

Analysis of third and one-third harmonic generation in lossy waveguides

Jianyu Zhang(张剑宇), Yunxu Sun(孙云旭), Qinghai Song(宋清海)

Citation: Chin. Phys. B . 2019, 28(6): 064206. **doi:** 10.1088/1674-1056/28/6/064206
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Mid-infrared supercontinuum generation and its application on all-optical quantization with different input pulses

Yan Li(李妍), Xinzhu Sang(桑新柱)

Chin. Phys. B . 2019, 28(5): 054206. **doi:** 10.1088/1674-1056/28/5/054206

Multiple Fano resonances in nanorod and nanoring hybrid nanostructures

Xijun Wu(吴希军), Ceng Dou(窦层), Wei Xu(徐伟), Guangbiao Zhang(张广彪), Ruiling Tian(田瑞玲), Hailong Liu(刘海龙)

Chin. Phys. B . 2019, 28(1): 014204. **doi:** 10.1088/1674-1056/28/1/014204

Generation of breathing solitons in the propagation and interactions of Airy-Gaussian beams in a cubic-quintic nonlinear medium

Weijun Chen(陈卫军), Ying Ju(鞠莹), Chunyang Liu(刘春阳), Liankai Wang(王连锴), Keqing Lu(卢克清)

Chin. Phys. B . 2018, 27(11): 114216. **doi:** 10.1088/1674-1056/27/11/114216

Surface plasmon polariton at the interface of dielectric and graphene medium using Kerr effect

Bakhtawar, Muhammad Haneef, B A Bacha, H Khan, M Atif

Chin. Phys. B . 2018, 27(11): 114215. **doi:** 10.1088/1674-1056/27/11/114215

Generation of femtosecond laser pulses at 263 nm by $K_3B_6O_{10}Cl$ crystal

Ning-Hua Zhang(张宁华), Shao-Bo Fang(方少波), Peng He(何鹏), Hang-Dong Huang(黄杭东), Jiang-Feng Zhu(朱江峰), Wen-Long Tian(田文龙), Hong-Ping Wu(吴红萍), Shi-Lie Pan(潘世烈), Hao Teng(滕浩), Zhi-Yi Wei(魏志义)

Chin. Phys. B . 2017, 26(6): 064208. **doi:** 10.1088/1674-1056/26/6/064208

Analysis of third and one-third harmonic generation in lossy waveguides*

Jianyu Zhang(张剑宇)[1], Yunxu Sun(孙云旭)[1,†], and Qinghai Song(宋清海)[2]

[1] Department of Electronic and Information Engineering, Harbin Institute of Technology (Shenzhen), Shenzhen 518055, China
[2] Department of Science, Harbin Institute of Technology (Shenzhen), Shenzhen 518055, China

(Received 15 January 2019; revised manuscript received 15 March 2019; published online 29 May 2019)

A comprehensive study on the requirements for the highly efficient third harmonic generation (THG) and its inverse process, one-third harmonic generation (OTHG), in lossy waveguides is proposed. The field intensity restrictions for both THG and OTHG caused by loss are demonstrated. The effective relative phase ranges, supporting the positive growth of signal fields of THG and OTHG are shrunken by the loss. Furthermore, it turns out that the effective relative phase ranges depend on the intensities of the interacting fields. At last, a modified definition of coherent length in loss situation, which evaluates the phase matching degree more precisely, is proposed by incorporating the shrunken relative phase range and the nonlinear phase mismatch. These theoretical analysis are valuable for guiding the experimental designs for highly efficient THG and OTHG.

Keywords: coupled mode theory, third harmonic generation, one-third harmonic generation, loss effects

PACS: 42.65.–k, 42.65.Wi, 42.65.Lm **DOI:** 10.1088/1674-1056/28/6/064206

1. Introduction

Nonlinear optical processes in optical microfibers have attracted significant interest because of the unique environment, including a tight confinement of light, long interaction lengths, and large manageable waveguide dispersion.[1–3] With a relatively low optical power, many enhanced nonlinear effects, such as supercontinuum generation,[4,5] bistability,[6,7] two-photon absorption,[8,9] third-harmonic generation,[10–13] and one-third harmonic generation,[13–17] have been demonstrated.

In this work, we focus on third-harmonic generation (THG) and one-third harmonic generation (OTHG) in optical microfibers, which are a pair of inverse processes and share the same phase matching condition that can be realized by engineering the waveguide dispersion in optical microfibers. The THG is widely used to produce novel wavelengths, which are attractive to optical logic unit,[18] material processing,[19] and so on. The OTHG, on the other hand, is a potential way to generate the triple photon states with possible applications in quantum cryptography, quantum information processing, and general experiments investigating the fundamental nature of quantum entanglement. The theoretical framework behind THG and OTHG in optical microfibers was developed initially by Grubsky and Savchenko in 2005.[10] Following, laser power and waveguide design parameters for high THG and OTHG efficiency were analyzed with the lossless approximation.[13,15] However, the optical attenuation is unneglectable and influences the nonlinear conversion in real experiments. Several theoretical researches on THG[20–22] and OTHG[14,17,20] have included the optical losses in their numerical calculations, but mainly aim at improving the corresponding conversion efficiency by designing the waveguide parameters. The influences of losses on the optical parameters, including power and phases, have not been systematically investigated for both THG and OTHG processes in microfibers.

Here, a general analysis on THG and OTHG in lossy waveguides is made to get a guideline for the experimental designs to achieve the high efficiencies. The required optical field conditions for THG and OTHG conversions in loss waveguides are deduced from the coupled mode equations. The requirements are quite different from those in the lossless situation. Accordingly, a modified definition of coherent length is proposed to evaluate the phase matching degree in loss situation more precisely. At last, some suggestions for improving the conversion efficiencies of THG and OTHG in lossy waveguides are made based on the analysis.

2. Coupled mode equations

The fields at ω and 3ω are assumed in different but single propagating modes of a waveguide as represented by the slowly varying complex amplitudes $A_m(z)$, $m = 1,3$ respectively. The following set of equations can then be derived,[10] with losses α_m, $m = 1,3$ included.

$$\frac{\partial A_1}{\partial z} = -\alpha_1 A_1 + \mathrm{i}\gamma_0 \left[\left(J_1 |A_1|^2 + 2J_2 |A_3|^2 \right) A_1 + J_3 (A_1^*)^2 A_3 \, \mathrm{e}^{\mathrm{i}\delta\beta z} \right], \tag{1a}$$

*Project supported by Shenzhen Municipal Science and Technology Plan Project, China (Grant Nos. JCYJ20160427183803458 and JCYJ20150403161923546).
†Corresponding author. E-mail: sunyunxu@hit.edu.cn
© 2019 Chinese Physical Society and IOP Publishing Ltd

$$\frac{\partial A_3}{\partial z} = -\alpha_3 A_3 + \mathrm{i}\gamma_0 \left[\left(6J_2 |A_1|^2 + 3J_5 |A_3|^2 \right) A_3 + J_3^* A_1^3 \mathrm{e}^{-\mathrm{i}\delta\beta z} \right]. \tag{1b}$$

The coefficients J_1, J_2, J_3, and J_5 are the modal overlap integrals, which respectively correspond to the self-phase modulation (SPM) of field at ω, cross-phase modulation (XPM) between fields ω and 3ω, nonlinear conversion between fields ω and 3ω, and SPM of field at 3ω. The waveguide nonlinear coefficient is $\gamma_0 = 2\pi n^{(2)}/\lambda_1$, where $n^{(2)}$ and λ_1 are the Kerr nonlinearity and vacuum wavelength of field at ω, respectively. The modal phase mismatch is $\delta\beta = \beta(3\omega) - 3\beta(\omega)$. The amplitudes ρ_m and phases ϕ_m of each field are incorporated in their complex amplitudes by the relations of $A_m = \rho_m \mathrm{e}^{\mathrm{i}\phi_m}$. Consequently, the coupling mode equations can be rewritten as:

$$\frac{\mathrm{d}\rho_1}{\mathrm{d}z} = -\alpha_1\rho_1 - \gamma_0 J_3 \rho_1^2 \rho_3 \sin\theta, \tag{2a}$$

$$\frac{\mathrm{d}\rho_3}{\mathrm{d}z} = -\alpha_3\rho_3 + \gamma_0 J_3 \rho_1^3 \sin\theta, \tag{2b}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \delta\beta + K(\rho_1, \rho_3, \theta), \tag{2c}$$

$$K(\rho_1, \rho_3, \theta) = \gamma_0 \left[3(2J_2 - J_1)\rho_1^2 + 3(J_5 - 2J_2)\rho_3^2 + \left(\rho_1^3 \rho_3^{-1} - 3\rho_1\rho_3 \right) J_3 \cos\theta \right]. \tag{2d}$$

Equations (2a) and (2b) respectively govern the amplitude evolutions of the fields at ω and 3ω along the propagating path. The first terms of these two expressions represent the linear loss rates, and the second terms stand for the nonlinear gain rates. The sign of the nonlinear gain term indicates the power flow direction, which is determined by the relative phase defined as $\theta = \delta\beta z + \phi_3 - 3\phi_1$. If $\sin\theta > 0$, it is THG process, otherwise OTHG. However, because of loss, the relative phase ranges for positive gain of THG and OTHG signal field become smaller, which are quantitatively demonstrated below. It shows that this phase range strongly depends on the intensities of the interacting fields. Equations (2c) describes the variation of relative phase along the propagating path, which arises from two terms: the modal dispersion and the nonlinear dispersion caused by Kerr effects. The latter term is given by Eq. (2d) as a function of intensities of the interacting fields and their relative phase. Based on Eqs. (2a)–(2d), the requirements of optical fields for efficient THG and OTHG conversions in lossy waveguides are investigated below.

2.1. THG process in lossy waveguides

As mentioned above, the field at ω transfers its energy to the field at 3ω when $\sin\theta > 0$, which is the THG process. This nonlinear transfer rate reaches its maximum at $\sin\theta = 1$, where the relative phase is the optimal value for THG process. In this part, we focus our discussion to THG with the relative phase satisfying $\sin\theta > 0$. Besides, the effective growth of THG signal field still needs the strengths of the two interacting fields to satisfy certain requirement.

Equation (2b) describes the change rate of THG signal field with respect to the propagating distance. At any location on the propagating path, the positive change rate requires the nonlinear transfer rate to be bigger than the attenuation rate, which is mathematically expressed as:

$$\gamma_0 J_3 \rho_1^3 \sin\theta > \alpha_3 \rho_3. \tag{3}$$

With the optimal relative phase value, equation (3) gives the intensity relationship between the two interacting fields, which supports the effective signal growth, as below:

$$\rho_3 < \frac{\gamma_0 J_3}{\alpha_3} \rho_1^3 = \rho_{3\max}. \tag{4}$$

This expression indicates that the efficient THG conversion needs the signal field to be lower than a maximum amplitude, which is directly proportional to the material nonlinearity γ_0, modal parameter J_3, and the third power of pump field amplitude, and is inversely proportional to the attenuation coefficient α_3. In lossless situation, the strength of signal field has no influence on the nonlinear energy transfer of THG process. This is the first influence of loss effect on THG conversion.

If the signal field just results from the THG conversion, rather than from the initial field, there is a maximum limit for the corresponding conversion efficiency with the definition of $\eta = P_{3\max}/P_1$, where $P_m = \rho_m^2$ are the power of the pump ($m = 1$) and signal ($m = 3$) fields, respectively,

$$\eta_{\max} = \left(\frac{\gamma_0 J_3}{\alpha_3} \right)^2 P_1^2. \tag{5}$$

As shown by Eq. (5), the conversion efficiency of the THG process is only limited by the square of the pump power in any lossy waveguides. Instead of facilitating the energy conversion of the THG process, the existence of third harmonic signal field decreases the percentage of the pump field that can be converted into the signal field. Accordingly, it is beneficial to enhance the THG conversion by using high nonlinearity material with small loss coefficient, choosing the interacting modes with bigger overlap integral and inputting the pump power as high as possible.

Even though the interacting fields satisfy the amplitude condition of Eq. (4), the positive growth of THG signal field still needs the relative phase to be within a certain range. At the beginning of this part, it has been mentioned that the relative phase for THG conversion ranges from 0 to π. However, due to the loss, the effective phase range for signal positive growth rate narrows symmetrically with respect to $\theta = \pi/2$.

According to Eq. (3), the effective phase range can be analytically derived as:

$$\Delta\theta_{\mathrm{THG}} = \pi - 2\arcsin\left(\rho_3/\rho_{3\max}\right). \tag{6}$$

Based on Eq. (6), the dependence of effective range of relative phase on the normalized signal amplitude is analytically calculated, which is shown as the curve in Fig. 1.

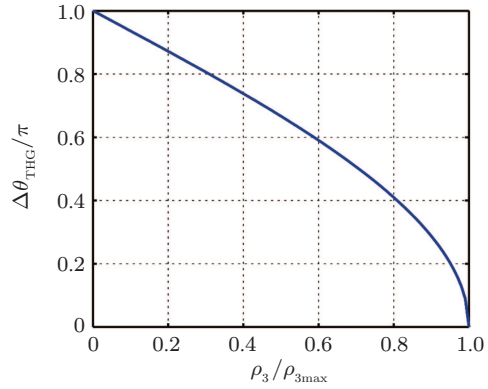


Fig. 1. Dependence of effective relative phase range of the THG process on the normalized signal amplitude.

For any given signal maximum amplitude, the effective phase range decreases with the increase of the signal amplitude. On the other hand, the effective phase range increases with the increase of signal maximum amplitude that is proportional to the third power of pump field amplitude, for any given signal amplitude. Thus, the effective phase range can be controlled by adjusting the intensities of the interacting fields.

2.2. OTHG process in lossy waveguides

The OTHG signal field evolution along the propagation path is described by Eq. (2a), in which the dependence of nonlinear transfer term on the relative phase is the contrary of the situation of THG process. Namely, the field at 3ω transfers its energy into the signal field at ω when $\sin\theta < 0$. This nonlinear transfer rate reaches its maximum at $\sin\theta = -1$, where the relative phase is optimal for OTHG process. To figure out the power dependence of OTHG process, we limit our discussion to the situation that the relative phase satisfies the condition of $\sin\theta < 0$.

At any location on the propagation path, the effective increase of the OTHG signal field requires the nonlinear transfer term to be larger than the loss term, which can be expressed as:

$$-\gamma_0 J_3 \rho_1^2 \rho_3 \sin\theta > \alpha_1 \rho_1. \tag{7}$$

With the optimal relative phase of $\sin\theta = -1$, the above relationship gives the lowest intensities of the two interacting fields to support the effective signal growth, as:

$$\rho_1\rho_3 > \frac{\alpha_1}{\gamma_0 J_3}. \tag{8}$$

This expression indicates that the efficient OTHG conversion requires the amplitude product of the pump and signal fields to be larger than a threshold value, which is directly proportional to the loss coefficient, and inversely proportional to the material nonlinearity and the modal parameter. For simplicity, we define the amplitude product as the parameter ζ by following:

$$\zeta = \rho_1\rho_3. \tag{9}$$

Then the threshold value can be defined as:

$$\zeta_{\min} = \frac{\alpha_1}{\gamma_0 J_3}. \tag{10}$$

Equation (9) shows the first limit set by loss effect to the OTHG conversion. For lossless situation, any nonzero signal and pump fields are possible to support the OTHG signal growth and they do not restrict each other like the situation shown in THG process. Therefore, the loss of waveguides sets an intensity threshold for the interacting fields to achieve the efficient OTHG. Furthermore, since the threshold is the product of the amplitudes of the interacting fields, a large pump power means the required OTHG signal filed can be weaker, and *vice versa*.

The second limit caused by loss effect is the shrinkage of the effective relative phase range. In lossless situation, the relative phase for the OTHG conversion ranges from $-\pi$ to 0 for any amplitude product. However, with the fields satisfying the threshold condition of Eq. (8), the effective relative phase range narrows symmetrically with respect to $\theta = -\pi/2$, which is deduced as:

$$\Delta\theta_{\mathrm{OTHG}} = \pi + 2\arcsin\left(-\zeta_{\min}/\zeta\right). \tag{11}$$

Similarly, the dependence of the effective relative phase range on the normalized amplitude product can be analytically calculated and shown by the curve in Fig. 2.

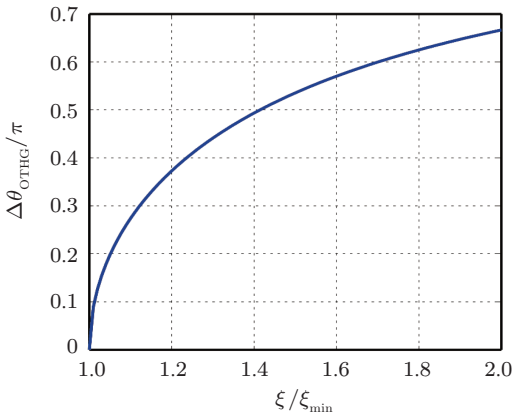


Fig. 2. Dependence of effective relative phase range for the OTHG process on the normalized amplitude product.

The effective relative phase range increases with the increase of the amplitude product larger than threshold, which can be realized by increasing either the pump field or the signal field, or both the two fields. It is different that the increase of the signal amplitude narrows rather than extends the effective range of the relative phase required by efficient THG conversion. The reason of this difference is that the THG is a spontaneous process, while the OTHG is a stimulated process.

2.3. Modified coherent length for THG and OTHG in lossy waveguides

The above analysis figures out the requirements of amplitude and relative phase to support the signal positive growth rate of the THG and OTHG processes, respectively. However, it is just one part of what makes for high conversion efficiency. The other part to acquire the efficient gain is a long effective propagation length in which the positive growth rate is maintained, as the final signal increases are the path integral of the amplitude growth rate governed by Eqs. (2a) and (2b).

Commonly, the effective propagation lengths are mainly restricted by the relative phase variation. It is easy to make the signal field amplitude far below the maximum value for THG process or to make the amplitude product larger than the threshold for OTHG process, while to keep the relative phase within the respective effective ranges are difficult because that the perfect phase matching is critical to be realized. The perfect phase matching can make the relative phase between the two interacting fields as a constant along the propagation path. But this situation occurs only when the modal dispersion fully compensates the nonlinear dispersion caused by optical Kerr effects, as shown by Eq. (2c). A more general situation is that the constant modal dispersion in uniform waveguides cannot totally compensates the field dependent nonlinear dispersion along the propagation path, as described by Eq. (2d).

A common parameter to quantitatively describe the phase mismatch is the coherent length, over which the relative phase is kept within the range for THG or OTHG process. In lossless situation, the definition of coherent length is the ratio of the relative phase range π to the modal phase mismatch $\delta\beta$. Accordingly, the coherent length is independent on the strengths of the interacting fields and only in reverse proportional to the modal phase mismatch. However, in consideration of loss effect, the effective relative phase ranges of the THG and OTHG processes depend on the intensities of the interacting fields and become smaller than π. Meanwhile, the variation of relative phase is also intensity dependent due to the nonlinear dispersion that is proportional to the strengths of the interacting fields.

Consequently, a modified definition of coherent length is proposed as:

$$L_{\mathrm{coh}} = \frac{\Delta\theta}{|\delta\beta + K|}, \tag{12}$$

in which the compressed relative phase range and the varied nonlinear phase mismatch are included. Accordingly, the nonzero coherent lengths for THG and OTHG processes firstly require the effective relative phase ranges that are fields dependent according to Eqs. (6) and (11), to be larger than zero. Then a longer coherent length can be obtained from a smaller total phase mismatch. Once the interacting fields dissatisfy the field amplitude relationship of Eq. (4) or Eq. (8), the effective ranges of relative phase and the consequent coherent lengths turn to zero, no matter how small the total phase mismatch is.

As expounded above, the perfect phase matching is hard to be maintained along the propagation path. Therefore, for a given total phase mismatch, the coherent length can be extended by enlarging the effective relative phase ranges. For the THG process, the effective relative phase range can be enlarged by reducing the normalized signal amplitude, as shown by Fig. 1. The effective relative phase range of the OTHG process, on the other hand, can be enlarged by increasing the normalized amplitude product, as shown by Fig. 2.

3. Conclusion

In conclusion, we have analyzed the optical field conditions supporting the THG and OTHG signal growth in lossy waveguides. The attenuation sets an upper limit for the THG signal field with the given pump field, and requires the amplitude product of interacting fields of the OTHG process to be larger than a threshold value. With the loss effects considered, the effective relative phase ranges for both THG and OTHG process are related to the strengths of the interacting fields. Based on our analysis, we proposed a modified definition of coherent length and pointed out that the phase matching condition also depends on the strengths of the interacting fields. Even with the severe phase mismatch, the signal increase for THG and OTHG can still be improved by enlarging the effective relative phase ranges. The work presented in this paper can offer a cogent theoretical explanation for the experimental research of THG and OTHG.

References

[1] Tong L M, Lou J Y and Mazur E 2004 *Opt. Express* **12** 1025
[2] Shahraam A V and Monro T M 2009 *Opt. Express* **17** 2298
[3] Wu X Q and Tong L 2013 *Nanophoton.* **2** 407
[4] Birks T A, Wadsworth W J and Russell P S 2000 *Opt. Lett.* **19** 1415
[5] Zhou G R, Feng G Y, Zhou H, *et al.* 2011 *Opt. Commun.* **284** 4769
[6] Vienne G, Li Y H, Tong L M, *et al.* 2008 *Opt. Lett.* **33** 1500
[7] Vienne G, Grelu P, Pan X Y, *et al.* 2008 *J. Opt. A: Pure Appl. Opt.* **10** 025303
[8] You H, Hendrickson S M and Franson J D 2008 *Phys. Rev. A* **78** 053803
[9] Hendrickson S M, Lai M M, Pittman T B, *et al.* 2010 *Phys. Rev. Lett.* **105** 173602
[10] Grubsky V and Savchenko A 2005 *Opt. Express* **13** 6798
[11] Grubsky V and Feinberg J 2007 *Opt. Commun.* **274** 447
[12] Coillet A and Grelu P 2012 *Opt. Commun.* **285** 3493
[13] Afshar V S, Lohe M A, Lee T, Monro T M and Broderick N G R 2013 *Opt. Lett.* **38** 329
[14] Huang T Y, Shao X G, Wu Z F, Lee T, Sun Y X, Lam H Q, Zhang J, Brambilla G and Ping S 2013 *Opt. Express* **21** 28403
[15] Sun Y X, Shao X G, Huang T Y, Wu Z F, Lee T, Perry S P and Brambilla G 2014 *J. Opt. Soc. Am. B* **31** 2142
[16] Hartley D, Lohe M A, Monro T M and Afshar V S 2015 *J. Opt. Soc. Am. B* **32** 379
[17] Zhang J Y, Sun Y X and Song Q H 2015 *Opt. Express* **23** 17407
[18] Schmid R P, Schneider T and Reif J 2002 *Opt. Commun.* **207** 155
[19] Endert H, Scaggs M, Basting D and Stamm U 1999 *J. Laser Appl.* **11** 1
[20] Khudus M I M A, Lee T, Huang T Y, Shao X G, Shum P and Brambilla G 2015 *Fiber Integr. Opt.* **34** 53
[21] Ismaeel R, Lee T, Ding M, Broderick N G R and Brambilla G 2012 *Opt. Lett.* **37** 5121
[22] Lee T, Broderick N G R and Brambilla G 2013 *J. Opt. Soc. Am. B* **30** 505