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General way to define tunneling time*

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With the development of attosecond science, tunneling time can now be measured experimentally with the attoclock technique. However, there are many different theoretical definitions of tunneling time and no consensus has been achieved. Here, we bridge the relationship between different definitions of tunneling time based on a quantum travel time in one-dimensional rectangular barrier tunneling problem. We find that the real quantum travel time $t_{\rm Re}$ is equal to the Bohmian time $t_{\rm Bohmian}$, which is related to the resonance lifetime of a bound state. The total quantum travel time $\tau_{\rm t}$ can perfectly retrieve the transversal time t_x and the Büttiker–Landauer time $\tau_{\rm BL}$ in two opposite limits, regardless of the particle energy.

Keywords: tunneling time, Büttiker–Landauer time, Bohmian time

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1. Introduction

The tunneling time problem is almost as old as quantum mechanics itself[1] and has been a subject of intense theoretical debate for many years.[2–15] Recently, progress in attosecond science has allowed for a measurement of tunneling time in the so-called attoclock experiments.[16–26] However, there are considerable controversies in the interpretation of the attoclock experiments. Some investigations have supported a nonzero tunneling delay time,[17,19,26] while others supported a zero tunneling delay time.[27–30]

On the theoretical side, there is still no consensus on the definition of tunneling time. The Larmor clock was proposed by Baz'[3] and Rybachenko[4] to measure tunneling time. This idea is to use spin polarized electrons and a potential barrier with a constant magnetic field inside, considering the rotation of the spin in the plane that is perpendicular to the magnetic field can define a time. Büttiker[6] recognized that the Larmor time is equal to the dwell time calculated by the ratio of integrated probability density over the barrier region to the incident flux. The identity relation between Larmor time and dwell time is also fulfilled for a potential barrier of arbitrary shape[11] and in the relativistic case.[10] Büttiker also pointed out that the main effect of the magnetic field in lamor precession is to align the spin with the field. Thus, the barrier preferentially transmits a particle with spin parallel to the magnetic field, which can be described by the Büttiker–Landauer time.[6] The Büttiker–Landauer time $\tau_{\rm BL}$[5] describes the time spent by the particle to travel from the entrance point x_1 to the exit point x_2 under the barrier $V(x)$. Within the Wentzel–Kramers–Brillouin (WKB) approximation, the general definition of Büttiker–Landauer time is $\tau_{\rm BL} = \int_{x_1}^{x_2} m/p(x){\rm d}x$,[5] where $p(x) = \sqrt{2m[V(x)-E]}$ is the momentum of particle with the kinetic energy E under the barrier. Afterwards, Sokolovski and Baskin constructed a single complex time by Feynman path-integral technique, which can elegantly combine the Larmor time and the Büttiker–Landauer time.[31] Very recently, we proposed a definition of quantum travel time which provides a reasonable interpretation of the tunneling delay time measured by attoclock experiment and bridges the gap between it and the Büttiker–Landauer time.[32] This quantum travel time can also provide a reasonable description in the case of very thin barrier where the Büttiker–Landauer time is not well defined.

Although there are many conflicting definitions of tunneling time in conventional quantum mechanics, Bohmian mechanics[33,34] privileges the time that the Bohmian trajectory spends between the entrance and exit points of the potential barrier.[35] Recently, it has been found that the Bohmian time does not correspond to the tunneling time, but agrees with the resonance lifetime of a bound state.[35]

In this paper, we further explore the relationship between the newly introduced quantum travel time with other definitions of tunneling time (Larmor time, Büttiker–Landauer time, and Bohmian time) in one-dimension rectangular tunneling process. In Section 2, we give a brief calculation about the one-dimension rectangular barrier tunneling and show the dif-

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ferent definitions of tunneling time (quantum travel time τ_t, t_{Re}, t_{Im}; the Larmor time t_{LM} and associated times (t_x, t_y, and t_z)). In Section 3, we investigate the relationship between different tunneling times and find that the real quantum travel time t_{Re} is equal to the Bohmian time $t_{Bohmian}$, and the total quantum travel time τ_t can bridge the connection between the time t_x and the Büttiker–Landauer time τ_{BL}.

2. Theoretical definition of tunneling time

2.1. Rectangular barrier tunneling

Consider a particle with kinetic energy $E = \hbar^2 k^2/2m$ moving along the x-axis and interacting with a rectangular barrier of height V_0 and width d centered at $x = 0$, as shown in Fig. 1. The wave function of the particle within different regions (I, II, and III) can be written as

$$\psi_{\rm I} = {\rm e}^{{\rm i}kx} + A{\rm e}^{-{\rm i}kx}, \quad x < -d/2, \tag{1}$$

$$\psi_{\rm II} = B{\rm e}^{\kappa x} + C{\rm e}^{-\kappa x}, \quad |x| \le d/2, \tag{2}$$

$$\psi_{\rm III} = D{\rm e}^{{\rm i}kx}, \quad x > d/2, \tag{3}$$

where $\kappa = \sqrt{2m[V_0 - E]}/\hbar$ and $k = \sqrt{2mE}/\hbar$; A, B, C, and D are coefficients determined by the continuity of wavefunction and its first derivative at the edge of the rectangular barrier. These coefficients can be obtained as[5]

$$A = R^{1/2}{\rm e}^{{\rm i}\Delta\phi}{\rm e}^{-{\rm i}\pi/2}{\rm e}^{-{\rm i}kd}, \tag{4}$$

$$B = \frac{\kappa + {\rm i}k}{2\kappa}{\rm e}^{{\rm i}kd/2}e^{-\kappa d/2}D, \tag{5}$$

$$C = \frac{\kappa - {\rm i}k}{2\kappa}{\rm e}^{{\rm i}kd/2}{\rm e}^{\kappa d/2}D, \tag{6}$$

$$D = T^{1/2}{\rm e}^{{\rm i}\Delta\phi}{\rm e}^{-{\rm i}kd}, \tag{7}$$

where T is the transmission probability

$$T = \left\{1 + \left[\left(\kappa^2 + k^2\right)^2/4k^2\kappa^2\right]\sinh^2(\kappa d)\right\}^{-1}, \tag{8}$$

and $\Delta\theta$ is the phase changed across the barrier with

$$\cot(\Delta\theta) = \frac{2k\kappa}{k^2 - \kappa^2}\coth(\kappa d). \tag{9}$$

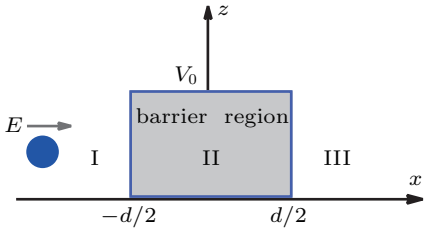


Fig. 1. Schematic diagram: a particle with kinetic energy E moves along the x-axis and interacts with a rectangular barrier with height V_0 and width d centered at $x = 0$.

2.2. Quantum travel time

In quantum mechanics, the momentum operator is well defined, thus the quantum travel time is introduced by analogy with the classical travel time[32]

$$\tau_t = \frac{m|x_2 - x_1|}{||\bar{p}||}, \tag{10}$$

where m is the mass of the particle and $||\cdots||$ means making the modula of "$\cdots$". The average momentum $\bar{p}$ of the particle during its stay within the barrier region (x_1, x_2) can be obtained by calculating the expected value of the momentum operator $\hat{p} = -{\rm i}\hbar\nabla$

$$\bar{p} = \frac{\int_{x_1}^{x_2}\psi^*(x)\hat{p}\psi(x){\rm d}x}{\int_{x_1}^{x_2}\psi^*(x)\psi(x){\rm d}x}, \tag{11}$$

where $\psi(x)$ is the wave function of the particle within the region (x_1, x_2).

By combining Eqs. (5)–(9) and substituting Eq. (2) into Eq. (11), we can obtain the average momentum of the particle within the barrier region[32]

$$\bar{p} = R(r, \Delta\theta)\hbar k + {\rm i}I(r, \Delta\theta)\hbar\kappa, \tag{12}$$

where

$$R(r, \Delta\theta) = \frac{2\kappa^2}{\kappa^2 - k^2}\frac{\left(\kappa^2 + k^2\right)rd}{\left(\kappa^2 + k^2\right)rd - 2k\cot(\Delta\theta)}, \tag{13}$$

$$I(r, \Delta\theta) = \frac{2k\kappa}{\kappa^2 - k^2}\frac{2k}{\left(\kappa^2 + k^2\right)rd - 2k\cot(\Delta\theta)}. \tag{14}$$

Here, $r = 4k^2\kappa^2\left[\left(\kappa^2 + k^2\right)^2\sinh^2(\kappa d)\right]^{-1}$ is the ratio of the transmission probability to the reflection probability. Substituting Eqs. (12)–(14) into Eq. (10), we can obtain

$$\tau_t = \frac{md}{\hbar\sqrt{R^2(r, \theta)k^2 + I^2(r, \theta)\kappa^2}}. \tag{15}$$

The average momentum divided by the mass of particle gives the average velocity of the particle within the barrier region. We define the average velocity as follows:

$$\bar{v} = \bar{v}_{\rm Re} + {\rm i}\bar{v}_{\rm Im}. \tag{16}$$

According to the definition of quantum travel time in Eq. (10), we can also define another two different times

$$t_{\rm Re} \equiv \frac{d}{\bar{v}_{\rm Re}}, \tag{17}$$

$$t_{\rm Im} \equiv \frac{d}{\bar{v}_{\rm Im}}. \tag{18}$$

We can easily obtain the relationship between the three times, $t_{\rm Re}$, $t_{\rm Im}$, and τ_t, from Eq. (16)

$$\left(\frac{1}{\tau_t}\right)^2 = \left(\frac{1}{t_{\rm Re}}\right)^2 + \left(\frac{1}{t_{\rm Im}}\right)^2. \tag{19}$$

2.3. Larmor time and associated times

Considering an incident particle with x-direction polarized spin tunneling through a barrier region within a constant z-direction magnetic field B, the particle will experience a Larmor precession.[6] For the transmitted wavefunction of the particle, we can calculate the expectation values of spin in three different directions: $< S_x >$, $< S_y >$, and $< S_z >$. The Larmor time $t_{\rm LM}$ is the average time spent by the particle inside the barrier, which is given by the degree of precession in the y-direction S_y[10,14]

$$t_{\rm LM} = t_y = \lim_{\omega_{\rm L}\to 0} < S_y > /(-\omega_{\rm L}/2), \tag{20}$$

where $\omega_{\rm L} = B/2$ is the Larmor frequency (the atomic units (a.u.) are used). By combining the definitions of r and $\cot(\Delta\theta)$, we can write the Larmor time $t_{\rm LM}$ as follows:

$$t_{\rm LM} = t_y = \frac{md}{\hbar k}\frac{\kappa^2 - k^2}{2\kappa^2}\frac{\left(\kappa^2 + k^2\right) rd - 2k\cot(\Delta\theta)}{\left(\kappa^2 + k^2\right) rd}T. \tag{21}$$

Since the particle tunneling through the barrier can also acquire a spin component parallel to the magnetic field,[6] the z-direction precession of spin $< S_z >$ can also define a time t_z[6,14]

$$t_z = \lim_{\omega_{\rm L}\to 0} < S_z > /(-\omega_{\rm L}/2). \tag{22}$$

This time t_z can be rewritten as

$$t_z = \frac{m}{\hbar\kappa^2}\frac{d(k^2 - \kappa^2)\cot^2(\Delta\theta) - 2k^2\kappa}{2k(r+1)\cot(\Delta\theta)}. \tag{23}$$

For the expectation value of spin in the x-direction $< S_x >$, a time t_x is also defined,[6] which satisfies the relationship as follows:

$$t_x = \sqrt{t_y^2 + t_z^2}. \tag{24}$$

3. Relationship between different tunneling times

3.1. The quantum travel times $t_{\rm Re}$, $t_{\rm Im}$, and $\tau_{\rm t}$

First, we discuss the behavior of the quantum travel time in two opposite limiting cases. In the limit of the very thin barrier $\kappa d \ll 1$, the coefficients on the right side of Eq. (12) can be approximated to $R(r,\Delta\theta) \approx 1$ and $I(r,\Delta\theta) \approx 0$. In the opposite limit of opaque barrier $\kappa d \gg 1$, we have $R(r,\Delta\theta) \approx 0$ and $I(r,\Delta\theta) \approx 1$. Accordingly, we can obtain

$$t_{\rm Re} \approx md/\hbar k, \quad \kappa d \ll 1, \tag{25}$$

$$t_{\rm Re} \approx \frac{mk_0^2}{8\hbar k\kappa^3}{\rm e}^{2\kappa d}, \quad \kappa d \gg 1, \tag{26}$$

$$t_{\rm Im} \approx \tau_{\rm BL}\frac{2\kappa}{k_0^2 d}, \quad \kappa d \ll 1, \tag{27}$$

$$t_{\rm Im} \approx \tau_{\rm BL}, \quad \kappa d \gg 1, \tag{28}$$

where $\tau_{\rm BL} = md/\hbar\kappa$ is the Büttiker–Landauer time. Thus, for the very thin barrier $\kappa d \ll 1$, the time $\tau_{\rm t}$ is mainly determined by the time $t_{\rm Re}$, i.e., $\tau_{\rm t} \approx t_{\rm Re} \approx md/\hbar k$, which means that a classical particle passes through a distance d with momentum $\hbar k$. While in the case of opaque barrier $\kappa d \gg 1$, $t_{\rm t} \approx t_{\rm Im}$ approaches the Büttiker–Landauer time $md/\hbar\kappa$ which is also obtained in the limit of opaque barrier.[5] These behaviors of the three quantum travel times can be clearly seen in Fig. 2.

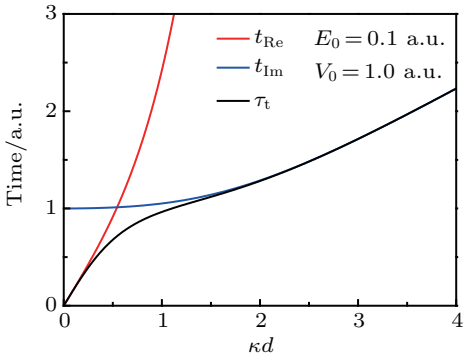


Fig. 2. A comparison between time $t_{\rm Re}$, $t_{\rm Im}$, and $\tau_{\rm t}$ as a function of κd with $E = 0.1$ a.u. and $V_0 = 1.0$ a.u. The atomic units are used with $e = m = \hbar = 1$.

3.2. The Bohmian time $t_{\rm Bohmian}$, the Lamor time $t_{\rm LM}$, and time $t_{\rm Re}$

The Bohmian time is defined as the time required for a Bohmian trajectory to pass the region between the two classical turning points x_1 and x_2[34,35]

$$t_{\rm Bohmian} = \frac{1}{J}\int_{x_1}^{x_2} |\psi_{\rm II}|^2 {\rm d}x, \tag{29}$$

where $J = \hbar kT/m$ is the stationary probability flux. After substituting Eq. (2) into Eq. (29), we obtain

$$\begin{aligned} t_{\rm Bohmian} &= t_{\rm Re} \\ &= \frac{md}{\hbar k}\frac{\kappa^2 - k^2}{2\kappa^2}\frac{\left(\kappa^2 + k^2\right) rd - 2k\cot(\Delta\theta)}{\left(\kappa^2 + k^2\right) rd}. \end{aligned} \tag{30}$$

The Bohmian time was found to be related to the resonance lifetime of a bound state,[35] which can be calculated from the resonance widths Γ taken from Ref. [36]. Comparing Eq. (21) with Eq. (30), we can obtain $t_{\rm Bohmian} = t_{\rm Re} = t_{\rm LM}/T = t_y/T$. Thus, the time $t_{\rm Re}$ is also related to the decay rates of quasi-stationary states and reflects the resonance lifetime.

3.3. The Büttiker–Landauer time $\tau_{\rm BL}$, the times t_x, and $\tau_{\rm t}$

The Büttiker–Landauer time $\tau_{\rm BL}$ is based on the onset of the "cross-over" regime between pure tunneling and tunneling while absorbing one or more photons from the oscillating field.[5] Büttiker and Landauer argued that this time $\tau_{\rm BL}$ can determine the actual barrier transversal time. In Section 3.1, we show that the time $\tau_{\rm t}$ is mainly determined by the time $t_{\rm Im}$ for an opaque barrier, which is equal to the Büttiker–Landauer time $\tau_{\rm BL}$. In Fig. 3, we show the comparison between $\tau_{\rm BL} = md/\hbar\kappa$ and $t_{\rm Im}$ for different kinetic energies E

of the particle. It can be seen that the time $t_{\rm Im}$ perfectly coincides with the Büttiker–Landauer time $\tau_{\rm BL}$ in the condition of $\kappa d > 3$. However, there is obvious difference between them in the region $\kappa d < 3$. The time $t_{\rm Im}$, in the limit of thin barrier, i.e., $d \to 0$, approaches a finite value $1/V_0$ regardless of the kinetic energy of the particle. This can be understood from the uncertainty relation of the time and energy $\Delta E \Delta t \sim \hbar$ considering that for a particle to pass through the barrier with a height V_0, the time it needs is about $1/V_0$. It is noted that the Büttiker–Landauer time $\tau_{\rm BL}$ is obtained in the opaque barrier approximation,[5] while in the limit of very thin barrier, there is no well-defined Büttiker–Landauer time.

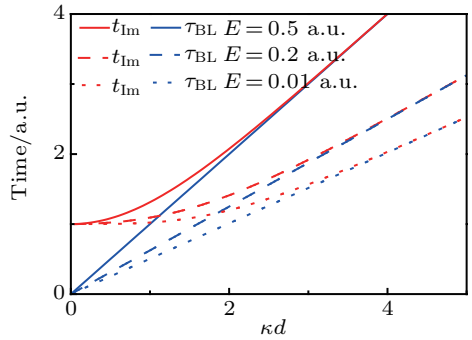


Fig. 3. A comparison between time $t_{\rm Im}$ and the Büttiker–Landauer time $\tau_{\rm BL}$ as a function of κd with $E = 0.5$ a.u. and $V_0 = 1.0$ a.u. The atomic units are used with $e = m = \hbar = 1$.

Büttiker proposed that the time t_x (Eq. (24)) can also be used as the barrier traversal time. In Fig. 4, we show a comparison between the times t_x (red line), $\tau_{\rm BL}$ (blue line), and $\tau_{\rm t}$ (black line) under different kinetic energies of the particle. Although both the time t_x and the Büttiker–Landauer time $\tau_{\rm BL}$ can be treated as barrier traversal time, there is a difference between them, especially when the dimensionless parameter κd is not very large. The difference increases as the kinetic energy of the particle decreases. It is interesting that the quantum travel time $\tau_{\rm t}$ can perfectly retrieve t_x and $\tau_{\rm BL}$ in two opposite limits, regardless of the particle energy. As seen in Fig. 4, $\tau_{\rm t}$ is equal to the time t_x when κd is small. And $\tau_{\rm t}$ has an asymptotic behavior similar to the time t_x, i.e., approaching the finite value $1/V_0$ in the limit that the kinetic energy of the particle and the width of the potential barrier tends to zero at the same time. In the opposite limit, $\tau_{\rm t}$ perfectly coincides with the Büttiker–Landauer time $\tau_{\rm BL}$ when $\kappa d > 3$. While in the range $1 < \kappa d < 3$, the value of time $\tau_{\rm t}$ just falls between the two times: t_x and $\tau_{\rm BL}$. The quantum travel time $\tau_{\rm t}$ is composed of two parts: time $t_{\rm Re}$ and time $t_{\rm Im}$. The time $t_{\rm Im}$ characterizes the quantum property of the tunneling process. The time $t_{\rm Re}$ reflects the time needed by the entire wavefunction of the particle to tunnel through the barrier, which includes the influence of the transmission probability. In the condition that the width of barrier approaches zero, the transmission probability tends to one. Thus, the time $t_{\rm Re}$ tends to the classical time, which means the time that a classical particle needs to pass through a distance d. The time t_x is also composed of two parts: t_y and t_z. The time t_y is related to the time $t_{\rm Re}$ ($t_{\rm Re} = t_y/T$). The time t_z is derived under infinitesimal field, which can be approximated to $\tau_{\rm BL}$ under opaque barrier approximation. While in our definition of quantum travel time, the time $t_{\rm Im}$ is equivalent to the Büttiker–Landauer time $\tau_{\rm BL}$ under opaque barrier approximation. Thus, we bridge the connection between the time t_x and the Büttiker–Landauer time $\tau_{\rm BL}$ through the quantum travel time $\tau_{\rm t}$.

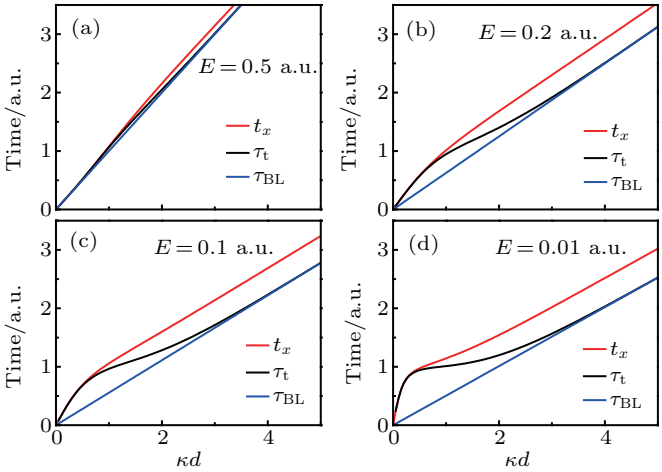


Fig. 4. The time t_x, the time $\tau_{\rm t}$, and the Büttiker–Landauer time $\tau_{\rm BL}$ as a function of κd with (a) $E = 0.5$ a.u., (b) $E = 0.2$ a.u., (c) $E = 0.1$ a.u., and (d) $E = 0.01$ a.u. The height of the potential barrier $V_0 = 1.0$ a.u. is the same in (a), (b), (c), and (d). The atomic units are used with $e = m = \hbar = 1$.

4. Conclusion

In conclusion, based on a new definition of quantum travel time, we bridge the connections between different tunneling times ($\tau_{\rm BL}$, $t_{\rm Bohmian}$, and t_x) in the one-dimensional rectangular barrier tunneling. The time $t_{\rm Re}$ is equal to the Bohmian time $t_{\rm Bohmian}$, which is related to the resonance lifetime of a bound state. The time $\tau_{\rm t}$ is a generalized traversal time which cannot only retrieve the Büttiker–Landauer time for $\kappa d > 3$ but also equal the time t_x when κd is small. In the case where the kinetic energy of the particle and the width of the barrier tend to zero at the same time, then time t_x and time $\tau_{\rm t}$ have the same limiting value $1/V_0$.

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