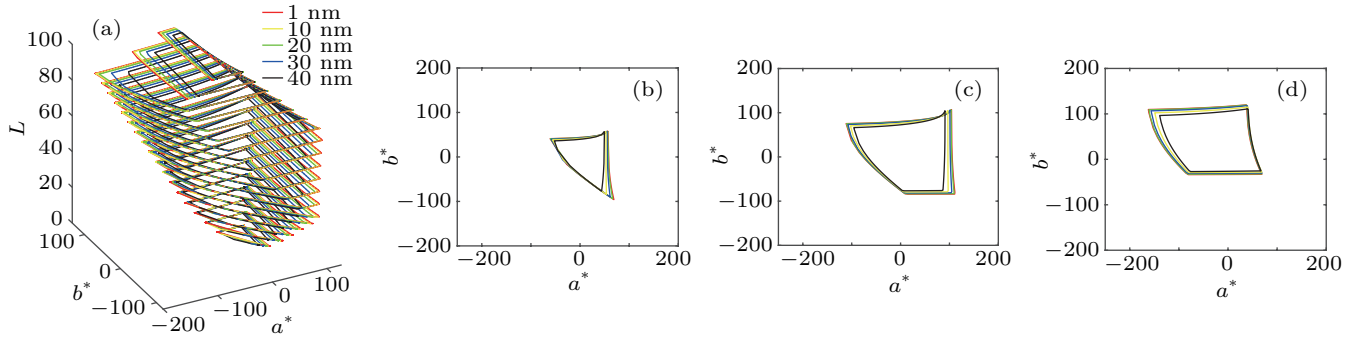


Fig. 7. Relationship between the Macadam color solid and spectrum width (a) and cross sections for $L = 20$, 50, and 80 (panels (b)–(d)).

Table 5. Relationship between the color gamut volume and spectrum width

$\Delta\lambda$/nm	$3P_{(630\text{ nm},532\text{ nm}.467\text{ nm})}/10^6$	1 nm/%
1	1.8470	100
10	1.8299	99.07
20	1.7667	95.65
30	1.6523	89.45
40	1.4973	81.07

We show that when spectral linewidth is 30 nm, the volume of chromatic stereoscopic is about 90% of that of 1 nm.

4. Conclusion

We have developed Macadam's theory to deal with RGB laser display. By substituting optimal RGB value for wavelength in traditional Macadam's theory, we obtain a developed Macadam's theory which is suitable for RGB laser display system. In this theory, we can calculate the influences of spectrum width to the volume of Rösch–Macadam color solid. The narrower spectrum width of RGB display system will lead to wider color gamut volume, and this theory will give a reference to the wavelength and spectrum width selection for display system to keep the balance between color gamut and problem of the speckle suppression.

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