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Dynamics of two levitated nanospheres nonlinearly coupling with non-Markovian environment*

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The dynamics of two nanospheres nonlinearly coupling with non-Markovian reservoir is investigated. A master equation of the two nanospheres is derived by employing quantum state diffusion method. It is shown that the nonlinear coupling can improve the non-Markovianity. Due to the sharing of the common non-Markovian environment, the state transfer between the two nanospheres can be realized. The entanglement and the squeezing of the individual mode, as well as the jointed two-mode are analyzed. The present system can be realized by trapping two nanospheres in a wideband cavity, which might provide a method to study adjustable non-Markovian dynamics of mechanical motion.

Keywords: non-Markovianity, levitated nanospheres, nonlinearity

PACS: 03.65.Yz, 42.50.Lc, 42.50.–p **DOI:** 10.1088/1674-1056/28/5/050302

1. Introduction

Trapping particles via an electromagnetic field is widely researched theoretically and experimentally because of its potential application in precise instrument and macroscopic quantum effects detection. For example, an optical tweezer is applied to capture and control cells in biological and medical research. The realization of Bose–Einstein condensation (BEC) and atom cooling have strict requirements for optical traps.[1–4] With the development of quantum techniques, the optical trap has been exploited in several research domains, such as generating squeezing states[5] and macroscopic superposition states of trapped matters,[6–8] entangling the trapped particles,[9,10] and cooling the trapped matter to ultra-low temperature.[11–14] In addition, the nonlinearity of the levitated system has been reported in Refs. [12]–[17]. The dynamics of a nanoparticle that nonlinearly couples to its reservoir has been theoretically studied in the Markovian regime.[18,19]

Non-Markovian theory plays an important role in treating a realistic system. The non-Markovian environment usually can keep the coherence and suppress the dissipation. The theory of open quantum system is researched through several perspectives, such as the projection operator theory,[20,21] Green's functions method,[22–24] perturbation theory,[25] and stochastic Schrödinger equations (SSEs) techniques.[26–30] However, these researches mostly investigate the system linearly coupling with its non-Markovian environment. However, the system coupled with its environment in nonlinear form remains under-studied.

In this paper we consider two nanospheres trapped by a continuous frequency field and we write the Hamiltonian of the system with a nonlinear coupling to its environment. By employing quantum state diffusion (QSD) method,[28–30] we derive a nonlinear master equation. We investigate the dynamics of the two levitated nanospheres with open quantum system techniques, and study the entanglement, squeezing of the two nanospheres, as well as the state transfer. We find that the nonlinear coupling can improve non-Markovianity.

2. Model and Hamiltonian

We consider two nanospheres trapped by a wideband cavity. The wideband cavity field can be considered as a non-Markovian environment of the two levitated nanospheres. As found in Refs. [5] and [17], the vibration frequency of the trapped nanospheres is on the order of kHz or MHz, which is smaller than the width of the trapping field. This means that we can treat the trapping fields as a wideband cavity. Recently, the crystal cavity with engineering modes, whose frequency is continuous, has been employed for trapping nanoparticles.[31] When acted upon by the optical forces, named the gradient force $F_{\rm grad}$ and the scattering force $F_{\rm scatt}$, the two nanospheres can be described as harmonic oscillators that linearly and nonlinearly couple to the wideband cavity field (the detail illustration can be seen in Ref. [6]). The Hamiltonian can be written as

$$H = \sum_k \Delta_k a_k^\dagger a_k + \sum_{i=1,2} \frac{\Omega_i}{2}(q_i^2 + p_i^2) + \sum_{k,i}(G_k^* a_k + G_k a_k^\dagger)(q_i - \gamma_i q_i^2), \quad (1)$$

where a_k ($a_k^\dagger$) is the annihilation (creation) operator of the k-th mode with detuning Δ_k, Ω_i is the levitated frequency, q_i (p_i) is

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position (momentum) operator of the i-th nanosphere, and G_k is the coupling between the k-th cavity field and nanospheres.

The Hamiltonian (1) describes the two nanospheres wrapped up by photon gas with a continuous spectrum distribution, which is analogous to Brownian motion. The system should be solved with non-Markovian theory. However, the third summation in Eq. (1) contains the nonlinear term in order of q_i which differs from the usual linear non-Markovian interaction. The system can not be solved with the general non-Markovian theory as done in Refs. [23] and [32].

To treat the nonlinear and non-Markovian dynamics, we employ QSD method. For convenience, we rewrite the Hamiltonian into three parts, the free energy of the environment $H_{\rm B}=\sum_k \Delta_k a_k^\dagger a_k$, the energy of the two nanospheres $H_{\rm s}=\sum_i \Omega_i/2(q_i^2+p_i^2)$, and the interaction part $H_{\rm int}=\sum_k(G_k^* a_k+G_k a_k^\dagger)L$, where $L=\sum_i q_i-\gamma_i q_i^2$. It is convenient to change into the interaction picture rotating with $H_{\rm B}$, then we have the time-dependent Hamiltonian

$$H_{\rm tot}(t)=H_{\rm s}+L\big(B(t)+B^\dagger(t)\big), \tag{2}$$

with

$$B(t)=\mathrm{e}^{\mathrm{i}H_{\rm B}t}\sum_k(G_k a_k^\dagger+G_k^* a_k)\mathrm{e}^{-\mathrm{i}H_{\rm B}t}. \tag{3}$$

The open quantum system obeys the non-Markovian quantum state diffusion equation

$$\partial_t|\Psi_t\rangle=H_{\rm tot}(t)|\Psi_t\rangle. \tag{4}$$

We assume the initial state of the system and environment as $|\Psi_0\rangle=|\psi_0\rangle\otimes|0_1\rangle\otimes\cdots\otimes|0_k\rangle\otimes$. The system state $|\psi_0\rangle$ is arbitrary, and the initial states of the environment are their ground states $|0_k\rangle$. The total state $|\Psi_t\rangle$ can be expanded as

$$|\Psi_t\rangle=\int\frac{\mathrm{d}^2z}{\pi}\mathrm{e}^{-|z|^2}|\psi_t(z^*)\rangle\otimes|z\rangle, \tag{5}$$

where $|z\rangle=|z_1\rangle\otimes|z_2\rangle\otimes\cdots\otimes|z_k\rangle\cdots$, and $\mathrm{d}^2z=\mathrm{d}^2z_1\mathrm{d}^2z_2\cdots\mathrm{d}^2z_k\cdots$ with identity for any Bargmann coherent state

$$I_k=\int\frac{\mathrm{d}^2z_k}{\pi}\mathrm{e}^{-|z_k|^2}|z_k\rangle\langle z_k|. \tag{6}$$

The stochastic state of system $\psi_t(z^*)=\langle z|\Psi_t\rangle$ obeys QSD equation

$$\partial_t\psi_t=-\mathrm{i}H_{\rm s}\psi_t+Lz_t^*\psi_t-L^\dagger\int_0^t\mathrm{d}s f(t-s)\frac{\delta\psi_t}{\delta z_{\rm s}^*}, \tag{7}$$

where

$$f(t-s)=\int\mathrm{d}\omega\mathcal{J}(\omega)\mathrm{e}^{-\mathrm{i}\omega(t-s)}=\sum_k G_kG_k^*\mathrm{e}^{-\mathrm{i}\Delta_k(t-s)} \tag{8}$$

is the correlation function of the bath. The variation in Eq. (7) can be taken place by some time-dependent operator O satisfying $\delta\psi_t/\delta z_{\rm s}^*=O(t,s,z^*)\psi_t$. Therefore, the non-Markovian QSD equation (7) is rewritten in a pseudo-time-local form

$$\partial_t\psi_t(z^*)=(-\mathrm{i}H_{\rm s}+Lz_t^*-L^\dagger\bar{O}(t,z^*))\psi_t(z^*), \tag{9}$$

where $\bar{O}(t,z^*)=\int_0^t f(t-s)O(t,s,z^*)\mathrm{d}s$. The operators $\bar{O}$ and O satisfy

$$\frac{\partial}{\partial t}O=[-\mathrm{i}H_{\rm s}+Lz_t^*-L^\dagger\bar{O},O]-L^\dagger\frac{\delta}{\delta z_{\rm s}^*}\bar{O}. \tag{10}$$

The reduced density operator of the open system can be obtained after partial trace operating over total state $|\Psi_t\rangle$, that is

$$\rho(t)=\mathrm{Tr}_{\rm B}[|\Psi_t\rangle\langle\Psi_t|]=\int\frac{\mathrm{d}^2z}{\pi}\langle z|\Psi_t\rangle\langle\Psi_t|z\rangle. \tag{11}$$

We denote the reduced density operator $\rho(t)=\mathcal{M}[|\psi_t\rangle\langle\psi_t|]$ with operator ensemble mean $\mathcal{M}[\cdot]=\int\mathrm{d}z^2/\pi\,\mathrm{e}^{-|z|^2}[\cdot]$ over all stochastic states. The master equation is obtained as

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S=-\mathrm{i}[H_{\rm s},\rho_S]+[L,\rho_S\bar{O}^\dagger]+[\bar{O}\rho_S,L^\dagger]. \tag{12}$$

However, it is still difficult to find the exact solution of O and $\bar{O}$ with Eq. (10), especially when the system contains nonlinear or multi-particle interaction. An alternative method is to expand it in terms of the weak coupling strength, i.e., $O(t,s,z^*)=\sum_n G^nO_n(t,s,z^*)$, where G represents the coupling strength. The lowest order of this operator can be written as

$$O(t,s,z^*)\approx O_0(t,s)=\mathrm{e}^{-\mathrm{i}H_{\rm s}(t-s)/\hbar}L\mathrm{e}^{\mathrm{i}H_{\rm s}(t-s)/\hbar} \tag{13}$$

by employing post-Markov approximation, which is valid when the square of the ratio of the environment memory time scale to system time scale is negligible.[33,34] The integrated operator should be expressed as

$$\begin{aligned}\bar{O}(t)\ =\ &\sum_i\big[(q_ig_{1i}+p_i\tilde{g}_{1i})\\&+\gamma_i(q_i^2g_{2i}+p_i^2\tilde{g}_{2i}-(p_iq_i+q_ip_i)h_i)\big],\end{aligned} \tag{14}$$

where the coefficients can be found in Appendix B. From Eq. (12), the master equation with nonlinear terms is obtained as

$$\begin{aligned}\dot{\rho}\ =\ &-\mathrm{i}[H_{\rm s},\rho]\\&+\sum_{i,j}\Big\{[q_i,\rho(q_jg_{1i}+p_j\tilde{g}_{1i})]+[(q_ig_{1i}+p_i\tilde{g}_{1i})\rho,q_j]\\&+\gamma_i\Big[q_j,\rho\big(g_{2i}q_i^2+p_i^2\tilde{g}_{2i})+(p_iq_i+q_ip_i)h_i\big)\Big]\\&+\gamma_j\Big[\big(g_{2i}^*q_i^2+p_i^2\tilde{g}_{2i}^*+(p_iq_i+q_ip_i)h_i^*\big)\rho,q_j\Big]\\&+\gamma_i\gamma_j\Big[q_j^2,\rho\big(g_{2i}q_i^2+p_i^2\tilde{g}_{2i}+(p_iq_i+q_ip_i)h_i\big)\Big]\\&+\gamma_i\gamma_j\Big[\big(g_{2i}^*q_i^2+p_i^2\tilde{g}_{2i}^*+(p_iq_i+q_ip_i)h_i^*\big)\rho,q_j^2\Big]\Big\},\end{aligned} \tag{15}$$

where γ_i is the label of nonlinear coupling. The contributions of the linear and nonlinear couplings can be discussed by choosing γ_i equal to zero or not. In addition, the method of the master equation deduction (15) can be easily extended to that with nonlinear coupling. If the two nanospheres interact with other fields except the non-Markovian reservoir, then one only need to add their interaction into the Hamiltonian and perform the above procedure again.

3. Squeezing of levitated nanospheres induced by the wideband cavity field

From the first line of the summation in Eq. (15), we can obtain an effective interaction between the two nanospheres resulting from the linear coupling, i.e., $H_{\rm eff} = -\sum_{i,j} {\rm Im}(g_{1i}) q_i q_j$, which can induce a self-squeezing and two-mode squeezing of the nanospheres.

Meanwhile, the nonlinear coupling between photon and phonon with $\gamma_i \neq 0$ induces another effective interaction

$$H_{\rm kerr} = \sum_{i,j} {\rm Im}(g_{2i}) \gamma_i \gamma_j q_i^2 q_j^2, \tag{16}$$

which describes a Kerr effect.

We would like to investigate the effect of nonlinear coupling under Lorentz spectrum $\mathcal{J}(\omega) = \frac{\Gamma_0 d_0^2}{(\omega - \Omega_{\rm CL})^2 + d_0^2}$, which is a typical example to analyze an open system thanks to its realism in experiment, where Γ_0 is the spectrum strength and d_0 is the spectrum width.

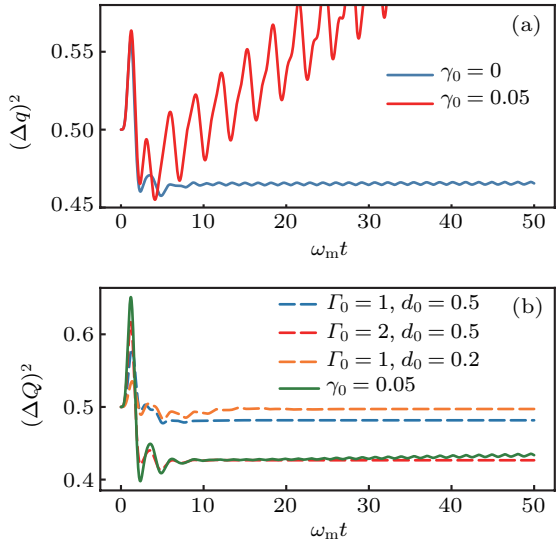


Fig. 1. The evolution of (a) single-mode fluctuation and (b) two-mode fluctuation of the levitated nanospheres. In panel (a), $\Gamma_0 = 2\omega_{\rm m}$, $d_0 = 0.5\omega_{\rm m}$, and $\Omega_{\rm CL} = \sqrt{2}\omega_{\rm m}$. In panel (b), for the green line, $\Gamma_0 = 2\omega_{\rm m}$ and $d_0 = 0.5\omega_{\rm m}$; for other dashed lines, $\gamma_i = \gamma_0 = 0$. We let $\Omega_i = \omega_{\rm m}$.

Squeezing can be quantified by the fluctuation of operators. The position fluctuation is defined as $(\Delta q)^2 = \langle q^2 \rangle - \langle q \rangle^2$. The two-mode fluctuation can be defined as $(\Delta Q)^2 = \langle q^2 \rangle - \langle q \rangle^2$, where $Q = \frac{1}{2}(q_1 + q_2)$. We choose $\Omega_1 = \Omega_2 = \omega_{\rm m}$ and assume that the initial states of the nanospheres are coherent states $|\alpha_i\rangle$ with $\alpha_1 = 1$ and $\alpha_2 = 0$. As shown in Fig. 1(a), the fluctuation $(\Delta q)^2$ with linear coupling attains a relative "stable" value (with weak amplitude oscillation). When $\gamma_0 = 0.05\omega_{\rm m}$, the single-mode squeezing finally disappears after long time evolution. For two-mode squeezing, there is little difference for $\gamma_0 = 0.05\omega_{\rm m}$ or not. Since the two-mode squeezing exhibits the correlation between the two nanospheres, the nonlinear coupling slightly affects the correlation. In addition, we can observe that the larger Γ_0, the stronger the two-mode squeezing, because Γ_0 is the spectrum strength. The spectrum width d_0 also has a little bit influence on the two-mode squeezing. One can see that the squeezing with $d_0 = 0.5\omega_{\rm m}$ is stronger than that with $d_0 = 0.2\omega_{\rm m}$ as shown in Fig. 1(b), but it does not mean the bigger d_0 the stronger squeezing. In our simulation, we find that for $d_0 = 3\omega_{\rm m}$, there is no squeezing. If $d_0 \to \infty$, $\mathcal{J}(\omega) = \Gamma_0$, then it is Markovian and the squeezing is obviously worse than that of non-Markovian.

4. Entanglement and state transfer

The two-mode squeezing represents the correlation between two nanospheres. Quantum entanglement, as an alternative correlation between two subsystems, can also be induced by effective interaction. The entanglement can be measured by negativity, which is defined as

$$E_{\rm N} = \max[0, -\ln 2\eta^-], \tag{17}$$

where $\eta^- = 1/\sqrt{2}\left[\Sigma(\boldsymbol{V}) - \sqrt{\Sigma(\boldsymbol{V})^2 - \det \boldsymbol{V}}\right]$ is the minimum symplectic eigenvalue of 2×2 block covariance matrix

$$\boldsymbol{V} = \begin{pmatrix} \boldsymbol{V}_{\rm A} & \boldsymbol{V}_{\rm C} \\ \boldsymbol{V}_{\rm C}^{\rm T} & \boldsymbol{V}_{\rm B} \end{pmatrix}$$

with element $V_{i,j} = \langle o_i o_j + o_j o_i \rangle / 2$ ($o_i = q_i, p_i$) and $\Sigma(\boldsymbol{V}) = \det \boldsymbol{V}_{\rm A} + \det \boldsymbol{V}_{\rm B} - 2 \det \boldsymbol{V}_{\rm C}$.

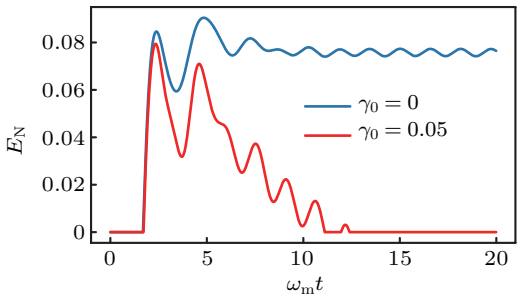


Fig. 2. The evolution of negativity where $\Gamma_0 = 2\omega_{\rm m}$ and $d_0 = 0.5\omega_{\rm m}$. Other parameters are the same as those in Fig. 1.

We plot the negativity in Fig. 2. The entanglement does not appear when $\omega_{\rm m} t < 2$ then increases fast(sudden birth) and stabilizes without nonlinear coupling. When the nonlinear coupling is included, the entanglement is always less than that

with $\gamma_0 = 0$, finally drops to zero, which means that the nonlinear coupling is not good for preserving the existence entanglement for a long time. This happens because the nonlinear coupling also induces an addition damping. We will explain this point later.

The effective interaction connects the two nanospheres, thus if the nanospheres are in different initial states $\rho_1(0), \rho_2(0)$, then the states $\rho_2(0)$ ($\rho_1(0)$) will transfer to $\rho_1(t)$ ($\rho_2(t)$) through effective interaction. We employ the conversion fidelity $\mathcal{F}(\rho_1, \rho_2) = \mathrm{Tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}$ to measure the efficiency of state transfer.[35] We plot the fidelity in Fig. 3. We can observe that the fidelity oscillates periodically with decreasing amplitude, whether or not the nonlinear coupling exists. When $\gamma_0 = 0.05\omega_\mathrm{m}$, the fidelity is obviously worse than that with $\gamma_0 = 0$. The maximum fidelity can reach 0.9, which is relative ideal, while the entanglement is weak (see Fig. 2) and the squeezing is not so deep (see Fig. 1). This happens because the equal frequency of the two nanospheres makes the effective interaction induced by linear coupling H_eff with beam split (BS) form, therefore the state transfer can be achieved while the squeezing and the entanglement are not so ideal.

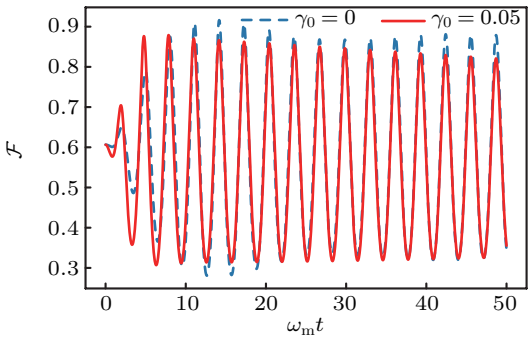


Fig. 3. The evolution of fidelity. Other parameters are the same as those in Fig. 1.

5. Non-Markovianity measure

We have shown that squeezing, entanglement, and state transfer between the two nanospheres can be achieved in our system. These properties result from their common coupling with the same non-Markovian environment; i.e., relating with the non-Markovianity in the present system. We now investigate the non-Markovianity of the system. For the non-Markovianity measure, several proposals[36–39] based on violation of dynamical semigroups or information backflow have been put forward. Here, we employ the measure of non-Markovianity proposed in Ref. [36]

$$\mathcal{N}(\Phi) = \max_{\rho_S^{1,2}} \int_{\sigma>0} \mathrm{d}t\,\sigma(t), \tag{18}$$

where $\sigma(t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}(\Phi_t\rho_S^1, \Phi_t\rho_S^2)$ is the derivative of trace distance between different initial states $\rho_S^{1,2}$ and

$$\mathcal{D}(\rho^1, \rho^2) = \frac{1}{2}\mathrm{Tr}|\rho^1 - \rho^2|, \tag{19}$$

with $|\rho| = \sqrt{\rho^\dagger\rho}$. Under this measure, $\mathcal{N}(\Phi) > 0$ implies that the dynamical map Φ is non-Markovian. We calculate the non-Markovianity measure by solving Eq. (15). As shown in Fig. 4(a), with the increase of γ_0, the non-Markovianity is improved whenever $d_0 = 0.25\omega_\mathrm{m}$ or $d_0 = 0.5\omega_\mathrm{m}$. Unfortunately, as we found in the previous section, the nonlinear coupling damages the squeezing, entanglement, and state transfer to some degree. This result conflicts with previous works, which declare that the information backflow could preserve those properties rather than damage them.[40–43] To uncover the reason for this, we plot the trace distance between different initial states $\mathcal{D}(\rho^1, \rho^2)$ in Fig. 4(b) with a random initial state pair where the initial state is the same for red and blue lines. After a long time evolution, the quasi-steady state may be achieved no matter what the initial state is. Therefore, the decline of $\mathcal{D}(\rho^1, \rho^2)$ for different initial state indicates the dissipation of the system. As shown in Fig. 4(b), when $\omega_\mathrm{m}t > 5$, a larger value of nonlinearity γ_0 will lead to a faster decline of $\mathcal{D}(\rho^1, \rho^2)$, which means that a larger value of γ_0 will lead to a larger dissipation rate. Thus, on the one hand the nonlinearity enhances the non-Markovianity, and on the other hand increases the loss rate. This is the reason why the squeezing and entanglement become bad after a long time evolution.

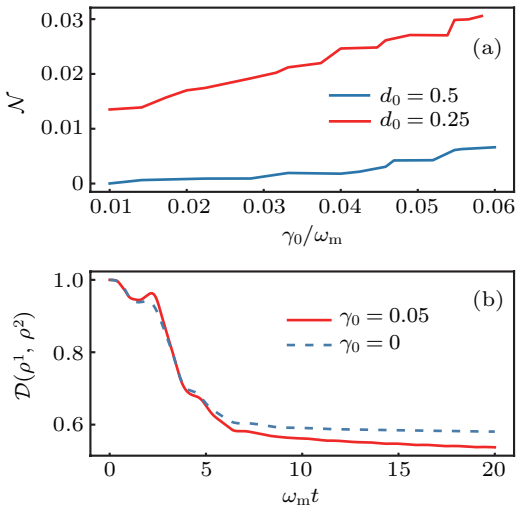


Fig. 4. (a) The non-Markovianity measure as a function of γ_0 where $\Gamma_0 = \omega_\mathrm{m}$. (b) The trace distance between two different initial states $\rho_S^{1,2}$ for $\Gamma_0 = 2\omega_\mathrm{m}$ and $d_0 = 0.5\omega_\mathrm{m}$. Other parameters are the same as those in Fig. 1.

6. Discussion and conclusion

We investigate two nanospheres that are trapped in a wideband cavity field, where the two nanospheres nonlinearly couple with the environment. The main contribution of this work is that we put forward a method to treat non-Markovian dynamics with nonlinear coupling. A master equation of the nonlinear non-Markovian system is derived, in which the effective linear and nonlinear interactions can be induced, which

can generate the squeezing, entanglement, and realize state transfer. We study the influence of this nonlinear coupling on the dynamics of the system. We find that the nonlinear coupling can improve non-Markovianity. After a long enough evolution, the nonlinear coupling will damage the squeezing and entanglement because the nonlinearity enlarges the loss rate. To compensate for the loss of the system, one can introduce another pumping field. In addition, one can see that the entanglement and the squeezing are not as good as the state transfer because the equal frequency of the two nanospheres results in the main interaction with beam split form. If one would like to obtain large entanglement and squeezing, then one could manipulate the two nanospheres with red and blue detunings, respectively. The spectrum of the environment depends on the distribution of the cavity field, which can be manipulated. Therefore, it might be a way to simulate the spectrum of the environment to study the non-Markovian system.

Acknowledgement

We would like to thank Xinyu Zhao and W. L. Li for helpful discussions.

Appendix A: The nonlinear coupling

The levitated nanospheres are trapped by an electromagnetic field. They receive the optical forces, namely the gradient force $F_{\rm grad}$ and the scattering force $F_{\rm scatt}$ according to the generation mechanism.[44] The $F_{\rm grad}$ and $F_{\rm scatt}$ can be described as

$$F_{\rm grad} = -k_{\rm trap} q, \quad F_{\rm scatt} = k_{\rm trap}\beta, \tag{A1}$$

where $k_{\rm trap} \propto |E|^2$ with E being the electromagnetic amplitude of the cavity field, β is a constant number for marking the scattering force, and q is the position of the nanosphere. The motion of the levitated nanosphere is simple harmonic oscillation where the gradient force acts as the restoring force.[17,44] The Hamiltonian of the levitated nanosphere can be written as

$$H = \frac{k_{\rm trap}}{2} q^2 + \frac{p^2}{2m} - \beta k_{\rm trap} q, \tag{A2}$$

where p is the momentum operator of the nanosphere, and m is the mass. Assuming $k_{\rm trap} = \chi a^\dagger a$ with a constant coefficient χ, where a ($a^\dagger$) is the annihilation (creation) operator of the cavity field, the stiffness factor $k_{\rm trap}$ can be expanded as $k_{\rm trap} = \chi(|\alpha|^2 + \alpha^* a + \alpha a^\dagger)$ with mean-field approximation $a \to \alpha + a$. The Hamiltonian of the nanosphere and cavity field can be rewritten as

$$H = \Delta a^\dagger a + \frac{\Omega}{2}(p^2 + q^2) - (G^* a + G a^\dagger)(q - \gamma q^2), \tag{A3}$$

where $\Omega = \chi|\alpha|^2/2$ is the frequency of the nanosphere, the operators q and p are changed to dimensionless ones with $q \to q/\sqrt{m\Omega}$ and $p \to \sqrt{m\Omega}\, p$, respectively, and the constant part $\chi|\alpha|^2 q$ inducing a displacement has been neglected, because it can be canceled just by displacing the zero-point of the oscillator. $G = \alpha\beta\chi/\sqrt{m\Omega}$ is the coupling between the nanosphere and cavity field, and $\gamma = \frac{1}{\beta}\frac{1}{\sqrt{m\Omega}}$ represents the nonlinear coupling strength. The first term is the energy of the cavity field with detuning $\Delta = \omega_{\rm c} - \omega_{\rm L}$ ($\omega_{\rm c}$ and $\omega_{\rm L}$ are the frequencies of the cavity field and the classical pumping field, respectively), the second term denotes the energy of the harmonic oscillator, and the third term describes the coupling between the cavity field and the mechanical oscillator.

Appendix B: Coefficients

The time-dependent coefficients of Eq. (14) can be calculated by Eq.(13) as

$$\begin{aligned} g_{ji} &= \int_0^t {\rm d}s f(s)\cos^j(\Omega_i s), \quad j = 0,1,2, \\ \tilde{g}_{ji} &= \int_0^t {\rm d}s f(s)\sin^j(\Omega_i s), \quad j = 1,2, \\ h_i &= \tilde{g}_1(\Omega_i \to 2\Omega_i), \end{aligned} \tag{B1}$$

where $f(t) = \int_0^\infty \frac{{\rm d}\omega}{2\pi}\mathcal{J}(\omega){\rm e}^{-{\rm i}\omega t}$. We can integrate Eq. (B1) to

$$\begin{aligned} g_{ji} &= \int_0^t {\rm d}s \int_0^\infty \frac{{\rm d}\omega}{2\pi}\mathcal{J}(\omega)\cos^j(\Omega_i s){\rm e}^{-{\rm i}\omega s}, \quad j = 0,1, \\ \tilde{g}_{1i} &= \int_0^t {\rm d}s \int_0^\infty \frac{{\rm d}\omega}{2\pi}\mathcal{J}(\omega)\sin(\Omega_i s){\rm e}^{-{\rm i}\omega s}. \end{aligned} \tag{B2}$$

To conveniently discuss the equation, we would like to divide the coefficients into imaginary and real parts $\mathcal{G}(\Omega_i,t) = \mathcal{R}(\Omega_i,t) + {\rm i}\mathcal{I}(\Omega_i,t)$ where $\mathcal{G}$ represents g_0, g_1, $\tilde{g}_1$.

For Lorentz form spectrum, the time correlation function is $f(t) = \frac{\Gamma_0 d_0}{2}{\rm e}^{-\tilde{d}_0 t}$, where $\tilde{d}_0 = d_0 + {\rm i}\Omega_{\rm CL}$, and the coefficients can be solved as

$$\begin{aligned} g_{0i} &= \frac{\Gamma_0 d_0}{2}\frac{1 - {\rm e}^{-\tilde{d}_0 t}}{\tilde{d}_0}, \\ g_{1i} &= \frac{\Gamma_0 d_0}{2}{\rm e}^{-\tilde{d}_0 t}\frac{-\tilde{d}_0\cos\Omega_i t + \tilde{d}_0\,{\rm e}^{\tilde{d}_0 t} + \Omega_i\sin\Omega_i t}{\Omega_i^2 + \tilde{d}_0^2}, \\ \tilde{g}_{1i} &= \frac{\Gamma_0 d_0}{2}{\rm e}^{-\tilde{d}_0 t}\frac{\Omega_i\,{\rm e}^{\tilde{d}_0 t} - \tilde{d}_0\sin\Omega_i t - \Omega_i\cos\Omega_i t}{\Omega_i^2 + \tilde{d}_0^2}. \end{aligned} \tag{B3}$$

When $\Omega_{\rm CL} \to 0$, $d_0 \to \infty$ corresponds to Markovian situation, then $g_0(t) = \Gamma_0/2$, $g_1(\Omega_i,t) = \Gamma_0/2$, $\tilde{g}_1(\Omega_i,t) = 0$, which means that there is no effective interaction except dissipation. In contrast, when $d_0 \ll \Omega_i$ the non-Markovian properties may occur, we can obtain

$$g_1(\Omega_i, t\to\infty) = \frac{\Gamma_0 d_0}{2}\frac{d_0(\Omega_{\rm CL}^2 + \Omega_i^2) + {\rm i}\Omega_{\rm CL}(\Omega_i^2 - \Omega_{\rm CL}^2)}{(\Omega_i^2 - \Omega_{\rm CL}^2)^2}$$

with $\Omega_{\rm CL} \neq \Omega_i$. As discussed in the main text, the imaginary part of $g_1(\Omega_i,t)$ generates the effective interaction between the two nanosphere.

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