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Tuning the intensity statistics of random speckle patterns*

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Speckle patterns are a fundamental tool in a variety of physical and optical applications. Here, we investigate a method of precisely tuning the intensity statistics of random speckle patterns into a desirable pattern that possesses the same spatial correlation length and similar statistics distribution. This tuning mechanism relies on the derivation of the transform function and transmission matrix, which achieves different contrasts while maintaining the same average value or energy level. The statistics properties of the generated speckle patterns are further investigated by analyzing the standard deviation under different fitting parameters. Precisely tuning the intensity statistics of random speckle patterns could be useful for both fundamental research and practical applications, such as microscopy, imaging, and optical manipulation.

Keywords: random speckle pattern, intensity statistics, local intensity transformation, optical manipulation

PACS: 78.20.hc, 78.20.Bh, 78.20.Ek, 78.68.+m **DOI:** 10.1088/1674-1056/28/5/057801

1. Introduction

Speckle patterns appear whenever a coherent wave interacts with a scattering or disordered medium.[1,2] In recent decades, the speckle phenomena have been employed in a variety of applications, such as speckle spectroscopy,[3] dynamic speckle illumination microscopy,[4,5] and super-resolution imaging.[6,7] Normally, the intensity statistics of random speckle patterns satisfies the Rayleigh distribution because its total field is the summation of a bunch of partial waves whose amplitudes are independently varied and phases are uniformly distributed within the range from 0 to 2π.[8–12] The wave–field joint probability density function (PDF) of this original speckle pattern is a negative exponential function, which means that with the increase of the intensity, its corresponding probability decays rapidly.[11–13] However, in many practical applications a non-Rayleigh intensity PDF statistics is very attractive and desirable. In previous studies,[14] the generated non-Rayleigh speckle patterns are either under-developed or partially coherent. Nieuwenhuizen *et al.*[15] addressed that multiple scattering could introduce mesoscopic correlations which modify Rayleigh statistics due to strong fluctuations in the total power transmitted through the medium. So far, the tailored speckle statistics has experimentally realized multiple negative-exponentially-like functions with different decaying rates, while the intensity PDFs of the generated speckle patterns are still monotonically decreasing.

In this work, we present a method of tuning the intensity statistics of random Rayleigh speckle patterns by a local intensity transformation according to the transform function. We start with the original speckle pattern whose intensity PDF satisfies the Rayleigh distribution statistics. Then, the local intensity transformation is deduced from the Gaussian distribution and performed to generate a desirable intensity PDF while maintaining the similar statistics properties such as average intensity value, spatial correlation length, and statistical distribution. Meanwhile, a thorough investigation of the statistical properties for four typical intensity PDFs is performed, which shows that this work could provide a flexible and versatile chance for a variety of optical applications in microscopy, imaging, and optical manipulation.

2. Mechanism of local intensity transformation

In our scheme, the system could be composed of a laser source, an iris, a spatial light modulator (SLM), a lens, and an image detector. The iris is employed to spatially filter the laser beam while the following lens transforms it into a parallel beam. This combination guarantees the generated Gaussian beam with random phases uniformly and vertically illuminating onto the SLM plate. The most important component is the SLM plate which could locally modulate the phases of the transmitted beam and consequently modify the corresponding intensity. Its operation region is a square array of macropixels, which further consists of a square array of smaller pixels. As the transparency of each pixel can be set by the software, the SLM plate could modulate the phases (intensity) of the transmitted beam at the pixel scale. These pixels function as scattering elements and correspond to the sources of different partial waves. The actual number of pixels which form the speckle patterns should satisfy the Nyquist sampling theorem:

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the spatial correlation length is at least twice of that of the adjacent sampling channels (pixels). The SLM is then set by a phase matrix, which modulates the phase front of the original beam and locally transforms the transmitted intensity. The intensity PDF of the Gaussian beam on the SLM complies with a Rayleigh distribution statistics of $P(I) = \exp[(-I/\langle I\rangle)/\langle I\rangle]$ ($\langle I\rangle$ denotes the average value of intensity), which shows an exponential decrease with the intensity. The transformation formula between the original and target intensity PDFs should be described as $P(I)\mathrm{d}I = F(I')\mathrm{d}I'$ for each sampling channel. To solve this equation, we employ the integral expression on both sides and normalize the original average intensity with $\langle I\rangle = 1$ to obtain

$$\int_0^I \mathrm{e}^{-I'}\,\mathrm{d}I' = \int_{I'_{\min}}^{I'} F(\tilde{I})\,\mathrm{d}\tilde{I}, \tag{1}$$

$$\langle I'\rangle = \int_{I'_{\min}}^{I'_{\max}} \tilde{I}'F(\tilde{I}')\,\mathrm{d}\tilde{I}'. \tag{2}$$

Here, I denotes the speckle intensity, while $I_{\min}$ and $I_{\max}$ are the minimum and maximum values of I. Evaluating these equations and solving for I' as a function of I could provide the key point for local intensity transformation. Although equations (1) and (2) have a set of open solutions, when setting the constraint condition of $\langle I'\rangle = \langle I\rangle$, these solutions would converge for a desirable intensity PDF while maintain the same average intensity or energy level. The flexible manipulation of the upper and lower limitations of Eq. (2) also allows us to relocate the operation range for different applications, such as the restriction for optical intensity.

Limited by the experimental imperfections, the theoretical calibration matrix T_{t} deviates from the actual matrix T_{a}. Their relationships with the original electric-field distribution E_{o} and the corresponding intensities of I_{d} and I_{e} are

$$I_{\mathrm{d}} = |T_{\mathrm{t}}\cdot E_{\mathrm{o}}|^2, \tag{3}$$

$$I_{\mathrm{e}} = |T_{\mathrm{a}}\cdot E_{\mathrm{o}}|^2. \tag{4}$$

The difference between the two intensity distributions could be obtained with the first order approximation of the matrices and the scalar quantities of $(I_{\mathrm{e}}-I_{\mathrm{d}})/(I_{\mathrm{d}})^{1/2} = 2\mathrm{Re}[(\Delta T E_{\mathrm{o}})^*]$, where ΔT is the deviation of the matrices and $\mathrm{Re}[\cdots]$ denotes its real part. The corresponding statistics distribution of $(I_{\mathrm{e}}-I_{\mathrm{d}})/(I_{\mathrm{d}})^{1/2}$ is

$$G(I_{\mathrm{e}},I_{\mathrm{d}}) = A\exp\left[\frac{-(I_{\mathrm{e}}-I_{\mathrm{d}})^2}{2aI_{\mathrm{d}}}\right], \tag{5}$$

where a is a fitting parameter that quantifies the experimental error and A is a normalized constant given by

$$\int_0^\infty G(I_{\mathrm{e}},I_{\mathrm{d}})\,\mathrm{d}I_{\mathrm{d}} = 1.$$

Consequently, we numerically simulate and perform a local intensity transformation of the original speckle pattern and convert it into a target redistributed pattern with a desirable intensity PDF $F(I_{\mathrm{e}})$

$$F(I_{\mathrm{e}}) = \int_0^\infty F(I_{\mathrm{d}})G(I_{\mathrm{e}},I_{\mathrm{d}})\,\mathrm{d}I_{\mathrm{d}}. \tag{6}$$

3. Statistics properties of the generated speckle patterns

According to the local intensity transformation method described above, we first investigate two typical kinds of intensity PDFs, which are very different from that of the Rayleigh statistics ($P(I/\langle I\rangle) = \exp[(-I/\langle I\rangle)/\langle I\rangle]$). Figure 1(a) shows the transform functions $I' = F(I)$ of the two target intensity PDFs, namely, $P(I/\langle I\rangle) = I/\langle I\rangle$ and $P(I/\langle I\rangle) = 1/2$ ($0 < I/\langle I\rangle < 2.4$), respectively. One could easily observe that each target speckle pattern corresponds to a fixed relationship between I' and I, and the transformation is nonlinear. In many previous experiments, it has already been verified that the phases of the generated patterns uniformly distribute within the range over $[0, 2\pi]$, which means that all the speckle patterns are fully developed. To check whether an additional spatial correlation length is introduced into the target speckle patterns during the transformation, we calculate spatial correlation function $C(\Delta r) = (\langle I'(r)I'(r+\Delta r)\rangle - \langle I'\rangle^2)/(\langle I'^2\rangle - \langle I'\rangle^2)$ of the original and two generated speckle patterns and plot them in Fig. 1(b). These curves indicate that both of the linear increasing and uniform intensity PDFs have the same correlation length of $\Delta r = 33$ μm, which means the fields at different positions are uncorrelated.

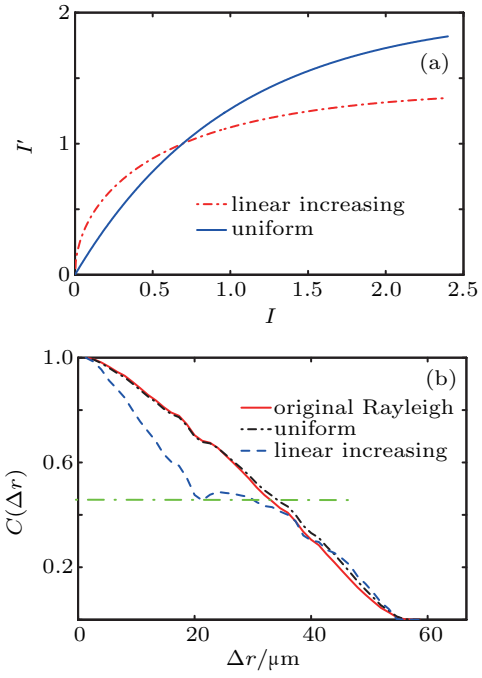


Fig. 1. (a) The local intensity transform functions $I' = F(I)$ between the original and two generated speckle patterns, (b) the curves of the spatial intensity correlation function $C(\Delta r)$ as a function of the distance Δr.

Figures 2(a) and 2(b) show the generated speckle patterns of the linear increasing and uniform intensity PDFs. Here, we define the speckle contrast as $C = (\langle I^2 \rangle / \langle I \rangle^2 - 1)^{1/2}$. From the intrinsic nature of intensity statistics, the highest, moderate, and lowest intensity points in the intensity PDF correspond to the center of bright dots, their edges, and the background of the speckle patterns, respectively. Therefore, we can deduce that in the case of the linear increasing intensity PDF (case 1), the number of lower intensity points is smaller than that of higher intensity points, which results in the overall speckle pattern shifting towards a higher intensity level with a lower contrast. In case 1, C is calculated to be 0.4625. In the case of the uniform intensity PDF (case 2), the number of the intensity points is independent of the intensity and the overall speckle pattern locates at the middle point of the intensity range, the contrast of the speckle should be higher, and it is found to be 0.8068. As the contrast in case 1 (Fig. 2(a)) is lower, the speckle edges become faint and the whole topology looks like a random interconnected web of dark channels. In case 2 (Fig. 2(b)), the higher contrast makes its edges sharper and the whole topology looks like a bundle of discrete bright dots. The rapid conversion of the contrast (variation as high as 42.7% in a controlled manner) between different types of topologies provides the possibility of controlling the motion or state of particles at a micro level.

By setting into the SLM plate, the transmission matrix is a fast way to transfer the original Rayleigh speckle into the target pattern. Normally, the measurement of transmission matrix could be realized by a common path interference method.[13,15–17] In the simulation here, we perform the local intensity transformation and normalize both of the intensity distributions to one. The corresponding transmission matrix could be constructed by the product of the generated intensity distribution and the inversed original intensity distribution.[3] Because the transmission matrix corresponds to the unique transfer function, it could be regarded as a fingerprint for certain transformations or adopted to other transformation systems.

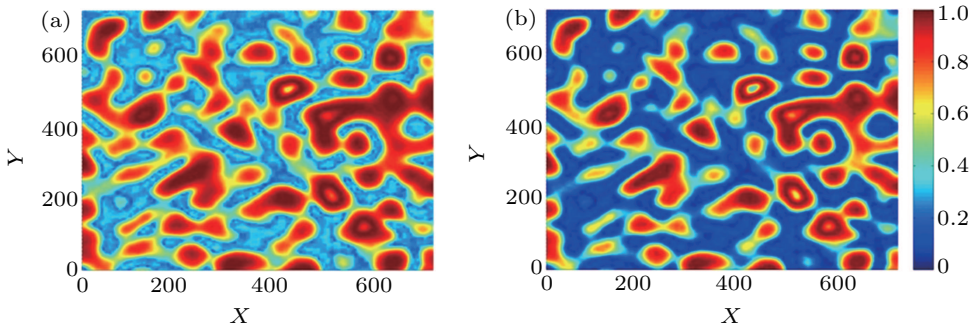


Fig. 2. The generated speckle patterns in the (a) linear increasing and (b) uniform intensity PDF cases.

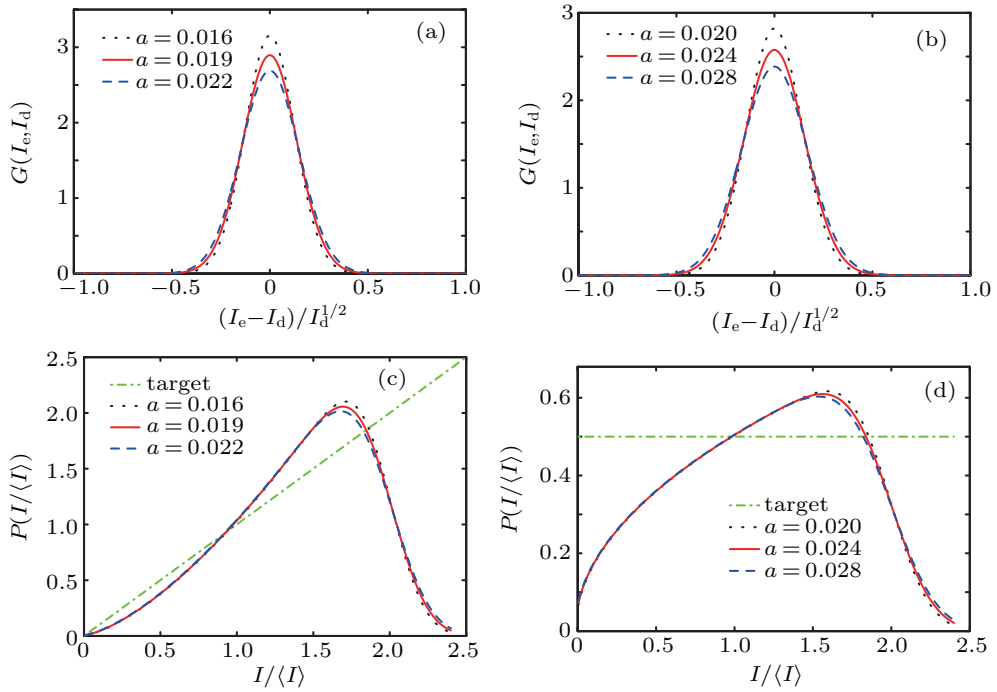


Fig. 3. The relationship of $G(I_e, I_d)$ with different a in the (a) linear increasing and (b) uniform intensity PDF cases, the target intensity PDFs $F(I/\langle I \rangle)$ with different a in the (c) linear increasing and (d) uniform intensity PDF cases.

To further investigate the statistical properties of this transformation method, we characterize both of the statistics distribution $G(I_e, I_d)$ and target speckle pattern $F(I_e)$ in detail. As $G(I_e, I_d)$ indicates the probability of the difference between the theoretical and actual intensity distributions. We conduct the nonlinear optimization algorithm to determine the optimal value of the fitting coefficient a, and its values for the linear increasing and uniform intensity PDFs are 0.019 and 0.024, respectively. Figures 3(a) and 3(b) illustrate the relationship of $G(I_e, I_d)$ with a, and a larger a makes the normalized Gaussian statistics distribution shorter and broader. Considering the errors between the theory and reality, the target intensity PDFs with different a values are obtained as shown in Figs. 3(c) and 3(d), and the three curves in each figure almost overlap. In case 1, the intensity PDF $F(I/\langle I\rangle)$ within low intensity range fits very well with the ideal curve (green line), and as $I/\langle I\rangle$ increases to 2, their difference becomes larger. In case 2, $F(I/\langle I\rangle)$ within the moderate intensity range fits well with the ideal curve (green line), while it rapidly drops to zero at the two ends. Since $G(I_e, I_d)$ is broader at larger I_e, the averaging effect is stronger, leading to a larger discrepancy at higher intensity. From these two cases, we can see that non-decreasing (linear increasing and uniform) intensity PDFs could be achieved within a finite intensity range. In Fig. 4, we calculate the standard deviation σ from the ideal case to quantify the fitting effect. The σ decreases slightly with the increase of a in case 1, while it is almost invariant in case 2, which proves that a has already been optimized. When the method is practically applied, a should come from the $G(I_e, I_d)$ fitting between the theoretical and the experimental data.

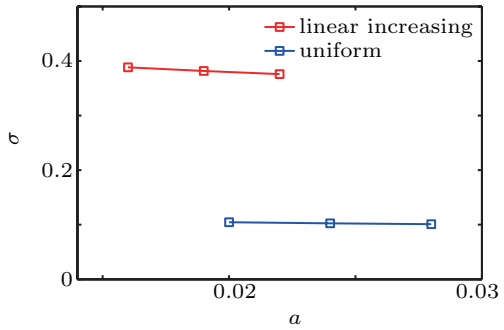


Fig. 4. The variations of the standard deviation σ with the fitting coefficient a in the two cases.

4. Transformation for the peak-shape intensity PDFs

For further extension, we also transplant this method to the half-peak (case 3) and full-peak (case 4) intensity PDFs, which are curves and different from those in Section 3. As shown in Fig. 5, the normalized $G(I_e, I_d)$ are plotted for both cases, where the optimal a equals to 0.036 (case 3) and 0.030 (case 4), respectively. The ideal intensity PDFs $F(I/\langle I\rangle) = 1-(I/\langle I\rangle)^2/6.25$, $F(I/\langle I\rangle) = 1-(I/\langle I\rangle - 1.5)^2/2.25$ (blue curves) and the corresponding generated intensity PDFs (red curves) are separately shown in Figs. 6(a) and 6(b). We can observe that in case 3, the large difference only emerges for $I/\langle I\rangle < 1$, and when $I/\langle I\rangle > 1$, the two curves fit well. The discrepancy at larger intensity is suppressed more due to stronger averaging effect. In case 4, a large difference emerges at higher intensity of $I/\langle I\rangle > 2$, while a fine fitting effect occurs at lower intensity where the discrepancy is suppressed by averaging. The corresponding σ values of the two cases are 0.2559 and 0.2515, respectively. Compared with the previous two cases, we can conclude that the target intensity PDF with a moderate value and relatively smooth envelope could have a better fitting performance. These investigations verify that when extending the application of local intensity transformation, one could obtain a desirable and reasonable intensity PDF from any generated speckle pattern.

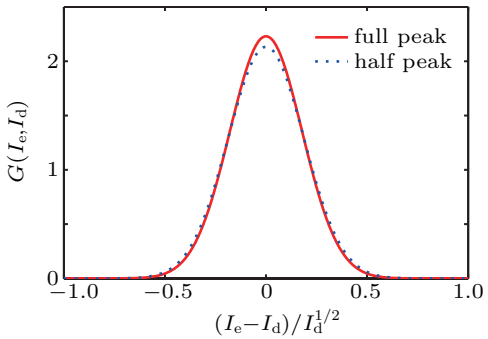


Fig. 5. The relationship of $G(I_e, I_d)$ with a in the half-peak and full-peak intensity PDF cases.

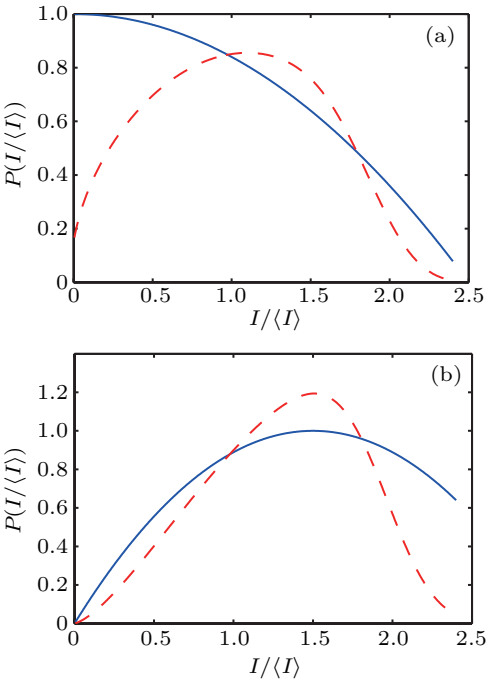


Fig. 6. The generated intensity PDFs with optimal a in the (a) half-peak and (b) full-peak cases.

5. Conclusion

We have investigated a method to precisely tune the intensity statistics of random speckle patterns. The generated speckle patterns indeed show radically different topologies but their spatial correlation length and statistical distribution maintain unchanged. The statistical properties (such as the contrast and standard deviation) of four typical intensity PDFs are thoroughly analyzed and compared with the ideal cases, which show a great potential for speckle transformation and manipulation. By precisely tuning the intensity statistics of random speckle patterns into a non-Rayleigh statistics, the generated patterns are of great importance for both fundamental research and practical applications.

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