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Atom interferometers with weak-measurement path detectors and their quantum mechanical analysis*

Zhi-Yuan Li(李志远)†

College of Physics and Optoelectronics, South China University of Technology, Guangzhou 510641, *China*

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According to the orthodox interpretation of quantum physics, wave-particle duality (WPD) is the intrinsic property of all massive microscopic particles. All gedanken or realistic experiments based on atom interferometers (AI) have so far upheld the principle of WPD, either by the mechanism of the Heisenberg's position-momentum uncertainty relation or by quantum entanglement. In this paper, we propose and make a systematic quantum mechanical analysis of several schemes of weak-measurement atom interferometer (WM-AI) and compare them with the historical schemes of strong-measurement atom interferometer (SM-AI), such as Einstein's recoiling slit and Feynman's light microscope. As the critical part of these WM-AI setups, a weak-measurement path detector (WM-PD) deliberately interacting with the atomic internal electronic quantum states is designed and used to probe the which-path information of the atom, while only inducing negligible perturbation of the atomic center-of-mass motion. Another instrument that is used to directly interact with the atomic center-of-mass while being insensitive to the internal electronic quantum states is used to monitor the atomic center-of-mass interference pattern. Two typical schemes of WM-PD are considered. The first is the micromaser-cavity path detector, which allows us to probe the spontaneously emitted microwave photon from the incoming Rydberg atom in its excited electronic state and record unanimously the which-path information of the atom. The second is the optical-lattice Bragg-grating path detector, which can split the incoming atom beam into two different directions as determined by the internal electronic state and thus encode the which-path information of the atom into the internal states of the atom. We have used standard quantum mechanics to analyze the evolution of the atomic center-of-mass and internal electronic state wave function by directly solving Schrödinger's equation for the composite atom-electron-photon system in these WM-AIs . We have also compared our analysis with the theoretical and experimental studies that have been presented in the previous literature. The results show that the two sets of instruments can work separately, collectively, and without mutual exclusion to enable simultaneous observation of both wave and particle nature of the atoms to a much higher level than the historical SM-AIs, while avoiding degradation from Heisenberg's uncertainty relation and quantum entanglement. We have further investigated the space–time evolution of the internal electronic quantum state, as well as the combined atom–detector system and identified the microscopic origin and role of quantum entanglement, as emphasized in numerous previous studies. Based on these physics insights and theoretical analyses, we have proposed several new WM-AI schemes that can help to elucidate the puzzling physics of the WPD of the atoms. The principle of WM-AI scheme and quantum mechanical analyses made in this work can be directly extended to examine the principle of WPD for other massive particles.

Keywords: wave-particle duality, atom interferometers, weak-measurement path-detector, quantum entanglement, Heisenberg's uncertainty relation

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1. Introduction

Quantum mechanics has had enormous success in handling the physics and chemistry of microscopic world in the past century. Basically, the theory system of quantum mechanics involves two parts, the first is the operational methodology and the second is the conceptual basis of physics. The operational methodology of quantum mechanics includes the wave function (which represents quantum states) and its statistical interpretation, Schrödinger's equation governing the evolution of wave functions and quantum states, and operators representing physical measurements against quantum states. Numerous experiences and practices have confirmed that this methodology works perfectly successfully. However, the conceptual basis of the physics of quantum mechanics has been a controversial issue since its foundation nearly over a century ago.

It is a general consensus that the old and well-known principle of wave-particle duality (WPD) (or more generally the principle of complementarity) stands at the central conceptual core of quantum physics. This principle states that all microscopic particles exhibit mutually exclusive behaviors of two intrinsic attributes of the wave nature and particle nature, namely: they behave either as wave or as particle, depending on how they are observed and measured, but never both.[1–8] This principle was set up by Bohr and his Copenhagen school colleagues at the start of quantum mechanics, around 1927, right at the first Einstein–Bohr debate on the conceptual basis of the newborn quantum mechanics. Numerous theoretical and experimental studies, either realistic or *gedanken*, have

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†Corresponding author. E-mail: phzyli@scut.edu.cn

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since been made to test the principle of WPD based on various interferometers, such as Mach–Zehnder interferometer (MZI) and Young's two-slit interferometer (YI), for either massive particles such as electrons, neutron, and atoms[9–19] or massless particles such as photons.[20–38] In some sense, it is a mystery why no one has ever been able to simultaneously observe the wave and particle behavior of microscopic particles, which strictly upholds the WPD principle — no matter how experimental technologies have developed and advanced beyond the era of the Einstein–Bohr debate. In particular, many single particle manipulation technologies, to which several Nobel prizes have been awarded,[39–41] have been invented, and many advanced optical and atom interferometers have been developed.[42–44] These new technologies should have stimulated new insights and ideas to shed a new light on this very old fundamental problem of how to explore and correctly understand the principle of WPD and other quantum physics conceptual issues. Nonetheless, little progress has been made in this area, despite these technological advancements.

The physics of why these conventional interferometers do not allow for simultaneous observation of the wave and particle nature of a microscopic particle has reached popular consensus after a long history of intensive and extensive theoretical and experimental discussions, debates, and investigation. Typical examples of investigation include the famous classical *gedanken* schemes such as Einstein's recoiling slit[1] or Feynman's light microscope,[2] where it is generally believed that Heisenberg's position–momentum uncertainty relation works to uphold the principle of WPD. More recently, several advanced *gedanken* and realistic schemes have been proposed and examined,[14–18,20–24] including Scully's micromaser-cavity atom interferometer[20] and the Rempe's optical-lattice Bragg-grating atom interferometer.[14] In these relatively new works, most researchers believed that Heisenberg's position–momentum uncertainty relation does not play an active role but rather another mechanism called quantum entanglement works to uphold the principle of WPD. Despite this fashionable conceptual innovation, controversies have occurred as to what physical mechanism, uncertainty principle versus quantum entanglement, upholds the principle of WPD in these new atom interferometers.[20–24] However, in spite of this discrepancy, there is one consensus: no one has challenged the principle of WPD or even thought of a new regime of microscopic world going beyond the principle of WPD, namely: a world where one can simultaneously observe the wave and particle nature of a microscopic particle. More recently, some conceptual innovations have been made in regard to the principle of WPD and other quantum physics conceptual issues.[27–38] For instance, whereas the classical wisdom shows via the so-called Wheeler's delayed-choice setup that a microscopic object such as a photon behaves either as wave or as particle,[3,9–20,25,26] some years ago several theoretical and experimental studies demonstrated via the so-called quantum delayed-choice architecture that a photon can lie in an intriguing quantum superposition state showing simultaneously partial wave and partial particle behaviors.[27–30,33] However, these architectures still do not allow for simultaneous observation of full wave and full particle behaviors, and thus still work to uphold the principle of WPD.

There are very rare, little, or essentially no true hard challenges to the principle of WPD in the long history of quantum physics. This situation continues today, even when so many advanced single-particle quantum control and measurement technologies have become a reality to provide powerful experimental means to thoroughly check this fundamental issue of quantum mechanics.[39–44] There may be several reasons for this situation. First, the foundation of quantum mechanics as settled down by the Copenhagen school pioneers of quantum mechanics is so fine to most contemporary scholars that there is no need to think over this problem or make any change. Second, even if there is conceptual issue in the physics foundation of quantum mechanics to some scholars, the operational methodology of quantum mechanics works so well in the microscopic world that there is no stringent demand to touch this foundation basis. Third, this conceptual issue about the physics foundation of quantum mechanics is so complicated and subtle, and the history is so long and obscure that it has become an extremely difficult task to resolve this long-standing puzzle. As a result, even those people who think that we need to rethink this conceptual issue are reluctant to pour their wisdom and enthusiasm into this old problem, seemingly in vain. Finally, for those who really think hard about this conceptual problem, the tradition of study and investigation is largely via conceptual or in some sense philosophical arguments rather than practical mathematical derivation and solution to specific *gedanken* or practical interferometer experiments based on strict quantum mechanical formulation. Yet, obviously this tradition has proven to be unable to find a promising route towards a new, deeper, and more delicate understanding of this complicated problem.

Recently, we revisited the working principle of MZI for photons in the framework of Wheeler's delayed-choice architecture using a different paradigm of study.[34–36] We investigated the wave function of photon travelling and evolving in the MZI and calculated the interference pattern (quantified by the value of fringe visibility V) and the which-path information (quantified by the value of path distinguishability D) by doing some simple homework of quantum mechanics. The calculation directly discloses fruitful information and allows us to judge whether it is possible, or to what extent one can make simultaneous observation of the wave and particle nature of photons in these schemes of MZI.[35] Based on the quanti-

tative analysis, a completely innovative scheme of interferometer called the weak-measurement (WM) MZI was proposed and the wave function solution was obtained by doing another quantum mechanics homework.[36] The result indicates that the power to simultaneously observe the wave and particle nature of photons can be significantly advanced (to a nearly perfect status) by using this new WM-MZI as compared with the conventional delayed-choice scheme of MZI, which has been intensively discussed by Wheeler and others.[36] In the current work, we aim to follow such a new paradigm of study and revisit the working principle of interferometers for massive particles, particularly atom interferometers, by doing the homework of quantum mechanics, which is now a much more complicated and troublesome task as compared with photon MZI but is still not an intractable task, where care and caution are the most that one needs to yield a correct and reasonable solution. We will propose a weak-measurement atom interferometer called WM-AI and make a standard quantum mechanical analysis on the wave function evolution in this WM-AI, as well as classical MZI and YI based on a full and strict solution to Schrödinger's equation. This study aims to shed a new light on the principle of WPD for massive particles as atoms in a more solid ground of quantum mechanics operation methodology.

The rest of this paper is arranged as follows. In Section 2, we make a quantum mechanical analysis of several classical AIs working in both the scheme of MZI and YI, such as Einstein's recoiling slit[1] or Feynman's light microscope.[2] In these AIs, the which-path detector is in direct interaction with the atom center-of-mass to identify its path unanimously and as a result disturb its wave function strongly and uncontrollably. We solve the evolution of the atom wave function input, within, and output from the interferometer, calculate the quantity V and D, discuss the perspective of simultaneous observation of the wave and particle nature of the atom using these interferometers, and analyze how the Heisenberg's position–momentum uncertainty relation works to uphold the principle of WPD. In Section 3, we systematically investigate the scheme of WM-AI as first proposed by Scully and coworkers, where the which-path detector is a micromaser cavity in direct strong and deterministic interaction with the outer Rydberg quantum state of the atom while in very weak indirect interaction with the atom's center-of-mass to disturb its wave function nearly negligibly. We make a quantum mechanical analysis over the wave function evolution, we discuss the consequent perspective of this WM-AI to observe the wave and particle nature of the atom, and compare with previous analyses made by Scully and coworkers. We then present an alternative scheme of MZI-type WM-AI based on a new architecture of interference pattern measurement and evaluation. In Section 4, we discuss a more practical scheme of WM-AI as first proposed by Rempe and coworkers, where the which-path detector is a standing-wave light Bragg grating to directly interact with the internal atomic quantum state of electrons and impose an optical force to cause splitting of the atom beam in different directions depending on the internal electronic state to identify the which-path information. Such an indirect interaction with the atom center-of-mass does not disturb its spatial wave function profile uncontrollably and maintain the coherence of the atomic beam travelling in different directions. We then make a quantum mechanical analysis over the atom center-of-mass wave function evolution and discuss the perspective of this WM-AI to observe the wave and particle nature of the atom. We further solve the wave function evolution of the internal electronic quantum states of the atom in both the real space and inner-state space when the atom travels through the interferometer and discuss its role in the atom which-path probe and interference pattern observation. We will point out that the internal quantum state of the atom also exhibit certain space distribution and interference patterns. When one attributes this pattern to the true atom's center-of-mass interference pattern, ambiguity will take place to drastically complicate and even mislead the interpretation of experimental observation in regard to the test of WPD. In Section 5, we further propose a general scheme of WM-AI that involves a crucial device called polarization beam splitter that allows us to encode the which-path information within the internal quantum state of the atom and other massive particles for evaluating the principle of WPD. Finally, in Section 6, we summarize this paper, echo several crucial discoveries made in this work, compare with previous schemes and analyses of the atom interferometers, and present a perspective of future studies in the research of WPD and other quantum physics conceptual issues.

2. Quantum mechanical analysis of classical atom interferometers

A schematic configuration of the well-established traditional AI in the scheme of MZI is illustrated in Fig. 1(a). It is similar to the classical MZI for photons, except that all the devices such as beam splitters, mirrors, path delayer, and interference pattern detector now must be replaced by atom optics devices.[42] This atom interferometer is thus called as MZI-AI. Two path detectors are inserted into the two arms of MZI-AI to probe the which-path information of the atoms. It is well-known that this classical MZI-AI does not allow us to simultaneously observe the wave and particle nature of a microscopic massless and massive particle, no matter how smart it is designed, like in the scheme of Wheeler's delayed-choice experiment.[35,36] Another popular traditional scheme to test the principle of WPD is the atomic two-slit YI (hereafter called

YI-AI) as illustrated schematically in Fig. 1(b). As a key composite, two path detectors (PDs) are placed around (either right before, or right after, or surrounding) the two slits of YI-AI to probe the which-path information of the atoms. Perhaps due to its simplicity in physics and geometry, this type of YI-AI dominantly becomes the historical *gedanken* experimental setups made by Einstein and Feynman to discuss the principle of WPD. However, numerous studies have clearly shown that this classical YI-AI also does not allow us to simultaneously observe the wave and particle nature of a microscopic massless and massive particle, no matter how smart it is designed.

The logic of reasoning and argument on how to uphold the principle of WPD in these historical MZI-AI and YI-AI setups is simple. If no path detector is used, either removing the path detector or turning them off in Figs. 1(a) and 1(b), then one can observe perfect interference pattern of the atom de Broglie wave. In contrast, once the path detector is in position and works to probe and identify unanimously the which-path information of the atom, the interference pattern disappears completely and immediately. The logic is indeed very simple, yet it works based on the fundamental assumption that the path detector, when it works, will inevitably cause an uncontrollable strong perturbation to the motion (more strictly, the wave function) of the atom. Then, if one wishes to shed a new light on deeper understanding the conceptual basis of quantum physics, it is worthwhile to revisit this assumption due to two reasons. The first is to have a more clarified picture of why and how the which-path probe perturb the atomic wave function, thus upholding the principle of WPD. The second and more important one is to see whether one can find a smarter AI setup via deliberate design to go beyond this principle of WPD. The lesson from our recent works on the design and analysis of photonic WM-MZI in Refs. [35] and [36] shows that it is indeed worthwhile to make a thorough analysis on this problem and nothing is impossible when new insights are poured into an issue. Indeed, the design of photonic WM-MZI is a direct inspiring point to simulate the study presented in the current work for atom interferometers: is it also possible to design a workable WM-AI?

The key to answer this question relies on the thorough analysis of the path-detector, as illustrated in Fig. 1(c), and its role in perturbing the motion and wave function of the atom during their mutual interaction for the which-path information to be recorded unanimously. This can only be done by deliberate mathematical and physical analyses, and, more importantly, made strictly in the framework of standard quantum mechanics methodology, the only working physical theory of the microscopic world. Nonetheless, before we do regular quantum mechanics homework of solving Schrödinger's equation and wave function evolution in these AI setups, we should first clarify several important conceptual issues so that it is relatively easier to reach a consensus after mathematical solution is found. First, we emphasize that the subject concerning us is the WPD of the atom itself, namely, about the which-path information and interference pattern of the atom, more precisely the atomic center-of-mass, rather than anything else, like the internal electrons and nucleus of the atom. This is the conceptual core of all discussions, either theoretical or experimental. Second, in principle, all wave and particle information has and should have already been involved within and can be extracted from the atom wave function following the well-established procedures of quantum mechanical operation to find the calculation and experiment outcome of physical quantities and operators (*e.g.*, the which-path distinguishability D and interference fringe visibility V). Third, the role of the internal electronic quantum states or the which-path detector against the atom should be taken into full account, but their contributions can be and should be handled reasonably and accurately without either underestimate or overestimate, by solving their evolution of quantum state wave function and influence to the atom de Broglie wave again in the framework of quantum mechanics. But great caution should be taken to insure that their wave function evolution is not meddled with the atom wave function either in the theoretical calculation or in the interpretation to experimental data about the wave and particle information, and that the physical and mathematical connection is reasonably interpreted. We will see that many of the previous realistic and *gedanken* experiments have problems in these issues.

Now we can begin our homework of quantum mechanical analysis. The quantum mechanics for the AI setup are illustrated in Fig. 1, which satisfies the following Schrödinger equation:

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(R,t) = H(R,t)\Psi(R,t). \tag{1}$$

Here, $H(R,t)$ is the Hamiltonian of the whole AI setup, $\Psi(R,t)$ is the wave function of the atom. The physical problem is to see how $\Psi(R,t)$ evolves in space–time when the atom beam (an ensemble of identical individual atoms) passes through the AI setup, one composite device after one composite device. All information should be encoded in $\Psi(R,t)$, but might be extracted appropriately and deliberately in various space–time domains. Obviously, this is the standard quantum scattering problem for atoms, and thus the determination of $\Psi(R,t)$ should closely rely on the boundary condition (at R_∞), as well as the initial condition (at t_0), which is written as the input wave function Ψ_{input} at the entrance port of the interferometer. In principle, the wave function $\Psi(R,t)$ anywhere within the interferometer and the output one Ψ_{output} can be calculated by solving Eq. (1), and it is the basis to analyze both the which-path information and interference pattern for testing the principle of WPD. These two properties can only be acquired via the

interaction of the atom with external detectors, whose subtle infrastructure details should determine the success or failure of the test.

For the AI illustrated in Fig. 1, the Hamiltonian can be written as

$$H(R,t) = H_0(R) + H_{\rm int}(R,t). \tag{2}$$

Here, $H_0(R)$ is the unperturbed transport Hamiltonian of free atom within the interferometer when the two external which-path detectors do not exist or do not work. For the MZI-AI setup as illustrated in Fig. 1(a), this Hamiltonian should involve all the composite devices of the interferometer implementing atom beam splitting, beam reflection, beam path delaying, beam combination, and other simple functionalities. The solution of Schrödinger's equation is straightforward within these composite devices against the wave function of incident atom beam, from which the transport behavior of the atom can be phenomenologically described by some transmission, reflection, and diffraction coefficients, similar to the situation in optics.[32,33] For the YI-AI setup as illustrated in Fig. 1(b), the situation is even relatively simpler.

Nonetheless, despite the difference in the configuration of MZI-AI and YI-AI setups, the most important issue that concerns us the most is the same, namely: how the originally incoming free-propagation atom interacts with the which-path detectors and how this interaction changes or disturbs the wave function of the atom. At this crucial stage of free atom beam passing through the which-path detector, the unperturbed Hamiltonian only contains the momentum energy term and reads $H_0(R) = H_{\rm free}(R) = P^2/2m_{\rm a} = -(\hbar^2/2m_{\rm a})(\partial^2/\partial R^2)$, where R is the space coordinate of the atom, or more precisely, the atom center-of-mass. The interaction Hamiltonian $H_{\rm int}(R)$ describes how the atom interacts with external detectors, either through direct interaction (called direct scheme) with the atom center-of-mass (abbreviated as "acm", and with coordinate R), called as $H_{\rm int}^{\rm acm}(R)$, or through indirect interaction (called the indirect scheme) with the internal electronic quantum state (abbreviated as "iqs", and with coordinate r) of the atom, called $H_{\rm int}^{\rm iqs}(R,r)$.

To quantitatively evaluate the principle of WPD in these AIs, we need to examine two relevant quantities. One quantity is the profile of the atom de Broglie wave interference pattern, which can be described quantitatively by the interference fringe visibility V, and the other quantity is the which-path information recording of the atom, which can be described by the path distinguishability D.[35,36] The estimate and calculation of V and D must be placed in the framework of quantum mechanics via solution of Eqs. (1) and (2) in various geometric and physical configurations of AI for the wave function $\Psi(R,t)$. The key is how to correctly solve $\Psi(R,t)$ and how to rationally interpret its physical meaning in connection with the interference pattern and which-path information. At first glance, this seems to be easy; however, recall of the history proves that this is a very subtle issue.

A close and careful analysis indicates that several famous historical AI setups such as Einstein's recoiling slit[1] or Feynman's light microscope[2] adopt the direct-scheme which-path detector to probe the path of the atom or other massive particles, similar to the AI setups that are schematically depicted in Fig. 1. In the direct scheme, the Hamiltonian is

$$H(R) = -\frac{\hbar^2}{2m_{\rm a}}\frac{\partial^2}{\partial R^2} + H_{\rm int}^{\rm acm}(R). \tag{3}$$

For the current quantum scattering physics problem, the core issue is the solution of the unknown transmission wave function $\psi_{\rm tran}(R)$ right after the which-path detector given the explicit form of both $H(R)$ and the incident wave function $\psi_{\rm inc}(R)$ right before the detector. Suppose that the incident atomic beam has good coherence, which is necessary for one to successfully observe the atom de Broglie wave interference pattern, we can then model $\psi_{\rm inc}(R) = \psi_0 {\rm e}^{{\rm i}kl}$ for simplicity but not without losing generality. The wave vector of the atom beam is related to the momentum of the atom P by $k = P/\hbar$, and the momentum energy (also the total energy) of the atom is $E = P^2/2m_{\rm a}$, with $m_{\rm a}$ being the mass of the atom. The de Broglie wavelength of the atom is $\lambda_{\rm a} = 2\pi/k = h/P$.

In the direct scheme, the which-path detector directly interacts with the atom center-of-mass to probe its path. To unanimously unveil the which-path information, the detector must be able to localize the atom within a sufficiently small size.[1,2] In Feynman's light microscope, this size is on the order of light wavelength, so that $\Delta z \approx \lambda_{\rm light}$. According to the Heisenberg's position–momentum uncertainty principle, the atom position localization will produce a momentum uncertainty on the order of $\Delta p \approx h/\Delta z \approx h/\lambda_{\rm light}$. This momentum uncertainty arises from the momentum kick transferred by the scattered photon, and it is so large that the interference pattern of the atom de Broglie wave will be smeared out completely. Such a well-known qualitative analysis was used by Bohr and his Copenhagen colleagues to safeguard the principle of WPD against Einstein.[1,2]

These qualitative physics analysis can be placed into a more formal framework of quantum mechanics by tracing back to the solution of wave function. Although the exact form of transmission wave function is hard to know without the knowledge of the exact form of $H_{\rm int}^{\rm acm}(R)$, a quantitative estimate can still be made. Suppose that the incident wave function $\psi_{\rm inc}(R) = \psi_0 {\rm e}^{{\rm i}kl}$ will become $\psi_{\rm tran}(R) = a\psi_0 {\rm e}^{{\rm i}kl+\Delta kd}$ after the atom passes through the which-path detector. By taking another approximation that the which-path detector only induces a loss of the atom beam intensity (via scattering or reflection) described by the coefficient $a < 1$ and a phase shift

described by the factor of $\delta\varphi = \Delta kd$, where $\Delta k = \Delta p/\hbar \approx 2\pi/\lambda_{\text{light}}$ is the wave vector change due to the momentum transfer during the atom–detector interaction, and d is the interaction path length. The energy transfer due to the interaction Hamiltonian $H_{\text{int}}^{\text{acm}}(R)$ is on the order of $\Delta E = (\Delta p)^2/2m_{\text{a}}$.

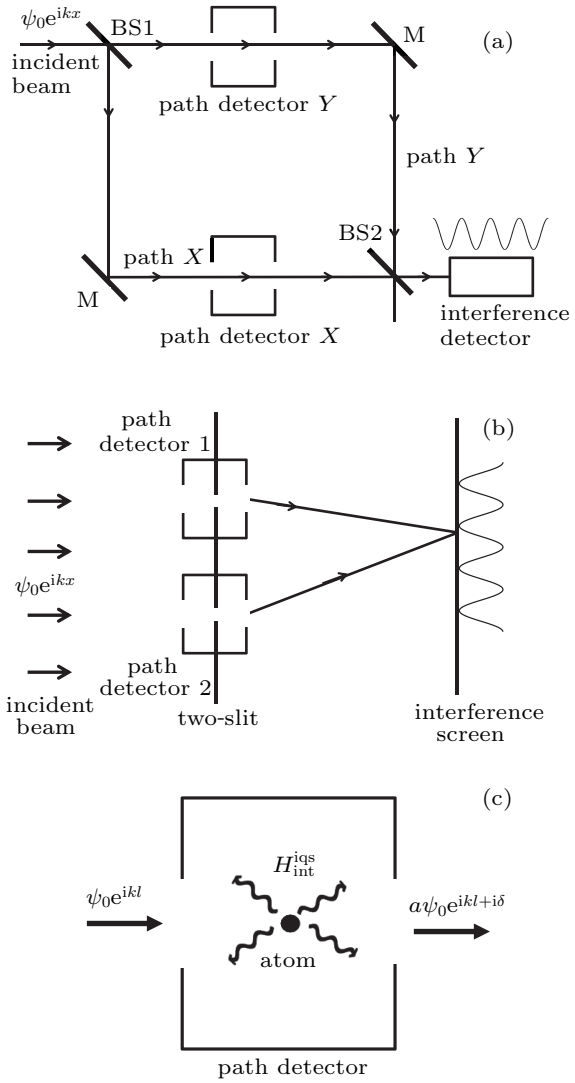


Fig. 1. Schematic illustration of classical strong-measurement atom interferometer used to test the WPD for atoms in the scheme of (a) Mach–Zehnder two-arm interferometer and (b) Young's two-slit interferometer involving the crucial composite device of (c) the strong-measurement path detector. The SM-AI includes historical schemes of AI such as Einstein's recoiling slit and Feynman's light microscope. Each SM-PD is designed to probe unanimously the passage of the atom via directly and strongly interacting with the atom center-of-mass, thus inducing an inevitable perturbation to the atom wave function as characterized by a random phase shift $\delta\varphi$ of large strength in each channel of AI. The action of SM-PD thus smears out the interference fringe pattern in the output observation screen, upholding the principle of WPD via the mechanism of the Heisenberg's position–momentum uncertainty relation.

In the MZI-AI and YI-AI setups as illustrated in Fig. 1, for the situations where the which-path detectors are removed, the relevant wave function probed by the interference pattern detector is given by

$$\begin{aligned}\psi(R) &= a(R)\psi_{1,\text{inc}}(R) + b(R)\psi_{2,\text{inc}}(R)\\ &= \psi_0\, e^{ikl}[a(R)\, e^{i\varphi_1} + b(R)\, e^{i\varphi_2}]. \end{aligned} \tag{4}$$

The consequent interference pattern at the observation screen is thus given by

$$I(R) = A|\psi(R)|^2 = A[a^2 + b^2 + 2ab\cos(\varphi_1 - \varphi_2)]. \tag{5}$$

Here, $a(R)$ and $b(R)$ are the single-arm or single-slit transmission coefficient in MZI-AI and YI-AI, respectively, φ_1 and φ_2 are the phase shift accumulated during the atom beam transport, and A represents the efficiency of the interference-pattern detector, which should be designed to have a signal response intensity $I(R)$ proportional to the atom population number varying in space $|\psi(R)|^2$ so that equation (5) holds true in high accuracy. Equation (5) clearly shows a perfect interference pattern varying in space and the fringe visibility is $V = 1$. In contrast, as no which-path detector is in place and thus there is no way to know the passing path of the atom, the path distinguishability is $D = 0$.

The wave function can also be estimated and calculated when adding the which-path detector. In early schemes like Einstein's recoiling slit or Feynman's light microscope, the atom–detector interaction will induce at least a random phase shift $\delta\varphi_1 = \Delta k_1 d$ and $\delta\varphi_2 = \Delta k_2 d$ in both paths of interferometer, and the relevant wave function up for detection by the interference-pattern detector is

$$\begin{aligned}\psi(R) &= a(R)\psi_{1,\text{tran}}(R) + b(R)\psi_{2,\text{tran}}(R)\\ &= \psi_0\, e^{ikl}[a_1 a(R)\, e^{i\varphi_1}\, e^{i\delta\varphi_1} + a_2 b(R)\, e^{i\varphi_2}\, e^{i\delta\varphi_2}]. \end{aligned} \tag{6}$$

By summing up the contributions from all of the possible random phase $\delta\varphi_1$ and $\delta\varphi_2$ as existing in practical experiments, $\delta\varphi_1$ and $\delta\varphi_2$ are so large that the signal intensity is

$$\begin{aligned}I(R) &= \overline{A|\psi(R)|^2}\\ &= A[a^2 + b^2 + 2ab\overline{a_1 a_2\cos(\varphi_1 - \varphi_2 + \delta\varphi_1 - \delta\varphi_2)}]\\ &= A(a^2 + b^2). \end{aligned} \tag{7}$$

Here, a reasonable ensemble average has been made to account for practical experiment observation. It is clear that the interference fringe completely disappears ($V = 0$) on the price of knowing unanimously the path the atom passes ($D = 1$). In terms of quantum interaction physics, the definite probe of the which-path information of the atom will inevitably cause strong perturbation to the motion of the atom and this is well reflected in the random phase of the wave function. This formal quantum mechanics analysis is in effect essentially identical to the more qualitative while simpler understanding based on the Heisenberg's uncertainty relation, which was dominantly adopted in the analysis over many classical schemes to test the principle of WPD.

This direct scheme is more or less a strong-measurement (SM) scheme, where the atom which-path information is acquired by directly localizing the position of the atom center-of-mass to a high precision in space, thus unavoidably causing

strong perturbation to the wave function and smearing out the interference of the atom de Broglie wave. However, is this awkward situation of SM-AI setup avoidable? The answer relies on whether one is willing to take an open vision and attitude, look into the warehouse of modern advanced atomic and optical fine measurement technologies and search for better ways to probe the path of the atom in MZI-AI and YI-AI that go beyond the constraint of the conventional wisdom rooted in the Einstein–Bohr debate. Anyhow, 90 years have passed and technologies have evolved that are far beyond the imagination of either Einstein or Bohr, as well as their contemporary pioneering colleagues of quantum physics.

3. Quantum mechanical analysis on WM-AI with micromaser-cavity WM-PD

3.1. New interferometer setup

A lesson from recent photonic version of MZI illustrated in Ref. [36] shows that one can take an alternative way of thinking. In Ref. [36], a simple modification is made upon the classical SM-MZI for photons so that a simple design of WM-MZI is generated and it allows for a much better power to simultaneously observe the particle and wave nature of photons. Similarly, a WM-AI can be designed and constructed, giving us a better power to simultaneously observe the particle and wave nature of the atoms and other massive particles. The strategy is to completely renunciate the above strong-measurement scheme adopted in the historical works[1,2] and replace it with the design and implementation of a weak-measurement path-detector (WM-PD) that can acquire as much as possible the overall information about the wave and particle nature of the atom. The WM-PD does not directly probe the exact spatial position of the atom, and thus there is no need to localize the probe light in Feynman's light microscope and atom in Einstein's recoiling slit to the extent that the Heisenberg's position–momentum uncertainty relation will play a crucial role. A prominent example of WP-PD is illustrated in Fig. 2(c), which is a high-Q micromaser cavity that can resonantly couple with the quantum transition between two high-orbital electronic states, $|a\rangle$ and $|b\rangle$ with an energy gap of E_{ba}, of a Rydberg atom, which is chosen as the microscopic object subject to the test of WPD principle. Note that this scheme was first proposed by Scully and his coworkers around 1991 but analyzed by them in a very different route of thinking.[20] The micromaser cavity is designed to ensure that all incoming Rydberg atoms at the excited state $|b\rangle$ will spontaneously decay to the lower state $|a\rangle$, emitting a microwave photon with a frequency of $\omega_{ba} = E_{ba}/\hbar$ when they pass through the cavity. By adopting this WM-PD, the classical MZI-AI and YI-AI setups as depicted in Figs. 1(a) and 1(b) are modified significantly to their evolution version WM-AI, as illustrated in Figs. 2(a) and 2(b), respectively.

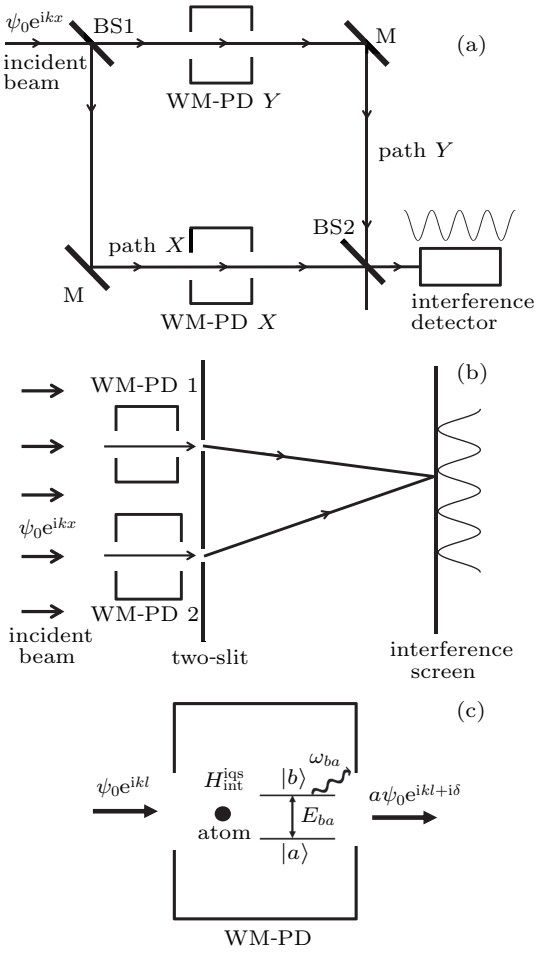


Fig. 2. Schematic illustration of weak-measurement atom interferometer used to test the WPD for Rydberg atoms with two high-orbital electronic states in (a) the MZI scheme and (b) YI scheme. (c) Illustration of WM-PD made from two high-Q micromaser cavities involving microwave photon interacting with the internal electronic quantum states. The WM-PD can unanimously probe the passage of an atom via sensing the spontaneously emitting microwave photon originated from down transition of bound electron from the excited state to the ground state. Because the indirect interaction of which-path recording has an energy scale much smaller than the atom center-of-mass momentum energy, the perturbation to the atom wave function and the random phase shift is negligible, so that the action of WM-PD still maintains a nearly perfect interference fringe pattern in the output observation screen. Thus, this WM-AI offers a superior power to simultaneously observe the wave and particle nature of the atom.

3.2. Quantum mechanical analysis of wave function evolution

In the most general situations, the WM-PD system is comprised of the atom (center-of-mass), high-orbital electrons, cavity photon, and their mutual interactions, and one needs to consider quantitatively the motion and wave function evolution of this coupled composite system. The system Hamiltonian has the following general form:

$$H_{\rm S} = H_{\rm acm} + H_{\rm photon} + H_{\rm electron} + H_{\rm int}^{\rm p.e.} + H_{\rm int}^{\rm a.e.} + H_{\rm int}^{\rm a.p.}, \quad (8)$$

where $H_{\rm acm}$ denotes the atom center-of-mass momentum energy, $H_{\rm photon}$ the cavity microwave photon energy, and $H_{\rm electron}$ the outer-orbital electron energy (modeled as two-level quantum states) surrounding the nucleus. $H_{\rm int}^{\rm p.e.}$ denotes the photon–

electron interaction (which is popularly called as light–atom or photon–atom interaction in general textbooks and literatures) accounting for the spontaneous emission of cavity microwave photon, $H_{\rm int}^{\rm a.e.}$ is the electron transition induced atom center-of-mass recoiling interaction, and $H_{\rm int}^{\rm a.p.}$ is the cavity microwave photon scattering induced atom center-of-mass recoiling interaction. At first glance, equation (8) describes a very complicated multi-scale and multi-freedom interacting system and is extremely hard to solve precisely. Yet, the underlying physics are not complicated. Obviously, the physics of this WM-PD is two levels. The first-level physics concerns the interaction of two-level electronic states with the cavity microwave photon, and it is through this interaction that the WM-PD acquires the which-path information of the atom: every time the excited-state atom passes through one cavity, it definitely spontaneously emits a microwave photon, which is also definitely absorbed and probed by the detection module associated with each cavity. Note that this interaction is strong and definite in terms of the exchange of energy and information between the cavity microwave photon and the two-level electronic quantum states. The second-level physics concerns the influence of the which-path detection process upon the atom center-of-mass motion.

Because the energy scale of the which-path detection is much smaller than the atom center-of-mass momentum energy, we can separate the two-level physics into two consecutive ones and solve them approximately but with a high precision using the concept of classical Born–Oppenheimer approximation method. Then, equation (8) becomes

$$H_{\rm level\ 1} = H_{\rm photon} + H_{\rm electron} + H_{\rm int}^{\rm p.e.}, \tag{9}$$

$$\begin{aligned} H_{\rm level\ 2} &= H_{\rm acm} = H_{0,\rm acm} + H_{\rm int}^{\rm a.e.} + H_{\rm int}^{\rm a.p.} \\ &= -\frac{\hbar^2}{2m_{\rm a}}\frac{\partial^2}{\partial R^2} + H_{\rm int}^{\rm iqs}(R,r). \end{aligned} \tag{10}$$

The solution of the electron–photon coupling problem as involved in Eq. (9) is straightforward as it has been well described in numerous textbooks of the atomic physics, quantum physics, and quantum optics. In each cavity (attached to one arm of MZI-AI and one slit of YI-AI), the general electron–photon quantum state is $\Psi = C_a|a\rangle|1\rangle + C_b|b\rangle|0\rangle$, which is subject to quantum state evolution due to photon–electron strong interaction when the atom passes through the cavity. A high-performance WM-PD requests that at the entrance of the cavity, $\Psi_{\rm in} = |b\rangle|0\rangle$ while at the exit of the cavity $\Psi_{\rm out} = |a\rangle|1\rangle$, and the definitely spontaneously emitted cavity microwave photon also becomes definitely absorbed and probed by the associated detector. The solution of the atom center-of-mass motion as involved in Eq. (10) is much more complicated. All the electron–photon coupling interaction, as well as the photon scattering interaction is described by the indirect coupling term $H_{\rm int}^{\rm iqs}(R,r)$, which should be related with the internal electronic quantum states of the atom. As the exact form of $H_{\rm int}^{\rm iqs}(R,r)$ is difficult to be known, the exact solution of Eq. (9) is not easy. But one thing is certain, the energy scale of $H_{\rm int}^{\rm iqs}(R,r)$ is at best on the order of $E_{ba} = \hbar\omega_{ba}$. So if one makes sure that the atom center-of-mass momentum energy is much larger than the energy of this microwave photon, then solution of Eq. (9) is also straightforward in terms of energy change and wave function change. Let $E_{\rm acm} = (\hbar k)^2/2m_{\rm a}$ and $E_{0,\rm acm} = (\hbar k_0)^2/2m_{\rm a}$ denote the momentum energy of the incoming and outgoing atoms, respectively. Because $|E_{\rm acm} - E_{0,\rm acm}| \le E_{\rm photon} = \hbar\omega_{ba}$, we find $\Delta k = |k - k_0| \le m_{\rm a}\omega_{ba}/\hbar k_0$, so that

$$\frac{\hbar\Delta k}{\hbar k_0} \le \frac{\hbar\omega_{ba}/2}{(\hbar k_0)^2/2m_{\rm a}} = \frac{E_{\rm photon}}{2E_{\rm atom}} \ll 1.$$

For an atom with a mass of nucleus number of 100 and a velocity of 100 m/s, and a microwave photon with frequency 1 GHz, we find

$$\begin{aligned} \frac{\Delta k}{k_0} &\le \frac{E_{\rm photon}}{2E_{\rm atom}} \\ &= \frac{6.63\times10^{-34}\times10^{8}}{100\times1.67\times10^{-27}\times(10^2)^2} = 4\times10^{-5} \ll 1. \end{aligned}$$

This energy change very small compared to the energy spectral width of atom beam in practical situations. This energy change will also reflect on the wave function change. Suppose that the incident wave function $\psi_{\rm inc}(R) = \psi_0\,{\rm e}^{{\rm i}kl}$ becomes $\psi_{\rm tran}(R) = a\psi_0\,{\rm e}^{{\rm i}kl+\Delta kd}$ after passing through the WM-PD. The random phase change is now

$$\delta\varphi = \Delta kd \le \frac{E_{\rm photon}}{2E_{\rm atom}} \times (k_0 d) = \frac{E_{\rm photon}}{2E_{\rm atom}}\varphi_0 \ll \varphi_0. \tag{11}$$

In practical experiments, this random phase change can be very small as compared with the transport phase of the atom beam, or even small compared with the phase fluctuation strength due to the finite energy spectral width of atom beam. As a result, the which-path detection via the WM-PD only causes a negligible influence to the energy and wave function of the atom center-of-mass. The detected interference as given by Eq. (7) is now

$$\begin{aligned} I(R) &= A[a^2 + b^2 + 2ab\overline{a_1 a_2}\cos(\varphi_1 - \varphi_2 + \delta\varphi_1 - \delta\varphi_2)] \\ &\approx A[a^2 + b^2 + 2ab\cos(\varphi_1 - \varphi_2)]. \end{aligned} \tag{12}$$

We see that the magnitude of the random phase $\delta\varphi_1$ and $\delta\varphi_2$ is so small that they will not perturb the atom center-of-mass wave function to wash out the interference fringe.

The physics involved in the WM-PD MZI-AI and YI-AI as illustrated in Fig. 2 are quite simple and free from ambiguity. The WM-PD acquires the definite which-path information by probing the internal quantum state transition definitely correlating with the cavity microwave photon, which is the intrinsic property of the WM-PD. The detection action of the WM-PD does not induce any remarkable influence to the

atom center-of-mass de Broglie wave function, and so does to the intrinsic interference pattern of the atom de Broglie wave. Similar to the WM-MZI for photons,[36] in these WM-AI setups we have used two sets of instrument to probe separately in space time the wave (interference pattern) and particle (which-path information) of the atom simultaneously. As their mutual influence can be minimized to the as low as possible level, the simultaneous observation of nearly perfect wave and particle nature of the atom becomes no longer an insurmountable obstacle.

3.3. Calculation of interference fringe visibility and which-path distinguishability

It is interesting to calculate quantitatively the interference fringe visibility V and the path distinguishability D of the WM-AI. The interference pattern is estimated from Eq. (12) as

$$\begin{aligned} I(R) &\approx A[a^2+b^2+2ab\cos(\delta\varphi_1-\delta\varphi_2)\cos(\varphi_1-\varphi_2) \\ &\quad -2ab\sin(\delta\varphi_1-\delta\varphi_2)\sin(\varphi_1-\varphi_2)] \\ &= 2A[1+\cos(\varphi_1-\varphi_2)\cos(2\delta\varphi) \\ &\quad +\sin(\varphi_1-\varphi_2)\sin(2\delta\varphi)], \end{aligned} \tag{13}$$

where the ideal situation $a=b=1$ has been assumed. The interference fringe visibility is simply $V^2=\cos^2(2\delta\varphi)\approx 1-(2\delta\varphi)^2\to 1$. The path distinguishability D is simply $D=1$ assuming a perfect performance of WM-PD. As a result, $V^2+D^2=2-(2\delta\varphi)^2\to 2\gg 1$. For comparison, in the scheme of WM-MZI for photons as discussed in Ref. [32], the interference fringe visibility is perfectly $V=1$, while the path distinguishability D can indefinitely approach $D\to 1$ so that $V^2+D^2\to 2\gg 1$. In the WM-AI, the key to reach an excellent power to acquire simultaneous wave and particle information of the atoms lies in the design and construction of a WM-PD that can probe definitely the which-path information of the atoms but only cause very small perturbation to the interference fringe visibility, while in the photonic WM-MZI, the key to reach an excellent power to acquire simultaneous wave and particle information of photons lies in the design and construction of a WM-PD that can probe definitely the interference fringe of photons but only cause very small perturbation to the which-path distinguishability. These studies clearly show that although it is not possible to acquire perfectly for both the wave and particle nature of either massive atoms or massless photons because measurement against these two fundamental physical quantities will induce inevitable perturbation to their motion, these perturbation can indeed be controllable and reduced to a very low level far beyond the conventional wisdom by adopting simple while somewhat smart designs of WM atomic and photonic interferometers. Moreover, the operation of these new interferometers does not violate any strict standard quantum mechanic law as quantized by Schrödinger's equation.

3.4. The role of quantum entanglement

Previously, we have only focused on the wave function evolution of the atom center-of-mass during its passage through the WM-PD under the interaction between the internal electronic state and the microwave cavity photon. The role of the which-path detection is eventually reflected on its influence on the atom center-of-mass wave function evolution, in particular the introduction of a random phase shift $\delta\varphi_1$ and $\delta\varphi_2$. This wave function will determine the interference pattern and the fringe visibility. It is also instructive to see the wave function evolution of the atom–electron–photon composite system (*i.e.*, atom center-of-mass, internal electronic states, and cavity microwave photon). Let the microwave photon state of the two cavities be denoted by $\psi_{\text{cavity},1}$ and $\psi_{\text{cavity},2}$, the internal electronic state denoted by $\psi_{\text{electron},1}$ and $\psi_{\text{electron},2}$, and the atom center-of-mass wave function denoted by $\psi_{\text{acm},1}(R)$ and $\psi_{\text{acm},2}(R)$ for the two channels of the atom passage. Then, according to the well-known operation methodology of orthodox quantum theory as adopted in Ref. [20], the total wave function of the coupled atom–electron–photon quantum system when the atom is within the cavity is given by

$$\begin{aligned} \Psi(R) &= \psi_{\text{acm},1}(R)\psi_{\text{electron},1}\psi_{\text{cavity},1} \\ &\quad +\psi_{\text{acm},2}(R)\psi_{\text{electron},2}\psi_{\text{cavity},2}. \end{aligned} \tag{14}$$

Following the route of the same operation methodology, the interference pattern is given by

$$\begin{aligned} I(\boldsymbol{R}) &= |\Psi(R)|^2=|\psi_1(R)|^2+|\psi_2(R)|^2 \\ &\quad +2\text{Re}[(\psi^*_{\text{electron},1}\psi_{\text{electron},2}) \\ &\quad \times(\psi^*_{\text{cavity},1}\psi_{\text{cavity},2})\psi^*_{\text{acm},1}(R)\psi_{\text{acm},2}(R)]. \end{aligned} \tag{15}$$

Before the atom goes into the micromaser cavity, we have $\psi_{\text{electron},1}=\psi_{\text{electron},2}=|b\rangle$, $\psi_{\text{cavity},1}=\psi_{\text{cavity},2}=|0\rangle|0\rangle$. When the internal quantum stat of the atom ψ_{electron} interacts with the micromaser cavity quantum state ψ_{cavity} so that the which-way information of the atom is unanimously transferred to and recorded by the which-way detector, one has relation that $\psi_{\text{electron},1}=\psi_{\text{electron},2}=|a\rangle$, $\psi_{\text{cavity},1}=|1\rangle|0\rangle$, $\psi_{\text{cavity},2}=|0\rangle|1\rangle$, so that $\psi^*_{\text{cavity},1}\psi_{\text{cavity},2}=0$, thus the interference term in Eq. (15) is equal to zero. Equation (15) now becomes

$$I(\boldsymbol{R})=|\Psi(R)|^2=|\psi_1(R)|^2+|\psi_2(R)|^2. \tag{16}$$

This means that the unanimous recording of the which-way information in the YI-AI will definitely sweep away the interference pattern for the atomic center-of-mass motion. This analysis popularly adopted in previous literatures by using the so-called orthodox methodology of quantum physics would lead to an outcome that is seemingly upholding the principle of WPD. However, a close look at this somewhat strange or even

ridiculous operation procedure of quantum physics methodology indicates that such a kind of interference pattern definition based on the atom–electron–photon system wave function would not be the true interference of the atom center-of-mass de Broglie wave, but rather a brutal artifact of the arbitrary application of the quantum mechanical methodology for the aim of some predesignated purposes. In practice, the outcome involved in Eqs. (14)–(16) is not equal to the observation of the atom center-of-mass interference pattern in usual interference screen as illustrated in Fig. 2(b), but should be acquired by an artificial correlation (via the coincidence counting technique popular adopted in quantum optics and quantum information sciences) of this atom center-of-mass interference pattern with the data set of micromaser cavity photon measurement.[34] In some sense, this quantum entanglement is just an artifact due to inappropriate implementation of the orthodox quantum mechanics operation rule in application to the interference pattern definition, observation, and interpretation. The claim that the principle of WPD is upheld via the mechanism of quantum entanglement between the which-path information and the which-path detector quantum state thus loses its physics ground.

To offer a clearer insight, we briefly discuss the evolution of the atom–electron–photon system wave function. Before the atom goes into the micromaser-cavity WM-PD, the system wave function is simply given by the direct wave function product of the three entities: atom, electron, and photon, which is

$$\begin{aligned}\Psi(R) &= \psi_{\rm acm}(R)\psi_{\rm electron}\psi_{\rm cavity}\\ &= \psi_{\rm acm,input}(R)\times|b\rangle\times(|0\rangle|0\rangle),\end{aligned}\tag{17}$$

because no interaction has begun. When the atom goes into the WM-PD, the system wave function is given by Eq. (14) due to the deterministic interaction between WM-PD and atom internal electronic states. After the atom passes through the WM-PD, the WM-PD and atom (together with its internal electron) will independently evolve in space–time. Then after the atom arrives at the interference screen and the microwave photon in the WM-PD is recorded, as obviously no interaction is ever in place at this event, the system wave function is given by

$$\begin{aligned}\Psi(R) &= \psi_{\rm acm}(R)\psi_{\rm electron}\psi_{\rm cavity}\\ &= \psi_{\rm acm,output}(R)\times|a\rangle\times(|0\rangle|0\rangle)\\ &= [a(R)\psi_{1,\rm tran}(R)+b(R)\psi_{2,\rm tran}(R)]\\ &\quad\times|a\rangle\times(|0\rangle|0\rangle),\end{aligned}\tag{18}$$

where we have reasonably assumed that the microwave photon has been annihilated due to the which-path information recording operation. The solution involved in Eqs. (17), (14), and (18) is a reasonable physical consequence of the fact that the atom–electron–photon system Hamiltonian takes very different forms in different space–time domains of the WM-AI, and thus the solution of system wave function must be highly space and time dependent. It is unphysical and wrong to use a single uniform wave function Eq. (14) to describe all events in all space–time domains of the WM-AI. These simple mathematical analyses again invalidate the previous claim that the quantum entanglement upholds the principle of WPD in this particular scheme of AI.

3.5. Alternative scheme of MZI-style WM-AI

Previously, we examined the interference pattern formed by the longitudinal component of the atom wave function, namely $\psi_1(R)=\psi_0\,{\rm e}^{{\rm i}kl+\varphi_1}$ *versus* $\psi_2(R)=\psi_0\,{\rm e}^{{\rm i}kl+\varphi_2}$. Lessons from photonic interferometers[35,36] show that there are other schemes to observe the interference pattern of the atom de Broglie wave. An example is illustrated in Fig. 3, where the second beam splitter of MZI-AI is replaced with an interference screen (IS) that is used to probe the interference pattern formed by the transverse component of the atom wave function, namely, $\psi_1(R)=\psi_0\,{\rm e}^{{\rm i}(k_{1x}x+k_{1y}y+\varphi_1)}$ *versus* $\psi_2(R)=\psi_0\,{\rm e}^{{\rm i}(k_{2x}x+k_{2y}y+\varphi_2)}$. For an IS 45° inclined with respect to the horizontal axis, when considering the random phase shift in the two arms similar to Eq. (13), the interference pattern is described by

$$\begin{aligned}I(R) &= 2A\Big[1+\cos(2\delta\varphi)\\ &\quad\times\cos\left(\frac{1}{\sqrt{2}}k_0x+\frac{1}{\sqrt{2}}k_0y+\varphi_1-\varphi_2\right)\Big].\end{aligned}\tag{19}$$

The operation principle is very similar to the setup illustrated in Fig. 2(a) for MZI-style WM-AI. In other words, if the MZI-AI in Fig. 2(a) works well, then the MZI-AI in Fig. 3 also works equally well.

It is worthwhile to say some more words about the practice perspective of WM-AI, as illustrated in Figs. 2 and 3. In these AI setups, one uses a microwave cavity WM-PD to probe the particle nature of the atoms and the interference-pattern detector to observe the wave nature of the atoms initially prepared in the high-orbital Rydberg electronic states. The operation principle of this WM-AI is very simple. The key ingredient to the success of these WM-PD AI setups is that the WM-PD imposes a very strong interaction with the internal quantum state of the atom via the photon–electron coupling to acquire definite which-path information and perfect path distinguishability D while the WM-PD imposes a very weak and almost negligible perturbation to the atom center-of-mass wave function, leading to almost non-degraded interference pattern and fringe visibility V. The condition for the WM-PD to work well is not stringent. First, the signal photon of the cavity must be much smaller in energy than the atom

momentum energy. This condition can be readily satisfied. Second, the quantum efficiency of WM-PD to probe a single signal photon should be sufficiently high. To achieve this stringent capability of single microwave photon detection, perhaps lessons and experiences from microwave telecommunication industries (*e.g.*, 4G and 5G technologies) and infrared single-photon detectors based on superconductivity or semiconductor avalanche photodiode (APD) technology can be learned and borrowed. Meanwhile, taking into account the state-of-the-art single-photon detection technologies, such as superconductor detector or semiconductor APD, one can turn to consider signal photon as the infrared and visible photons, instead of the microwave photons, to facilitate a much higher detection efficiency, but at the same time, adopting an atom or other particle system involving transition of two internal quantum states matching with the energy of infrared and visible signal photon. To ensure that the action of which-path information detection does not influence much the atom center-of-mass wave function, the energy of infrared or visible photon must still be much smaller than the momentum energy of the atom, namely, atom has a much higher motion speed.

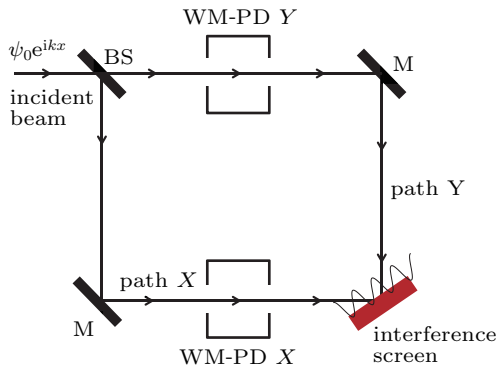


Fig. 3. A modified scheme of MZI-type WM-AI where the wave nature of the atoms is probed via observation of the transverse interference pattern while the particle nature of the atoms is probed by the two microwave-cavity WM-PDs. The two atom beams will cross at the site where the interference-screen detector of the atom is located and will in principle form an interference pattern in their wave functions.

4. Quantum mechanical analysis on WM-AI with Bragg-grating WM-PD

4.1. Setup for observing wave nature of the atom and wave function evolution

Due to the practical difficulty of the micromaser-cavity WM-AI, as discussed in the previous section, we turn now to consider other more practically available schemes to construct the WM-PD and WM-AI. An example is the WM-AI setup illustrated in Fig. 4 and realized experimentally by Rempe and coworkers in 1998,[14] where the microscopic object that is subject to the test of WPD is a ^{85}Rb atom beam. As illustrated in Fig. 4(a), the atom involves three atomic energy levels (more precisely, three outer-orbital electronic quantum states), including one upper optically excited state $|e\rangle$ and two lower hyperfine states $|2\rangle$ and $|3\rangle$. Transition can occur between $|2\rangle$ and $|3\rangle$ by shining microwave pulses. As illustrated in Fig. 4(b), the atomic beam is sent to pass through two one-dimensional (1D) standing-wave light beams, where the periodic optical field with intensity distribution $I(z) = I_0 \cos^2(k_{\text{light}} z)$ (1D optical lattice) imposes a light shift potential in the form of $U(z) = U_0 \cos^2(k_{\text{light}} z)$ to the atom and induces Bragg scattering and diffraction of the incoming atom beam into various orders (different transport directions).[43,44] Here, k_{light} is the wave vector of laser light beam. This 1D optical lattice, when deliberately designed, serves as a 50:50 beam splitter (BS) for an atom beam. The two 1D optical lattices basically serve as BS1 and BS2 in Figs. 1(a) and 2(a). An important thing is that the 1D optical-lattice Bragg-grating atomic beam splitters can be appropriately designed to ensure that the beam splitting action does not cause any loss to the coherence of the atom beam, much like an optical beam splitter does to photons. Moreover, this beam splitter does not change the internal quantum state of the atom. It is thus seen that the AI setup illustrated in Fig. 4(b) is basically an MZI-type AI. Yet, considering the finite atomic beam diameter and the resulting diffraction effect, the AI setup in Fig. 4(b) still involves some elements of YI-type AI, as illustrated in Figs. 1(b) and 2(b). Therefore, this AI setup can be named an MZ-YI mixture-type AI.

It is interesting to first analyze the evolution of the atomic wave function and see its power to test the principle of WPD of the atom using this mixture-type AI. Suppose that the incoming atom beam A is described by the spatial wave function of the atom center-of-mass as $\psi_{\text{A}}(R) = \psi_0 S_{\text{A}}(x,z)\, e^{i\boldsymbol{k}\cdot\boldsymbol{R}} = \psi_0 S_{\text{A}}(x,z)\, e^{i(k_x x - k_z z)}$ when making some certain approximation over the beam spatial profile and energy spectral profile of the atom. The internal electronic quantum state is supposed to lie in the ground state $|2\rangle$, so that the atom–electron composite wave function of the incoming atom beam A is given by $\Psi_{\text{A}}(R,r) = \psi_{\text{A}}(R)|2\rangle = \psi_0 S_{\text{A}}(x,z)\, e^{i(k_x x - k_z z)}|2\rangle$. The function $\psi_0\, e^{i(k_x x - k_z z)}$ describes the de Broglie wave of the atom beam with (k_x, k_z) being its propagation wave vector, while the function $S_{\text{A}}(x,z)$ describes the envelope profile of the finite-diameter atom beam, *e.g.*, a Gaussian atom beam. When passing through the first 1D optical-lattice beam splitter, beam A is Bragg-diffraction split into beam B and C, whose state evolutions are described by the composite wave function

$$\begin{aligned} \Psi_{\text{B}}(R,r) &= (\psi_0/\sqrt{2}) S_{\text{B}}(x,z)\, e^{i(-k_x x - k_z z)}|2\rangle, \\ \Psi_{\text{C}}(R,r) &= (\psi_0/\sqrt{2}) S_{\text{C}}(x,z)\, e^{i(k_x x - k_z z)}|2\rangle. \end{aligned} \tag{20}$$

When beams B and C proceed to transport and pass through the second 1D optical-lattice beam splitter, Bragg diffraction takes place once again, leading to four beams D–G. Their

atom–electron composite wave functions are described by

$$\begin{aligned}
\Psi_{\rm D}(R,r) &= (\psi_0/2)S_{\rm D}(x,z)\,{\rm e}^{{\rm i}(-k_x x-k_z z)}|2\rangle,\\
\Psi_{\rm E}(R,r) &= (\psi_0/2)S_{\rm E}(x,z)\,{\rm e}^{{\rm i}(-k_x x-k_z z)}|2\rangle,\\
\Psi_{\rm F}(R,r) &= (\psi_0/2)S_{\rm F}(x,z)\,{\rm e}^{{\rm i}(k_x x-k_z z)}|2\rangle,\\
\Psi_{\rm G}(R,r) &= (\psi_0/2)S_{\rm G}(x,z)\,{\rm e}^{{\rm i}(k_x x-k_z z)}|2\rangle. \qquad (21)
\end{aligned}$$

The envelope functions $S_{\rm B}(x,z)$–$S_{\rm G}(x,z)$ describe the diffraction effect of these finite-diameter atom beams. For simplicity of discussion, it is assumed here that all the envelope profile functions (from $S_{\rm A}(x,z)$ to $S_{\rm G}(x,z)$) are the same as $S(x,z)=1$. Note that in all these Bragg scattering of the atom beam, the internal quantum state maintains unchanged. In principle, there is no way to discern the which-path information of each atom detected at the cross-over region of the pair of parallel beam D and E, and at the cross-over region of another pair of parallel beam F and G. As a result, interference fringe was observed in the spatial overlap region of beams D and E, as well as of beams F and G.[14] According to the popular procedure of quantum mechanics operation methodology and considering the atomic beam diffraction effect, the corresponding interference pattern is described by

$$\begin{aligned}
I_{\rm DE}(R) &= A|\Psi_{\rm D}(R,r)+\Psi_{\rm E}(R,r)|^2 = I\cos(kx+\varphi_1),\\
I_{\rm FG}(R) &= A|\Psi_{\rm F}(R,r)+\Psi_{\rm G}(R,r)|^2 = I\cos(kx+\varphi_2). \qquad (22)
\end{aligned}$$

Here, I, k, φ_1, φ_2 are parameters describing the atom beam interference pattern and they are determined by specific experimental setup. One can observe that in both cross-over regions, the interference pattern has a perfect fringe visibility $V=1$ but completely null path distinguishability $D=0$. This result is of course in accordance with the principle of WPD. In other words, this mixture-type AI setup, although technically beautifully and elegantly designed and constructed, does not arm one with a better power of simultaneous observation of the wave and particle nature of the atom beyond that allowed by the principle of WPD.

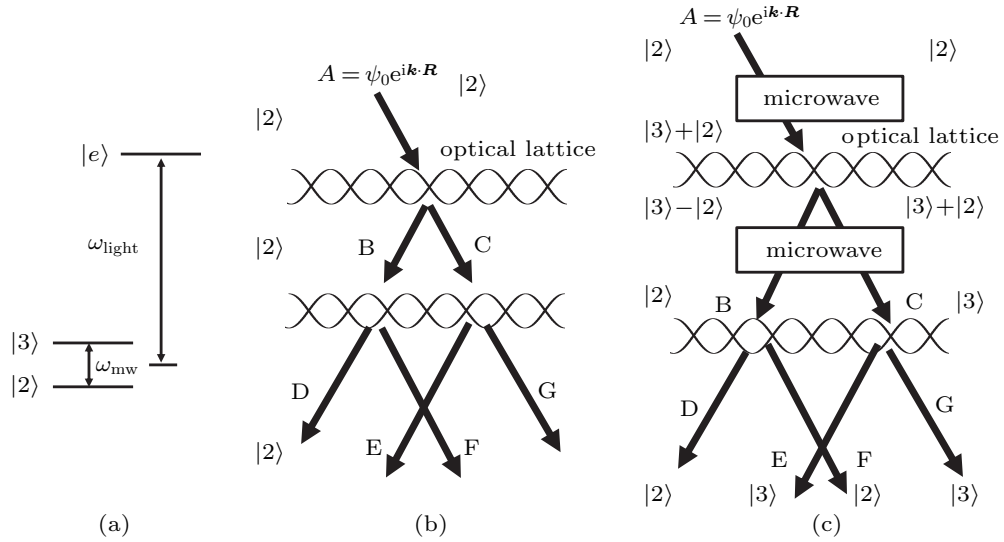


Fig. 4. Illustration of a MZI-YI mixture-type WM-AI involving two 1D optical-lattice Bragg gratings to serve as the first and second atomic beam slitter and two microwave pulses to prepare the bound electron in a specific hyperfine state to offer which-path information of the atom, and with both operations not degrading the coherence of the atom beam. (a) Relevant energy level of the Rydberg atom consisting of two ground hyperfine electronic states and one upper excited state; (b) the setup used to measure the wave nature of the atom, which involves only two atomic beam splitters but without applying the two microwave pulses to deliberately prepare the internal bound electron in a desirable hyperfine state; (c) the setup used to measure the particle nature of the atom, which involves both beam splitters and two microwave pulses to offer the which-path information of the atom via the internal quantum state. As the atom center-of-mass wave function is nearly not influenced by the microwave-pulse which-path probe action with an energy scale much smaller than the atom momentum energy, the WM-PD can unanimously monitor the passage of each atom so that the path distinguishability is $D=1$, and the interference fringe visibility still maintains a nearly perfect level of $V\to 1$. Therefore, this setup can reach an index of $V^2+D^2\to 2\gg 1$ and offer a superior power to simultaneously observe the wave and particle nature of the atoms.

4.2. WM-AI setup with WM-PD and wave function evolution

The situation is very different when the internal quantum state of the atom beam is encoded and correlated with its path of transport.[14] This is implemented by shining two microwave pulses to the atom beams during their transport, as depicted in Fig. 4(c).[14] These two microwave pulses will strongly modify the internal quantum state of the atom so that they encounter a definite desirable transition but only cause negligible degradation to the atom beam coherence, or mathematically the atom beam center-of-mass wave function. These two microwave pulses together with the two 1D optical-lattice Bragg grating serve the role of WM-PD, and this mixture-type AI is a workable scheme of WM-AI.

According to Ref. [14], via such an encoding operation, the atom–electron composite wave function evolves as fol-

lows:

$$\begin{aligned}
\Psi_{\rm A} &= \psi_0 \,{\rm e}^{\,{\rm i}(k_x x - k_z z)}|2\rangle \to \Psi_{\rm A} \\
&= (\psi_0/\sqrt{2})\,{\rm e}^{\,{\rm i}(k_x x - k_z z)}(|3\rangle + |2\rangle), \\
\Psi_{\rm B} &= (\psi_0/2) S_{\rm B}(x,z)\,{\rm e}^{\,{\rm i}(-k_x x - k_z z)}(|3\rangle - |2\rangle) \to \Psi_{\rm B} \\
&= (\psi_0/\sqrt{2}) S_{\rm B}(x,z)\,{\rm e}^{\,{\rm i}(-k_x x - k_z z)}|2\rangle, \\
\Psi_{\rm C} &= (\psi_0/2) S_{\rm C}(x,z)\,{\rm e}^{\,{\rm i}(k_x x - k_z z)}(|3\rangle + |2\rangle) \to \Psi_{\rm C} \\
&= (\psi_0/\sqrt{2}) S_{\rm C}(x,z)\,{\rm e}^{\,{\rm i}(k_x x - k_z z)}|3\rangle,
\end{aligned}$$

when the two microwave pulses are exerted to the two lower electronic states together with the first 1D optical-lattice beam splitter. After passing the two optical-lattice beam splitters and two encoding microwave pulses, the final four atom beams are described by

$$\begin{aligned}
\Psi_{\rm D}(R,r) &= (\psi_0/2) S_{\rm D}(x,z)\,{\rm e}^{\,{\rm i}(-k_x x - k_z z)}|2\rangle, \\
\Psi_{\rm E}(R,r) &= (\psi_0/2) S_{\rm E}(x,z)\,{\rm e}^{\,{\rm i}(-k_x x - k_z z)}|3\rangle, \\
\Psi_{\rm F}(R,r) &= (\psi_0/2) S_{\rm F}(x,z)\,{\rm e}^{\,{\rm i}(k_x x - k_z z)}|2\rangle, \\
\Psi_{\rm G}(R,r) &= (\psi_0/2) S_{\rm G}(x,z)\,{\rm e}^{\,{\rm i}(k_x x - k_z z)}|3\rangle. \qquad (23)
\end{aligned}$$

For simplicity, it is also assumed that all the envelope profile functions (from $S_{\rm A}(x,z)$ to $S_{\rm G}(x,z)$) are the same as $S(x,z)=1$. Furthermore, according to the popular procedure of quantum mechanics operation methodology, the corresponding interference pattern between parallel beams D and E and between parallel beams F and G are described by

$$\begin{aligned}
I_{\rm DE}(R) &= A|\Psi_{\rm D}(R,r) + \Psi_{\rm E}(R,r)|^2 \\
&= I_0 + I_1 \cos(kx + \varphi_1)\langle 2|3\rangle = I_0, \\
I_{\rm FG}(R) &= A|\Psi_{\rm F}(R,r) + \Psi_{\rm G}(R,r)|^2 \\
&= I_0 + I_1 \cos(kx + \varphi_2)\langle 2|3\rangle = I_0. \qquad (24)
\end{aligned}$$

It is seen clearly that the interference fringe completely disappears in both pairs of parallel beams, due to the negative modulation of the internal quantum state encoded which-path information against the spatial wave function of the atom beam. This result again seems to perfectly echo with the principle of WPD, namely: now that there is some way to identify the which-path information of the atoms (particle nature), one completely loses the power to observe an interference pattern of the atoms (wave nature).[14] The two setups in Figs. 4(b) and 4(c) thus each can probe perfectly one side of coin of the wave–particle entity of the atom, either wave or particle, at one time, but never both simultaneously. In some sense, these two setups can be combined together to form a delayed-choice scheme of AI similar to the classical delayed-choice MZI as discussed by Wheeler and others,[3] although nobody has made such an attempt.

4.3. Rigorous quantum mechanical analysis

Notice that these analyses are based on popular methodology of quantum mechanics, in particular pointing to the principle of WPD. According to the authors of Ref. [14], these theoretical analyses and results were in good agreement with their experimental observations. These results could also be attributed to the entanglement or correlation between the which-path information and the atom transport. More specifically, because the which-path information has been encoded within the internal quantum state for two arms of the atom beam and can be definitely known, the interference fringe must be smeared out according to the principle of WPD. This quantum entanglement enforced compliance with WPD is very different from the classical Heisenberg's position–momentum uncertainty relation enforcement of WPD in methodology but is exactly the same in effect; *i.e.*, both unable to simultaneously observe the wave and particle nature of the atoms.

Nonetheless, a lesson from the analysis made in Section 3 indicates that it deserves a more careful, close, and delicate examination on the physics of this atom interferometer, on its operation principle and details, on how theoretical and experimental analyses were made, and on how they reached good agreement in Ref. [14]. The basic principle of the mixture-type AI setup illustrated in Fig. 4 is the same as in Figs. 1–3. The problem of WPD should only be examined within the standard quantum mechanics, as described in Eqs. (1) and (2). The Hamiltonian for the atom–electron composite system under the interaction of 1D optical-lattice field and microwave pulse field reads

$$H_{\rm S} = H_{\rm acm} + H_{\rm int}^{\rm a.l.} + H_{\rm electron} + H_{\rm int}^{\rm e.m.} + H_{\rm int}^{\rm a.e.}, \qquad (25)$$

where $H_{\rm acm}$ denotes the ^{85}Rb atom center-of-mass momentum energy. $H_{\rm int}^{\rm a.l.}$ is the effective Hamiltonian accounting for the light shift interaction between atom and 1D optical lattice, and it basically originates from the standing-wave light (at frequency $\omega_{\rm light}$) induced transition between the excited electronic state $|e\rangle$ and the two lower states $|2\rangle$ and $|3\rangle$. This effective Hamiltonian accounts for the Bragg scattering and diffraction of the atom beam and the consequent beam splitting functionality. $H_{\rm electron}$ is the free Hamiltonian for the two lower states $|2\rangle$ and $|3\rangle$, $H_{\rm int}^{\rm e.m.}$ describes the coupling between the microwave pulse field at frequency $\omega_{\rm mw}$ and the two lower electronic states $|2\rangle$ and $|3\rangle$ accounting for the modification of the internal electronic quantum states by microwave pulse, and $H_{\rm int}^{\rm a.e.}$ is the electron transition induced atom center-of-mass recoiling interaction.

Before we solve the complicated Eq. (25), we make some notes. First, the light shift interaction $H_{\rm int}^{\rm a.l.}$ between atom and 1D optical lattice is strong enough to change the transport path of the atom but only implements simple functionality of beam splitting and will not destroy the coherence of the atom center-of-mass wave function. Second, the atom center-of-mass transport is only very weakly influenced by the transition between the electronic quantum states $|2\rangle$ and $|3\rangle$ induced

by the microwave pulse because the atom momentum energy is far larger than the microwave photon energy. Third, the physical effect and consequence of the light shift interaction $H_{\rm int}^{\rm a.l.}$ is sensitive to the internal electronic states encoded in $|2\rangle$ and $|3\rangle$. Through this correlation (or called quantum entanglement by numerous authors), the which-path information is automatically encoded within the internal electronic states. This physical picture means that the quantum problem described by Eq. (25) also involves two-level physics. The first-level physics concerns the interaction of the lower two-level electron states with the microwave pulse field to prepare an appropriate electronic ground state. Through this interaction, the WM-PD acquires the which-path information of the atom through definite correlation between the transport path and the internal electronic quantum states. The second-level physics concerns the influence of the correlation interaction of the internal electronic quantum states upon the atom transport and its wave function. By using the classical Born–Oppenheimer approximation concept with a high precision, these two-level physics can be separately considered as follows:

$$H_{\rm level\ 1} = H_{\rm electron} + H_{\rm int}^{\rm e.m.}, \tag{26}$$

$$H_{\rm level\ 2} = H_{\rm acm} = H_{0,\rm acm} + H_{\rm int}^{\rm a.l.} + H_{\rm int}^{\rm a.e.} = -\frac{\hbar^2}{2m_{\rm a}}\frac{\partial^2}{\partial R^2} + H_{\rm int}^{\rm iqs}(R,r). \tag{27}$$

Note that the Hamiltonian of electron-optical lattice-microwave system as expressed in Eq. (25) has been written in the laboratory coordinate. This involves the atom center-of-mass momentum energy term that accounts for the fact that the electron transports along with the atom center-of-mass besides its own orbital motion surrounding the atom nucleus. Meanwhile, the Hamiltonian of electron-microwave system as expressed in Eq. (26) has been written in the atom center-of-mass coordinate (*i.e.*, assuming atom is in static state), while the Hamiltonian of the atom as expressed in Eq. (27) has also been written in the laboratory coordinate.

4.4. Wave function evolution of the atom center-of-mass and internal electronic state

The general solution to the electronic quantum physics as described in Eq. (26) can be written as

$$\Psi_{\rm electron}(r) = \sum_i a_i \psi_i(r) \tag{28}$$

in the atom center-of-mass coordinate. Yet, in the laboratory coordinate where the electron moves along with the atom center-of-mass within the mixture-type WM-AI illustrated in Figs. 4(b) and 4(c), we find

$$\Psi_{\rm electron}(R,r) = \sum_i a_i \Psi_{{\rm acm},i}(R)\psi_i(r). \tag{29}$$

This composite wave function just describes the translational motion of electron along with its mother atom nucleus as described by $\Psi_{{\rm acm},i}(R)$, as well as its orbital motion surrounding the nucleus as described by $\psi_i(r) = |e\rangle,\ |2\rangle,\ |3\rangle$, which is the so-called internal electronic quantum state of the atom. Considering the experimental configuration of the mixture-type WM-AI in Fig. 4, the electronic wave function is only those expressed in Eqs. (21) and (23) for the situation with and without the microwave pulse interaction against the internal electronic quantum state of the atom. Therefore, the quantum interference as expressed in Eqs. (22) and (24)is just the interference of the de Broglie wave of internal electrons of the atom, rather than the pure atom center-of-mass de Broglie wave.

The interference pattern of the desirable atom center-of-mass in principle should be traced back to the atom wave function, whose evolution must be solved via Eq. (27). Assuming that the perturbation of the interaction of internal electronic quantum state with the 1D optical lattice and microwave pulse field is very small in energy scale when compared with the atom center-of-mass momentum energy, the wave function should generally be

$$\Psi_{\rm acm}(R) = \sum_i a_i \Psi_{{\rm acm},i}(R)\,{\rm e}^{{\rm i}\varphi_i}\,{\rm e}^{{\rm i}\delta\varphi_i}, \tag{30}$$

where we have considered the phase shift φ_i during the transport of the atom center-of-mass, as well as the random phase shift $\delta\varphi_i$ due to uncontrollable perturbation for each path the atom takes, when considering the atomic core recoiling effect induced by the external optical and microwave interaction. For the setup to measure the wave nature (interference fringe pattern) of the atom as illustrated in Fig. 4(b), the wave function for the four channels of the atomic beam is given by

$$\begin{aligned}
\Psi_{\rm acm,D} &= (\psi_0/2)S_{\rm D}(x,z)\,{\rm e}^{{\rm i}(-k_x x - k_z z)+{\rm i}\varphi_{\rm D}+{\rm i}\delta\varphi_{\rm D}},\\
\Psi_{\rm acm,E} &= (\psi_0/2)S_{\rm E}(x,z)\,{\rm e}^{{\rm i}(-k_x x - k_z z)+{\rm i}\varphi_{\rm E}+{\rm i}\delta\varphi_{\rm E}},\\
\Psi_{\rm acm,F} &= (\psi_0/2)S_{\rm F}(x,z)\,{\rm e}^{{\rm i}(k_x x - k_z z)+{\rm i}\varphi_{\rm F}+{\rm i}\delta\varphi_{\rm F}},\\
\Psi_{\rm acm,G} &= (\psi_0/2)S_{\rm G}(x,z)\,{\rm e}^{{\rm i}(k_x x - k_z z)+{\rm i}\varphi_{\rm G}+{\rm i}\delta\varphi_{\rm G}}.
\end{aligned} \tag{31}$$

Meanwhile, for the setup measuring the particle nature (which-path information) of the atom as illustrated in Fig. 4(c) and in reference to Eq. (23), the wave function for the four channels of the atomic beam is still given by Eq. (31).

Considering this specific situation of the atom center-of-mass wave function evolution, the internal electronic quantum states encoding the which-path information of the atom are given by

$$\begin{aligned}
\Psi_{\rm electron,D} &= (\psi_0/2)S_{\rm D}(x,z)\,{\rm e}^{{\rm i}(-k_x x - k_z z)+{\rm i}\varphi_{\rm D}+{\rm i}\delta\varphi_{\rm D}}|2\rangle,\\
\Psi_{\rm electron,E} &= (\psi_0/2)S_{\rm E}(x,z)\,{\rm e}^{{\rm i}(-k_x x - k_z z)+{\rm i}\varphi_{\rm E}+{\rm i}\delta\varphi_{\rm E}}|2\rangle,\\
\Psi_{\rm electron,F} &= (\psi_0/2)S_{\rm F}(x,z)\,{\rm e}^{{\rm i}(k_x x - k_z z)+{\rm i}\varphi_{\rm F}+{\rm i}\delta\varphi_{\rm F}}|2\rangle,\\
\Psi_{\rm electron,G} &= (\psi_0/2)S_{\rm G}(x,z)\,{\rm e}^{{\rm i}(k_x x - k_z z)+{\rm i}\varphi_{\rm G}+{\rm i}\delta\varphi_{\rm G}}|2\rangle,
\end{aligned} \tag{32}$$

for the setup measuring the wave nature (interference fringe pattern) of the atom as illustrated in Fig. 4(b), while they are given by

$$\begin{aligned}
\Psi_{\text{electron,D}} &= (\psi_0/2)S_{\text{D}}(x,z)\,\mathrm{e}^{\mathrm{i}(-k_x x-k_z z)+\mathrm{i}\varphi_{\text{D}}+\mathrm{i}\delta\varphi_{\text{D}}}|2\rangle,\\
\Psi_{\text{electron,E}} &= (\psi_0/2)S_{\text{E}}(x,z)\,\mathrm{e}^{\mathrm{i}(-k_x x-k_z z)+\mathrm{i}\varphi_{\text{E}}+\mathrm{i}\delta\varphi_{\text{E}}}|3\rangle,\\
\Psi_{\text{electron,F}} &= (\psi_0/2)S_{\text{F}}(x,z)\,\mathrm{e}^{\mathrm{i}(k_x x-k_z z)+\mathrm{i}\varphi_{\text{F}}+\mathrm{i}\delta\varphi_{\text{F}}}|2\rangle,\\
\Psi_{\text{electron,G}} &= (\psi_0/2)S_{\text{G}}(x,z)\,\mathrm{e}^{\mathrm{i}(k_x x-k_z z)+\mathrm{i}\varphi_{\text{G}}+\mathrm{i}\delta\varphi_{\text{G}}}|3\rangle \qquad (33)
\end{aligned}$$

for the setup measuring the particle nature (which-path information) of the atom as illustrated in Fig. 4(c). It can be clearly seen that the wave function of internal electronic state as displayed in Eqs. (32) and (33) is the composite of the atom center-of-mass wave function (describing the spatial motion of electron together with the atom) and the inner electronic state (describing the electronic motion around the atom nucleus). Therefore, this wave function has involved the same spatial motion information (and thus which-path information and interference pattern) as the atom wave function and is depicted in Eq. (31). This close connection and correlation is the physics basis for using the internal quantum state to serve as the WM-PD to probe the which-path information, while avoiding any remarkable perturbation and influence to the atom center-of-mass motion wave function and thus the interference pattern observation. However, this connection and correlation (or identification in some sense) will cause serious ambiguity to the interpretation of experimental data if caution is not taken because of ignorance in physics.

Now, we turn back to calculation of the interference pattern and which-path information in terms of new atom wave function. Because $\delta\varphi_i$ is negligible, the interference patterns under both situations of applying and not applying the microwave pulses are both written as

$$\begin{aligned}
I_{\text{DE}}(R) &= A|\Psi_{\text{acm,D}}(R)+\Psi_{\text{acm,E}}(R)|^2\\
&= I_0+I_1\cos(kx+\varphi_1),\\
I_{\text{FG}}(R) &= A|\Psi_{\text{acm,F}}(R)+\Psi_{\text{acm,G}}(R)|^2\\
&= I_0+I_1\cos(kx+\varphi_2). \qquad (34)
\end{aligned}$$

Equation (34) tells us that in both situations of encoding or not encoding the which-path information within the internal electronic quantum state, the atom center-of-mass interference shows effectively the same good interference fringe because atom motion does not suffer from remarkable random phase shift during its transport in this WM-AI. Therefore, it is feasible to use the scheme of mixture-type WM-AI with WM-PD to simultaneously observe the wave and particle nature of the atom, and this is obviously in drastic contrast with the conclusion made in Ref. [14].

4.5. Origin of quantum entanglement enforcing WPD

Comparison of our previous theoretical analyses against the theoretical analyses made in Ref. [14] has raised very different results and insights about the interference of the atom beam when its transport is encoded or not encoded with a which-path mark within the internal electronic quantum state of the atom. In Ref. [14], the atom motion is described by the center-of-mass wave function encoded with the internal electronic quantum state, as illustrated explicitly in Eqs. (21)–(24). However, in our analysis, this composite wave function just describes the overall motion of the internal electron in space; *i.e.*, both its translational motion accompanying with its mother atom nucleus and its orbital motion surrounding the nucleus, but not the pure atom center-of-mass motion wave function. In essence, it is this wave function that is responsible for the interference of the atom de Broglie wave (not the internal electrons).

A strict and reasonable argument would justify that if one wants to test the principle of WPD of atoms, then what one needs to do about both theoretical calculations and experimental observations (including data acquisition, processing, analysis, and conclusion) must be based on the microscopic wave function of the atoms; *i.e.*, $\Psi_{\text{acm,D}}-\Psi_{\text{acm,G}}$. All the external influences, no matter whether large or small, against the atoms must be eventually traced back to their influence to this wave function. In particular, the random phase shift $\delta\varphi_i$, and quantum mechanics, in the formulation of Schrödinger's equation, offers a reliable and accountable tool to quantitatively and accurately evaluate these influences. This is basically why we use Eqs. (30), (31), and (34) to calculate the interference pattern. We can also handle the principle of WPD of electrons, which are bound electrons within atoms rather than free electrons in usual interferometers (like free atoms in current atom interferometers), although, of course it is relatively unusual to talk about WPD for bound electrons. We then need to trace back to the wave function of electrons, which are approximately Eqs. (32) and (33) when we only consider the influence from the atom recoiling motion and neglect other influences originated from uncontrollable microwave and light interaction with the three electronic quantum states ($|2\rangle$, $|3\rangle$, and $|e\rangle$). In this situation, the interference pattern arising from the bound electron should be

$$\begin{aligned}
I_{\text{DE}}(R) &= A|\Psi_{\text{electron,D}}(R,r)+\Psi_{\text{electron,E}}(R,r)|^2\\
&= I_0+I_1\cos(kx+\varphi_1)\langle 2|2\rangle\\
&= I_0+I_1\cos(kx+\varphi_1),\\
I_{\text{FG}}(R) &= A|\Psi_{\text{electron,F}}(R,r)+\Psi_{\text{electron,G}}(R,r)|^2\\
&= I_0+I_1\cos(kx+\varphi_2)\langle 2|2\rangle\\
&= I_0+I_1\cos(kx+\varphi_2), \qquad (35)
\end{aligned}$$

for the wave-nature probe setup as displayed in Fig. 4(b), and

$$I_{\mathrm{DE}}(R) = A|\Psi_{\mathrm{electron,D}}(R,r)+\Psi_{\mathrm{electron,E}}(R,r)|^2 \quad (36)$$
$$= I_0 + I_1\cos(kx+\varphi_1)\langle 2|3\rangle = I_0,$$
$$I_{\mathrm{FG}}(R) = A|\Psi_{\mathrm{electron,F}}(R,r)+\Psi_{\mathrm{electron,G}}(R,r)|^2$$
$$= I_0 + I_1\cos(kx+\varphi_2)\langle 2|3\rangle = I_0, \quad (37)$$

for the particle-nature probe setup as displayed in Fig. 4(c). It can thus be seen that the two setups are complementary with each other in testing the principle of WPD for electrons: the first is able to probe perfectly the wave nature (interference pattern) but completely loses the power to probe the particle nature (which-path information) of electron, while the second is able to probe the full particle nature while completely unable to observe the wave nature of electron. This mixture-type AI armed with WM-PD still does not go beyond the principle of WPD, or the principle of complementarity, as analyzed and emphasized in Ref. [14].

Having analyzed the wave function evolution of the atom center-of-mass motion and internal electron quantum states, we now go on to briefly discuss how to do experiments to measure both the interference patterns of the atom and electron and their which-path information (identical for both atom and electron because they are bound together) in accordance with the previous theoretical analyses. As noticed in our previous work,[34] it is critical to design and use appropriate instruments to implement the measurement against a specific physical quantity, either the interference pattern or the which-path information in the current problem. For instance, reference[42] has discussed a large amount of fine technologies (*e.g.*, atom detectors based on hot-wire ionizer and ion collector) to precisely observe the interference pattern of the atomic de Broglie wave, which is just the atom number density (*i.e.*, atom population, proportional to the probability intensity $|\Psi_{\mathrm{acm}}(R)|^2$) as a function of its spatial position R.

We turn to discuss the experimental work on the test of WPD principle for atom as presented in Ref. [14], in particular the observation of interference pattern of the atom de Broglie wave under the setups illustrated in Fig. 4. In this work, the authors observed appearance of fine interference fringe pattern for the setup as displayed in Fig. 4(b), and complete disappearance of interference fringe pattern for the setup as displayed in Fig. 4(c). Because of this seemingly good agreement between theory and experiment, the authors claimed that their experiments would still uphold the principle of WPD and complementarity, this time not through the mechanism of Heisenberg's position–momentum uncertainty but through the mechanism of quantum entanglement. According to the authors, "the mere fact that which-way information is stored in the detector and could be read out already destroys the interference pattern" no matter whether or not one really reads the which-way information, namely by probing whether the atom is in the $|2\rangle$ state or in the $|3\rangle$ state.

A close analysis over the experimental technology presented in Ref. [14] shows that the interference pattern was probed by fluorescence pattern of the atom beam. Because the probe light is near resonance with the upper and lower electronic states as depicted in Fig. 4(a), this fluorescence observation technique in fact probes the composite atom–electron wave function, or equivalently the electronic wave function in the laboratory coordinate space, and their interference pattern as described by Eqs. (35) and (36). To observe the true interference pattern of the atom de Broglie wave, one should adopt techniques that are not sensitive to the internal electronic quantum state but only sensitive to the atom center-of-mass; for instance, the hot-wire ionizer, as described in Ref. [42].

5. Other architecture designs for WM-AI and WM-particle interferometers

The mixture-type WM-AI illustrated in Fig. 4 has involved two major elements of technology to implement the fundamental purpose of testing the principle of WPD for atoms. The first is the use of microwave pulses to prepare the internal electronic state of the atom into a predesignated quantum state that has definite and unique correlation with the path of motion that the atom takes so that the which-path information is encoded within the internal electronic state. The second is the use of two 1D optical-lattice Bragg gratings to serve as the atomic beam splitter that can selectively separate the incoming atom beam into different directions dependent on the internal electronic state of the atom while maintain the good coherence of the atomic beam by imposing as small as possible a perturbation to the atomic beam wave function. The success of this WM-AI for testing the principle of WPD completely originates from these two technological innovations. After almost 20 years, these detection technologies should have evolved to an even much higher level so that we are now much better at performing again similar experiments to test the principle of WPD.

When we are only concerned with one specific channel of interference pattern observation, such as beams D and E or beams F and G, then the functionality of the mixture-type WM-AI can be modeled and summarized into a simplified version of the MZI-type WM-AI, as schematically illustrated in Fig. 5. This simple simplification and extension would result in a new scheme of WM-AI that can describe a much broader range of AI scheme than the WM-AI, as illustrated in Fig. 4. The crucial composite device in the new scheme of WM-AI is designated as the polarization beam splitter (PBS), which would split the incoming atomic beam into the direct transmission path (path Y) for atoms with the internal electronic quantum state φ_1 and into the 90° reflection path (path X) for atoms

with the internal electronic quantum state φ_2. If the incident atom beam is in the electronic state of $\varphi_1+\varphi_2$, after passing through the PBS, it would be split into two beams by 50:50, one along the path Y and in the electronic state φ_1, while one along the path X and in the electronic state φ_2. Obviously, the PBS in the current MZI-AI is similar in functionality to the well-known true PBS, which is a popular optical device in doing quantum optics and quantum information experiments for photons. In fact, we directly borrow the popular nomenclature in optics and photonics and apply it to atomic physics in the current work. Of course, the concept of PBS can apply to any particles other than atoms with two internal quantum states.

Obviously, the major difference of the WM-AI scheme in Fig. 5 from the WM-AI scheme illustrated in Figs. 2(a) and 3 is that the PBS is used to replace the beam splitter. For an incoming atom beam that is described by the atom–electron composite wave function as $\Psi(R,r)=\psi_0\,\mathrm{e}^{\mathrm{i}kx}[\varphi_1(r)+\varphi_2(r)]$, the beam transporting along the path X carries the atom–electron wave function as $\Psi_x(R,r)=a_x\psi_0\,\mathrm{e}^{\mathrm{i}kx}\varphi_1$, while another beam transporting along the path Y carries the atom–electron wave function as $\Psi_y(R,r)=a_y\psi_0\,\mathrm{e}^{\mathrm{i}ky}\varphi_2$. Note that an appropriate design of this PBS should ensure that the internal state selective beam splitting operation of the PBS does not cause any loss to the coherence of the atom beam, much like its optical counterpart does to photons.

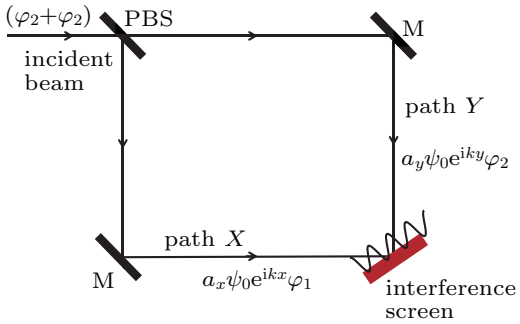


Fig. 5. A general scheme of MZI-type WM-AI involving an internal quantum state correlated beam splitter as the WM-PD. The wave nature of the atoms, the interference pattern, is probed via observation of the transverse interference pattern while the particle nature of the atoms, the which-path information, is probed automatically via the PBS. The incident beam is described by a composite wave function $\psi_0\,\mathrm{e}^{\mathrm{i}kx}(\varphi_1+\varphi_2)$, which involves the usual coherent spatial wave function $\psi_0\,\mathrm{e}^{\mathrm{i}kx}$ but dressed with a coherent superposition of internal quantum state φ_1 and φ_2. The PBS selectively transmits the beam component with state φ_2 along the path Y, while it reflects the beam component with state φ_1 along the path X, thus automatically offering the which-path information. The transverse interference pattern arising from the two atom beam cross over is recorded by a deliberately designed interference screen that is only sensitive to the atom center-of-mass spatial state ψ_0, but is insensitive to the internal electronic state φ_1 and φ_2. This simple scheme of PBS WM-AI can also offer a superior power to simultaneously observe the wave and particle nature of the atoms, and can be applicable to other massive particles.

According to the standard interpretation of the principle of WPD, with Ref. [14] serving as a representative example, because the internal quantum state (φ_1 or φ_2) has provided a natural way to identify the which-path information of each atom, in principle there should be no way to observe the interference pattern of the atom at the IS. In this situation, the principle of WPD is upheld by the entanglement or correlation between the internal quantum state encoded which-path information and the atom motion. Once this which-path information is erased, the interference pattern is recovered.[14,15,20] These physical arguments can be placed in more mathematical terms. According to Refs. [14], [15], and [20] the interference pattern is described by

$$\begin{aligned} I(R) &= A|\Psi(R,r)|^2 = A|\Psi_x(R,r)+\Psi_y(R,r)|^2 \\ &= A[a_x^2+a_y^2+2a_xa_y\cos(kx-ky+\varphi)\langle\varphi_1|\varphi_2\rangle]. \end{aligned} \tag{38}$$

If the which-path information is definitely encoded in the internal quantum state of the atom, then $\langle\varphi_1|\varphi_2\rangle=0$, the interference fringe disappears. However, if the which-path information is missing so that $\varphi_1\approx\varphi_2=\varphi$ and $\langle\varphi_1|\varphi_2\rangle=1$, then the interference fringes recovers. According to the literature, the principle of WPD is upheld by the entanglement between the internal quantum state of the atom and its transport path along the two arms of AI. Because the PBS only causes beam splitting of the atom beam through interaction with the internal quantum states, it does not induce any degradation of the spatial coherence of the atom beam due to the inevitable introduction of uncontrollable random phase shift $\delta\varphi$ to the wave function.

Our systematic analyses and discussions in Section 4 show that the physics represented in Eq. (38) is indeed the interference for the internal state rather than the atom de Broglie wave. This equation can be regarded as the convolution of the interference pattern formed by the atom center-of-mass de Broglie wave in the laboratory space and the interference pattern formed by the internal electronic state within the subatomic space, with the latter being the modulation factor to the former. Because of the negative modulation factor, the interference fringe pattern of the atom center-of-mass de Broglie wave seemingly disappears. However, this result does not represent the true disappearance of interference fringe for the atom center-of-mass de Broglie wave, which should be described by the following formula:

$$\begin{aligned} I(R) &= A|\Psi(R)|^2 = A|\Psi_x(R)+\Psi_y(R)|^2 \\ &= A[a_x^2+a_y^2+2a_xa_y\cos(kx-ky+\varphi)]. \end{aligned} \tag{39}$$

Equation (39) means that due to the maintenance of coherence of the two atomic beams, the interference fringe pattern appears with a visibility close to $V=1$. Because one can automatically identify the which-path information of each atom through their internal electronic state, the path distinguishability is $D=1$. As a result of $V^2+D^2\to 2\gg 1$, this general PBS-type WM-AI as depicted in Fig. 5 can allow for a much higher power to simultaneously observe the wave and particle

nature of the atom than previous schemes of AI either working in the principle of Heisenberg's uncertainty relation or the principle of quantum entanglement. Of course, to acquire such a superior power experimentally, an appropriate observation technique that is only sensitive to the atom's center-of-mass but is insensitive to the internal quantum state φ_1 and φ_2 must be designed, chosen, and used with great caution.

The working principle of the PBS-type WM-AI as depicted in Fig. 5, the optical-lattice Bragg-grating WM-AI as illustrated in Fig. 4, and the microcavity-type WM-AI as presented in Fig. 2 can be applied not only to atoms but also applicable to other types of massive particles (such as electrons, neutrons, molecules, *etc*.). These massive particles should have certain internal quantum states (*e.g*., spins and magnetic moments for electrons and neutrons) that can be probed with high sensitivity by some specific deliberately designed external advanced detectors. In addition, all of the detection operation energy with the internal quantum states should be set to be arbitrarily weak compared with the momentum energy of massive particles under the test of the WPD principle. Simple analyses, similar to those presented in the previous two sections, will automatically point to a simple conclusion that these WM-particle interferometers can arm people with an unprecedented power to simultaneously observe the wave and particle nature of all these massive particles. This would far exceed many historical interferometers governed either by the principle of Heisenberg's uncertainty relation or by the principle of quantum entanglement. However, whether or not these interferometers work with massless particles such as photons remains an unresolved issue that needs additional systematic studies.

This discussion raises another issue about the relation between the concept of identical particles and the check of WPD. In many classical two-slit or two-arm interferometer experiments of particles such as electrons and neutrons, it is generally agreed that if the particles in two channels are identical, namely, if there is no way to identify their path information, then a perfect interference pattern is observed. However, this condition of an identical particle is not necessary when considering their internal freedoms, such as spins or others. In practical electrons and neutrons interference experiments, the incoming electron or neutron beam upon two-slit or two-arm interferometers does not have identical spins or other internal freedoms because no special means and measures are adopted to guarantee this condition. Strictly speaking, the incident electron and neutron beams are not identical particles in two channels, but all experiments have shown that this non-identical particle situation does not influence the observation of perfect interference pattern. The reason for this is simple: the interference pattern detectors in these interferometers are insensitive to these internal freedoms. In other words, it is possible to only observe the interference pattern of electron and neutron center-of-mass while completely neglecting their internal freedoms. One can either simply neglect these internal freedoms and miss the which-path information (like in classical interferometers) or deliberately utilize them to serve as powerful which-path detectors and tell which-path the particle goes (like in our current work), although neither these operations nor measures influence the essence of wave nature of the particles.

6. Summary

In summary, we have presented a systematic theoretical analysis in the framework of quantum mechanics upon several historical *gedanken* or realistic atomic interferometers to test the principle of WPD and complementarity, including Einstein's recoiling slit, Feynman's light microscope, Scully's micromaser-cavity interferometer, and Rempe's optical-lattice Bragg-grating interferometer. These interferometers are in various operation schemes such as MZI-AI, YI-AI, or MZI-YI mixture-type AI. No matter how smart and delicate they were, they were long believed to strictly uphold the principle of WPD — either by the mechanism of the Heisenberg's position–momentum uncertainty relation or quantum entanglement. All practical experiments have so far confirmed the orthodox interpretation of quantum physics that WPD is the intrinsic property of all massless and massive microscopic particles and no one is able to simultaneously observe the wave and particle nature of these microscopic objects.

To examine and evaluate these schemes of AI from a new point of view and with a hope to shed some new light on these old and somewhat unresolved fundamental quantum physics problems, we commit to present a reasonable and trustful quantum theoretical analysis strictly in the framework of orthodox quantum mechanics methodology by using Schrödinger's equation to evaluate the time–space evolution of wave function under a specific model of Hamiltonian. Because a practical AI involves microscopic entities such as the atom center-of-mass and internal electrons bound to the nucleus, and their interaction with various composite devices to implement measurement of the wave and particle behavior of the atom, it is important to take some necessary steps to reasonably solve this fundamental problem. The first step is to construct a reasonable model of Hamiltonian and wave function to correctly describe relevant interaction and measurement processes. The second step is to solve the time–space evolution of various wave functions and identify their contribution to the observation of wave (interference pattern) and particle (which-way information). The third step is to correctly calculate the quantitative index of the interference fringe visibility V and the which-path distinguishability D for atom (more accurately, the atom center-of-mass de Broglie wave) and take caution not to

mix with other microscopic entities, such as the internal bound electron quantum states and the which-path detector quantum states. The fourth and final step is to draw a complete and balanced picture about the ultimate power of a specific AI to observe the wave and particle nature of the atoms, and then tell whether it upholds or breaks down the principle of WPD.

Following this logic of reasoning and examination, we have first chosen to handle the historical YI-type AI schemes, such as Einstein's recoiling slit and Feynman's light microscope. Because these *gedanken* schemes of AI use an external position detector that directly interacts with the atom center-of-mass to determine unanimously the which-path information of the atom, they can be categorized as the SM-AI involving an SM-PD. For these AIs, we have introduced a quantum mechanical model that involves a Hamiltonian composed of the atom center-of-mass momentum energy term $H_{\rm free}(R)$ and the which-path measurement interaction Hamiltonian $H_{\rm int}^{\rm acm}(R)$. The solution of Schrödinger's equation in a reasonable approximation has introduced a random phase shift $\delta\varphi$ of significant strength to the free-transport wave function of the atom beam for each channel (one slit in YI-AI or one arm in MZI-YI), and the superposition of these two randomly fluctuated wave functions in the observation screen will inevitably result in an interference pattern of null visibility, namely: $V = 0$ when $D = 1$. Meanwhile, when no which-path information is probed by external detectors so that $H_{\rm int}^{\rm acm}(R)$ disappears, the superposition of the two unperturbed free-transport wave functions in the observation screen will result in an interference pattern of perfect visibility, namely, $V = 1$ but $D = 0$. Thus, this type of SM-AI involving SM-PD perfectly echoes with the principle of WPD, which is upheld by the mechanism of the Heisenberg's position–momentum uncertainty relation in these AIs.

Having noticed that the SM-AI is unable to arm one with the power to simultaneously observe the wave and particle nature of the atom, we turn to another category of AI scheme called WM-AI and see whether these AI setups can allow for a better power to go beyond the limitation of WPD. By introducing WM-PD into the AI setup, we can now probe the which-path information by making the which-path detector only interact with the internal electronic quantum state of the atom (described by Hamiltonian $H_{\rm int}^{\rm iqs}(R,r)$) with a very weak energy scale (*e.g.*, microwave photon energy) that is far smaller than the atom's center-of-mass momentum energy. The detection operation of the which-path information will cause deterministic internal transition of electronic quantum states but will only have a negligible influence on the free-transport atom center-of-mass wave function and also their interference pattern. In principle, these WM-AIs are able to offer us with a much higher capability of simultaneously observing the wave and particle behavior of the atom.

Following such an intuitive insight of physics, we consider two WM-AI schemes . The first, as first analyzed by Scully and coworkers in 1991, is designed to test the principle of WPD for a Rydberg atom with two high-orbital electronic states. The most crucial part of this setup is two high-Q micromaser cavities to serve as the WM-PD, which can probe the passage of an atom via sensing the spontaneously emitting microwave photon originated from down transition of bound electron from the excited state to the ground state. We have considered the influence from various factors from the composite atom–electron–cavity system and built a quantum mechanical model involving all these features. The solution of Schrödinger's equation to this complicated quantum mechanical problem indicates that in a reasonable approximation, this problem could be separated and simplified into a two-level quantum physics problem because of the weak perturbation nature of the which-path detector against the atom center-of-mass motion via the emission and probe of microwave photon. The first-level physics handles the deterministic probe of microwave photon by the micromaser-cavity which-path detector and deals with the microwave photon interaction with the internal electronic quantum states. The second-level physics deals with the microwave photon emission and detection induced recoiling perturbation to the atom center-of-mass wave function. Our analysis for a practical energy scale of the atom momentum energy and microwave photon energy indicates that the weak perturbation induced random phase shift to the atom wave function in two channels is so small that it only has a very minute degraded influence on the interference pattern and the fringe visibility. As a result, while the WM-PD can unanimously monitor the passage of each atom so that the path distinguishability is $D = 1$, the interference fringe visibility still maintains a nearly perfect level of $V \to 1$. Thus, we see that this WM-AI in both the MZI and YI configurations can achieve $V^2 + D^2 \to 2 \gg 1$ and thus offer a much higher power to simultaneously observe the wave and particle nature of the atoms than classical schemes of SM-AI as Einstein's recoiling slit and Feynman's light microscope.

The second scheme of WM-AI, which is an MZI-YI mixture-type AI and was first analyzed by Rempe and coworkers in 1998, uses an atom of two ground hyperfine electronic states and one upper excited state for testing the principle of WPD. The crucial parts of this setup are two microwave pulses to prepare the bound electron in a specific hyperfine state, and two 1D optical-lattice Bragg gratings to serve as the first and second atomic beam slitter. Importantly, the two atomic beam splitters can selectively split the incoming atomic beam into different directions dependent on the internal electronic quantum state, thus the which-path information of the atom is automatically encoded within the internal state. Equally importantly, the operations of the which-path information encoding and beam splitting do not cause any considerable perturba-

tion and degradation to the atomic beam coherence. Thus, the combinative action of beam splitters and microwave pulses serves as the WM-PD for the incoming atom beam. We then consider the influence from various factors of the composite interacting system involving the atom, beam splitters, and microwave pulses and build a quantum mechanical model in combination with the solution of Schrödinger's equation. In addition, under the reasonable two-level physics model, we are able to solve the time–space evolution of both the atom center-of-mass wave function and the internal electronic state bound to atom. The results of such a thorough and systematic quantum mechanical analysis show that the microwave pulse interaction with the internal electronic state only induces a very weak recoiling perturbation and thus a negligible random phase shift to the atom wave function in two channels. Consequently, this operation of which-path information encoding and detection only cause very minute degradation to the interference pattern and the fringe visibility of the atom's de Broglie wave. This analysis indicates that while the WM-PD can unanimously monitor the passage of each atom so that the path distinguishability is $D = 1$, the interference fringe visibility still maintains a nearly perfect level of $V \to 1$. Therefore, we see that this MZI-YI mixture-type WM-AI can also reach an index of $V^2 + D^2 \to 2 \gg 1$ and thus offer a superior power to simultaneously observe the wave and particle nature of the atoms.

Our quantum mechanical analyses clearly show that the well-established mechanism of the Heisenberg's position–momentum uncertainty relation governing the SM-AI as Einstein's recoiling slit and Feynman's light microscope does not play an active role in these two WM-AI schemes. This observation is in accordance with previous theoretical and experimental studies by the Scully group and the Rempe group. However, there exists a drastic difference between our current quantum mechanical analysis and the studies made by these two groups, who claimed that in their WM-AI schemes the principle of WPD is still strictly upheld via the mechanism of quantum entanglement. To clarify the role of quantum entanglement, we investigate the space–time evolution of the internal state of the atom as well as the combined atom-detector system, which allows us to identify the microscopic origin of quantum entanglement, whose role has been emphasized in numerous previous literature. Our quantum mechanical analysis shows that if one only focuses on the atom center-of-mass de Broglie wave and looks at its interference pattern formation following the formula as $I_{\mathrm{DE}}(R) = A|\Psi_{\mathrm{acm,D}}(R) + \Psi_{\mathrm{acm,E}}(R)|^2$, as should be done for the aim to test the principle of WPD for the atom, then one will also find that the mechanism of quantum entanglement does not play an active role in these two WM-AI schemes . Nonetheless, if one does not take caution and make reasonable constraint to limit the interpretation only to the atom center-of-mass as the single object to test the WPD but rather expands with arbitrariness their perspective to the internal electronic states or the detector quantum states as the object to test the WPD following the formula as $I_{\mathrm{DE}}(R) = A|\Psi_{\mathrm{electron,D}}(R,r) + \Psi_{\mathrm{electron,E}}(R,r)|^2$, then one will naturally introduce the mechanism of quantum entanglement, either between the atom center-of-mass which-path information with the which-path micromaser-cavity detector in Scully's scheme or between the atom center-of-mass which-path information with the internal electronic state which-path detector in Rempe's scheme. By doing these operations and manipulations over calculation and experiment, the principle of WPD is still upheld. However, from the point of view of rigorous scholarship, this route of reasoning and argument is out of logic to a large extent and will inevitably lead to an unphysical artifact in understanding the fundamental physics problem of WPD.

The critical key towards the superior power of the proposed and analyzed WM-AI setups in simultaneous observation of the wave and particle nature of the atom largely lies in the design and introduction of the WM-PD. This instrument deliberately interacts with the atomic internal freedoms to unanimously probe the which-path information of the atom, while only inducing negligible perturbation of the atomic center-of-mass motion. Another instrument directly interacting with the atomic center-of-mass while insensitive to the internal freedoms is used to monitor the atomic center-of-mass interference pattern. These two sets of instruments work separately and collectively to nearly perfectly enable simultaneous observation of both the wave and the particle nature of the particles, while avoiding the complicated role of Heisenberg's uncertainty relation and quantum entanglement. The operation of WM-PD does not break down the coherent superposition of the atom center-of-mass transport state and turns it into an incoherent mixture state, thus we can eventually maintain a good atom-of-mass interference pattern. However, the price is that the operation will destroy the coherent superposition of the WM-PD quantum state (in either the internal electron of the atom or in the path detector), but fortunately this will not influence the atom center-of-mass quantum state and its interference pattern. Note that the current WM-PD has a very different operation principle than classical nondemolition measurement scheme.[42]

Based on these physics insights and quantum mechanical analyses that were found using this technique, we are able to further propose a fruitful variety of WM-AI schemes that can help to elucidate the puzzling physics of WPD for atoms. Because the key of this class of new concept WM-AI is to use two sets of detection instrument to probe the wave (interference pattern) and particle (which-path information) behaviors separately, simultaneously, and without mutual repulsion and

disturbance in different space–time domains, the principle and infrastructure of WM-AI can be straightforwardly extended to design and construct other types of weak-measurement particle interferometers working on different energy scales, different particles (electrons, neutrons, protons, atoms, molecules, *etc*.), difference internal states (electron, nuclear spin, *etc*.), and different path detectors. We hope that the new physics insights and analytical methodologies presented in this work will help to shed a new light on the exploration, understanding, and elucidation of the principle of WPD for microscopic objects and will also place quantum physics on a more solid conceptual ground.

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