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# Entropy squeezing for three-level atom interacting with a single-mode field*

Fei-Fan Liu(刘非凡), Mao-Fa Fang(方卯发)†, and Xiong Xu(许雄)

*Synergetic Innovation Center for Quantum Effects and Application, Key Laboratory of Low-dimensional Quantum Structures and Quantum Control of Ministry of Education, School of Physics and Electronics, Hunan Normal University, Changsha* 410081, *China*

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The entropy squeezing for a three-level atom interacting with a single-model field is studied. A general definition of entropy squeezing for three-level atom is given according to entropic uncertainty relation of three-level system, and the calculation formalism of entropy is derived for a cascade three-level atom. By using numerical calculation, the entropy squeezing properties of a cascade three-level atom are examined. Our results show that, three-level atom can generate obvious entropy squeezing effect via choosing appropriate superposition state of three-level atom. Our results are meaningful for preparing three-level system information resources with ultra-low quantum noise.

**Keywords:** variance squeezing, entropy squeezing, three-level atom

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## 1. Introduction

The precision measurement of observable quantities in quantum physical experiments is greatly restricted by quantum noises (quantum uncertainty[1]) of quantum systems. Hence, breaking through the restriction of quantum noises and realizing quantum physical experiments with ultra-low quantum noise is a strong expectation for physical scientists. As we know, the suppression of quantum noises in quantum systems via squeezing effects is one of the most outstanding achievements in modern quantum optics over the last few years. Therefore, many squeezing effects have been investigated, such as the atomic squeezing. It should be pointed out that quantum information entropy is a accurate measure of the atomic squeezing, and one of our authors has defined the entropy squeezing (ES) for a two-level atom by using quantum information entropy according to entropic uncertainty relation based on two conjugate observables,[2] which overcomes the limitation of the definition of squeezing via the variance of the system observable according to Heisenberg uncertainty relation. For more than a decade, the entropy squeezing (ES) properties of quantum system have been investigated widely. Such as, Yu and Fang have studied the steady and optimal entropy squeezing of a two-level atom with quantum-jump-based feedback and classical driving in a dissipative cavity and provided a scheme of generating steady and optimal entropy squeezing in the two-level system;[3] Wang has investigated sustained optimal entropy squeezing of a two-level atom via non-Hermitian operation and provided a scheme for generating the sustained optimal entropy squeezing of the atom via non-Hermitian operation;[4] Xiao has studied the entropy squeezing of a two-level system by detuning in non-Markovian environments and shown that the atomic entropic squeezing can be protected for a long time when both the non-Markovian effect and detuning are present simultaneously;[5] the entropy squeezing of a two-level atom in Jaynes–Commings model has been reported widely;[6] Zou has studied the quantum entropic uncertainty relation and entanglement witness in the two-atom system coupling with the non-Markovian environments.[7,8] However, these results are obtained only in the case of two-level atom, the entropy squeezing properties of three-level atom have not yet been involved until now.

On the other hand, as we know, three-level system has more obvious advantages than two-level system for the representation of quantum information,[9,10] the key distribution scheme of a three-level entanglement system is more effective than the two-level system for preventing eavesdropping in cryptography,[11] and three-level quantum systems have more obvious quantum nonlocality than the two-level system.[12] Therefore, three-level system with ultra-low quantum noise is a more useful information resource, which means that the study of entropic squeezing of three-level system becomes very meaningful in quantum information processing. In this paper, we study the entropy squeezing of three-level atom interacting with a single-mode field. The outline of this paper is as follows. In Section 2, we give a general definition of entropy squeezing for three-level atom by using quantum information entropy according to entropic uncertainty relation of three-level atom based on two conjugate observables. In section 3, the reduced density matrix is obtained, and the calculation formalism of entropy is derived for a cascade three-

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†Corresponding author. E-mail: mffang@hunnu.edu.cn

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level atom. We analyze the properties of entropy squeezing of a cascade three-level atom by using numerical calculation in section 4, while in section 5 a brief summary is presented.

## 2. The general definition of entropy squeezing for a three-level atom

Since the entropy squeezing of three-level atom has not been discussed before, we first give a general definition of the entropy squeezing for a three-level atom. A three-level atom can be described by dipole operators[13,14]

$$\begin{aligned} S_x &= \frac{1}{2}(D_+ + D_-), \\ S_y &= \frac{1}{2\mathrm{i}}(D_+ - D_-), \\ S_z &= \frac{1}{2}(D_+D_- - D_-D_+) \end{aligned} \tag{1}$$

with

$$\begin{aligned} D_+ &= \alpha_{21}|2\rangle\langle 1| + \alpha_{32}|3\rangle\langle 2| + \alpha_{31}|3\rangle\langle 1|, \\ D_- &= (D_+)^\dagger, \end{aligned} \tag{2}$$

where $(D_+)^\dagger$ is the conjugate transpose of $D_+$; $\alpha_{21}$, $\alpha_{32}$, and $\alpha_{31}$ are dimensionless constants. The spin operators can satisfy the commutation $[\hat{S}_x, \hat{S}_y] = \mathrm{i}\hat{S}_z$ and the uncertainty relation is given by

$$\triangle S_x \triangle S_y \geq \frac{1}{2}|\langle S_z\rangle|, \tag{3}$$

where the variance $\triangle S_\alpha = \sqrt{\langle S_\alpha^2\rangle - \langle S_\alpha\rangle^2}$, and $\langle S_z\rangle$ is the expectation of the observable $S_z$.

For three complementary observables $S_\alpha$, $(\alpha = x, y, z)$ of a three-level atom, the corresponding information entropies can be defined as[15]

$$H(S_\alpha) = -\sum_{i=1}^{3} P_i(S_\alpha)\ln P_i(S_\alpha), \quad (\alpha \equiv x, y, z), \tag{4}$$

where $P_i(S_\alpha) = \langle\psi_{\alpha i}|\rho|\psi_{\alpha i}\rangle$, $i = 1, 2, 3$, and $|\psi_{\alpha i}\rangle$ is an eigenvector of the operator $S_\alpha$. Alberto *et al.*[16] derived entropic uncertainty relations for three observables systems of dimensions $d = 3$

$$H(S_x) + H(S_y) + H(S_z) \geq 2. \tag{5}$$

This inequality can be also written as

$$\delta H(S_x)\delta H(S_y) \geq \frac{e^2}{\delta H(S_z)}, \tag{6}$$

where

$$\delta H(S_\alpha) \equiv \exp[H(S_\alpha)], \quad (\alpha \equiv x, y, z). \tag{7}$$

We define the squeezing of the atom by Eq. (6). The fluctuations in the component $S(\alpha)$ are said to be squeezed in entropy if the information entropy $H(S_\alpha)$ satisfies the condition

$$E(S_\alpha) = \delta H(S_\alpha) - \frac{e}{\sqrt{\delta H(S_z)}} < 0, \quad (\alpha \equiv x \text{ or } y). \tag{8}$$

Equation (8) is the central result of this paper. From Eq. (8) we find that the optimal value the entropy squeezing factor $E(S_\alpha)$ is $1 - e/\sqrt{3} \approx -0.569$.

For the convenience of the following discussion, we also give the definition of variance squeezing based on the Heisenberg uncertainty relation. When the variance in the component $S_\alpha$ satisfies the condition

$$\Delta S_\alpha < \left(\frac{1}{2}|\langle S_z\rangle|\right)^{1/2}, \quad (\alpha \equiv x \text{ or } y), \tag{9}$$

or

$$V(S_\alpha) = \Delta S_\alpha - \left(\frac{|\langle S_z\rangle|}{2}\right)^{1/2} < 0, \quad (\alpha \equiv x \text{ or } y). \tag{10}$$

Fluctuations in the component $S_\alpha$ are said to be squeezed.

We are interested in the case that equation (10) is trivial when $\langle S_z\rangle = 0$ (as $\Delta S_\alpha \geq 0$) and fails to provide any useful information while the equation (8) is non-trivial and can provide a useful information.

## 3. The reduced density matrix and entropic calculation formalism of a cascade three-level atom

In this section, we consider a cascade three-level atom interacting with a single-mode field. The transition frequencies between $|1\rangle$ and $|2\rangle$, $|3\rangle$, and $|2\rangle$ are respectively related to a single-mode field with a frequency $\omega$, and the transition between $|1\rangle$ and $|3\rangle$ is electrical dipole forbidden, that is $\alpha_{31} = 0$ in Eq. (2). The Hamiltonian of the interaction between the atom and the field is given by

$$H = H_0 + H_\mathrm{I}, \tag{11}$$

with

$$H_0 = \sum_{\alpha=1}^{3} \omega_\alpha C_\alpha^\dagger C_\alpha + \omega_\alpha a^\dagger a, \tag{12}$$

$$H_I = g_1(aC_3^\dagger C_2 + a^\dagger C_2^\dagger C_3) + g_2(aC_2^\dagger C_1 + a^\dagger C_1^\dagger C_2), \tag{13}$$

where $a$ and $a^\dagger$ are the annihilation and creation operators of the field with the frequency $\omega$, $\omega_\alpha$ is the eigenenergy of level $|\alpha\rangle$ of the atom, $g_j$ $(j = 1, 2)$ is a coupling constant of the atom and single-mode field. $C_\alpha$ and $C_\alpha^\dagger$ $(\alpha = 1, 2, 3)$ correspond to the Fermi annihilation and creation operators of the atomic energy level $\alpha$, respectively, which follows the rules

$$C_\alpha C_\beta^\dagger = 0, \quad C_\alpha C_\alpha^\dagger C_\beta = C_\beta, \quad (\alpha \neq \beta). \tag{14}$$

However, for the sake of simplicity, we consider the resonance condition between the three-level atom and the field, namely,

$\omega_3-\omega_1=2\omega$. In the interaction picture, the Hamiltonian is given by

$$H^{\rm I}=g_1\left(aC_3^{\dagger}C_2\,{\rm e}^{{\rm i}\Delta t}+a^{\dagger}C_2^{\dagger}C_3\,{\rm e}^{-{\rm i}\Delta t}\right)+g_2\left(aC_2^{\dagger}C_1\,{\rm e}^{-{\rm i}\Delta t}+a^{\dagger}C_1^{\dagger}C_2\,{\rm e}^{{\rm i}\Delta t}\right),\tag{15}$$

where $\Delta=\omega-(\omega_2-\omega_1)=(\omega_3-\omega_2)-\omega,(\Delta\ll\omega)$. We suppose that the initially field is the coherent state $|\alpha\rangle=\sum_n F_n|n\rangle$ with $F_n=\exp\left[(-1/2)|\alpha|^2\right]\alpha^n/\sqrt{n!}$, $\alpha=|\alpha|\exp({\rm i}\beta)$, and the initially atom is the coherent superposition state of three levels. If we assume at time $t=0$ the three-level atom and the field are decoupled, the initial state vector of system can be written as

$$|\Psi_{\rm AF}^{\rm I}(0)\rangle=\sum_n(F_n\cos\theta|3,n\rangle+F_n\sin\theta\sin\varphi|2,n\rangle+F_n\sin\theta\cos\varphi|1,n\rangle).\tag{16}$$

For the initial condition Eq. (16), the solution of the Schrodinger equation in interaction picture is given by

$$|\Psi_{\rm AF}^{\rm I}(t)\rangle=\sum_n(a_n(t)|3,n\rangle+b_n(t)|2,n\rangle+c_n(t)|1,n\rangle),\tag{17}$$

where

$$\begin{aligned}a_n(t)&=-{\rm i}g_1\,{\rm e}^{{\rm i}\Delta t}\sqrt{n+1}\left[\frac{Y_{n+1}}{Z_{n+1}}\sin(Z_{n+1}t)-\frac{X_{n+1}}{Z_{n+1}^2}\cos(Z_{n+1}t)\right]-F_{n+1}\sin\theta\sin\varphi,\\ b_n(t)&=Y_n\cos(Z_nt)+\frac{X_n}{Z_n}\sin(Z_nt),\\ c_n(t)&=-{\rm i}g_2\,{\rm e}^{{\rm i}\Delta t}\sqrt{n}\left[\frac{Y_{n-1}}{Z_{n-1}}\sin(Z_{n-1}t)-\frac{X_{n-1}}{Z_{n-1}^2}\cos(Z_{n-1}t)\right]-F_{n+1}\sin\theta\sin\varphi,\end{aligned}\tag{18}$$

with

$$\begin{aligned}X_n&=-{\rm i}g_1\,{\rm e}^{-{\rm i}\Delta t}\sqrt{n}F_{n-1}\cos\theta-{\rm i}g_2\,{\rm e}^{-{\rm i}\Delta t}\sqrt{n+1}F_{n+1}\sin\theta\cos\varphi,\\ Y_n&=F_n\sin\theta\sin\varphi,\\ Z_n&=\sqrt{g_1^2n+g_2^2(n+1)}.\end{aligned}\tag{19}$$

According to Eq. (17), the reduced density matrix of the three-level cascade atom can be expressed as

$$\begin{aligned}\rho_{\rm A}^{\rm I}(t)&={\rm Tr_F}|\Psi_{\rm AF}^{\rm I}(t)\rangle\langle\Psi_{\rm AF}^{\rm I}(t)|\\ &=\begin{pmatrix}\sum\limits_n a_na_n^* & \sum\limits_n a_nb_n^* & \sum\limits_n a_nc_n^*\\ \sum\limits_n b_na_n^* & \sum\limits_n b_nb_n^* & \sum\limits_n b_nc_n^*\\ \sum\limits_n c_na_n^* & \sum\limits_n c_nb_n^* & \sum\limits_n c_nc_n^*\end{pmatrix}\\ &=\begin{pmatrix}\rho_{11} & \rho_{12} & \rho_{13}\\ \rho_{21} & \rho_{22} & \rho_{23}\\ \rho_{31} & \rho_{32} & \rho_{33}\end{pmatrix},\end{aligned}\tag{20}$$

where $\rho_{11}$, $\rho_{22}$, $\rho_{33}$ are the atomic populations in energy level $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively.

In order to obtain the entropic calculation formalism of the spin operators of the cascade three-level atom, we give the concrete form of the spin operators as

$$\begin{aligned}S_x&=\begin{pmatrix}0 & \sqrt{2}/2 & 0\\ \sqrt{2}/2 & 0 & \sqrt{2}/2\\ 0 & \sqrt{2}/2 & 0\end{pmatrix},\\ S_y&=\begin{pmatrix}0 & -\sqrt{2}/2{\rm i} & 0\\ \sqrt{2}/2{\rm i} & 0 & -\sqrt{2}/2{\rm i}\\ 0 & \sqrt{2}/2 & 0\end{pmatrix},\\ S_z&=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}.\end{aligned}\tag{21}$$

Using Eqs. (4), (20) and (21), we obtain the entropic calculation formalism of the spin operators $S_x$, $S_y$, $S_z$ of the cascaded three-level atom

$$\begin{aligned}H(S_x)&=H(S_{x_1})+H(S_{x_2})+H(S_{x_3}),\\ H(S_y)&=H(S_{y_1})+H(S_{y_2})+H(S_{y_3}),\\ H(S_z)&=H(S_{z_1})+H(S_{z_2})+H(S_{z_3}),\end{aligned}\tag{22}$$

where $H(S_{\alpha_i})=-P_i(S_\alpha)\log_2P_i(S_\alpha)$, $(\alpha=x,y,z,\ i=1,2,3)$, and

$$\begin{aligned}H(S_{x_1})&=-\left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{2}\rho_{33}+\frac{\sqrt{2}}{2}{\rm Im}(\rho_{21})+\frac{1}{2}{\rm Im}(\rho_{13})+\frac{\sqrt{2}}{2}{\rm Im}(\rho_{23})\right]\\ &\quad\times\log_2\left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{2}\rho_{33}+\frac{\sqrt{2}}{2}{\rm Im}(\rho_{21})+\frac{1}{2}{\rm Im}(\rho_{13})+\frac{\sqrt{2}}{2}{\rm Im}(\rho_{23})\right],\\ H(S_{x_2})&=-\left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{2}\rho_{33}-\frac{\sqrt{2}}{2}{\rm Im}(\rho_{21})+\frac{1}{2}{\rm Im}(\rho_{13})-\frac{\sqrt{2}}{2}{\rm Im}(\rho_{23})\right]\end{aligned}$$

$$\times \log_2 \left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{2}\rho_{33}-\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{21})+\frac{1}{2}\mathrm{Im}(\rho_{13})-\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{23})\right],$$

$$H(S_{x_3}) = -\left(\frac{1}{2}\rho_{11}-\frac{1}{2}\rho_{31}-\frac{1}{2}\rho_{23}+\frac{1}{2}\rho_{33}\right)\log_2\left(\frac{1}{2}\rho_{11}-\frac{1}{2}\rho_{31}-\frac{1}{2}\rho_{23}+\frac{1}{2}\rho_{33}\right), \tag{23}$$

$$H(S_{y_1}) = -\left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{4}\rho_{33}+\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{21})-\frac{1}{2}\mathrm{Re}(\rho_{13})-\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{23})\right]$$

$$\times \log_2 \left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{4}\rho_{33}+\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{21})-\frac{1}{2}\mathrm{Re}(\rho_{13})-\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{23})\right],$$

$$H(S_{y_2}) = -\left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{4}\rho_{33}-\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{21})-\frac{1}{2}\mathrm{Re}(\rho_{13})+\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{23})\right]$$

$$\times \log_2 \left[\frac{1}{4}\rho_{11}+\frac{1}{2}\rho_{22}+\frac{1}{4}\rho_{33}-\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{21})-\frac{1}{2}\mathrm{Re}(\rho_{13})+\frac{\sqrt{2}}{2}\mathrm{Im}(\rho_{23})\right],$$

$$H(S_{y_3}) = -\left(\frac{1}{2}\rho_{11}+\frac{1}{2}\rho_{31}+\frac{1}{2}\rho_{23}+\frac{1}{2}\rho_{33}\right)\log_2\left(\frac{1}{2}\rho_{11}+\frac{1}{2}\rho_{31}+\frac{1}{2}\rho_{23}+\frac{1}{2}\rho_{33}\right), \tag{24}$$

$$H(S_{z_1}) = -\rho_{11}\log_2\rho_{11}, \quad H(S_{z_2}) = -\rho_{22}\log_2\rho_{22}, \quad H(S_{z_3}) = -\rho_{33}\log_2\rho_{33}. \tag{25}$$

Employing the reduced operator given by Eq. (20) and the entropic calculation formalism of the spin operators given by Eqs. (22)–(25), we investigate the properties of the entropy squeezing of the cascade three-level atom interacting with a single-mode field in the next section.

## 4. Numerical results and discussion

In this section, we examine numerical results for the atomic entropy squeezing. For comparison, we also consider the atomic variance squeezing and atomic inversion.

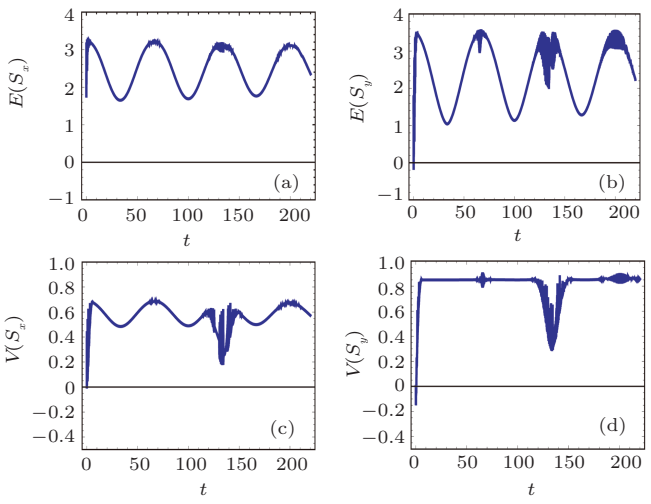



**Fig. 1.** The time evolution of the squeezing factors of a cascade three-level atom. The atom is initially in a superposition state of $|3\rangle$ and $|1\rangle$ with $\theta = 0$, $\varphi = \pi/2$, and the field is the coherent state with initial average photon number $\bar{n} = |\alpha|^2 = 225$; (a) entropy squeezing factor $E(S_x)$; (b) entropy squeezing factor $E(S_y)$; (c)variance squeezing factor $V(S_x)$; (d) variance squeezing factor $V(S_y)$.

The time evolution of the squeezing factors $E(S_x)$, $E(S_y)$, $V(S_x)$, and $V(S_y)$ are shown in Fig. 1, while the atomic inversion $\langle S_z\rangle$ is plotted in Fig. 4(a), for the atom initially in the excited state ($\theta = 0$, $\varphi = \pi/2$) and the field in a coherent state with average photon number $\bar{n} = 225$ and phase $\beta = 0$. We can see from Fig. 1(a) and Fig. 1(c) that both $E(S_x)$ and $V(S_x)$ predict no squeezing in the variable $S_x$ when the atom is initially in the excited state, while figures 1(b) and 1(d) display that $E(S_y)$ and $V(S_y)$ exhibit entropy squeezing only in initial time of the time evolution, which means that, the cascade three-level atom initially in excited state can not generate entropy squeezing except for it in initial time. In addition, from Fig. 4(a), it can be seen that in collapse region of atomic inversion, $\langle S_z\rangle$ evolves to zero. Therefore, Heisenberg uncertainty relation shows triviality, and variance squeezing can not reflect accurately the quantum fluctuations of the observable variables of three-level atom in collapse region of atomic inversion.

Figure 2 displays the entropy and variance squeezing for the cascade three-level atom initially in the superposition state of $|3\rangle$ and $|1\rangle$ with $\theta = \pi/4$, $\varphi = 0$, and the field is the coherent state with the average photon number $\bar{n} = 225$, while the atomic inversion $\langle S_z\rangle$ is plotted in Fig. 4(b). We see from Figs. 2(a) and 2(c) that both $E(S_x)$ and $V(S_x)$ predict no squeezing in the variable $S_x$, while figures 2(b) and 2(d) show that there are great differences between $E(S_y)$ and $V(S_y)$: the former shows entropy squeezing in certain time ranges whilst the latter predicts no variance squeezing at all. This result stems from the fact that, $V(S_y)$, as we see in Fig. 2(d), does not exhibit any variance squeezing since the atomic inversion $\langle S_z\rangle$ evolves to values very close to zero, showing in the Fig (b). In this case, the value of $V(S_y)$ has no particular significance and the information provided by Heisenberg uncertainty relation is not helpful.

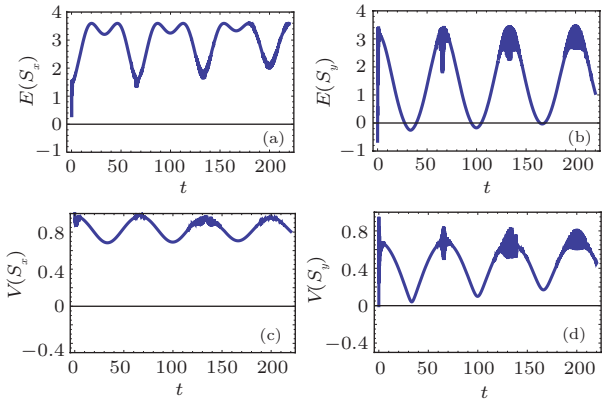



**Fig. 2.** The time evolution of the squeezing factors of a cascade three-level atom. The atom is initially in the superposition state of $|3\rangle$ and $|1\rangle$ with $\theta = \pi/4$, $\varphi = 0$, and the field is the coherent state with initial average photon number $\bar{n} = |\alpha|^2 = 225$; (a) entropy squeezing factor $E(S_x)$; (b) entropy squeezing factor $E(S_y)$; (c) variance squeezing factor $V(S_x)$; (d) variance squeezing factor $V(S_y)$.

The entropy and variance squeezing factors are presented in Fig. 3 for the atom is initially in the coherent superposition state with $\theta = \pi/3$, $\varphi = \arcsin(\sqrt{6}/3)$ and the field is the coherent state with $\bar{n} = 225$. It can be seen that from Figs. 3(a) and 3(b) that there are alternate entropy squeezing in the atomic variable components $S_x$ and $S_y$, whereas $V(S_x)$ and $V(S_y)$, as illustrated in Figs. 3(c) and 3(d), display no variance squeezing. This result also stems from the fact that the atomic inversion $\langle S_z \rangle$ evolves to values very close to zero, showing in the Fig. 4(c), and the variance squeezing can not provide any useful squeezing information.

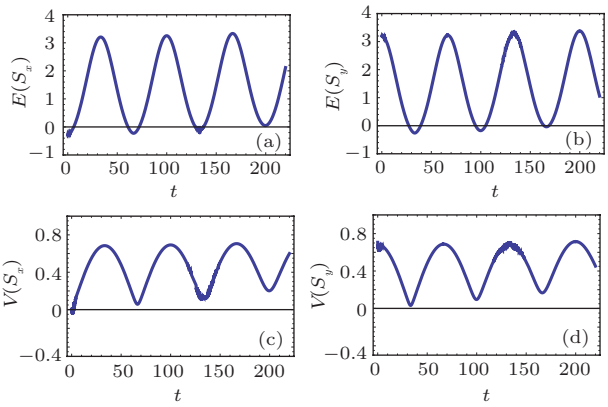



**Fig. 3.** The time evolution of the squeezing factors of a three-level atom. The atom is in a coherent superposition state with $\theta = \pi/3$, $\varphi = \arcsin(\sqrt{6}/3)$, and the field is the coherent state with initial average photon number $\bar{n} = |\alpha|^2 = 225$; (a) entropy squeezing factor $E(S_x)$; (b) entropy squeezing factor $E(S_y)$; (c)variance squeezing factor $V(S_x)$; (d) variance squeezing factor $V(S_y)$.

From Figs. 1, 2, and 4, it can be seen that, the both squeezing factor and atom inversion exhibit violent oscillation with high frequency in some time interval in time evolution. The emergence of this phenomenon is a result generated by the superposition of Rabi oscillation with different frequency of the cascade three-level atom under coherent field driving.[13]

It is worth pointing out that, the results above obtained for the entropy squeezing for a cascade three-level atom interacting with a single-model field is based on a universally adapted spin operator given by Eq. (21), but if different form spin operator is adapted, the regularity of entropy squeezing for the cascade three-level atom is different according to the independence of squeezing definition for the different operator.[13]

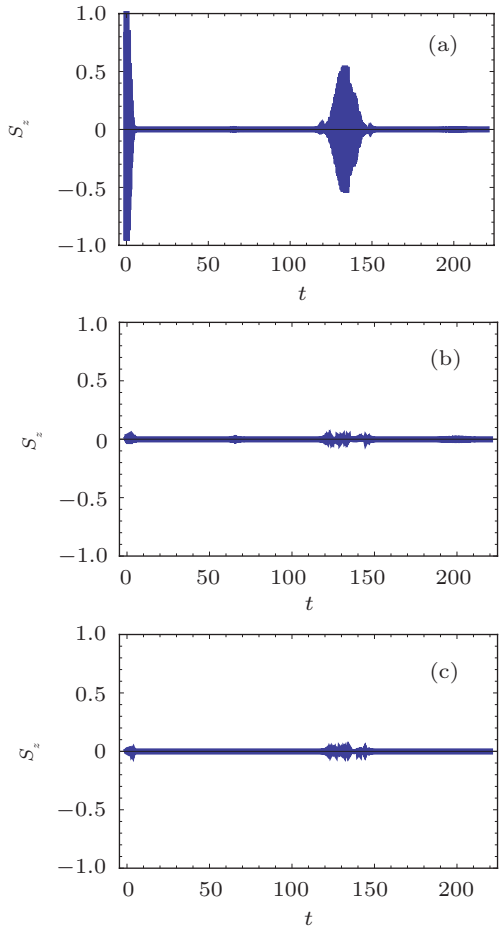



**Fig. 4.** The time evolution of the atom inversion $\langle S_z \rangle$ for a cascade three-level atom. In these pictures, the field is the coherent state with initial average photon number $\bar{n} = |\alpha|^2 = 225$. (a) The atom is initially in a ground state with $\theta = \pi/2$, $\varphi = 0$; (b) The atom is initially in superposition state of $|3\rangle$ and $|1\rangle$, $\theta = \pi/4$, $\varphi = 0$; (c) The atom is in a coherent superposition state with $\theta = \pi/3$, $\varphi = \arcsin(\sqrt{6}/3)$.

## 5. Conclusion

In conclusion, we have studied the entropy squeezing properties for a cascade three-level atom interacting with a single-model field. First, a general definition of entropy squeezing of a three-level atom is given by using quantum information entropy according to entropic uncertainty relation of three-level atom. Secondly, the reduced density matrix is obtained, and the calculation formalism of entropy is derived for a cascade three-level atom. Finally, we analyze the properties of entropy squeezing of a three-level cascade atom by using numerical calculation. Our results show that, a cascaded three-level atom interacting with a single-model field can generate obvious entropy squeezing effect via choosing appropriate superposition state of three-level atom. Our results are meaningful for preparing three-level system information resources with ultra-low quantum noise.

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