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Research on co-propagation of QKD and classical communication by reducing the classical optical power*

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We investigate the crosstalk noise, especially the spontaneous Raman scattering, in the optical fiber of a co-propagation system between quantum key distribution (QKD) and classical communications. Although many methods have been proposed, such as increasing the wavelength spacing and narrowband filtering technique, to suppress Raman scattering noise, these methods greatly affect the performance of QKD. One way to solve the obstacle restricting the co-existence is to decrease the classical signal power. Based on the high gain of the gated avalanche photodiode and pulse position modulation, we demonstrate that the co-propagation system works effectively with only a small effect on long-haul fibers, which has great significance for the practical widespread commercialization of QKD.

Keywords: QKD, co-propagation, spontaneous Raman scattering, gated APD, modulation formats

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1. Introduction

Quantum key distribution (QKD)[1] is an unconditional secure communication technique based on physical principles.[2,3] The theoretical research has relatively mature conclusions and corresponding experimental verifications.[4–6] However, due to the high cost and low key generation rate, QKD is currently used in major trunk resources such as the Beijing–Shanghai Trunk Line, Tokyo Group Network and other government agencies and financial center institutions. For most people, QKD is still a costly and inefficient communication technique. In order to highlight the advantages of QKD on the premise that its performance is acceptable compared with existing encryption communication systems, QKD should have a breakthrough in the cost and key generation rate.

One way to implement QKD is to realize single-fiber transmission with classical communication through wavelength division multiplexing (WDM). The obstacle to the co-propagation system is that strong classical signals can seriously affect the weak quantum signals. The classical optical power is 6 to 7 orders of magnitude higher than that of QKD, causing weak quantum signals to be inundated with the crosstalk noise generated by the classical bright light. The general solutions of reducing crosstalk noise include improving the isolation of quantum signals and classical signals, passing the narrow-band filtering technique, and improving the receiver sensitivity to decrease the classical optical power.[7–11] However, by increasing the channel spacing, by placing the quantum signal in the O-band and the classical channel in the C-band, for example, the maximum transmission distance of QKD is greatly shortened due to the high loss characteristics of the O-band. Narrow-bandwidth filtering or multi-stage filtering makes the system more complex and inevitably introduces losses and even filters out some of the useful signals. For classic transceivers, the receiver has a deterministic detector sensitivity to satisfy system bit error rate requirements.

We proposed a more practical method to create a co-propagation system between QKD and classical communication by reducing the classical optical power based on gated avalanche photodiodes (APDs)[12–15] and pulse position modulation (PPM).[16–18] On one hand, based on the high gain of the gated APD, the sensitivity in the classical receiver is ultra-high. Thus, the classic optical power can be reduced by 2 to 3 orders of magnitude, resulting in very weak crosstalk, especially spontaneous Raman scattering noise (SpRS).[19] On the other hand, PPM can further reduce the limit of the sensitivity of the classical receiver. Combining the high gain characteristics of APD and the advantages of PPM, we have verified that it is a more practical and cost-effective method to integrate QKD with classical communication in the existing classical optical communication system with acceptable loss.

This paper is organized as follows. In Section 2, we test the crosstalk noise produced by the classical signals on quantum channels as a function of classical optical power. In order to reduce the classical optical power, we compare several detectors, and finally the gated APD is found to be more suitable for the co-propagation system. We introduce PPM in Sec-

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tion 3. The simulation results show that PPM has higher power efficiency and better performance than on–off keying (OOK). The discussion and conclusion can be found in Section 4.

2. Spontaneous Raman scattering noise and gated avalanche photodiodes

In order to deal with the impairments of QKD caused by the presence of classical signals, we need to try to reduce this crosstalk. The classical optical power should meet the detector sensitivity requirements. However, the higher the classical optical power, the greater the linear and nonlinear noise that affects the quantum channel. There are additional pseudo photons in the quantum channel, such as linear crosstalk, four-wave mixing (FWM), SpRS, Rayleigh scattering, and dispersion. By properly selecting the classical channel separation, the influence of FWM on the quantum channel can be minimized. Sufficient dense wavelength division multiplexer (DWDM) channel isolation can reduce linear crosstalk and Rayleigh scattering. Dispersion is generally severely affected in extremely high-speed communication, and can be improved by applying dispersion compensation. Therefore, in the actual co-propagation system, SpRS is the main crosstalk noise.

The SpRS is produced by non-elastic interaction in the fiber, resulting in a broad spectrum spanning around 200 nm. The SpRS is approximately proportional to the incident light power and the SpRS detection counts can be calculated by[20]

$$p^{\mathrm{FR}} = \frac{I\,\mathrm{e}^{-\alpha L} L \beta(\lambda)\Delta\lambda}{E_{\mathrm{photon}}} T_{\mathrm{d}} \eta_{\mathrm{d}}, \tag{1}$$

$$\mathrm{p}^{\mathrm{BR}} = \frac{I(1-\mathrm{e}^{-2\alpha L})\beta(\lambda)\Delta\lambda}{2\alpha E_{\mathrm{photon}}} T_{\mathrm{d}} \eta_{\mathrm{d}}, \tag{2}$$

where index FR stands for forward SpRS, BR for backward, I denotes the classic optical power, α indicates the fiber attenuation coefficient, L indicates the length of the fiber, $\beta(\lambda)$ is the effective Raman scattering cross-section, $\Delta\lambda$ denotes the quantum receiver bandwidth, T_{d} is the detector gating interval, and η_{d} is the detector efficiency. Here, E_{photon} can be calculated using

$$E_{\mathrm{photon}} = \frac{hc}{\lambda}, \tag{3}$$

where h is the Planck constant and c is the speed of light.

We measured the relationship between the crosstalk noise of the quantum channel and the classical channel. In the experiment, according to the International Telecommunication Union (ITU) grid spaced by DWDM components, we set the quantum channel to 1550.12 nm and the classical channel to 1550.92 nm. The classical signal is generated by a laser source triggered directly by a pulse signal with a trigger rate of 1.25 GHz/32 and wavelength of 1550.92 nm. The transmission and reflection ends of the insertion loss of DWDM are approximately 1.5 dB and the isolation of adjacent channels is measured to be 40 dB. The forward and backward crosstalk noise of the quantum channel is counted by superconducting nanowire single-photon detectors (SNSPD) with a dark count of 100 Hz and a detection efficiency of 60%. The experimental data, as shown in Fig. 1, are consistent with the theoretical SpRS simulation and only the previous part has deviation due to the dark count of SNSPD. Therefore, reducing the incident optical power can reduce crosstalk noise, while the classical optical power has a limit on the sensitivity of the receiver. Generally, the average number of photons of a quantum signal is less than one. Therefore, the SpRS should be reduced to the dark count level of QKD. However, the typical classic transceiver detection sensitivity is not enough. We have studied several commonly used photodetectors: the photomultiplier tube (PMT), semiconductor APDs, and SNSPDs.

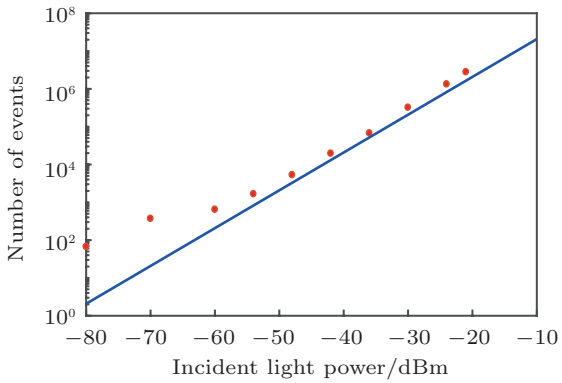


Fig. 1. The number of events of the crosstalk (points) in the quantum receiver as a function of the receiver power of the classical optical signals and the theoretical SpRS simulation (line). The classical signal and the quantum signal are multiplexed by a DWDM with 100-GHz compatible channel spacing. The red points represent the crosstalk noise of the classical signal with a frequency of 40 MHz to the quantum channel, which is detected by the SNSPD, and is consistent with the simulation. Here, $\beta(\lambda) = 2.1 \times 10^{-9}$ $(\mathrm{km \cdot nm})^{-1}$, $T_{\mathrm{d}} = 1 \times 10^{-9}$ s, and $\eta_{\mathrm{d}} = 0.2$.

The sensitivity of both APDs and PMTs can reach the single-photon counting level. However, PMT has been severely limited in its application due to its magnetic field sensitivity and the complex structure within the vacuum vessel. Compared with the PMT, the APD has an all-solid structure and high quantum efficiency, and can maintain a good signal-to-noise ratio at high gain. At the same time, APDs also have the characteristics of fast response, low sensitivity to magnetic field, small size, light weight, and low power consumption.

In APDs, photons are absorbed by the depletion layer when exposed to the incident light. Since the photon energy is greater than the band-gap energy, electron–hole pairs are generated. The internal electric field in the depletion region drives the electrons to the n+ side when reverse-biased, while it also moves holes toward the p+ side. The carriers in the electric field are accelerated then get enough energy and collide with the crystal lattice. This collision results in the generation of another electron–hole pair, while losing some of the carriers' kinetic energy. This process is called impact ionization. Under the action of a high electric field, the generated

electron–hole pairs obtain enough energy to continue the impact ionization process, which is called an avalanche. As the bias voltage increases, the APD multiplication gain increases sharply until the device reaches the point where large short-circuit currents occur. The voltage at which the multiplication gain is nearly infinity is called the breakdown voltage. When the reverse-bias voltage remains below the breakdown voltage, and the photogenerated charge is amplified with a finite multiplication gain, the APD operates in linear mode. In this mode, the avalanche multiplication process induces a measurable current by amplifying the negligible current generated by the photons. Despite amplification, this multiplier gain is still low. Therefore, APDs operating in a linear mode are limited in detecting extremely low light. On the other hand, when the reverse-bias voltage is higher than the breakdown voltage, the impact ionization process will continue until the tube burns, unless terminated by external conditions. This mode is called the Geiger mode, where APD has a self-sustaining avalanche effect and its multiplication gain approaches infinity. This ensures that the detector is sufficient to detect a single photon.

In the co-propagation system, we proposed that the high-speed APD gated by a sine wave is used in the receiver. There are several advantages: for the APD itself, the gate signal allows the APD to alternately operate in the Geiger and linear modes. When the sine signal is at the peak, the APD gain is sufficient to detect the incident photons. When the sine signal is at the trough, the lower bias voltage makes the APD return to the state to be detected more quickly. In terms of the characteristics of the sine wave itself, it only includes the fundamental frequency and its higher harmonic components after the differential response of the APD, which is easily filtered by the band-stop filter. From the perspective of the bit error rate (BER) at the detection, gating can filter out a part of the noise and reduce the BER in the receiver.

We also discussed the relationship between the randomness of APD gain and the BER. The photons absorbed in the active region of the APD may generate primary carriers, and the carrier-induced avalanche process has randomness. Due to the randomness of the multiplication gain, when the incident light is weak, photons may not be successfully detected and cause a bit error. As the incident light power increases, the multiplication gain gradually becomes deterministic so that the BER is decreased. A certain power of incident light is needed to ensure the stability of the gain of the APD to meet the BER requirements of the system. Figure 2 illustrates the schematic of the gated APD. The gated signal is generated by a signal generator with the operating frequency at 1.25 GHz. Then it is coupled with the direct current (DC) voltage by a bias tree and outputs to the APD as the reverse voltage. The APD we test is the LSIAPD-50 from Beijing Lightsensing Technologies Ltd, which can be used for single-photon detection with higher gain than ordinary APDs. It is operated in linear mode for classical optical detection. The capacitive response noise generated by the gated signal is filtered by the band-stop filter and the low-pass filter. The out-of-band rejection ratio of the filter is better than 60 dB. Then the avalanche signal is amplified, discriminated, and shaped by the amplifying and shaping circuit. Based on this gated APD, we tested the relationship between the incident light power and detection sensitivity. The classical signal with a frequency of 40 MHz is triggered by a pulse signal. The experimental data show that a BER of 10^{-9} for classical communications using the OOK modulation format is achieved with an average photon number of 150 photons/pulse in the receiver based on gated APD.

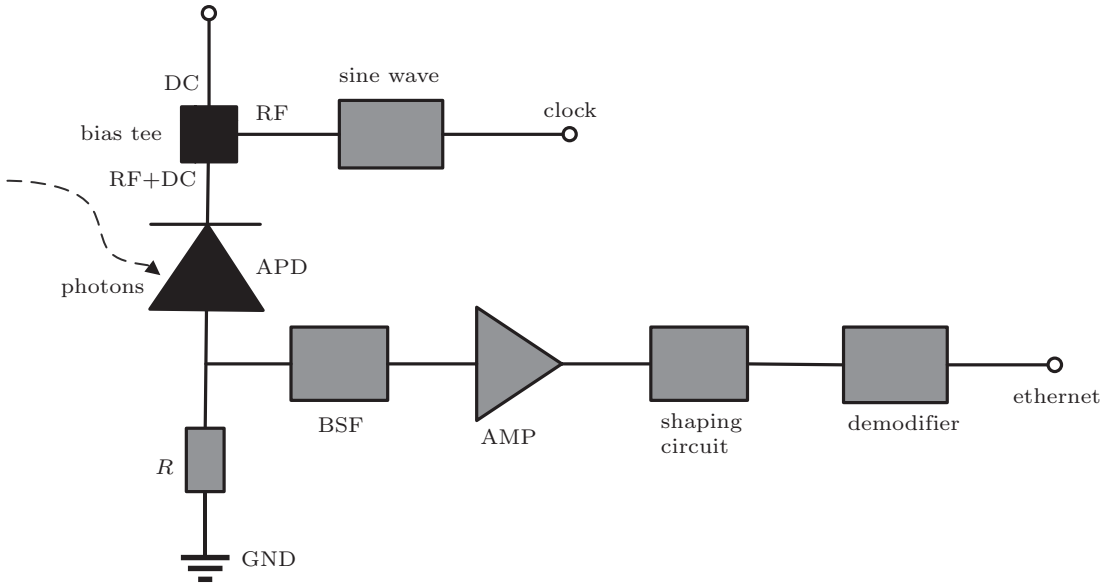


Fig. 2. The schematic diagram of the sinusoidally gated APD. DC: direct current, RF: radio frequency, APD: avalanche photodiode, BSF: band-stop filter, and AMP: amplifier.

The gain-bandwidth product of a finished APD is limited. In the classical communication system, because of the relatively high bandwidth, the APD operates in a linear mode with relatively high gain. Such detectors are not sufficient to detect optical signals at the single-photon level. By changing the APD working mode, we can make the APD work alternately in the linear mode and the Geiger mode. Using the Geiger mode to approximate the infinite gain to detect weaker signals, we achieved co-propagation of QKD and classical communication at relatively high communication bandwidths. On the other hand, SNSPD performs better than APD, with higher detection efficiency, lower dark count rate, and very accurate time resolution. However, SNSPD needs to work at very low temperatures to ensure superconductivity, so the cost is much higher than APD. Obviously, for our co-propagation system, the more practical and effective detection solution uses the gated APD.

3. Co-propagation of QKD and PPM classical channel

Figure 3 shows the multiplexing schematic of the QKD and classical channel. Using gated APD can allow weak light signals for classical communication, resulting in greatly reduced crosstalk noise levels. From another perspective, modulation and demodulation techniques are often the most economical means to increase the co-propagation system capacity. The encoding format is closely related to the receiver sensitivity. We research several modulation and detection techniques to improve spectral efficiency and reduce the limit of the sensitivity of the receiver. The receiver sensitivity is a function of the modulation format for the classical communication.

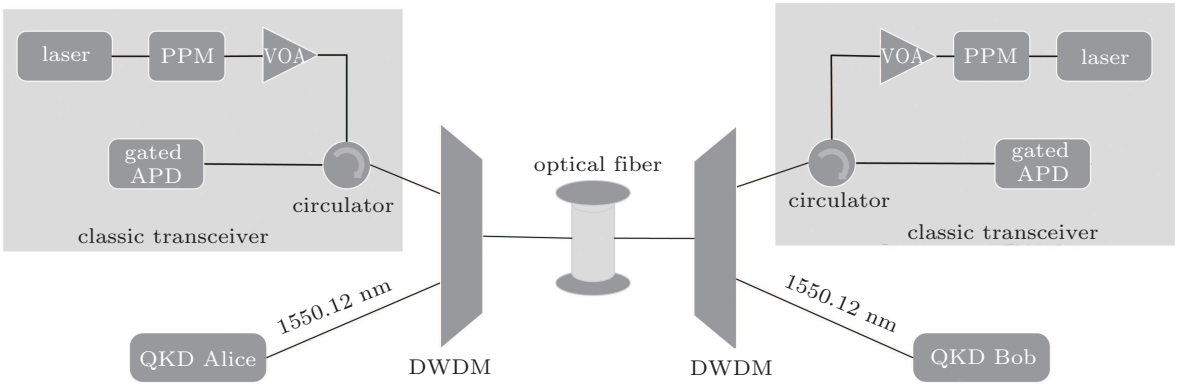


Fig. 3. Multiplexing schematic diagram of QKD and classical channel. The classic transmitter includes a laser, a modulator (PPM), a variable optical attenuator (VOA), and the classical receiver includes a gated APD. The duplex mode is implemented by the circulator. The QKD and classical channel are wavelength multiplexed using a DWDM.

As the simplest modulation scheme, OOK uses binary "1"s and "0"s to indicate the presence or absence of pulses. It uses on and off optical signals to transmit information. PPM is a quadrature modulation technique, in which the pulse position is changed to transmit information. In L time slots, each symbol includes $L-1$ "off" pulses and one "on" pulse, and the number of time slots is fixed. The binary n-bit data set is mapped to one of L time slots, that is to say the light pulse is sent during the slot. The transmitted bit is $\log_2 L$ for each symbol and the transmitted pulse shape is

$$s_n(t) = \begin{cases} P, & s\tau < t < (s+1)\tau, \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

where P is the average power of the pulse signal, s represents the pulse position, and τ is the time slot width.

We analyze the average transmit power of the above modulation method when the peak power is the same. This assumes the same probability of "0" and "1". So the average transmit power of OOK is $P_{\text{OOK}} = 0.5P$. For PPM, $P_{\text{PPM}} = P/2^n$. Thus, as shown in Fig. 4, the average power requirement of PPM is lower than that of OOK for the same peak power and given n. In our multiplexed system, the classic optical power is limited, and the gated APD operating frequency is limited, currently on the order of GHz. Under this premise, PPM can achieve a high data transmission rate with a small repetition frequency at a given average optical power, which can make up for the defect that the gated APD operating frequency is relatively low and the optical power is relatively weak.

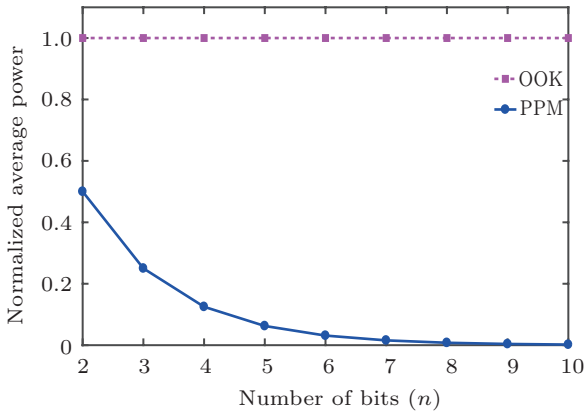


Fig. 4. The comparison of normalized average power requirement.

From Section 2, most of the crosstalk noise is composed of the SpRS of classical signals. Assuming that only SpRS,

we simulate the effects that the OOK and PPM modulation formats of classical communication with gated APD detection affect QKD system. In the QKD system, the detector counts caused by SpRS are independent of the quantum signal, which can be classified as background light counts. Combining GLLP with the decoy method, the formula for the key generation rate of QKD[21] is

$$R \geq q\{-Q_\mu f\left(E_\mu\right) H_2\left(E_\mu\right) + Q_1\left[1 - H_2\left(e_1\right)\right]\}, \quad (5)$$

where $q = 1/2$ for the BB84 protocol, μ denotes the intensity of signal states, Q_μ is the gain of signal states, E_μ is the quantum bit error rate (QBER), f is the inefficiency of error correction, Q_1 is the gain of single-photon states, and e_1 is the estimated error rate of single-photon states. The QBER is given by

$$E_\mu = \frac{1}{Q_\mu}\left[e_0 Y_0 + e_{\mathrm{d}}\left(1 - \mathrm{e}^{-\eta\mu}\right)\right], \quad (6)$$

where $e_d = 0.5\%$, $e_0 = 1/2$ denotes that the background is random, and Y_0 is the background rate, which includes the detector dark count P_{d} and the count generated by spontaneous Raman scattering noise

$$Y_0 = p_{\mathrm{d}} + p^{\mathrm{BR}} + p^{\mathrm{FR}}. \quad (7)$$

Figure 5 shows the key generation rates of QKD in the co-propagating system. We set the average photon numbers of the signal and weak decoy states to 0.48 and 0.05, respectively. The detectors work in a gated mode with a detection efficiency of 20% and a dark count rate of 6×10^{-6} per clock cycle. As shown in Fig. 5, the effect of crosstalk noise on QKD in short-range communication is negligible, with only a small effect on long-haul fibers. Moreover, PPM has a higher power efficiency and better performance than OOK.

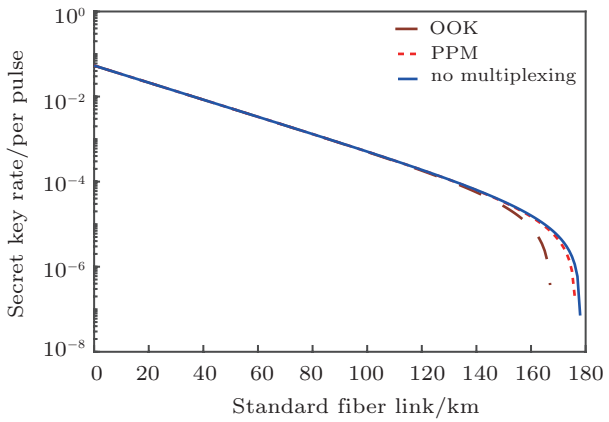


Fig. 5. Simulated QKD secure key rates with quantum signals co-propagating with classical channel, using Eq. (5). The minimum classical optical power is the experimental data obtained by measuring the classical optical power and detector sensitivity according to Section 2. Combined with the formulas (1) and (2), that is, the function of Raman scattering and classical optical power, the crosstalk noise can be calculated. Only SpRS is considered. The line in the figure represents OOK, PPM, and no multiplexing from left to right.

The OOK and PPM are both intensity modulation with direct detection (IM/DD) methods, which are easy and cost-effective to implement. In addition to IM/DD modulation, coherent modulation depends not only on the presence or absence of energy, but also on phase information. It also has a relatively high detection sensitivity. However, coherent modulation needs to be implemented using heterodyne or homodyne down-conversion by a local oscillator laser, and at the cost of increased implementation complexity. Considering the secure transmission distance of QKD, whether it is IM/DD or coherent modulation, the maximum transmission distance of the multiplexed system is limited only by QKD instead of classic communication. Therefore, considering only cost and device complexity, PPM is more practical and cost-effective for our co-propagation system with gated APD in the classical receiver.

4. Conclusion

In conclusion, using the high gain of the gated APD, the sensitivity in the classical receiver is greatly improved. By reducing the classical light power by two to three orders of magnitude, resulting in reduced various linear and nonlinear crosstalk, QKD can transport in a relatively pure environment with substantially negligible degradation. At the same time, the PPM modulation format is used to further reduce the limit of the sensitivity in the receiver with better performance.

There are also disadvantages in using the gated APD to improve the sensitivity of the detector. As the data bandwidth is larger, the existing gated APD will not be able to meet the higher bandwidth communication due to the effects of the dark count and the after pulse. At present, the optimal gated APD can only reach the GHz level. In combination with PPM modulation, we do not discuss other factors such as group velocity dispersion, polarization mode dispersion, and self-phase modulation, which are not negligible in higher data bandwidth transmission systems. Therefore, there are still obstacles in the co-propagation of QKD and ultra-high-speed communication systems while it is enough for the access network users.

Returning to cost, our co-propagation technique can solve the crosstalk noise without complicated changes to the multiplexing system. Therefore, it is a relatively cost-effective and practical technique. We demonstrate that the co-propagation system, based on a gated APD classical receiver and PPM modulation format, works effectively with only a negligible effect on the security key generation rate and transmission distance of QKD. The technique is practical and cost-effective for integrating a multi-channel coexistence network of QKD and classical communications, which brings QKD close to practical widespread commercialization.

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