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A novel multi-cavity Helmholtz muffler*

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A novel multi-cavity Helmholtz muffler is proposed. The multi-cavity Helmholtz muffler is composed of steel structures and silicone membranes. With suitable construction, the Helmholtz muffler can be designed to exhibit negative mass density in low frequency, and the muffling frequency can be adjusted when we change the internal structure of the cavity, which will be very attractive for noise control. In this paper, we investigate the influence of the membranes and the cavities on noise reduction characteristics with theoretical calculations and simulations. The results show that the numbers of membranes and the volumes of the cavities can have a great effect on the position of the muffling frequency. The number of cavities can have a great effect on the width of the muffling frequency (reduce the noise by 10 dB). With different combinations of the membranes and cavities, we can get different muffling frequencies, which can meet different muffling demands in practical applications and is more flexible than the traditional Helmholtz cavity.

Keywords: multi-cavity, Helmholtz muffler, negative mass density, noise reduction

PACS: 43.50.+y, 43.40.+s, 43.30.+m **DOI:** 10.1088/1674-1056/28/5/054303

1. Introduction

In modern naval warfare, submarines[1] play an indispensable role, and anti-submarine detection has developed towards low frequency.[2] Controlling the low-frequency noise to avoid being detected by the enemy is the key to improving the stealth performance of the submarine.[3] However, it is difficult to control the noise of submarines from the source for many complex reasons, designing a kind of muffler[4] with low frequency noise reduction characteristic is very important.

Because there are many limitations in the practical engineering application of traditional materials, artificial composite materials with special physical properties have become the focus of attention.[5] In the last 20 years, artificial composite materials have expanded into the field of dynamics, used for flexible regulation of mechanical waves, such structures are called phononic crystals.[6–8] Then, in the modern industrial equipment and products of miniaturization, integration, and lightweight backdrop, people have made further development of the new type of artificial microstructure feature size of subwavelength, later known as acoustic metamaterial.[9–12] The equivalent physical properties of acoustic metamaterials exhibit either 'single negative' or 'double negative' properties,[13] which perform as low frequency sound absorption and low frequency sound insulation in the process of sound wave propagation. Guided by this theory, many researchers have designed many devices to filter wave or reduce noise in low frequency. Fang *et al.*[14] designed a kind of Helmholtz cavity, they showed that the equivalent volume modulus of the whole structure is negative when the sound frequency is in the bandgap. In his paper, the Helmholtz cavity is arranged perpendicular to the direction of sound propagation, the sound waves can trigger fluid movements in the neck of each cavity, when the excitation frequency is close to the characteristic frequency of the cavity, the response of the volume modulus would be excited, it is related to $1/\left(\omega_0^2-\omega^2\right)$. So, when the movement of the neck fluid column changes from the same phase to the different phase, the negative volume modulus will show up. Ding *et al.*[15] constructed a one-dimensional periodic structure on the basis of Helmholtz resonator theory and carried out a corresponding experiment, they proved that there would be a sharp attenuated absorption peak when sound waves pass through it. Lee *et al.*[16] constructed an acoustic metamaterial structure with double negative properties by using the thin membrane and the edge cavity. Shen *et al.*[17] constructed a Helmholtz muffler which is applied in a seawater pipe system based on phononic crystal theory. The Helmholtz cavity is placed periodically in the seawater pipe. They further optimized the range or bandgap through adjusting the structural parameters of the Helmholtz cavity. This design has an important application potential for the acoustic attenuation and vibration reduction to a seawater pipeline.

However, many acoustic metamaterials with either single negative or double negative[18] properties cannot reduce noise in a relatively wider frequency range although they have one or more muffling peaks. Besides, the position of the muffler frequency cannot be regulated systematically in many metamaterials. We design a new kind of metamaterial with multi-cavity Helmholtz muffler with membrane structure in this paper. We

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also investigate the influence of the number of membranes and the volumes of the Helmholtz cavities on the frequency of noise reduction. By changing the number of the rubber and the volume of the cavity, we can obtain different muffling frequencies and different widths of the muffling frequency. We can also rough and fine adjust the peak of the muffling frequency to meet different muffling demand. When we place the muffler in the water, increase the number of Helmholtz cavities and stack more layers of silicone membranes, the range of frequency to reduce noise can become wider and show the tendency to move to lower frequencies. This new kind of muffler provides a new idea for the design of underwater acoustic structures.

2. Modeling and theoretical discussion

The Helmholtz cavity in Ref. [14] showed that this metamaterial has a negative volume modulus around 32 kHz, the negative volume modulus is the same as the negative mass density. It can also turn sound waves into falling waves, so the transmission loss around this frequency shows up. The acoustic metamaterial can have a very good acoustic absorption around 32 kHz. In our research, we proposed a new concept of multi-cavity Helmholtz muffler with membrane structure. We added several cavities in our muffler structure on the basis of the original Helmholtz cavity, and made the position and width of the sound absorption frequency adjustable by changing the number of membranes and the volumes of the cavities. The new structure can meet different muffling demands in the practical application and it is more flexible than the traditional Helmholtz cavity.

In order to study the influence of the membrane (silicone) on the negative mass density of the Helmholtz cavity, we add different numbers of membranes in the Helmholtz cavity ($n \geq R_0$), the model is shown in Fig. 1, where $l_1 = 5$ mm, $l_2 = 20$ mm, $l_3 = 10$ mm, $a = 20$ mm, $r = 30$ mm, $t = 1$ mm, and the thickness of the whole steel structure is $t = 1$ mm.

Figure 2 shows the simplified model of each cavity, as the volume of the neck is much smaller than that of the cavity, the neck fluid can therefore be approximately considered as incompressible. In contrast, the fluid in the cavity is considered compressible, and it can provide a reverse resilience when the fluid is pressurized. According to the above description, we can consider the fluid in the neck as mass and the fluid in the cavity as spring. Besides, the membrane we add in the cavity should also be considered as mass.

For studying the mass–spring unit cell in Fig. 2, the equations of motion of each mass are

$$k_0(u_1 - u_0) = (-\mathrm{i}w)^2 m_0 u_0,$$
$$k_i(u_{i+1} - u_i) + k_{i-1}(u_{i-1} - u_i) = (-\mathrm{i}w)^2 m_i u_i,$$
$$k_n(U - u_n) + k_{n-1}(u_{n-1} - u_n) = (-\mathrm{i}w)^2 m_n u_n, \quad (1)$$

where i indicates the i-th unit cell ($i \in \{1, \ldots, n-1\}$), m_i is the mass of neck fluid and each membrane, k_i is the elastic stiffness of each cavity, u_i is the displacement of each mass block, and U is the displacement of the whole Helmholtz cavity. We can rewrite Eq. (1) in a matrix form

$$\boldsymbol{K}\boldsymbol{u} + \boldsymbol{B} = \boldsymbol{M}\boldsymbol{u}, \quad (2)$$

where the displacement vector $\boldsymbol{u}$, the output vector $\boldsymbol{B}$, the stiffness matrix $\boldsymbol{K}$, and the mass matrix $\boldsymbol{M}$ are defined as

$$\boldsymbol{u} = \begin{pmatrix} u_0 & u_1 & \cdots & u_n \end{pmatrix}^{\mathrm{T}}, \quad \boldsymbol{B} = \begin{pmatrix} 0 & \cdots & 0 & k_n U \end{pmatrix}^{\mathrm{T}}, \quad (3)$$

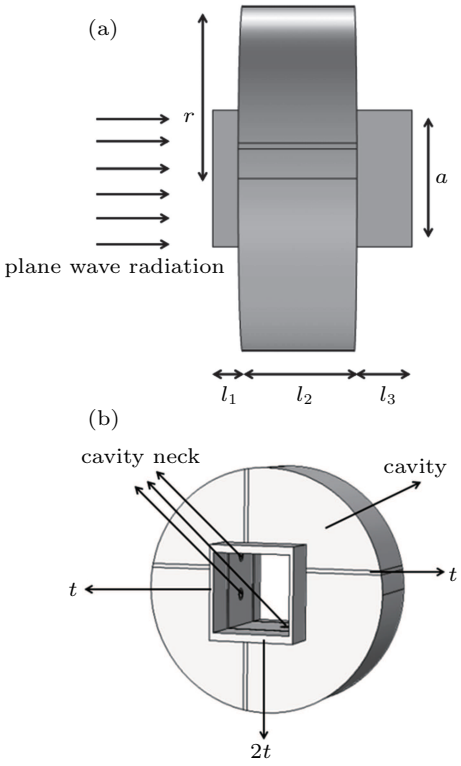


Fig. 1. The model of the multi-cavity Helmholtz muffler: (a) the left view, (b) the front view.

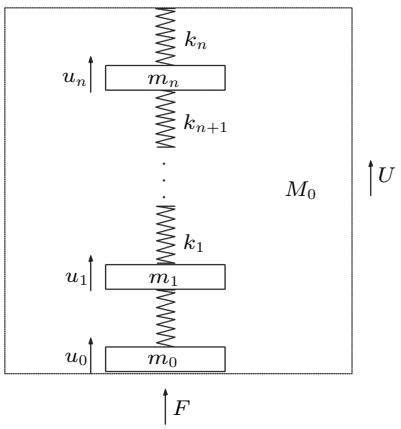


Fig. 2. The simplified model of each cavity.

$$\boldsymbol{K}=\begin{bmatrix} -k_0 & k_0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ k_0 & -(k_1+k_0) & k_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & k_1 & -(k_2+k_1) & k_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & & \ddots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & k_{n-2} & -(k_{n-1}+k_{n-2}) & k_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & k_{n-1} & -(k_n+k_{n-1}) \end{bmatrix}_{n+1},$$

$$\boldsymbol{M}=\begin{bmatrix} a_0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \cdots & 0 \\ 0 & \cdots & a_i & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & a_n \end{bmatrix}_{n+1}, \tag{4}$$

where $a_0=(-\mathrm{i}w)^2 m_0$ and $a_i=(-\mathrm{i}w)^2 m_i$.

We use MATLAB (R2016a) to compute the relationship between the displacement u_i of each mass block and the displacement U of the whole Helmholtz cavity, equation (2) can be rewritten as

$$\boldsymbol{u}=(\boldsymbol{M}-\boldsymbol{K})^{-1}\boldsymbol{B}. \tag{5}$$

For the whole structure we provide a vertical upward force F in order to calculate the equivalent mass M_{eff}, then Newton's second law can be expressed as

$$F=M_{\mathrm{eff}}\ddot{X}, \tag{6}$$

where M_{eff} is the effective mass, and X is the displacement of the whole structure,

We add up all equations in Eq. (1) and obtain

$$k_n(U-u_n)=(-\mathrm{i}w)^2[m_0u_0+m_1u_1+\cdots+m_nu_n]. \tag{7}$$

For the whole structure (the cavity, neck, and membrane are excluded), then Newton's second law can be expressed as

$$F-k_n(U-u_n)=(-\mathrm{i}w)^2 M_0U, \tag{8}$$

where F is the external excitation force and M_0 is the mass of the whole structure (the cavity, neck, and membrane are excluded). After substituting Eq. (7) into the system of equation (8), the equation of motion can be written in the form

$$F=(-\mathrm{i}w)^2[m_0u_0+m_1u_1+\cdots+m_nu_n+M_0U]. \tag{9}$$

Compared with Eq. (6), M_{eff} can be defined as

$$M_{\mathrm{eff}}=M_0+m_0\frac{u_0}{U}+\frac{m_1u_1+m_2u_2+\cdots m_nu_n}{U}. \tag{10}$$

We consider three different cases about this muffler ($n=0$, 1, 2), and put Eq. (5) into Eq. (10), the effective mass M_{eff} can be expressed as

$$M_{\mathrm{eff}}=M_0+\frac{m_0}{1-\frac{w^2}{w_0^2}},\quad (n=0),$$

$$M_{\mathrm{eff}}=M_0+\frac{m_0+m_1\left(1-\frac{w^2}{w_0^2}\right)}{1+\frac{w^4}{w_0^2w_1^2}-\frac{w^2}{w_1^2}-2\frac{w^2}{w_0^2}},\quad (n=1),$$

$$M_{\mathrm{eff}}=M_0+\frac{m_0+m_1\left(1-\frac{w^2}{w_0^2}\right)+m_2\left(1-2\frac{w^2}{w_0^2}-\frac{w^2}{w_1^2}+\frac{w^4}{w_0^2w_1^2}\right)}{1-3\frac{w^2}{w_0^2}-2\frac{w^2}{w_1^2}-\frac{w^2}{w_2^2}+2\frac{w^4}{w_0^2w_1^2}+2\frac{w^4}{w_0^2w_2^2}+\frac{w^4}{w_1^2w_2^2}-\frac{w^6}{w_0^2w_1^2w_2^2}},\quad (n=2). \tag{11}$$

The effective mass density ρ_{eff} can be expressed as

$$\rho_{\mathrm{eff}}=\frac{M_{\mathrm{eff}}}{\rho V}, \tag{12}$$

where $w_0=\frac{1}{2\pi}\sqrt{\frac{k_0}{m_0}}$, $w_1=\frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}$, and $w_2=\frac{1}{2\pi}\sqrt{\frac{k_2}{m_2}}$ are the natural frequencies of the neck fluid and each membrane, V and ρ are the volume and density of the structure, m_i, k_i ($i=0,1,2$) are the equivalent mass and equivalent stiffness, which can be expressed as

$$m_0=\mathrm{S}_0\times L_0\times\rho_{\mathrm{water}},\quad m_i=V_i\times\rho_{\mathrm{rubber}},$$
$$K_i=\frac{\mathrm{S}_0^2\times v^2\times\rho_{\mathrm{water}}}{V_j}, \tag{13}$$

where V_i, V_j are the volumes of the membrane and the cavity, respectively, and S_0 is the area of the neck liquid.

We consider one of the Helmholtz cavities which is shown in Fig. 3. All the modeling and simulation are done by the FEM software COMSOL Multiphysics. The material parameters are listed in Table 1, and the parameters are measured after modeling in the software COMSOL. The bottom radius and the height of the neck liquid are $r=1$ mm and $L_0=5$ mm. We calculate the ρ_{eff} of the three different cases about this Helmholtz cavity ($n=0$, 1, 2) by the software MATLAB, the results are shown in Fig. 4. When we increase the layers of rubber in this Helmholtz cavity, the frequency of negative mass density decreases from 998.14 Hz ($n=0$) to 387.63 Hz ($n=2$). The reason for this is that the natural frequency of the neck liquid and rubber decreases, so the resonance frequency becomes lower. The sound waves would at-

tenuate greatly at the frequencies where negative mass density occurs based on the Helmholtz cavity single negative theory, which we will also verify in later simulations. So, we can choose different layers of rubber according to the requirement of different muffling frequencies. It has a profound influence on the structure of submarine construction.

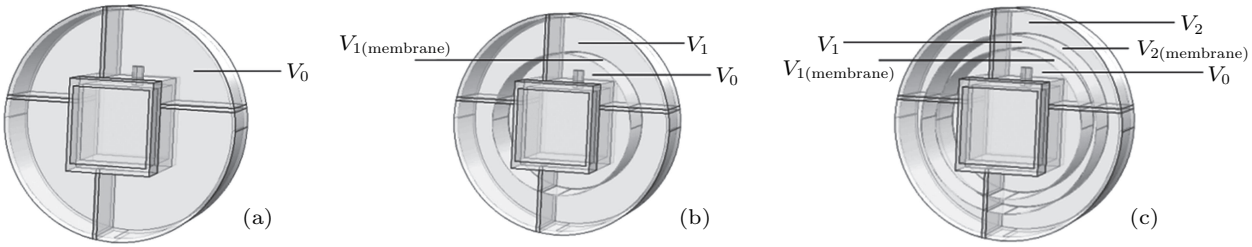


Fig. 3. The internal structure of one of the Helmholtz cavities: (a) the number of the membrane is $n = 0$, (b) $n = 1$, (c) $n = 2$.

Table 1. The material parameters.

Component	Young's modulus/GPa	Poisson's ratio	Density/kg·m^{-3}	Sound velocity/m·s^{-1}
Steel (structure)	200.6	0.3	7780	–
Water (in cavity)	–	–	1000	1450
Membrane (silicone)	2×10^{-4}	0.5	980	–
	Volume ($n=0$)/m^3	Volume ($n=1$)/m^3	Volume ($n=2$)/m^3	
Steel (structure)	2.41×10^{-6}	2.41×10^{-6}	2.41×10^{-6}	
Water (in cavity)	$V_0 = 9.742\times10^{-6}$	$V_0 = 2.844\times10^{-6}$ $V_1 = 6.791\times10^{-6}$	$V_0 = 2.844\times10^{-6}$ $V_1 = 2.247\times10^{-6}$ $V_2 = 4.415\times10^{-6}$	
Silicone (membrane)	–	$V_1 = 1.069\times10^{7}$	$V_1 = 1.069\times10^{-7}$ $V_2 = 1.296\times10^{-7}$	

In order to further verify the feasibility of the analytical solution after simplifying the model, the node-based smoothed finite element method (NS-FEM) is used to conduct a comparative analysis of the muffler structure.[19]

Applying the divergence theorem, the smoothed strain gradient matrix $\boldsymbol{B}_i(x_k)$ on cell Ω^k can be obtained as follows:

$$\begin{aligned}\boldsymbol{B}_i(x_k) &= \frac{1}{V^{(k)}}\int_{\Gamma^{(k)}} n^{(k)}(x)N_i(x)\,\mathrm{d}\Gamma \\ &= \begin{bmatrix} b_{ix}(x_k) & 0 & 0 \\ 0 & b_{iy}(x_k) & 0 \\ 0 & 0 & b_{iz}(x_k) \\ b_{iy}(x_k) & b_{ix}(x_k) & 0 \\ 0 & b_{iz}(x_k) & b_{iy}(x_k) \\ b_{iz}(x_k) & 0 & b_{ix}(x_k) \end{bmatrix},\end{aligned} \tag{14}$$

with

$$b_{ih}(x_k) = \frac{1}{V^{(k)}}\sum_{n=1}^{N} N_i\left(X_i^{\mathrm{GP}}\right) n_{ih}^{(k)} A_i^{(k)}, \quad (h = x,y,z), \tag{15}$$

where $\Gamma_i^{(k)}$ is the boundary of the domain Ω^k, N is the total number of boundary areas of $\Gamma_i^{(k)}$, and X_i^{GP} is the central point (Gaussian point) of the boundary area $\Gamma_i^{(k)}$ whose area and outward unit normal are denoted as $A_i^{(k)}$ and $n_{ih}^{(k)}$, respectively.

The stiffness matrix can be assembled as

$$K_{ij} = \sum_{k=1}^{N_n} K_{ij}^{(k)}, \tag{16}$$

where $K_{ij}^{(k)}$ is the stiffness matrix associated with node k and is calculated by

$$K_{ij}^{(k)} = \int_{\Omega^k} \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{D} \boldsymbol{B}_j \mathrm{d}\Omega = \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{D} \boldsymbol{B}_j V^{(k)}. \tag{17}$$

The vibration equation can be written as

$$M\ddot{u} + Ku = F, \tag{18}$$

$$M\ddot{u} + K\frac{\ddot{u}}{(-\mathrm{i}w)^2} = F. \tag{19}$$

In Eqs. (6) and (12), M_{eff} and ρ_{eff} can be defined as

$$M_{\mathrm{eff}} = \frac{F}{\ddot{u}} = M - \frac{K}{w^2}, \tag{20}$$

$$\rho_{\mathrm{eff}} = \frac{|M_{\mathrm{eff}}|}{\rho V}. \tag{21}$$

We calculate the relationship between ρ_{eff} and the frequency ω by the software MATLAB. The results are also shown in Fig. 3, which have good agreement with the analytical solution. The small error of the resonance peak frequency by NS-FEM method is mainly derived from the selection of nodes at the fluid–solid coupling interface and the number of selected nodes.

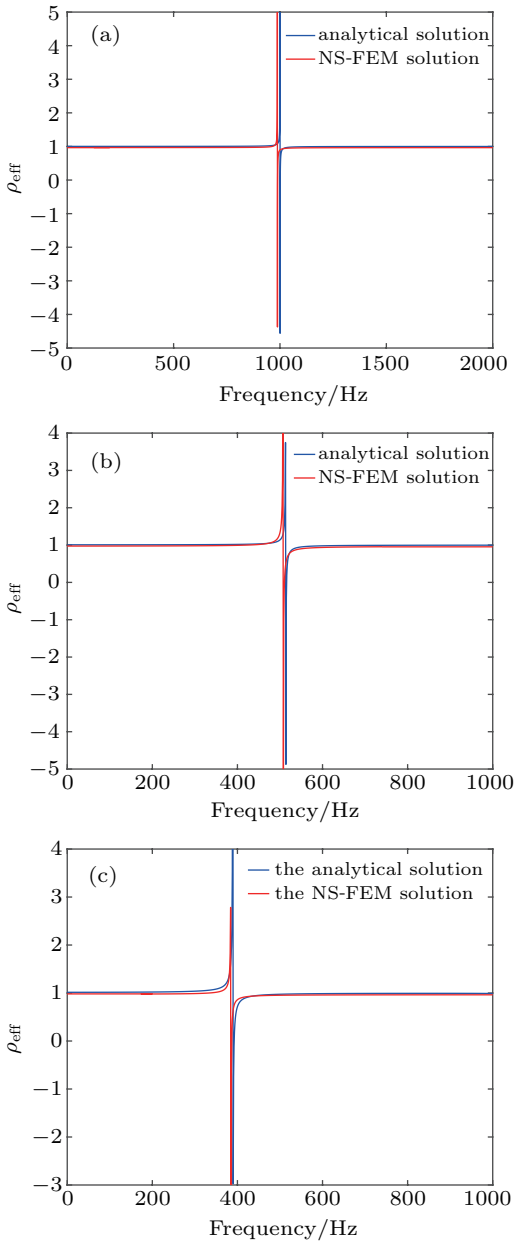


Fig. 4. The dimensionless equivalent density in the muffler with a single Helmholtz cavity: (a) the number of the membranes is $n = 0$, (b) $n = 1$, (c) $n = 2$.

3. Simulation

The above-mentioned analytical examples have shown that the frequency of negative mass density can be changed by choosing different numbers of membranes. In this section, we consider the influence of the Helmholtz cavity on the muffling frequencies. Figure 5 shows the Helmholtz muffler with different numbers of cavities (only shows the situation of no membrane). We applied the plane wave radiation on one side and measured the transmission loss of sound on the other side by the software COMSOL. Because there is fluid–solid coupling in the whole muffler, the physical fields of the steel structure and the membrane structure are set as solid mechanics (solid), and the physical fields of the cavity and the pipe are set as acoustic pressure (acpr). The schematic diagram is shown in Fig. 1(a), the transmission loss is defined as follows:[4]

$$\mathrm{TL} = L_{\mathrm{i}} - L_{\mathrm{t}} = 10\lg\frac{I_{\mathrm{i}}}{I_{\mathrm{t}}}, \tag{22}$$

where L_{i} and L_{t} are the power levels of the incident sound and transmission sound, I_{i} and I_{t} are the powers of the incident sound and transmission sound

$$I_{\mathrm{i}} = \int_{S_{\mathrm{p}}} \frac{|P_{\mathrm{i}}|^2}{2\rho c_0}\,\mathrm{d}s, \tag{23}$$

$$I_{\mathrm{t}} = \int_{S_{\mathrm{p}}} \frac{|P_{\mathrm{t}}|^2}{2\rho c_0}\,\mathrm{d}s, \tag{24}$$

where P_{i} and P_{t} are the incident sound pressure and transmission sound pressure, S_{p} is the surface area of the inlet and outlet, and the transmission coefficient of sound intensity is defined as follows:

$$t_{\mathrm{I}} = \frac{I_{\mathrm{t}}}{I_{\mathrm{i}}} = \frac{|P_{\mathrm{t}}|^2/2\rho c_0}{|P_{\mathrm{i}}|^2/2\rho c_0} = \frac{|P_{\mathrm{t}}|^2}{|P_{\mathrm{i}}|^2}. \tag{25}$$

If the area of pipe inlet and outlet is the same, the relationship of transmission loss and transmission coefficient can be defined as follows:

$$\mathrm{TL} = 10\lg\left(t_{\mathrm{I}}^{-1}\right). \tag{26}$$

We calculate the transmission loss by the software COMSOL, the single-cavity simulation results are shown in Fig. 6. We consider the number of membranes in the cavity is $n = 0$, 1, 2, respectively, the muffling frequency has a good agreement with what we calculate in the previous section, and we can see that the muffling frequency decreases as the number of membranes increases. It can help us to get relatively lower muffling frequencies. However, we can find that the range of the muffling frequency is very narrow regardless of the number of membranes. If we want to achieve 10-dB muffling efficiency, the range of frequency is only around 50 Hz, which is unfavorable to the stealth performance of the submarine. So, we need to widen the range of the muffling frequency to make it harder for submarines to be detected by enemies.

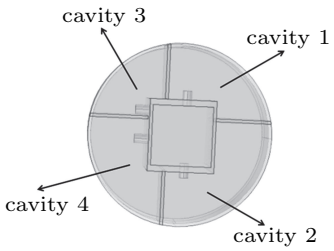


Fig. 5. Helmholtz muffler with different numbers of cavity.

Figure 7 shows the transmission loss of a multi-cavity Helmholtz muffler (the schematic diagram is depicted in Fig. 5). We find that when we add a cavity, the range of muffling frequency can increase by around 50 Hz, and when we

open all four cavities, the frequency range increases up to over 200 Hz. Besides, the increased frequency region is always near the original one where we open only one cavity, so, increasing the cavity only widens the range of frequencies without much effect on the position of the frequencies. Moreover, the volume of the cavity has a slight effect on the attenuation peak and this effect is much smaller than the membranes. So, we can use the membrane to roughly adjust the sound attenuation frequency and use the cavity to fine adjust the sound attenuation frequency. The volume sizes of four cavities are $V_2 > V_4 > V_1 > V_3$ (depicted in Fig. 5), we can find in Fig. 7 that the larger the cavity volume, the smaller the frequency of the attenuation peak , and if the volumes of two cavities are similar, the attenuation peak will be very close or even overlap. So that if there are enough cavities, there will be no attenuation valley, it is helpful to obtain a continuous range of muffling frequency.

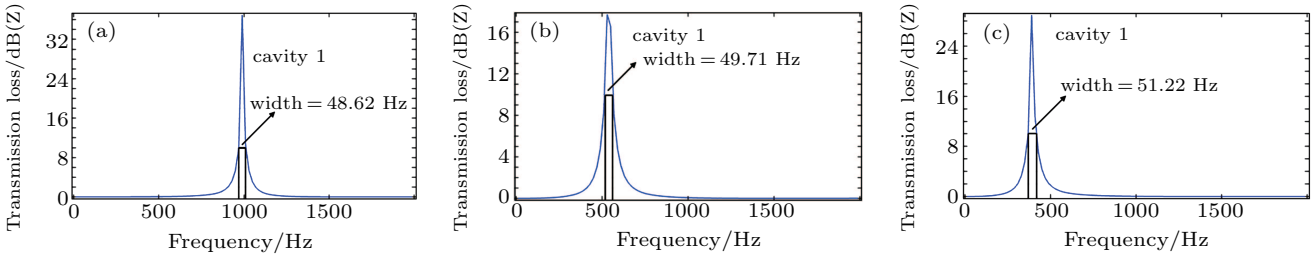


Fig. 6. Transmission loss of single-cavity Helmholtz muffler: (a) the number of membrane $n = 0$, (b) $n = 1$, (c) $n = 2$.

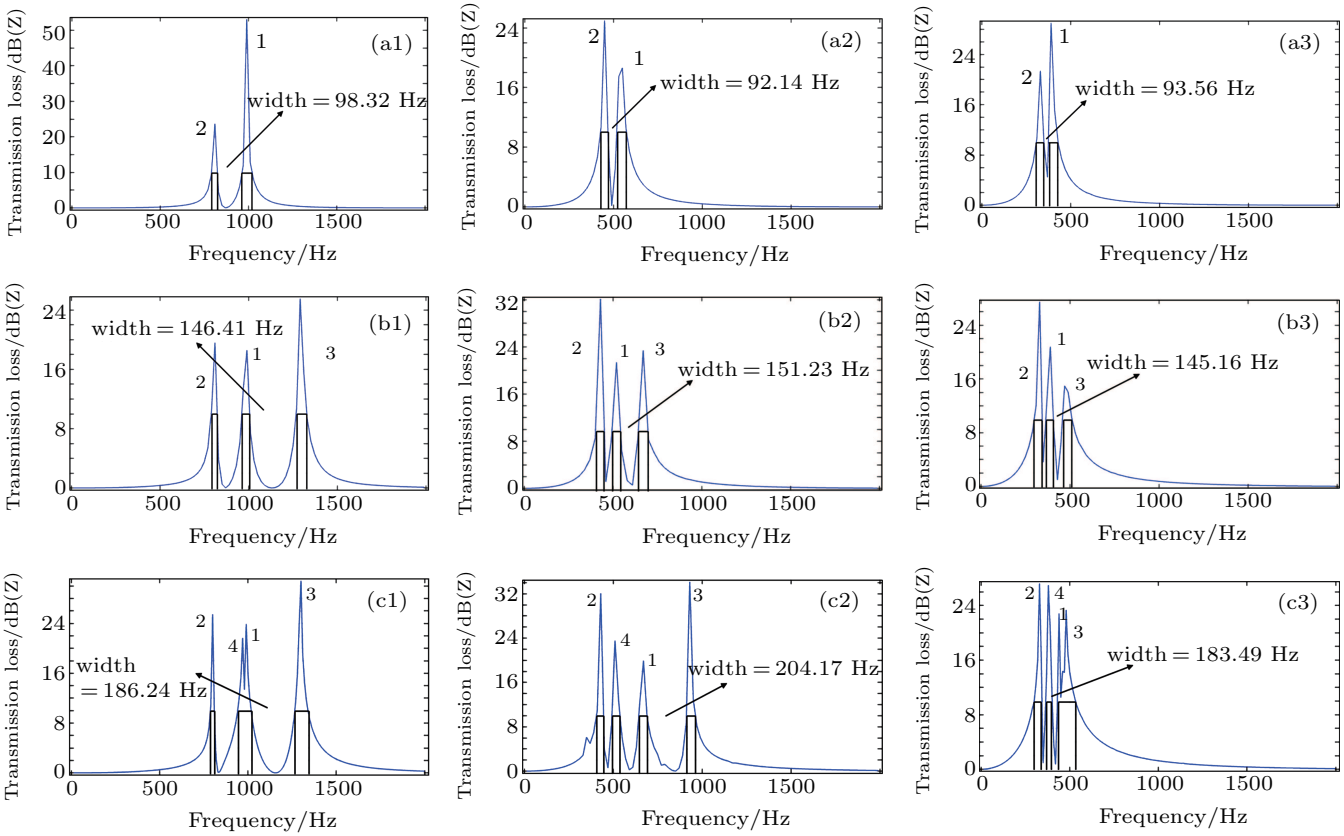


Fig. 7. Transmission loss of multi-cavity Helmholtz muffler with (a) two cavities, (b) three cavities, (c) four cavities, the number of membranes is $n = 0$, 1, 2, and the order is the same as that in Fig. 6.

From the simulation, we can draw the conclusion that the number of rubbers determines the position of the muffling frequency and the volume of cavity can fine adjust the muffling frequency, while the number of cavities determines the range of muffling frequency. This discovery provides us more choices for constructing the submarine muffler.

It can be seen that the attenuation peak will be very close or even overlap when the volumes of two cavities are similar. We make $V_1 = V_2 \approx V_3 = V_4 = 9.874 \times 10^{-6}$ m^3 (depicted in Fig. 8), and keep the material parameters and incentive conditions the same as before. We add one rubber in the multi-cavity Helmholtz muffler, the simulation results are showed in Fig. 9. We can find that the four peaks of transmission loss on behalf of four cavities are overlap completely into only one peak, and the frequency which makes the transmission loss below 10 dB becomes continuous, ranges from near 450 Hz

to 700 Hz, the width is about 250 Hz. The results have more advantages in application than the previous. First, the muffler can get a continuous frequency to absorb wave, and there is no wave absorption trough in the enemy's detection frequency. Second, the width of frequency to reduce noise is relatively wider than the muffler with different volumes. It is more beneficial to widen the range of low frequency noise reduction. We also investigated the influence of volumes on the peak frequency of the transmission loss and the influence of volumes on the width of frequency to reduce noise below 10 dB. In the following simulation, we change the volumes of the multi-cavity (with one rubber added) but keeping the volume of the four cavities the same, the results are depicted in Fig. 10. We find that the volume and frequency are inversely proportional in Fig. 10(a), when the volume is the maximum ($V_1 = V_2 \approx V_3 = V_4 = 9.874$ mm^3), the peak frequency of the transmission loss gets the minimum, which is near 500 Hz, and the peak frequency can get 5000 Hz when the volume is near 10000 mm^3. Moreover, there is the interesting phenomenon that when the volume ranges from 5000 mm^3 to 10000 mm^3, the frequency curve drops very slowly, which only goes down from 1000 Hz to 500 Hz, it also verifies the conclusion in the previous simulation that the volume of cavity has little influence on the general position of the muffling frequency but can fine adjust the muffling frequency. In Fig. 10(b), we can see that the width of frequency to reduce noise below 10 dB has nothing to do with the volume of the cavities, which means that we can fine adjust the muffling frequency without having to consider the decrease of the width of noise reduction frequency.

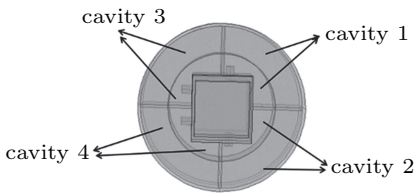


Fig. 8. The muffler with four equal volume cavities.

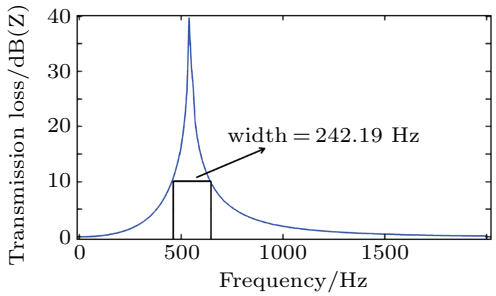


Fig. 9. Transmission loss of the muffler with four equal volume cavities.

From what has been discussed above, we can obtain three effective methods to regulate the position and width of the sound absorption frequency of this muffler, the first is in order to get the continuous and wider muffling frequency, we can make the volume of the four cavities equal, the second is we can change the number of rubbers which are added into the multi-cavity to change the general position of the muffling frequency, the third is we can change the volumes of multi-cavity between 5000 mm^3 to 10000 mm^3 to fine adjust the muffling frequency.

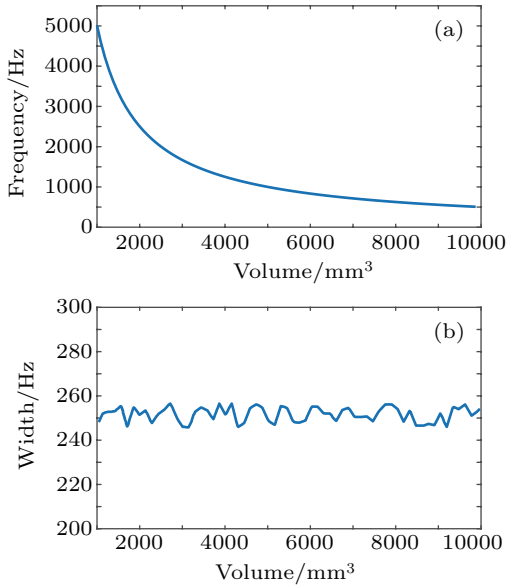


Fig. 10. (a) The influence of volumes on the peak frequency of the transmission loss, (b) the influence of volumes on the width of noise reduction frequency below 10 dB.

4. Conclusion

We introduced a multi-cavity Helmholtz muffler with membrane structure and investigated the influence of the number of Helmholtz cavities and silicone membranes on noise reduction characteristics. In this paper, the relationship between the equivalent negative mass density and the number of membranes was deduced based on the equivalent model, and the muffling frequency has a good agreement with the simulation. We found that the position of the muffling frequency is determined by the number of membranes, with the increase of the number of the membranes in the cavity, the frequency may decrease greatly, which provides us with a way to roughly regulate the position of the muffling frequency. In contrast, the volume of cavity can slightly affect the muffling frequency, when we increase the volume from 5000 mm^3 to 10000 mm^3, the muffling frequency only goes down from 1000 Hz to 500 Hz, which states that we can use the volume of cavity to fine tune the muffling frequency. Moreover, when we make the volumes of the four cavities equal, the muffling frequency becomes continuous and the width becomes wider, and the muffling frequency can also decrease with the increase of the volumes. Besides, the width of the frequency has nothing to do with the volume, so we can fine adjust the muffling frequency without

having to consider the decrease of the width of noise reduction frequency. By changing the number of rubbers and the volume of the cavity, we can get different positions of muffling frequency and different widths of muffling frequency. We can also rough and fine adjust the peak of the muffling frequency to meet different muffling demands, which means in modern naval warfare, we can design different kinds of muffler structures according to the noise frequency of the submarine itself, and so make the submarine avoid being detected by an enemy's radar. The theoretical and simulation investigations in our work may provide guidelines for designing novel muffling structures for submarine.

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