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Negativity of Wigner function and phase sensitivity of an SU(1,1) interferometer*

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Both the negativity of Wigner function and the phase sensitivity of an SU(1,1) interferometer are investigated in this paper. In the case that the even coherent state and squeezed vacuum state are input into the interferometer, the Heisenberg limit can be approached with parity detection. At the same time, the negativity volume of Wigner function of detection mode comes entirely from the input state and varies periodically with the encoding phase. In addition, the negativity volume of Wigner function is positively correlated with the phase sensitivity of the SU(1,1) interferometer. The positive correlation may mean that the non-classicality indicated by negative Wigner function is a kind of resource that can verify some related research results of phase estimation.

Keywords: SU(1,1) interferometer, Wigner function, phase sensitivity

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1. Introduction

Quantum metrology is a field of exploiting quantum properties to achieve a higher precision in the measurement of a physical parameter.[1,2] Here, quantum state, measurement device, and method of detection contribute to improving the precision. As a typical high-precision measurement device based on optics, the SU(1,1) interferometer (SU(1,1)I)[3–6] consists of two optical parametric amplifiers,[7] two mirrors and a phase shifter, which is similar with Mach–Zehnder interferometer (MZI) but provides a gain factor $[2\sinh^2(g)(2\sinh^2(g)+2)]^{-1/2}$ over the MZI.[6,8] In addition, researchers have described the structural changes[9–11] and the loss effects of the interferometer,[5,12–14] which has led to the gradual perfection of the phase estimation based on an optical system.

The non-classicality of quantum states, such as entanglement and squeezing, is helpful to improve the accuracy of phase estimation in the SU(1,1)I. For example, in Ref. [15] the thermal state and the squeezed vacuum state as inputs of an SU(1,1)I can approach the Heisenberg limit. There are other combinations of input states beating the standard quantum limit and approaching the Heisenberg Limit, such as coherent state and squeezed vacuum state,[16,17] coherent state and displacement squeezed vacuum state,[8,18] and so on. The accuracy of phase estimation utilizing the SU(1,1)I has also been realized experimentally.[19–23] For instance, an interferometer with the parametric amplifier has proved that the improvement of phase sensitivity[19] is due to the enhanced input intensity. Recently, the absolute sensitivity of SU(1,1)-type interferometer has been investigated and obtained 3-dB improvement[23] over the MZI. The detection method is another factor contributing to achieving better phase sensitivity. Different input states always go with their own optimal measurement methods.[16,24,25] In Ref. [16], for the input combinations, coherent state and squeezed vacuum state, and coherent state and vacuum state, the performance of parity detection is better than intensity detection and homodyne detection, while for the input combination of two coherent states, the parity detection is worse.

The Wigner function is another useful tool to describe a quantum system. In Ref. [26] a new method of coherent mode decomposition is proposed to determine the mode functions and the weights without solving the eigenvalue problem of a large matrix. The phase sensitivity with parity detection is also analyzed by Wigner function in Refs. [8], [17], [27], and [28]. The non-classicality of quantum system can be charactered by the negative Wigner function.[29–31] In addition, quantum interference pattern of Wigner function is used to measure entanglement between two modes of vortex states.[32] Hence, we raise a bold question: could the negativity of Wigner function of quantum state provide certain benefits for the phase estimation of the SU(1,1)I?

In this paper, we propose a combination of even coherent state and squeezed vacuum state as inputs of the SU(1,1)I, which can approach the Heisenberg limit under parity detec-

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tion when the gain factors of two optical amplifiers are equal. Compared with other combinations of input states, the phase sensitivity is better when even coherent state and squeezed vacuum state are used as input states. In addition, the negativity volume of Wigner function is completely from the input state, which is due to the intrinsic characteristics of the SU(1,1)I. At the same time, the behavior of the phase sensitivity and the negativity volume of Wigner function with the encoding phase is observed in contrast. Its variation with the encoding phase is positively correlated. This relationship may mean the non-classicality indicated by negative Wigner function is a kind of resource that can verify the research results of the phase estimation.

This paper is organized as follows. In Section 2, an SU(1,1)I is proposed to take even coherent state and squeezed vacuum state as inputs, and its phase sensitivity is analyzed by the Wigner function of the output field. The variation between the negativity volume of Winger function and the phase sensitivity is compared in Section 3. Finally, a summary is provided in Section 4.

2. Phase sensitivity of an SU(1,1)I taking even coherent state and squeezed vacuum sate as inputs

2.1. Winger function of output field

We start with study on the SU(1,1)I , as shown in Fig. 1. After the first OPA, only the path a passes through a phase shift ϕ. In the SU(1,1)I, the relationship between input and output variables of the whole interferometer can be described as

$$\begin{pmatrix} \alpha_{\rm i} \\ \beta_{\rm i}^* \end{pmatrix} = M^{-1} \begin{pmatrix} \alpha_{\rm f} \\ \beta_{\rm f}^* \end{pmatrix}. \tag{1}$$

The transformation matrix of the SU(1,1)I is

$$M = M_{\rm OPA2} M_{\phi} M_{\rm OPA1}, \tag{2}$$

where $M_{{\rm OPA}j} = \begin{pmatrix} u_j & v_j \\ v_j^* & u_j \end{pmatrix}$ is the transformation of OPAj, $u_j = \cosh(g_j)$, $v_j = {\rm e}^{{\rm i}\theta_j}\sinh(g_j)$, g_j and θ_j are used to denote the strength and the phase shift in the OPAs ($j = 1$, 2). $M_{\phi} = \begin{pmatrix} {\rm e}^{{\rm i}\phi} & 0 \\ 0 & 1 \end{pmatrix}$ represents the phase-shift transformation. Here, the balanced configuration corresponds to $g_1 = g_2 = g$ and $\theta_2 - \theta_1 = \pi$. The optimal phase sensitivity can be obtained under the balance of the SU(1,1)I, so we focus on the balanced situation.

Now, let us consider even coherent state and squeezed vacuum state, $|\psi_e\rangle \otimes |\psi_\xi\rangle$, as inputs in the SU(1,1)I. The corresponding Winger function can be written as[33]

$$\begin{aligned} W_{|\psi_{\rm e}\rangle} &= \frac{2N^2}{\pi}\{{\rm e}^{-2|\alpha_0-\alpha_{\rm i}|^2} + {\rm e}^{-2|\alpha_0+\alpha_{\rm i}|^2} \\ &\quad + ({\rm e}^{-2|\alpha_0|^2}\,{\rm e}^{2(\alpha_0^*-\alpha_{\rm i}^*)(\alpha_0+\alpha_{\rm i})} + {\rm c.c})\}, \end{aligned} \tag{3}$$

$$W_{|\psi_\xi\rangle} = \frac{2}{\pi}{\rm e}^{-2|\beta_{\rm i}|^2\cosh(2r)+(\beta_{\rm i}^2+\beta_{\rm i}^{*2})\sinh(2r)}, \tag{4}$$

respectively, where $|\psi_{\rm e}\rangle = N(|\alpha_0\rangle + |-\alpha_0\rangle)$ with $N = [2(1 + {\rm e}^{-2|\alpha_0|^2})]^{1/2}$, $\alpha_0 = \sqrt{n_{\rm c}}\,{\rm e}^{-{\rm i}\varphi_1}$. When even coherent state and squeezed vacuum state pass through the SU(1,1)I, the Winger function of output field is

$$W_{\rm f}(\alpha_{\rm f}, \beta_{\rm f}) = W_{\rm f}[\alpha_{\rm f}(\alpha_{\rm i}, \beta_{\rm i}), \beta_{\rm f}(\alpha_{\rm i}, \beta_{\rm i})]. \tag{5}$$

Combined with Eqs. (1)–(5), the Winger function of output field of SU(1,1)I is given by

$$\begin{aligned} &W_{\rm f}(\alpha_{\rm f}, \beta_{\rm f}) \\ &= \frac{4N^2}{\pi^2}{\rm e}^{-2S_1^2\cosh(2r)+(S_1^2+S_1^{*2})\sinh(2r)}\{{\rm e}^{-2|\alpha_0-S_2|^2} \\ &\quad + {\rm e}^{-2|\alpha_0+S_2|^2} + \left({\rm e}^{-2|\alpha_0|^2}\,{\rm e}^{2\left(\alpha_0^*-S_2^*\right)(\alpha_0+S_2)} + {\rm c.c}\right) \end{aligned} \tag{6}$$

with

$$\begin{aligned} S_1 &= [{\rm e}^{{\rm i}\theta_1}(1-{\rm e}^{{\rm i}\phi})\cosh(g)\sinh(g)]\alpha_{\rm f}^* \\ &\quad + [\cosh^2(g) - {\rm e}^{{\rm i}\phi}\sinh^2(g)]\beta_{\rm f}, \end{aligned} \tag{7}$$

$$\begin{aligned} S_2 &= [{\rm e}^{-{\rm i}\phi}\cosh^2(g) - \sinh^2(g)]\alpha_{\rm f} \\ &\quad - [{\rm e}^{{\rm i}\theta_1}(1-{\rm e}^{-{\rm i}\phi})\cosh(g)\sinh(g)]\beta_{\rm f}^*. \end{aligned} \tag{8}$$

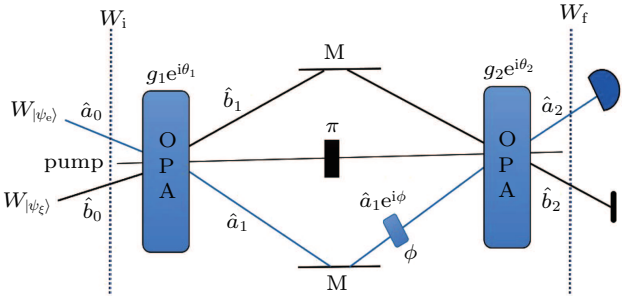


Fig. 1. Schematic diagram of an SU(1,1)I. $W_{\rm i}$ and $W_{\rm f}$ correspond to Wigner functions of input and output fields. $g_{1(2)}$ and $\theta_{1(2)}$ describe the strength and the phase shift in the OPA process 1(2), respectively.

2.2. Phase sensitivity with parity detection

The important character of the accuracy of phase estimation in optical interferometer is phase sensitivity $\Delta\phi$, which can be characterized using the error propagation formula

$$\Delta\phi^2 = \frac{\langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2}{\left(\partial_\phi\langle\hat{O}\rangle\right)^2}. \tag{9}$$

The operator $\hat{O}$ is observable for detection. Now, we adopt parity detection, where the parity operator on the output mode a can be defined as $\hat{\Pi}_a = (-1)^{\hat{a}_2^+\hat{a}_2}$. Owing to the integration property of the characteristic function for the parity operator, the average value $\langle\hat{\Pi}_a\rangle$ can be written as[8]

$$\langle\hat{\Pi}_a\rangle = \frac{\pi}{2}\int W_{\rm f}(0,\beta_{\rm f})\,{\rm d}\beta_{\rm f}\,{\rm d}\beta_{\rm f}^*. \tag{10}$$

By substituting Eq. (6) into Eq. (10), we can get

$$\langle\hat{\Pi}_a\rangle = \frac{4N^2({\rm e}^{J_1} + {\rm e}^{J_2})}{\sqrt{p_5}}, \tag{11}$$

with

$$J_1 = -2|\alpha_0|^2 + p_1(p_3 - p_4)/p_2, \tag{12}$$

$$J_2 = p_1(p_3 + p_4)/p_2, \tag{13}$$

$$p_1 = \frac{1}{4}e^{-2i(\theta_1+\varphi_1)}(-1+e^{i\phi})^2|\alpha_0|^2\sinh^2(2g), \tag{14}$$

$$\begin{aligned} p_2 = {} & \frac{1}{64}[(-1+e^{i\phi})^2 - 2(-1+e^{i\phi})^2\cosh(4g)\cosh^2(r) \\ & + (1+6e^{i\phi}+e^{2i\phi})\cosh(2r)]^2 \\ & + [e^{i\phi}\cosh^2(g) - \sinh^2(g)]^2 \\ & \times [\cosh^2(g) - e^{i\phi}\sinh^2(g)]^2\sinh^2(2r), \end{aligned} \tag{15}$$

$$\begin{aligned} p_3 = {} & [e^{2i\phi} + e^{4i(\theta_1+\varphi_1)}]\cosh^4(g)\sinh(2r) \\ & + \frac{1}{2}\{2[1+e^{2i(2\theta_1+\phi+2\varphi_1)}]\sinh^4(g) \\ & - e^{i\phi}[1+e^{4i(\theta_1+\varphi_1)}]\sinh^2(2g)\}\sinh(2r), \end{aligned} \tag{16}$$

$$\begin{aligned} p_4 = {} & \frac{1}{2}e^{2i(\theta_1+\varphi_1)}\{-2(1+e^{2i\phi})\cosh^2(r)\sinh^2(2g) \\ & + 2e^{i\phi}[2\cosh^4(g)\cosh(2r) \\ & + 2\cosh(2r)\sinh^4(g) + \sinh^2(2g)]\}, \end{aligned} \tag{17}$$

$$\begin{aligned} p_5 = {} & 4e^{-2i\phi}\left\{\frac{1}{64}[(-1+e^{i\phi})^2 \right. \\ & - 2(-1+e^{i\phi})^2\cosh(4g)\cosh^2(r) \\ & + (1+6e^{i\phi}+e^{2i\phi})\cosh(2r)]^2 \\ & + [e^{i\phi}\cosh^2(g) - \sinh^2(g)]^2 \\ & \left. \times [\cosh^2(g) - e^{i\phi}\sinh^2(g)]^2\sinh^2(2r)]\right\}. \end{aligned} \tag{18}$$

When we take the average value $\langle \hat{\Pi}_a \rangle$ into Eq. (9) and simplify the expression through a bunch of tedious simplifications, the phase sensitivity is found to be optimal point at $\phi = 0$ and is given by

$$\Delta\phi = \left(\frac{m_1\,\mathrm{csch}^2(2g)}{m_2\cosh(2r) + m_3\sinh(2r)}\right)^{1/2}, \tag{19}$$

with

$$m_1 = 2e^{2i(\theta_1+\varphi_1)}(1+e^{2|\alpha_0|^2}), \tag{20}$$

$$m_2 = 2e^{2i(\theta_1+\varphi_1)}\left[1 - 2|\alpha_0|^2 + e^{2|\alpha_0|^2}(1+2|\alpha_0|^2)\right], \tag{21}$$

$$m_3 = (1+e^{2|\alpha_0|^2})\left[e^{2i(\theta_1+\varphi_1)} + (1+e^{4i(\theta_1+\varphi_1)})\right]|\alpha_0|^2. \tag{22}$$

From Eq. (19), the phase sensitivity is related to not only the intensity $|\alpha_0|^2$ and the phase angle φ_1 of input state but also the phase shift θ_1 of OPA$_1$. After some simple simulation calculations, the phase sensitivity is optimized with phase angle $\theta_1 = 0$, $\varphi_1 = 0$.

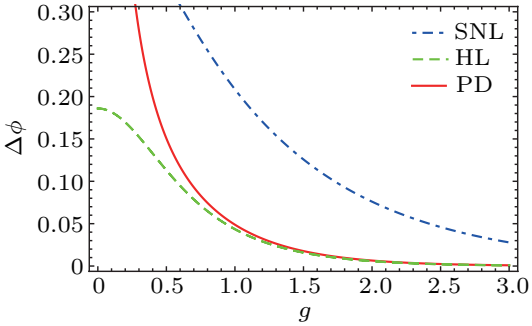


Fig. 2. Comparison of the phase sensitivity with the Heisenberg limit (HL) and the shot noise limit (SNL). Parameters: $|\alpha_0| = 2$, $r = 1$.

Then, in Fig. 2, the optimal phase sensitivity is compared with the Heisenberg limit (HL) and the shot noise limit (SNL), which are related to the total photon number and displayed as

$$\Delta\phi_{\mathrm{HL}} = \frac{1}{N_{\mathrm{Tot}}} = \frac{1}{(1+2\sinh^2(g))[n_1+\sinh^2(r)] + 2\sinh^2(g)}, \tag{23}$$

$$\Delta\phi_{\mathrm{SNL}} = \frac{1}{\sqrt{N_{\mathrm{Tot}}}} = \frac{1}{\sqrt{(1+2\sinh^2(g))[n_1+\sinh^2(r)] + 2\sinh^2(g)}}, \tag{24}$$

where $n_1 = |\alpha_0|^2(1-e^{-2|\alpha_0|^2})/(1+e^{-2|\alpha_0|^2})$ is the average photon number in the mode a of the input states. The phase sensitivity becomes better gradually and remains below the shot noise limit with the gain coefficient g increasing. When $g > 1.9$ with $|\alpha_0| = 2$, $r = 1$, the optimal phase sensitivity goes close to the Heisenberg limit.

To analyze the effects of different combinations of input states on the accuracy of phase estimation, the phase sensitivities with parity detection are compared among different combinations of input states, as shown in Table 1. According to Table 1, the phase sensitivities with parity detection are expressed as the presence of a constant N_{c}, which is caused by the amplification process of OPAs. It is possible from the comparison that the phase sensitivity of the input combination of squeezed vacuum state and even coherent state is better than those of other combinations. To clarify this point, the phase sensitivities of the input combinations including squeezed vacuum states is chosen to be plotted with gain coefficient g in Fig. 3. It is true that the phase sensitivity of $|\psi_{\mathrm{e}}\rangle \otimes |\psi_\xi\rangle$ as inputs is better than that for other input combinations, even the combination of coherent state and squeezed vacuum state $|\psi_\alpha\rangle \otimes |\psi_\xi\rangle$. And the phase sensitivity of the input combination $|0\rangle \otimes |\psi_\xi\rangle$ is the worst. In brief, even coherent state can replace coherent state when $g \leq 1.4$ as input of the SU(1,1)I to improve the accuracy of phase estimation when the gain coefficient g is not large enough.

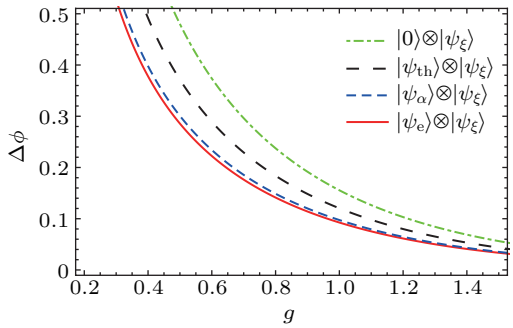


Fig. 3. The sensitivity of phase estimation with different combinations including squeezed vacuum states of input states as a function of gain coefficient g. Total photon number $N = 2.14$.

Table 1. Different combinations of input states and their phase sensitivities with parity detection.

Input states	Phase sensitivities
$\|0\rangle\otimes\|0\rangle$	$[N_{\rm c}]^{-1/2}$
$\|0\rangle\otimes\|\psi_\xi\rangle$	$[N_{\rm c}(1+n_\xi)]^{-1/2}$
$\|0\rangle\otimes\|\psi_{\rm e}\rangle$	$[N_{\rm c}(1+n_{\rm e})]^{-1/2}$
$\|\psi_\alpha\rangle\otimes\|\psi_\xi\rangle$	$[N_{\rm c}(1+n_\xi+n_\alpha {\rm e}^{2r})]^{-1/2}$ Ref. [16]
$\|\psi_{th}\rangle\otimes\|\psi_\xi\rangle$	$[N_{\rm c}(1+n_\xi+n_{th}+2n_\xi n_{th})]^{-1/2}$ Ref. [15]
$\|\psi_{\rm e}\rangle\otimes\|\psi_\xi\rangle$	$\{N_{\rm c}\{[\frac{1}{2}+n'_{\rm e}\sinh(2r)]+[\frac{1}{2}+n'_{\rm e}+\frac{-2n'_{\rm e}}{1+{\rm e}^{2n'_{\rm e}}}]\cosh(2r)\}\}^{-1/2}$

Note that (i): $|\psi_\xi\rangle$ squeezed vacuum state; $|\psi_\alpha\rangle$ coherent state; $|\psi_{\rm th}\rangle$ thermal state; $|\psi_{\rm e}\rangle$ even coherent state. (ii) constant parameter $N_{\rm c} = 2\sinh^2(g)[2\sinh^2(g)+2]$, $n'_{\rm e} = |\alpha_0|^2$. (iii) row 6: the parity detection with phase angle $\varphi_1 = 0$.

3. Comparison between the phase sensitivity and negativity volume of the Winger function

An improvement in the accuracy of phase estimation is usually attributed to the non-classicality in quantum states, such as the negative Wigner function. In the following, we will analyze comparatively the phase sensitivity and the negativity volume of output Wigner function. The negativity volume of Wigner function[30] is defined as

$$V = \frac{1}{2}\int (|W_{\rm af}(\alpha_{\rm f},\alpha_{\rm f}^*)| - W_{\rm af}(\alpha_{\rm f},\alpha_{\rm f}^*))\,{\rm d}\alpha_{\rm f}{\rm d}\alpha_{\rm f}^*, \qquad (25)$$

where $W_{\rm af}(\alpha_{\rm f}) = \int W_{\rm f}(\alpha_{\rm f},\beta_{\rm f})\,{\rm d}\beta_{\rm f}{\rm d}\beta_{\rm f}^*$. By combining Eq. (6) and Eq. (25), we plot the negativity volume of Wigner function *versus* the encoding phase in Fig. 4. As can be seen from Fig. 4, the negativity volume of Wigner function maximizes at the point of $\phi = 0$ and varies periodically with the encoding phase. In Fig. 4(a), it is shown that the increasing of the intensity of even coherent state as input can enable the maximum negativity volume of Wigner function at the point of $\phi = 0$ rising. Compared with Fig. 4(b) and Fig. 4(c), the maximum of the negativity volume of Wigner function remains unchanged for different squeezing parameters and gain coefficients when the average photon number of the input states is fixed, which displays that the negativity of the output field completely comes from the input state.

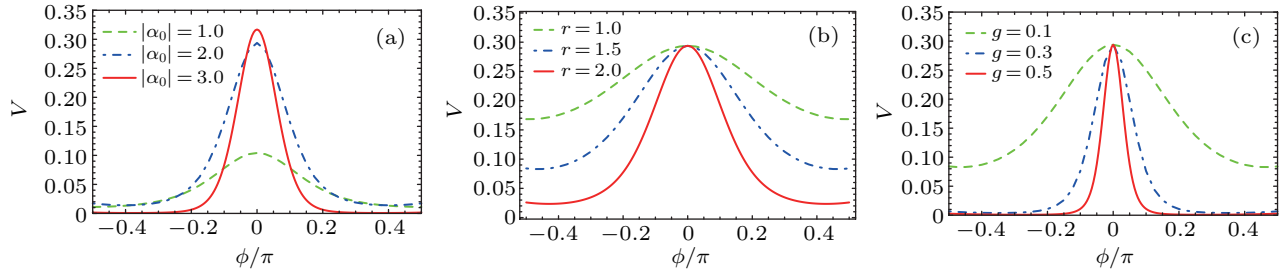


Fig. 4. The negativity volume of Winger function *versus* the encoding phase (a) with different $|\alpha_0|$, parameters: $r = 1.5$, $g = 0.2$; (b) with different squeezing parameters r, parameters: $|\alpha_0| = 2$, $g = 0.2$; (c) with different gain coefficients g, parameters: $|\alpha_0| = 2$, $r = 1.5$.

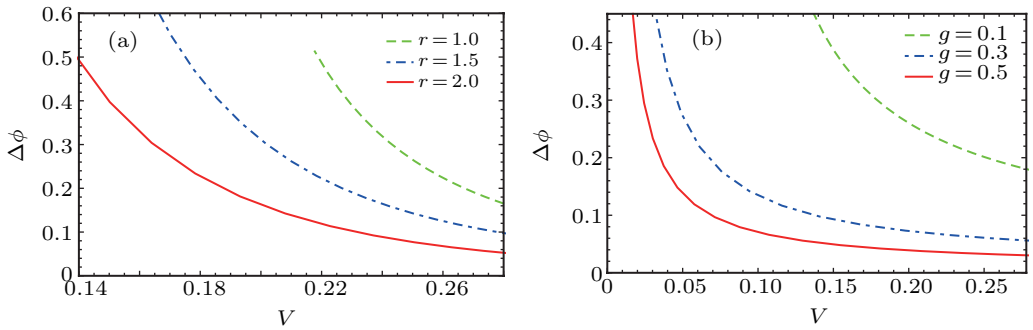


Fig. 5. Sensitivity of phase estimation *versus* the negativity volume of Winger function on the output field of mode a with the encoding phase ranging from 0 to $\pi/4$. (a) parameters: $g = 0.1$, $|\alpha_0| = 2$. (b) parameters: $r = 1.5$, $|\alpha_0| = 2$.

The negativity of the output field varies periodicity with the encoding phase for squeezing parameters and gain coefficients, which is determined by the intrinsic properties of the SU(1,1)I. Interestingly, the researchers have also found that the phase sensitivity varies periodically with the encoding phase. To see more clearly that the relation between the phase sensitivity and the negativity of Wigner function with certain input state, we will now analyze them comparatively.

Due to periodicity and symmetry of the encoding phase, we perform numerical simulation to obtain Fig. 5 in the range of $\phi = 0$ to $\phi = \pi/4$ based on Eq. (11) and Eq. (25). Here, one can see clearly in Fig. 5(a) and Fig. 5(b) that the phase sensitivity is positively correlated with the negativity volume of the Winger function. This positive correlation with different squeezing parameters and gain coefficients indicates that the negativity volume of the Wigner function can be used as a verification for the variation of phase sensitivity with parameters.

4. Conclusions

In this work, we have investigated the phase sensitivity and the negativity of Winger function in the SU(1,1)I with even coherent state and squeezed vacuum state as inputs under parity detection. Compared with other combinations of input states, the phase sensitivity for even coherent state and squeezed vacuum state is optimal, by which the Heisenberg limit can be approached under appropriate parameters. The negativity volume of Winger function in the detection mode comes entirely from the input state and varies periodically with the encoding phase. Through numerical simulation, it can be found that the phase sensitivity is positively correlated with the negativity volume of the Winger function, which provides a supplementary verification of phase estimation.

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