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Ratchet transport of overdamped particles in superimposed driven lattices*

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Ratchet transport of overdamped particles is investigated in superimposed driven lattices using Langevin dynamics simulations. It is found that noise can strongly affect the transport of the particles. When lattices driving dominates the transport, the noise acts as a disturbance of the directed transport and slows down the average velocity of the particles. When the driving phase has less impact on particle transport, Gaussian white noise can play a positive role. By simply modulating these two parameters, we can control efficiency and the direction of the directed currents.

Keywords: Gaussian white noise, superimposed driven lattices, rectification

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1. Introduction

In recent years, the rachet effect in driven lattice potentials — which can evoke directed particle transport from an unbiased nonequilibrium system — has been a popular research topic.[1–8] The rachet effect is the phenomenon of driving thermal Brownian particles into directed currents by breaking certain space–time symmetries in the absence of a net force. Broadly speaking, there are four types of rachet models: rocking ratchets,[9,10] flashing ratchets,[11–13] correlation ratchets,[14–17] and entropic ratchets.[18–21]

Particles can be pumped in periodic asymmetric potential by switching the sawtooth dielectric potential on and off.[22] Further study finds current reversal occurring in deterministic ratchets, which is related to bifurcation from a chaotic and periodic regime with the change of an exponent and a control parameter.[23] A symmetric periodic potential and a zero-mean friction force produced by two counterpropagating lattice beams can also break the temporal symmetry of the system and induce directed motion, which has been demonstrated experimentally.[24] It has been theoretically and experimentally demonstrated that the interaction between deterministic driving and fluctuations produces the resonance which is related to driving frequency. The direction of the transport is determined by the driving frequency.[25] Based on the mechanism of ratchet effect, optical lattices can be used to trap colloidal Brownian particles and induce systematic transport in experiments.[26–28] Brownian particles can be transported in a two-dimensional (2D) coupled flashing ratchet driven by Gaussian noise.[12] In the anisotropic structures, white thermal noise together with periodic driving force can generate a nonzero macroscopic velocity.[9] Rectification phenomenon specific to high-dimensional rocking ratchets is determined theoretically and experimentally.[29,30] As for deterministic optical rocking ratchets, the direction of current can be changed by the asymmetry of the potential.[31] Experiments show that dissipation can induce currents for cold atoms in a space–time symmetric potential.[32] In a 2D ac-driven lattice, changing the structure of the lattice in the direction perpendicular to the applied driving force can induce current reversals.[33] Another study finds out using a periodically shaken 2D dissipative lattice can separate particles in specific directions by their physical properties.[34]

Recently, Mukhopadhyay and coworkers superimposed a symmetric oscillating lattice as "substrate lattice" on a second oscillating lattice as "carrier lattice" to manipulate particle transport in real time. By switching the carrier lattice on (off), directed current can be accelerated (freezed). Directed current can be slowed down by changing the value of a phase difference.[35] However, noise is not considered in this work. As we know, noise plays an important role in rachet transport. In this paper, we study the directed transport of overdamped particles in the superimposed driven lattices and focus on finding how noise affects the rectification.

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2. Model and methods

We consider N noninteracting overdamped particles moving in a periodic potential $V(x,t) = V_s(x,t) + V_c(x,t)$ as shown in Fig. 1. Here, V_s represents substrate lattice and V_c represents carrier lattice.[35] The V_s and V_c are

$$V_s(x,t) = V_s \cos^2\{k[x + d\cos(\omega t)]\}, \quad (1)$$

$$V_c(x,t) = V_c \cos^2\{k[x + 2d\cos(2\omega t + \phi)]\}, \quad (2)$$

where k is the wave number. The oscillation amplitude and frequency of carrier lattice are twice as large as those of substrate lattice. Each particle obeys the overdamped Langevin equation

$$\gamma \frac{dx}{dt} = \frac{\partial V(x,t)}{\partial x} + \sqrt{2D_0}\xi(t), \quad (3)$$

where x is the position of the particle and γ is the friction coefficient; D_0 is the noise intensity; and the Gaussian white noise is $\xi(t)$ with zero mean and unit variance. The average velocity in the asymptotic long-time regime can be obtained from the formula,

$$V = \frac{1}{N}\sum_{i=1}^{N} \lim_{t\to\infty} \frac{x_i(t) - x_i(0)}{t}. \quad (4)$$

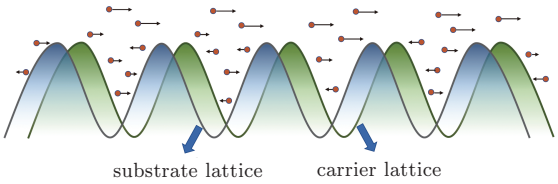


Fig. 1. Schematic illustration of particles moving in superimposed driven lattices called substrate lattice and carrier lattice.

3. Results and discussion

In the simulation, the integration time step is set as 10^{-2} and the total integration time is set as 10^4. Unless otherwise noted, our simulations are performed under the conditions of the parameter sets: $V_s = V_c = 2.0$, $k = 1.0$, $N = 10^4$, and $\gamma = 1.0$. Our simulation results do not depend on time step and integration time when $dt < 0.05$ and $t > 100$. We vary D_0, d, ω, and ϕ and calculate the average velocity V of passive particles. The simulation results are shown in the following Figs. 2–7.

Figure 2 shows the average velocity V as a function of translational diffusion coefficient D_0 for different ϕ. When $\phi = \pi$, the directed motion cannot be observed. For $\phi = \pi/2$ and $3\pi/2$, the directed current is most significant and the average velocity V decreases monotonically when D_0 increases. In these cases, lattices driving dominates the transport, and transports the particles efficiently. With the increase of D_0, particles diffuse faster and faster randomly, which means the noise acts as a disturbance and is against the directed transport. For $\phi = \pi/2$ and $3\pi/2$, the average forces provided by lattices are completely opposite in a time period 2π, which indicates that the time cumulative effects of lattices are completely opposite and induces opposite currents. For $\phi = \pi/4$ and $5\pi/4$, there exists an optimal value of D_0 ($D_0 < 0.1$) at which V reaches its maximal. For $\phi = \pi/4$ and $5\pi/4$, the rachet effect becomes faint, and the particles diffuse faster and are easier to escape from the well with the D_0 gradually increasing from 0. When particles can just enough pass through the well, V reaches to a peak which is concerned with the oscillation characteristics of the lattices. When $D_0 \to \infty$, the rachet effect disappears and the particle starts the chaotic motion.

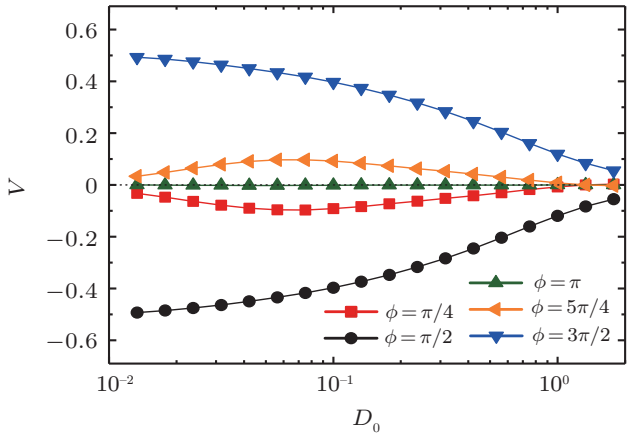


Fig. 2. Average velocity V as a function of noise intensity D_0 for different ϕ at $d = \pi$ and $\omega = 1.0$.

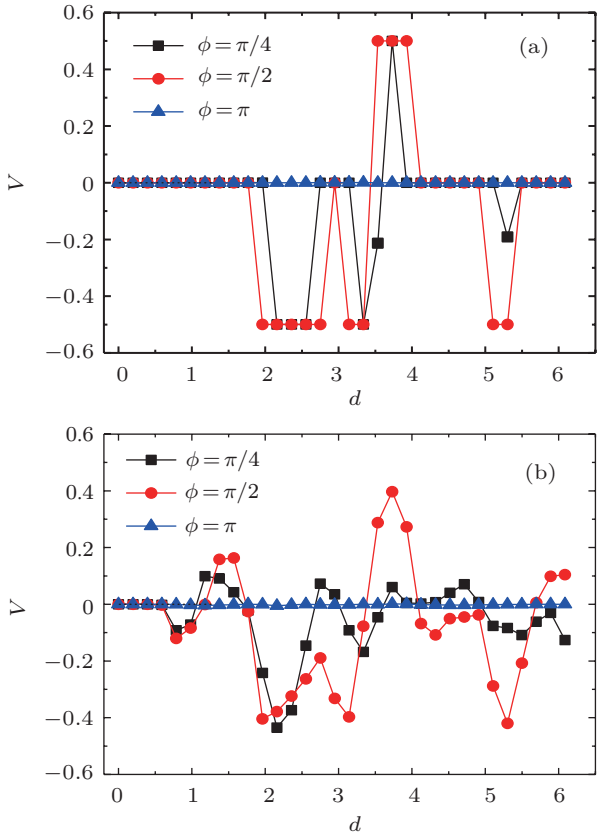


Fig. 3. Average velocity V as a function of oscillation amplitude d for different ϕ. (a) Without white noise. (b) With white noise. The other parameters are $D_0 = 0.1$ and $\omega = 1.0$.

Figure 3 shows the average velocity V as a function of d with or without white noise. As shown in Fig. 3, when $d \to 0$, oscillation amplitude is so small that lattices driving disappears. When $d \to \infty$, the oscillation amplitude is too large, thus there is not enough time for particles to diffuse away. From Fig. 2, we can find out when lattices driving dominates the transport, noise acts as a disturbance of the directed transport and slows down the average velocity of the particles, and shown in Fig. 3. For example, when $\phi = \pi/2$ and $d = 3.7$, V takes its maximum up to 0.5 without noise (see Fig. 3(a)), but it only takes 0.4 for overdamped particle (see Fig. 3(b)). When the rachet effect becomes faint, particles diffuse faster with noise. Therefore, noise facilitates the directed transport such as the curve at $\phi = \pi/4$ and $d = 4.7$. In addition, when $\phi = \pi$, the system is time-reversal symmetry with or without noise. Only the Gaussian white noise cannot induce a particle current. In Fig. 3(a), when d is appropriate, the oscillating lattices show the feature of resonance and cause the jump of data. The resonance condition is determined by d, ω, and ϕ, as shown in Figs. 3, 5, and 6. In particular, we further verify that particle number N has almost no impact on our results as shown in Fig. 4.

Figure 5 displays the average velocity V as a function of ω with or without white noise. When $\omega \to 0$, the lattices stop oscillating and the particle pump is not allowed. When $\omega \to \infty$, the lattices oscillate so fast that the particles cannot diffuse away. In Fig. 5(a), particles are transported to positive direction only by lattices driving at appropriate ω, while in Fig. 5(b), there also exists noise driving, thus these two kinds of driving will compete. For $\phi = 3\pi/2$, particles take the maximal absolute average velocity when oscillation frequency ω is nearly 1 with or without noise, which means lattices driving dominates the transport and noise disturbs the directed motion. Especially, when ω is much smaller (or larger) than 1, the effect of lattices driving recedes and noise driving can facilitate the transport. For $\phi = \pi/4$, we can arrive at the same conclusions. Therefore, the value range of ω allowing for the directed current is broadened with the white noise (see Fig. 5(b)), and ω can be the switch of the current reversal.

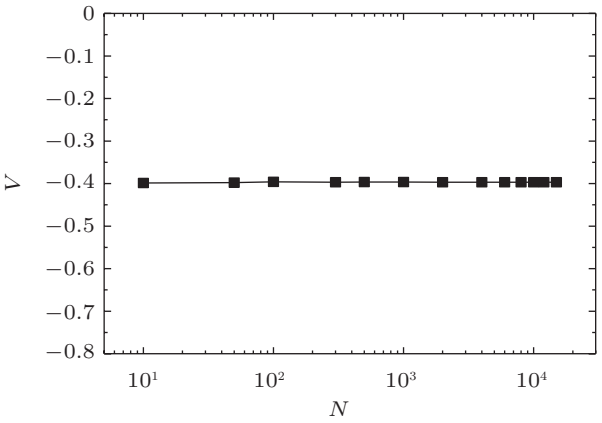


Fig. 4. Average velocity V as a function of particle number N at $\phi = \pi/2$, $d = \pi$, $D_0 = 0.1$, and $\omega = 1.0$.

Figure 6 describes the average velocity V as a function of ϕ with or without white noise. As the figures show, all curves are antisymmetric about π. Therefore, ϕ can be the switch to control the direction of the current. When $d = 3\pi/4$, π, and $5\pi/4$, oscillation amplitude reaches a appropriate value and lattices driving dominates the transport. In these cases, noise seems to slow down the directed currents. However, when $d = \pi/2$ and $3\pi/2$, no current occurs without the noise. The oscillation amplitude is out of limits, thus white noise plays an important role on directed transport when lattices driving is ineffective.

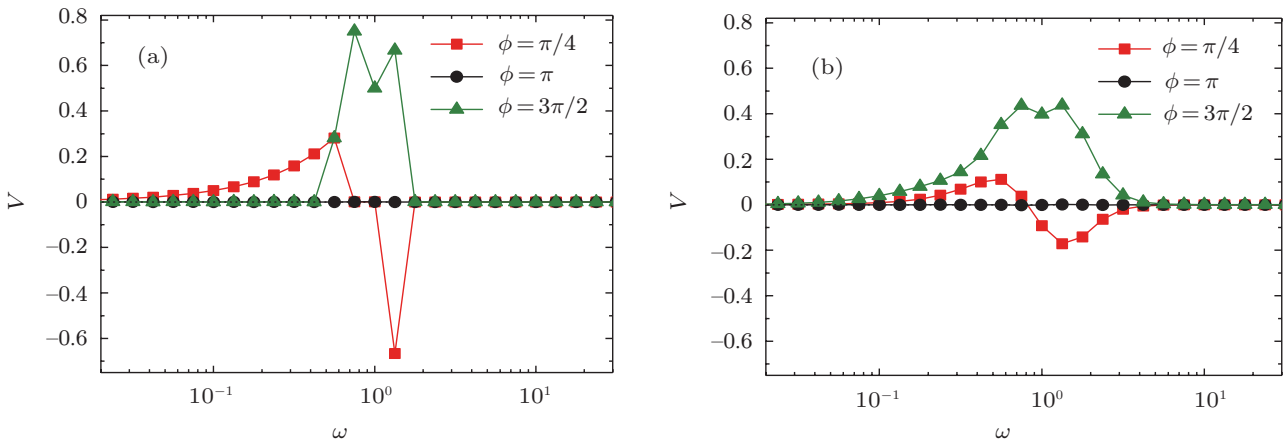


Fig. 5. Average velocity V as a function of oscillation frequency ω for different ϕ. (a) Without white noise. (b) With white noise. The other parameters are $D_0 = 0.1$ and $d = \pi$.

To study more details of the dependence on these parameters, we calculate the average velocity V as a function of d and ϕ in Fig. 7(a). When $d = d_1$, average velocity V is non-negative for $0 < \phi < \pi$ and non-positive for $\pi < \phi < 2\pi$, thus ϕ can switch the direction of the current. For $\phi = \phi_1$, it changes its direction twice when d is increased from zero. Figure 7(b) shows

the contour plots of average velocity V as a function of ω and ϕ. For $\omega = \omega_1$, the average velocity is positive when $\phi < \pi$ and negative when $\phi > \pi$. For $\phi_1 < \phi < \phi_2$ and $\phi_3 < \phi < \phi_4$, it changes its direction once when ω is increased from zero.

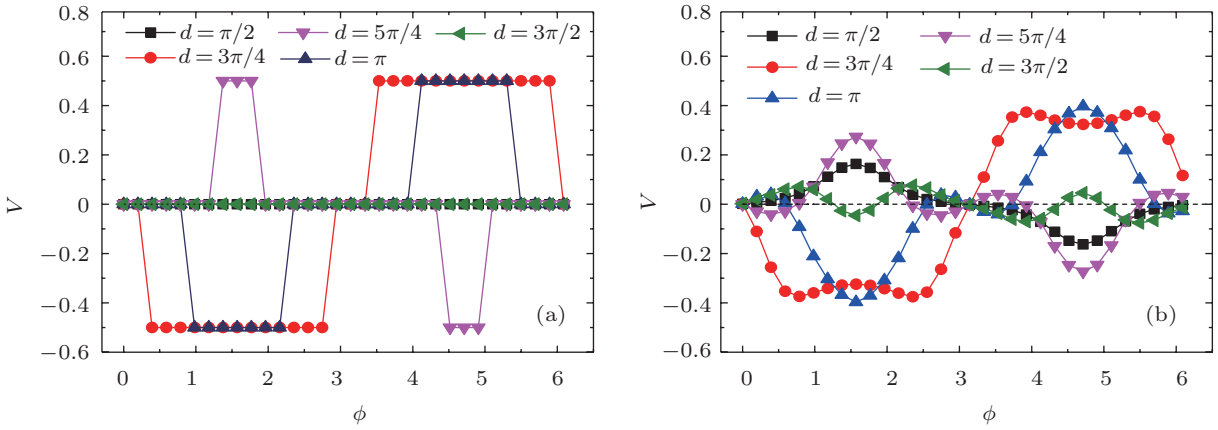


Fig. 6. Average velocity V as a function of driving phase ϕ for different d. (a) Without white noise. (b) With white noise. The other parameters are $D_0 = 0.1$ and $\omega = 1.0$.

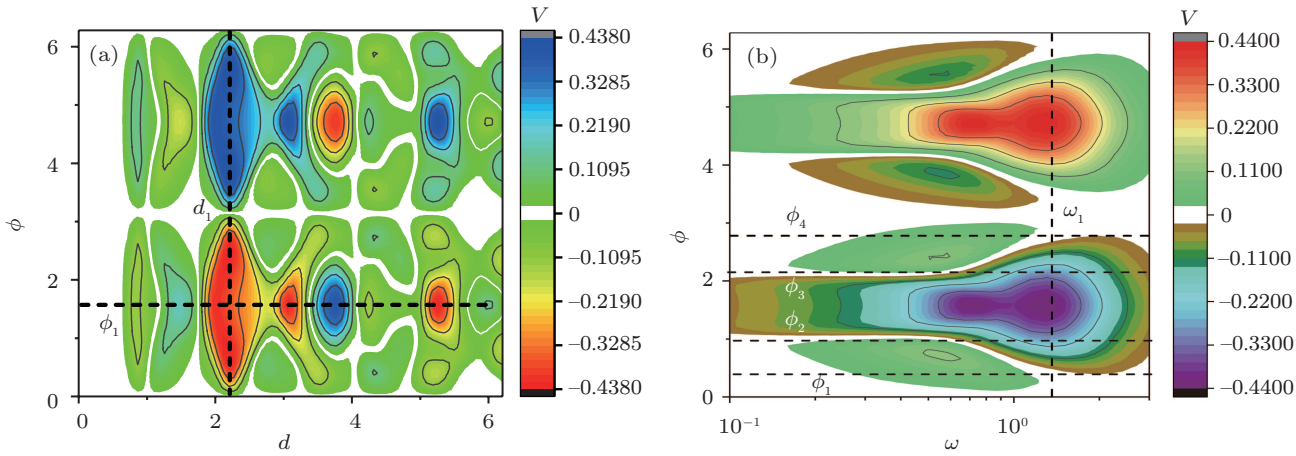


Fig. 7. (a) Contour plots of the average velocity V as a function of d and ϕ with white noise at $D_0 = 0.1$ and $\omega = 1.0$. (b) Contour plots of the average velocity V as a function of ω and ϕ with white noise at $D_0 = 0.1$ and $d = \pi$.

4. Conclusion

To summarize, we study the transport of noninteracting particles moving in superimposed driven lattices. We calculate the dependence of noise intensity D_0, oscillation amplitude d, oscillation frequency ω and driving phase ϕ on the directed currents. Base on Mukhopadhyay and coworkers' study, we further explore three aspects in this work.

(i) We consider noise in this model and focus on the difference between transporting particles in superimposed driven lattices without and with noise.

(ii) We find that noise plays an important role in superimposed driven lattices and get some interesting result.

(iii) We also provide contour plots of the average velocity and show the dependence of these parameters collectively and directly.

It is found that the influence of noise driving on the rectification is relative to the driving lattices. When lattices driving dominates the transport, the noise acts as disturbance of the directed transport and slows down the average velocity of the particles. When the driving phase has less impact on particle transport, Gaussian white noise can play a positive role. By simply modulating these two parameters, we can control efficiency and the direction of the directed currents.

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