ISSN 1674-1056
http://cpb.iphy.ac.cn
http://iopscience.iop.org/cpb
中国物理 B
Chinese Physics
中国物理 Chinese Physics B

Dynamics of Airy beams in parity-time symmetric optical lattices

Rui-Hong Chen(陈睿弘), Wei-Yi Hong(洪伟毅)

Citation: Chin. Phys. B . 2019, 28(5): 054202. **doi:** 10.1088/1674-1056/28/5/054202
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Reflected and transmitted powers of arbitrarily polarized plane waves at interface between chiral and achiral media

Yan-Feng Lu(路彦峰), Yi-Ping Han(韩一平)
Chin. Phys. B . 2019, 28(2): 024202. **doi:** 10.1088/1674-1056/28/2/024202

Propagation characteristics of oblique incidence terahertz wave through non-uniform plasma

Antao Chen(陈安涛), Haoyu Sun(孙浩宇), Yiping Han(韩一平), Jiajie Wang(汪加洁), Zhiwei Cui(崔志伟)
Chin. Phys. B . 2019, 28(1): 014201. **doi:** 10.1088/1674-1056/28/1/014201

Bifurcated overtones of one-way localized Fabry–Pérot resonances in parity-time symmetric optical lattices

Fatma Nafaa Gaafer, Yaxi Shen(沈亚西), Yugui Peng(彭玉桂), Aimin Wu(武爱民), Peng Zhang(张鹏), Xuefeng Zhu(祝雪丰)
Chin. Phys. B . 2017, 26(7): 074218. **doi:** 10.1088/1674-1056/26/7/074218

Relation between Airy beams and triple-cusp beams

Ren Zhi-Jun, Li Xiao-Dong, Jin Hong-Zhen, Chen Chen, Wu Jiang-Miao
Chin. Phys. B . 2014, 23(8): 084202. **doi:** 10.1088/1674-1056/23/8/084202

Propagation of Airy beams from right-handed material to left-handed material

Lin Hui-Chuan,Pu Ji-Xiong
Chin. Phys. B . 2012, 21(5): 054201. **doi:** 10.1088/1674-1056/21/5/054201

Dynamics of Airy beams in parity–time symmetric optical lattices

Rui-Hong Chen(陈睿弘)[1] and Wei-Yi Hong(洪伟毅)[2,†]

[1] *Guangzhou Key Laboratory for Special Fiber Photonic Devices and Applications, South China Normal University, Guangzhou* 510631, *China*

[2] *Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials and Devices, South China Normal University, Guangzhou* 510631, *China*

(Received 22 July 2018; revised manuscript received 2 February 2019; published online 19 April 2019)

We investigate the dynamics of airy beams propagating in the parity–time (*PT*) symmetric optical lattices in linear and nonlinear regimes, respectively. For the linear propagation, the position of the channel guided by the *PT* lattice can be shifted by tuning the lattice frequency. The underlying physical mechanism of this phenomenon is also discussed. An interesting phenomenon is found in the nonlinear regime in that the airy beam becomes a tilt channel with several Rayleigh lengths. These findings create new opportunities for optical steering and manipulations.

Keywords: parity–time symmetry, Airy beam, nonlinear Schrödinger equation, optical lattices

PACS: 42.25.Bs, 42.60.Jf, 43.25.+y **DOI:** 10.1088/1674-1056/28/5/054202

1. Introduction

According to the traditional theory of quantum mechanics, there is a basic assumption that all operators corresponding to observable physical quantities are Hamiltonian with real eigenvalues. However, in 1998, Bender and Boettcher[1] showed in a seminal work that this assumption needs to be further refined and introduced the concept of parity–time (*PT*-symmetric) complex-valued potentials. They found that non-Hermitian Hamiltonians can still have entirely real eigenvalue spectra provided that these Hamiltonians respect the *PT* symmetry. Even though examples of such operators have been known for a long time, Bender and Boettcher's (1998) discovery had profound implications because it suggested a possibility of *PT*-symmetric modification of the conventional quantum mechanics that considers observables as Hermitian operators in the Hilbert space L^2.

It was also demonstrated that these types of Hamiltonians can undergo a phase transition above a critical threshold; i.e., a spontaneous *PT* symmetry breaking. Above this transition point, the eigenvalue spectrum is no longer entirely real but is rather partially complex.[2–4] So for a general Hamiltonian, one proves that a necessary condition for the Hamiltonian to be *PT*-symmetric is $V(x) = V^*(-x)$,[1,5–7] where $\hat{p}$ and $\hat{x}$ denote the momentum and position operators, respectively. That is to say, *PT*-symmetry requires that the real part of the complex potential involved must be an even function of position whereas the imaginary component should be odd.[6]

In the theoretical arena, the early promotive work of Musslimani *et al.*[5] studied the effect of Kerr nonlinearity on the unique beam dynamics in *PT*-symmetric complex-valued periodic optical potentials; i.e., the formation of nonlinear self-trapped modes, alias optical solitons in both one-dimensional (1D) and two-dimensional (2D) *PT*-symmetric synthetic linear optical lattices. The study of *PT*-symmetric linear optical lattices[6,7] has also attracted much attention during the past few years. Beam dynamics in *PT*-symmetric complex-valued periodic lattices can exhibit unique characteristics, such as double refraction, power oscillations, nonreciprocal diffraction patterns, and so on.[6] Furthermore, gap solitons in parity–time complex-valued periodic optical lattices with the real part of the linear lattice potential having the shape of a double-periodic function (a super lattice potential) were studied by Zhu *et al.*[8] Guo and his scientific research team's finding provided the first example of embedded solitons in 2D *PT* periodic systems.[9] From both theoretical and experimental points of view, the investigation of beam dynamics in nonlinear symmetric systems is essential. Konotop *et al.* comprehensively reviewed recent progress on nonlinear properties of *PT*-symmetric systems.[10] In their complete work, nonlinear *PT*-symmetric systems arising from various physical disciplines are presented, nonlinear properties of these systems are thoroughly elucidated, and relevant experimental results are described.

Recently, the dynamics of beams in *PT*-symmetric optic lattices have been extensively studied,[6,8–13] but there many directions are still to be explored, especially the dynamics of special nondiffracting beams in *PT*-symmetric optic lattices have seldom been studied. Jones took the Mathieu function as an example to show the use of equivalent Hermitian Hamiltonian for *PT*-symmetric sinusoidal optical lattices.[14] To the best of our knowledge, the dynamics of Airy beam in *PT*-symmetric optical lattices have never been studied. Compared

†Corresponding author. E-mail: hongwy@m.scnu.edu.cn

© 2019 Chinese Physical Society and IOP Publishing Ltd

to Mathieu beams, in addition to its quasi-diffraction-free [15] and self-healing properties,[16] the Airy beam has the singular properties of self-bending propagation. Christodoulides first experimentally implemented an Airy beam in 2007.[17,18] Due to its special properties, the Airy beam quickly attracted the attention of researchers.[19–31] In this paper, the dynamics of airy beams propagating in the PT-symmetric optical lattices are investigated both in linear and nonlinear regimes, respectively. For the linear propagation, it is shown that the position of the channel guided by the PT lattice can be shifted via tuning the lattice frequency. The underlying physical mechanism of this phenomenon is also discussed. An interesting phenomenon is found in the nonlinear regime—the airy beam becomes a tilt channel with several Rayleigh lengths.

2. Theoretical model and numerical results

We consider the nonlinear Schrödinger equation with a complex potential modulated by the refractive index and the gains and/or losses in transverse direction. The normalized equation can be described in the form

$$\mathrm{i}\frac{\partial U}{\partial z}+\frac{\partial^2 U}{\partial x^2}-V(x)U+\Delta nU=0,$$

where x and z, respectively, represent the transverse and longitudinal coordinates, U is the complex waveform, and Δn is the change of the nonlinear refractive index. To satisfy the requirement of $V(x)=V^*(-x)$, which is a necessary but not a sufficient condition for the eigenvalue spectrum to be real, we assume $V(x)=A[\cos^2(\omega x)+\mathrm{i}V_0\sin(2\omega x)]$ ($A=4$), where ω is the modulate period and V_0 is the gain or loss coefficient.

In this paper, the input Airy beams propagating in the PT-symmetric lattices can be expressed as $U=u(x,0)=\sqrt{P\mu(\alpha)}Ai(x)\exp(\alpha x)$, where $Ai(x)$ is the Airy function, α is the decay parameter, and $\mu(\alpha)$ is a normalized coefficient. For more convenient discussion, we set $\alpha=0.06$.

2.1. Linear regime

To see how the PT symmetric optical lattices impact on the Airy beam, we first consider the propagation of the Airy beam in the PT lattices in the linear regime by setting $\Delta n=0$. The evolutions of the beam intensity for the cases of $V_0=0.2$, $V_0=0.5$, $V_0=0.7$, and $V_0=1$ are respectively shown in Figs. 1(a)–1(d). The first two bands in the corresponding band structures are also presented in Figs. 1(e)–1(h). For the particular potential considered in our work, the threshold is found to be $V_0^{\mathrm{th}}=0.5$. The band structure is entirely real when $V_0<V_0^{\mathrm{th}}$, and some bands start to merge together and form a oval-like structure when $V_0>V_0^{\mathrm{th}}$, as shown in the figures. For the case of $V_0=0.2$, the forbidden gaps are open, the acceleration of the beam can still be observed while a clear channel with fluctuating energy is guided. At the threshold ($V_0=V_0^{\mathrm{th}}$), clear secondary emissions are observed during discrete diffraction, which is due to the unfolding of the nonorthogonal Floquet–Bloch modes. When V_0 exceeds the threshold, for instance, $V_0=0.7$ in Fig. 1(g), the secondary emissions start to be guided by the lattices. For a larger value of $V_0=1$, the beam is entirely guided by the lattices forming a main channel and a series of sub-channels, of which the lengths are longer the 10 Rayleigh length of the incident beam.

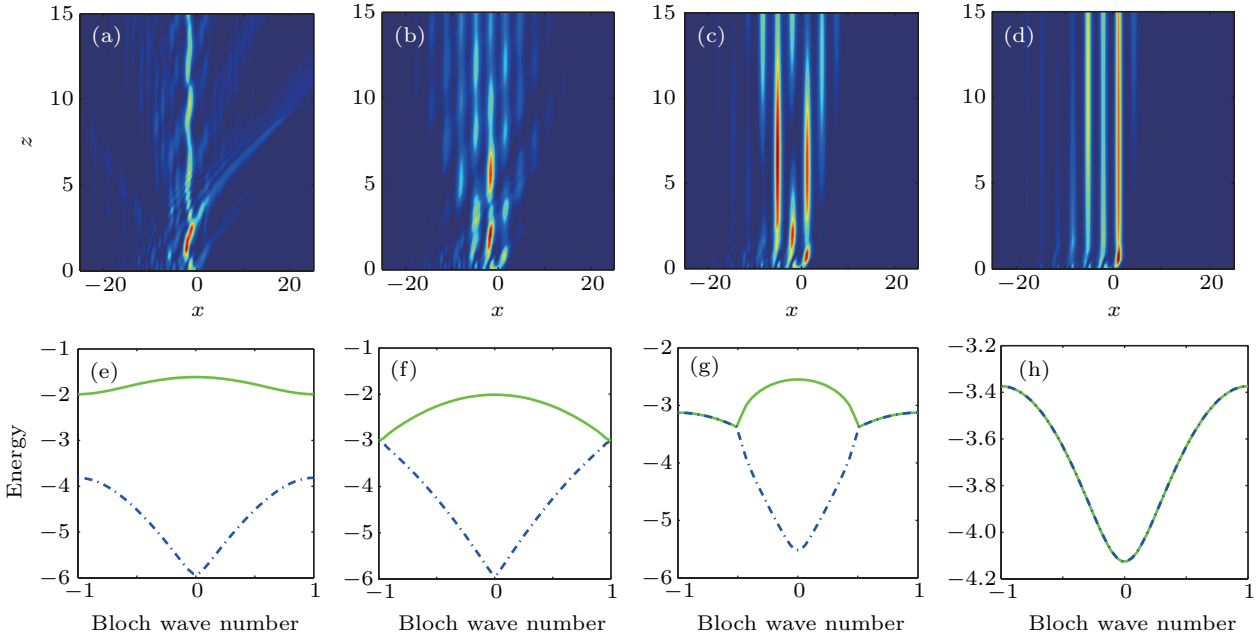


Fig. 1. The evolutions of the Airy beam in the PT lattices in the linear regime with (a) $V_0=0.2$, (b) $V_0=0.5$, (c) $V_0=0.7$, and (d) $V_0=1$. The band structures (first two bands) for the cases of (e) $V_0=0.2$, (f) $V_0=0.5$, (g) $V_0=0.7$, and (h) $V_0=1$. The other parameter is set as $\omega=1$.

Next, we consider the evolutions of the Airy beam in the lattices with different lattice frequencies under the condition that V_0 is much larger than the threshold. Note that the threshold decreases when reducing the lattice frequencies. An interesting phenomenon is found that the main channel guided by the lattice is shifted to the minus direction of the transverse coordinate x, as depicted in Fig. 2. In detail, the main channel is at $x = 3$, $x = 4.5$, and $x = 7$ for the cases of $\omega = 0.8$, $\omega = 0.5$, and $\omega = 0.3$, respectively. The intuitive demonstration for this phenomenon is shown in Fig. 3. The solid and dotted lines in the figure are, respectively, the real and imaginary parts of $V(x)$. It is shown that the location of the main channel is near the loop(s), which is within the positive portion of the imaginary part of $V(x)$, marked by the arrow in the figure. Therefore, such loop(s) gains energy prior to other loops and is guided to become the main channel. For the case of lower frequency, such as $\omega = 0.3$, more loops gain energy at the beginning of the propagation and merge to a channel, of which the width is also larger.

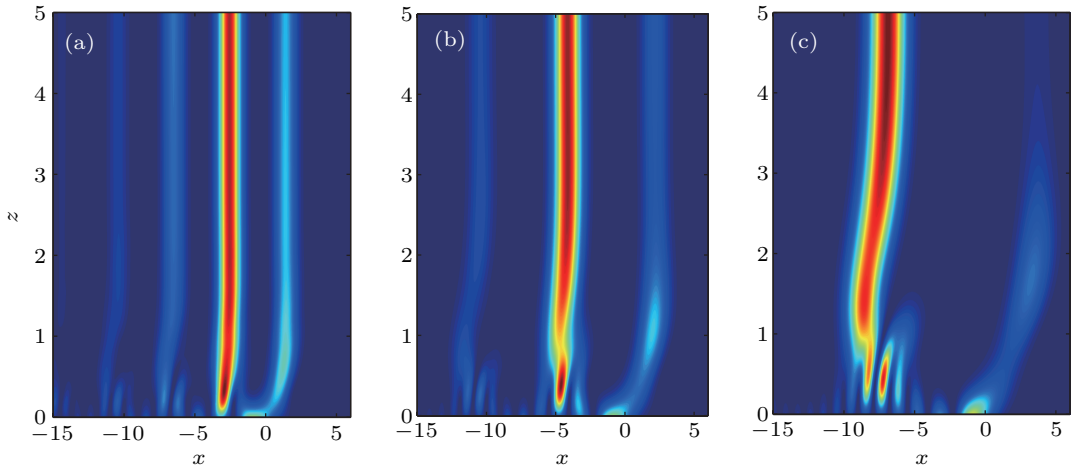


Fig. 2. The evolutions of the Airy beam in the *PT* lattices for the cases of (a) $\omega = 0.8$, (b) $\omega = 0.5$, and (c) $\omega = 0.3$. V_0 is fixed to 1.

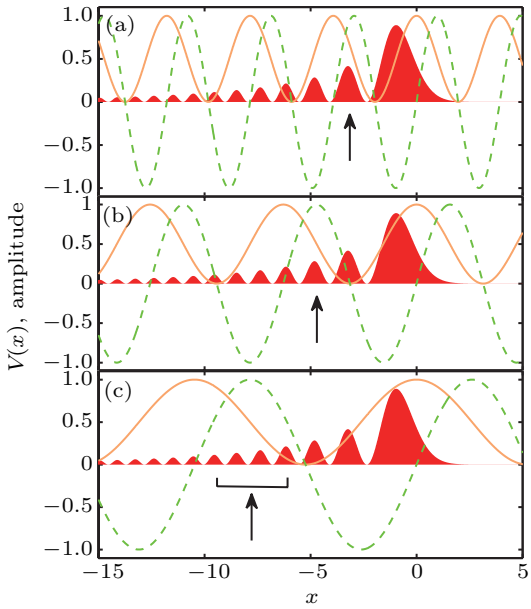


Fig. 3. The real (solid line), imaginary (dotted line) parts of $V(x)$ and the amplitude of the Airy beam for the cases of (a) $\omega = 0.8$, (b) $\omega = 0.5$, and (c) $\omega = 0.3$. The other parameters are the same as those in Fig. 2.

2.2. Nonlinear regime

If the nonlinearity is considered, the propagation of the Airy Beam in the *PT* lattices becomes rather complicated. The results are shown in Figs. 4(a)–4(c) for $P = 1$, $P = 5$, and $P = 9$, with $P = \int_{\infty}^{\infty} |u(x,0)|^2 \mathrm{d}x$ the energy of the initial beam. As P increases, the channel guided by the lattices is shortened and tilted. For some proper energy, for instance, $P = 5$, a clear tilted channel with the length of approximate 3 Rayleigh lengths of the initial beam is observed. As P increases, the channel becomes narrow due to the focusing nonlinearity, and its power also becomes fluctuant. More details for such a channel are depicted in Fig. 5. Figure 5(a) shows the distributions of the beam intensity at different propagation distances. It is shown that the intensity profile keeps unchangeable with approximately 5% of the intensity fluctuation. The total power of the channel as a function of the propagation distance z for different P is also presented in Fig. 5(b). The power of the channel rapidly increases for the cases of different P at the beginning of the propagation, and sharply declines for the case of $P = 5$. Figure 5(c) shows the root-mean-square (RMS) width of the beam as a function of z for different P. For the case of $P = 5$, the RMS width rapidly decreases to approximate 0.55 and then stays unchanged until $z = 5.2$, where the channel diffuses.

The tilted channel is also manipulated by the frequency of the lattice. The results are shown in Figs. 6(a)–6(c) for the cases of different frequencies. For the case of higher frequency where the channel is less shifted to the minus direction of x, the channel is shorter and less tilted, and secondary emissions are observed. In this case, a distinct channel with the length longer than 2 Rayleigh lengths of the initial beam could hardly be observed by adjusting P and V_0 according to our simulation.

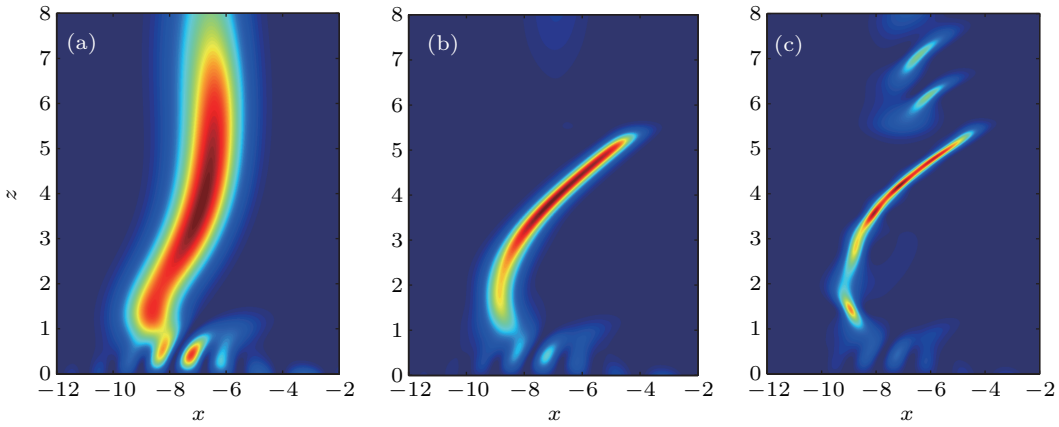


Fig. 4. The evolutions of the Airy beam in the *PT* lattices for (a) $P = 1$, (b) $P = 5$, and (c) $P = 9$. Other parameters are set as $V_0 = 1$ and $\omega = 0.3$ in the simulation.

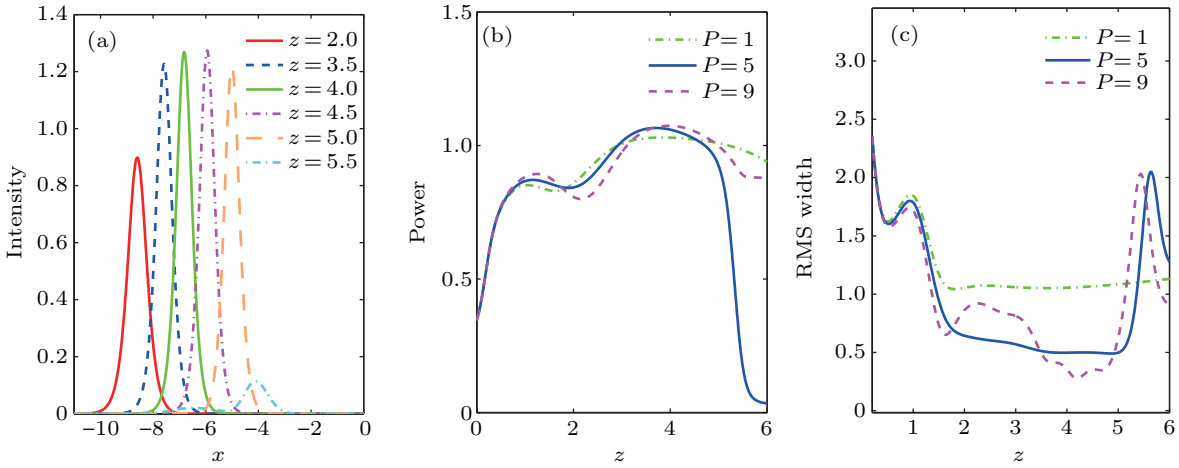


Fig. 5. (a) The distributions of the beam intensity at different propagation distances. (b) The total power and (c) the RMS width of the channel as functions of the propagation distance z for different P. All the parameters are the same as those in Fig. 4(b).

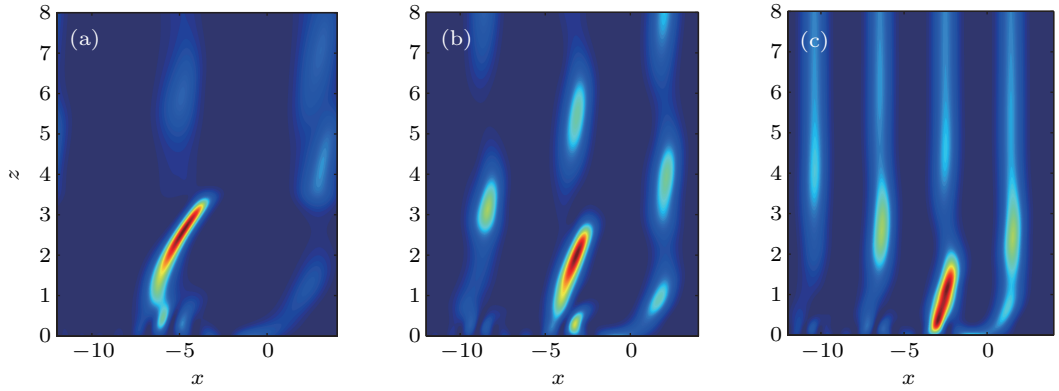


Fig. 6. The evolutions of the Airy beam in the *PT* lattices for (a) $\omega = 0.4$, (b) $\omega = 0.6$, and (c) $\omega = 0.8$. Other parameters are set as $V_0 = 1$ and $P = 5$ in the simulation.

3. Conclusion

We investigate the dynamics of airy beams propagating in the parity–time symmetric optical lattices in both linear and nonlinear regimes. For the linear propagation, it is shown that the position of the channel guided by the *PT* lattice can be shifted via tuning the lattice frequency, which is due to the energy gain of the selective sub-loop(s). An interesting phenomenon is found in the nonlinear regime that the airy beam turns to be a tilt channel with several Rayleigh lengths. These findings create new opportunities for optical steering and manipulations.

References

[1] Bender C M and Boettcher S 1998 *Phys. Rev. Lett.* **80** 5243
[2] Ahmed Z 2001 *Phys. Lett. A* **282** 343
[3] Bender C M, Brody D C and Jones H F 2003 *Am. J. Phys.* **71** 1095
[4] Bender C M, Brody D C, Jones H F and Meister B K 2007 *Phys. Rev. Lett.* **98** 040403

[5] Musslimani Z H, Makris K G, El-Ganainy R and Christodoulides D N 2008 *Phys. Rev. Lett.* **100** 030402
[6] Makris K G, El-Ganainy R, Christodoulides D N and Musslimani Z H 2008 *Phys. Rev. Lett.* **100** 103904
[7] Makris K G, El-Ganainy R, Christodoulides D N and Musslimani Z H 2010 *Phys. Rev. A* **81** 063807
[8] Zhu X, Wang H, Zheng L X, Li H and He Y J 2011 *Opt. Lett.* **36** 2680
[9] Guo D, Xiao J, Li H and Dong L 2016 *Opt. Lett.* **41** 4457
[10] Konotop V V, Yang J and Zezyulin D A 2016 *Rev. Mod. Phys.* **88** 035002
[11] Dai C Q and Wang Y Y 2014 *Laser Phys.* **24** 035401
[12] Liu J B, Xie X T, Shan C J, Liu T K, Lee R K and Wu Y 2015 *Laser Phys.* **25** 015102
[13] He Y J, Zhu X, Mihalache D, Liu J L and Chen Z X 2012 *Phys. Rev. A* **85** 013831
[14] Jones H F 2011 *J. Phys. A* **44** 345302
[15] Broky J, Siviloglou G A, Dogariu A and Christodoulides D N 2008 *Opt. Express* **16** 12880
[16] Siviloglou G A, Broky J, Dogariu A and Christodoulides D N 2008 *Opt. Lett.* **33** 207
[17] Siviloglou G A, Broky J and Dogariu A 2007 *Phys. Rev. Lett.* **99** 213901
[18] Siviloglou G A and Christodoulides D N 2007 *Opt. Lett.* **32** 979
[19] Zhang P, Prakash J, Zhang Z, Mills M S, Efremidis N K, Christodoulides D N and Chen Z G 2011 *Opt. Lett.* **36** 2883
[20] Zheng Z, Zhang B F, Chen H, Ding J and Wang H T 2011 *Appl. Opt.* **50** 43
[21] Baumgartl J, Mazilu M and Dholakia K 2008 *Nature Photon.* **2** 675
[22] Polynkin P, Kolesik M and Moloney J 2009 *Phys. Rev. Lett.* **103** 123902
[23] Polynkin P, Kolesik M, Moloney J V, Siviloglou G A and Christodoulides D N 2009 *Science* **324** 229
[24] Rose P, Diebel F, Boguslawski M and Denz C 2013 *Optics and Photonics News* **24** 45
[25] Li J X, Fan X L, Zang W P and Tian J G 2011 *Opt. Lett.* **36** 648
[26] Li J X, Zang W P and Tian J G 2010 *Opt. Express* **18** 7300
[27] Choi D, Lee K, Hong K, Lee I M, Kim K Y and Lee B 2013 *Opt. Express* **21** 18797
[28] Panagiotopoulos P, Abdollahpour D, Lotti A, Couairon A, Faccio D, Papazoglou D G and Tzortzakis S 2012 *Phys. Rev. A* **86** 013842
[29] Driben R, Konotop V V and Meier T 2014 *Opt. Lett.* **39** 5523
[30] Pasiskevicius V 2009 *Nature Photon.* **3** 374
[31] Xu Y Q, Zhou G Q, Zhang L J and Ru G Y 2015 *Laser Phys.* **25** 085005